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A Delay-Dependent Approach to Robust H_∞ Control for Uncertain Stochastic Systems with State and Input Delays

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Abstract In this paper, the problem of delay-dependent robust H_∞ control for uncertain stochastic systems with state and input delays is investigated. The time delays are assumed to be bounded and time varying and the uncertainties are assumed to be norm bounded. By using the Lyapunov functional method, a new delay-dependent robust H_∞ control scheme is presented in terms of linear matrix inequalities (LMIs). Some numerical examples are given to illustrate the effectiveness of the proposed approach.

Keywords H_∞ control · Stochastic systems · Input delay · LMIs

1 Introduction

Time delays often arise in many dynamic systems, and they are often a source of instability. Thus, considerable attention has been paid to time-delay systems and some quite significant results have been reported; see for example, [1, 2, 10, 11] and the

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references therein. Recently, by using the linear matrix inequality (LMI) approach, the H_∞ control theory for time-delay systems has been investigated widely; see, e.g., [12, 14, 28]. It has been shown that H_∞ control is closely associated with many robustness problems such as robust stabilization of uncertain systems. However, when parameter uncertainty appears in the plant modeling, the standard H_∞ theory [13] cannot provide guaranteed H_∞ performance as well as stability of the closed-loop system. This has motivated the study of the robust H_∞ control problem, and numerous results on both continuous-time and discrete-time systems have been reported in the literature; see, e.g., [3, 7, 9, 18, 31] and the references therein.

In recent years, the problem of stability analysis and controller design of stochastic systems has been an important topic of control theory since stochastic modeling has come to play an important role in many branches of science and engineering applications. Many results on stability analysis, controller synthesis and filtering design for stochastic time-delay systems have been developed; see, e.g., [4–6, 8, 16, 19–26, 29]. For instance, by using the Moon's inequalities method, the investigation of exponential stability in the mean square sense has been reported in [4] for stochastic systems with multiple delays. In [24], the authors dealt with robust stability and stabilization problems for a class of stochastic time-delay interval systems with nonlinear disturbances by developing delay-dependent analysis techniques. More recently, the problems of robust H_∞ control for uncertain stochastic time-delay systems have been investigated. For example, the authors considered the problem of robust H_∞ control for uncertain stochastic systems with state delay, and a delay-independent H_∞ control scheme was proposed in [27]. In [5], the Moon's inequalities approach was applied to investigate the problem of delay-dependent robust stochastic stabilization and H_∞ control for stochastic systems with norm-bounded uncertainties and state delay. However, none of the aforementioned works took the effect of the control input delays into account when the controllers were designed for stochastic systems.

Yet input delays are often encountered in control systems because of the transmission of measurement information. Especially in networked control systems, sensors, controllers and plants are often connected over a net medium. Hence, it is quite meaningful to study the effect of the input delay in the design of controllers. Some controller design schemes have been proposed for linear systems with input delay [30]. However, to the best of the authors' knowledge, providing less conservative delay-dependent stability criteria and designing controllers for uncertain stochastic systems with both state and input delays to achieve desired performance are still open problems.

Motivated by this observation, this paper deals with the problem of delay-dependent robust H_∞ control for uncertain stochastic systems with state and input delays. The time delays are assumed to be bounded and time varying. Based on the Lyapunov–Krasovskii functional method and the free-weighting matrix method, a delay-dependent robust H_∞ control scheme is proposed in terms of LMIs. Some numerical examples are used to illustrate the effectiveness of the proposed design method.

Notation The following notation will be used throughout this paper. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that the $X - Y$ is positive-semidefinite (respectively, positive-definite); M^{T} denotes the transpose of the matrix M; I stands for the identity matrix with appropriate dimension; $L_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$; $|\cdot|$ refers to the Euclidean vector norm; $\|\cdot\|_2$ stands for the usual $L_2[0,\infty)$ norm; $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ is a probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. the filtration contains all P-null sets and is right continuous); and $L^2_{\mathcal{F}_0}([-\sigma,0];\mathbb{R}^n)$ denotes the family of all $\mathcal{F}_0$-measurable $C([-\sigma,0];\mathbb{R}^n)$-valued random variables $\xi = \{\xi(\theta): -\sigma \leq \theta \leq 0\}$ such that $\sup_{-\sigma\leq\theta\leq 0}\mathcal{E}|\xi(\theta)| < \infty$, where $\mathcal{E}(\cdot)$ stands for the mathematical expectation. Matrices, if not explicitly stated, are assumed to have compatible dimensions. " $*$ " means a block that is readily inferred by symmetry.

2 Problem Formulation

Consider the following uncertain stochastic system with state delay and input delays:

$$\begin{aligned}\mathrm{d}x(t) = {} & \big[\big(A + \Delta A(t)\big)x(t) + \big(A_\tau + \Delta A_\tau(t)\big)x\big(t-\tau(t)\big) + \big(B + \Delta B(t)\big)u(t) \\ & + \big(B_\tau + \Delta B_\tau(t)\big)u\big(t-\tau(t)\big) + B_v v(t)\big]\,\mathrm{d}t \\ & + \big[\big(E + \Delta E(t)\big)x(t) + \big(E_\tau + \Delta E_\tau(t)\big)x\big(t-\tau(t)\big) + E_v v(t)\big]\,\mathrm{d}\omega(t), \end{aligned} \tag{1}$$

$$z(t) = Cx(t) + C_\tau x\big(t-\tau(t)\big) + Du(t) + D_\tau u\big(t-\tau(t)\big), \tag{2}$$

$$x(t) = \phi(t), \qquad u(t) = \varphi(t), \quad \forall t \in [-\sigma, 0], \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ denotes the control input, and $v(t) \in \mathbb{R}^p$ stands for the disturbance input which belongs to $L_2[0,\infty)$. $z(t) \in \mathbb{R}^q$ means the controlled output, $\omega(t)$ is a one-dimensional Brownian motion satisfying $\mathcal{E}\{\mathrm{d}\omega(t)\} = 0$ and $\mathcal{E}\{\mathrm{d}\omega(t)^2\} = \mathrm{d}t$, and $\tau(t)$ is a time-varying bounded delay and satisfies $0 < \tau(t) \leq \sigma$, $\dot{\tau}(t) \leq h < \infty$. A, A_τ, B, B_τ, B_v, E, E_τ, E_v, C, C_τ, D and D_τ are known real constant matrices of appropriate dimensions. $\Delta A(t)$, $\Delta A_\tau(t)$, $\Delta B(t)$, $\Delta B_\tau(t)$, $\Delta E(t)$ and $\Delta E_\tau(t)$ are time-varying parameter uncertainties which are of the following form:

$$\begin{aligned}&\begin{bmatrix}\Delta A(t) & \Delta A_\tau(t) & \Delta B(t) & \Delta B_\tau(t) & \Delta E(t) & \Delta E_\tau(t)\end{bmatrix} \\ &\quad = HF(t)[N_a \quad N_{a\tau} \quad N_b \quad N_{b\tau} \quad N_e \quad N_{e\tau}],\end{aligned} \tag{4}$$

where H, N_a, $N_{a\tau}$, N_b, $N_{b\tau}$, N_e and $N_{e\tau}$ are real constant matrices with appropriate dimensions. $F(t)$ is an unknown time-varying matrix function and satisfies the following inequality:

$$F^{\mathrm{T}}(t)F(t) \leq I. \tag{5}$$

It is assumed that all the elements of $F(t)$ are Lebesgue measurable. $\Delta A(t)$, $\Delta A_\tau(t)$, $\Delta B(t)$, $\Delta B_\tau(t)$, $\Delta E(t)$ and $\Delta E_\tau(t)$ are said to be admissible if both (4) and (5) hold. $\phi(t) \in C([-\sigma,0];\mathbb{R}^n)$ and $\varphi(t)$ denote the initial function.

Before stating our main results, we first introduce the following concepts.

Definition 1 [27] The nominal system (1) and (3) with $u(t) = 0$ and $v(t) = 0$ is said to be mean-square stable if for any $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that $\mathcal{E}\{|x(t)|^2\} < \varepsilon$ when

$$\sup_{-\sigma \le s \le 0} \mathcal{E}\{|\phi(s)|^2\} < \delta(\varepsilon).$$

Particularly, if

$$\lim_{t\to\infty} \mathcal{E}\{|x(t)|^2\} = 0$$

for any initial conditions, then this stochastic system (1) and (3) with $u(t) = 0$ and $v(t) = 0$ is said to be mean-square asymptotically stable. The uncertain stochastic system in (1) and (3) is said to be robustly stochastically stable if the system associated to (1) and (3) with $u(t) = 0$ and $v(t) = 0$ is mean-square asymptotically stable for all admissible uncertainties satisfying (4)–(5).

Definition 2 [27] Given a scalar $\gamma > 0$, the uncertain stochastic system (1)–(3) with $u(t) = 0$ is said to be robustly stochastically stable with disturbance attenuation γ if it is robustly stochastically stable and under zero initial conditions

$$\|z(t)\|_{E_2} < \gamma \|v(t)\|_2 \tag{6}$$

is satisfied for all nonzero $v(t) \in L_2[0, \infty)$ and all admissible uncertainties satisfying (4)–(5), where

$$\|z(t)\|_{E_2} = \left(\mathcal{E}\left\{\int_0^\infty |z(t)|^2 \,\mathrm{d}t\right\}\right)^{\frac{1}{2}}. \tag{7}$$

The following lemmas are essential for the proofs in the sequel.

Lemma 1 [17] *For any constant matrix $M > 0$, any scalars a and b with $a < b$, and a vector function $x(t) : [a, b] \to \mathbb{R}^n$ such that the integrals concerned are well defined, the following holds*:

$$\left[\int_a^b x(s)\,\mathrm{d}s\right]^{\mathrm{T}} M \left[\int_a^b x(s)\,\mathrm{d}s\right] \le (b-a)\int_a^b x^{\mathrm{T}}(s) M x(s)\,\mathrm{d}s.$$

Lemma 2 [19] *Let M, E and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \le I$. Then, the following inequality holds for any $\varepsilon > 0$*:

$$MF(t)E + E^{\mathrm{T}}F^{\mathrm{T}}(t)M^{\mathrm{T}} \le \varepsilon MM^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E.$$

3 H_∞ Performance Analysis

By $u(t) = Kx(t)$, the closed-loop stochastic system consisting of (1) and (2) can be rewritten in the following form:

$$\mathrm{d}x(t) = \left[\bar{A}_K x(t) + \bar{A}_{\tau K} x\big(t-\tau(t)\big) + B_v v(t)\right]\mathrm{d}t \\ + \left[\bar{E}x(t) + \bar{E}_\tau x\big(t-\tau(t)\big) + E_v v(t)\right]\mathrm{d}\omega(t), \tag{8}$$

$$z(t) = C_K x(t) + D_{\tau K} x\big(t-\tau(t)\big) \tag{9}$$

where $\bar{A}_K = A + BK + \Delta A(t) + \Delta B(t)K$, $\bar{A}_{\tau K} = A_\tau + B_\tau K + \Delta A_\tau(t) + \Delta B_\tau(t)K$, $\bar{E} = E + \Delta E(t)$, $\bar{E}_\tau = E_\tau + \Delta E_\tau(t)$, $C_K = C + DK$ and $D_{\tau K} = C_\tau + D_\tau K$. In this section, a delay-dependent approach is proposed to solve the problem of robust stochastic stabilization with disturbance attenuation level. To this end, we first assume that the feedback gain matrix K is known. For robust H_∞ performance analysis of the system (8)–(9), we have the following result.

Theorem 1 *Consider the closed-loop system* (8)–(9). *For given scalars* $\gamma > 0$, $\sigma > 0$, h *and feedback gain* K, *the stochastic system* (8) *and* (9) *is robustly stochastically stable with disturbance attenuations* γ *for any* $\tau(t)$ *satisfying* $0 < \tau(t) \le \sigma$, $\dot{\tau}(t) \le h < \infty$, *if there exist matrices* $P > 0$, $Q_p > 0$, $R > 0$, N_p, G_p, M_q *and* S_q $(p = 1, 2,\ q = 1, 2, 3)$, *as well as positive scalars* ε_1 *and* ε_2 *such the following LMI holds*:

$$\begin{bmatrix} \phi_{11} & \phi_{12} & -G_1 & \phi_{14} & \phi_{15} & -N_1 & -G_1 & \phi_{18} & \phi_{19} & M_1H & S_1H \\ * & \phi_{22} & -G_2 & \phi_{24} & \phi_{25} & -N_2 & -G_2 & \phi_{28} & \phi_{29} & M_2H & S_2H \\ * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 & \phi_{48} & 0 & M_3H & 0 \\ * & * & * & * & \phi_{55} & 0 & 0 & \phi_{58} & 0 & 0 & S_3H \\ * & * & * & * & * & -\frac{1}{\sigma}R & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\sigma}R & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{10}$$

where

$$\phi_{11} = Q_1 + Q_2 + N_1 + N_1^{\mathrm{T}} + M_1(A+BK) + (A+BK)^{\mathrm{T}}M_1^{\mathrm{T}} + S_1E + E^{\mathrm{T}}S_1^{\mathrm{T}} \\ + \varepsilon_1(N_a + N_bK)^{\mathrm{T}}(N_a + N_bK) + \varepsilon_2 N_e^{\mathrm{T}}N_e,$$

$$\phi_{12} = -N_1 + N_2^{\mathrm{T}} + G_1 + M_1(A_\tau + B_\tau K) + (A+BK)^{\mathrm{T}}M_2^{\mathrm{T}} + S_1E_\tau + E^{\mathrm{T}}S_2^{\mathrm{T}} \\ + \varepsilon_1(N_a + N_bK)^{\mathrm{T}}(N_{a\tau} + N_{b\tau}K) + \varepsilon_2 N_e^{\mathrm{T}}N_{e\tau},$$

$$\phi_{14} = P - M_1 + (A+BK)^{\mathrm{T}}M_3^{\mathrm{T}}, \qquad \phi_{15} = -S_1 + E^{\mathrm{T}}S_3^{\mathrm{T}},$$

$$\phi_{18} = M_1B_v + S_1E_v, \qquad \phi_{19} = C^{\mathrm{T}} + K^{\mathrm{T}}D^{\mathrm{T}},$$

$$\phi_{22} = -(1-h)Q_1 - N_2 - N_2^{\mathrm{T}} + G_2 + G_2^{\mathrm{T}} + M_2(A_\tau + B_\tau K) + (A_\tau + B_\tau K)^{\mathrm{T}}M_2^{\mathrm{T}} \\ + S_2E_\tau + E_\tau^{\mathrm{T}}S_2^{\mathrm{T}} + \varepsilon_1(N_{a\tau} + N_{b\tau}K)^{\mathrm{T}}(N_{a\tau} + N_{b\tau}K) + \varepsilon_2 N_{e\tau}^{\mathrm{T}}N_{e\tau},$$

$$\phi_{24} = -M_2 + (A_\tau + B_\tau K)^{\mathrm{T}}M_3^{\mathrm{T}}, \qquad \phi_{25} = -S_2 + E_\tau^{\mathrm{T}}S_3^{\mathrm{T}},$$

$$\phi_{28} = M_2 B_v + S_2 E_v, \qquad \phi_{29} = C_\tau^{\mathrm{T}} + K^{\mathrm{T}} D_\tau^{\mathrm{T}}, \qquad \phi_{44} = \sigma R - M_3 - M_3^{\mathrm{T}},$$
$$\phi_{48} = M_3 B_v, \qquad \phi_{55} = P - S_3 - S_3^{\mathrm{T}}, \qquad \phi_{58} = S_3 E_v.$$

Proof The proof is twofold: we first show that (6) is satisfied under the given conditions, and then prove that system (8) with $v(t) = 0$ is robustly stochastically stable. First, we define two new state variables,

$$y(t) = \bar{A}_K x(t) + \bar{A}_{\tau K} x\big(t - \tau(t)\big) + B_v v(t), \tag{11}$$

$$g(t) = \bar{E} x(t) + \bar{E}_\tau x\big(t - \tau(t)\big) + E_v v(t). \tag{12}$$

Then, the closed-loop system (8) can be represented as

$$\mathrm{d}x(t) = y(t)\,\mathrm{d}t + g(t)\,\mathrm{d}\omega(t). \tag{13}$$

Now, choose the Lyapunov–Krasovskii functional as follows:

$$V(x_t, t) = x^{\mathrm{T}}(t) P x(t) + \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s) Q_1 x(s)\,\mathrm{d}s + \int_{t-\sigma}^{t} x^{\mathrm{T}}(s) Q_2 x(s)\,\mathrm{d}s + \int_{-\sigma}^{0} \int_{t+s}^{t} y^{\mathrm{T}}(\theta) R y(\theta)\,\mathrm{d}\theta\,\mathrm{d}s, \tag{14}$$

where $x_t = \{x(t+\theta) : -2\sigma \le \theta \le 0\}$. Then, by Itô's formula we can obtain the stochastic differential as [27]

$$\mathrm{d}V(x_t, t) = \mathcal{L}V(x_t, t)\,\mathrm{d}t + 2x^{\mathrm{T}}(t) P g(t)\,\mathrm{d}\omega(t),$$

where

$$\begin{aligned}
\mathcal{L}V(x_t, t) &= 2x^{\mathrm{T}}(t) P y(t) + g^{\mathrm{T}}(t) P g(t) + x^{\mathrm{T}}(t)[Q_1 + Q_2]x(t) \\
&\quad - \big(1 - \dot{\tau}(t)\big) x^{\mathrm{T}}\big(t - \tau(t)\big) Q_1 x\big(t - \tau(t)\big) - x^{\mathrm{T}}(t-\sigma) Q_2 x(t-\sigma) \\
&\quad + \sigma y^{\mathrm{T}}(t) R y(t) - \int_{t-\sigma}^{t} y^{\mathrm{T}}(s) R y(s)\,\mathrm{d}s \\
&\le 2x^{\mathrm{T}}(t) P y(t) + g^{\mathrm{T}}(t) P g(t) + x^{\mathrm{T}}(t)[Q_1 + Q_2]x(t) \\
&\quad - (1 - h) x^{\mathrm{T}}\big(t - \tau(t)\big) Q_1 x\big(t - \tau(t)\big) - x^{\mathrm{T}}(t-\sigma) Q_2 x(t-\sigma) \\
&\quad + \sigma y^{\mathrm{T}}(t) R y(t) - \int_{t-\sigma}^{t-\tau(t)} y^{\mathrm{T}}(s) R y(s)\,\mathrm{d}s \\
&\quad - \int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s) R y(s)\,\mathrm{d}s.
\end{aligned} \tag{15}$$

Then, it follows from Lemma 1 and $0 < \tau(t) \le \sigma$ that

$$-\int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s) R y(s)\,\mathrm{d}s \le -\frac{1}{\sigma} \left[\int_{t-\tau(t)}^{t} y(s)\,\mathrm{d}s\right]^{\mathrm{T}} R \left[\int_{t-\tau(t)}^{t} y(s)\,\mathrm{d}s\right], \tag{16}$$

$$-\int_{t-\sigma}^{t-\tau(t)} y^{\mathrm{T}}(s)Ry(s)\,\mathrm{d}s \le -\frac{1}{\sigma}\left[\int_{t-\sigma}^{t-\tau(t)} y(s)\,\mathrm{d}s\right]^{\mathrm{T}} R\left[\int_{t-\sigma}^{t-\tau(t)} y(s)\,\mathrm{d}s\right]. \quad (17)$$

Now, define the new vector $e_1(t)$ as

$$e_1^{\mathrm{T}}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau(t)) \end{bmatrix}.$$

For free-weighting matrices N and G, the following equalities hold:

$$\eta_1(t) = 2e_1^{\mathrm{T}}(t)N\Big(x(t) - x\big(t-\tau(t)\big) - \int_{t-\tau(t)}^{t} y(s)\,\mathrm{d}s - \int_{t-\tau(t)}^{t} g(s)\,\mathrm{d}\omega(s)\Big) = 0, \quad (18)$$

$$\eta_2(t) = 2e_1^{\mathrm{T}}(t)G\Big(x\big(t-\tau(t)\big) - x(t-\sigma) - \int_{t-\sigma}^{t-\tau(t)} y(s)\,\mathrm{d}s - \int_{t-\sigma}^{t-\tau(t)} g(s)\,\mathrm{d}\omega(s)\Big) = 0, \quad (19)$$

where $N = [N_1^{\mathrm{T}}, N_2^{\mathrm{T}}]^{\mathrm{T}}$, $G = [G_1^{\mathrm{T}}, G_2^{\mathrm{T}}]^{\mathrm{T}}$with appropriate dimensions.

Similarly, for matrix $M = [M_1^{\mathrm{T}}, M_2^{\mathrm{T}}, M_3^{\mathrm{T}}]^{\mathrm{T}}$ and $S = [S_1^{\mathrm{T}}, S_2^{\mathrm{T}}, S_3^{\mathrm{T}}]^{\mathrm{T}}$ with compatible dimensions, the following equalities hold:

$$\eta_3(t) = 2e_2^{\mathrm{T}}(t)M\big(\bar{A}_K x(t) + \bar{A}_{\tau K} x\big(t-\tau(t)\big) + B_v v(t) - y(t)\big) = 0, \quad (20)$$

$$\eta_4(t) = 2e_3^{\mathrm{T}}(t)S\big(\bar{E}x(t) + \bar{E}_\tau x\big(t-\tau(t)\big) + E_v v(t) - g(t)\big) = 0, \quad (21)$$

where

$$e_2^{\mathrm{T}}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau(t)) & y^{\mathrm{T}}(t) \end{bmatrix},$$
$$e_3^{\mathrm{T}}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau(t)) & g^{\mathrm{T}}(t) \end{bmatrix}.$$

Let $\mathcal{F}(\mathrm{d}\omega(t)) = -2e_1^{\mathrm{T}}(t)N\int_{t-\tau(t)}^{t} g(s)\,\mathrm{d}\omega(s) - 2e_2^{\mathrm{T}}(t)G\int_{t-\sigma}^{t-\tau(t)} g(s)\,\mathrm{d}\omega(s)$. Note that the mathematical expectation of $\mathcal{F}(\mathrm{d}\omega(t))$ equals $\mathcal{E}\mathcal{F}(\mathrm{d}\omega(t)) = 0$. Then, combining (15)–(21) gives

$$\mathcal{E}\mathcal{L}V(x_t, t) \le \xi^{\mathrm{T}}(t)[\Lambda_1 + \Delta\Lambda_1 + \Delta\Lambda_2]\xi(t),$$

where

$$\Lambda_1 = \begin{bmatrix} \tilde{\phi}_{11} & \tilde{\phi}_{12} & -G_1 & \phi_{14} & \phi_{15} & -N_1 & -G_1 & \phi_{18} \\ * & \tilde{\phi}_{22} & -G_2 & \phi_{24} & \phi_{25} & -N_2 & -G_2 & \phi_{28} \\ * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 & \phi_{48} \\ * & * & * & * & \phi_{55} & 0 & 0 & \phi_{58} \\ * & * & * & * & * & -\frac{1}{\sigma}R & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\sigma}R & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\Delta\Lambda_1=(\sigma_{ij})_{8\times 8},\quad \Delta\Lambda_2=(c_{ij})_{8\times 8},$$

$$\begin{aligned}\xi^{\mathrm{T}}(t)=\big[&x^{\mathrm{T}}(t)\quad x^{\mathrm{T}}(t-\tau(t))\quad x^{\mathrm{T}}(t-\sigma)\quad y^{\mathrm{T}}(t)\quad g^{\mathrm{T}}(t)\\ &\textstyle\int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)\,\mathrm{d}s\quad \int_{t-\sigma}^{t-\tau(t)} y^{\mathrm{T}}(s)\,\mathrm{d}s\quad v^{\mathrm{T}}(t)\big]\end{aligned}$$

with

$$\begin{aligned}
\tilde{\phi}_{11}&=Q_1+Q_2+N_1+N_1^{\mathrm{T}}+M_1(A+BK)+(A+BK)^{\mathrm{T}}M_1^{\mathrm{T}}+S_1E+E^{\mathrm{T}}S_1^{\mathrm{T}},\\
\tilde{\phi}_{12}&=-N_1+N_2^{\mathrm{T}}+G_1+M_1(A_\tau+B_\tau K)+(A+BK)^{\mathrm{T}}M_2^{\mathrm{T}}+S_1E_\tau+E^{\mathrm{T}}S_2^{\mathrm{T}},\\
\tilde{\phi}_{22}&=-(1-h)Q-N_2-N_2^{\mathrm{T}}+G_2+G_2^{\mathrm{T}}+M_2(A_\tau+B_\tau K)+(A_\tau+B_\tau K)^{\mathrm{T}}M_2^{\mathrm{T}}\\
&\quad+S_2E_\tau+E_\tau^{\mathrm{T}}S_2^{\mathrm{T}},\\
\sigma_{11}&=M_1\big(\Delta A(t)+\Delta B(t)K\big)+\big(\Delta A(t)+\Delta B(t)K\big)^{\mathrm{T}}M_1^{\mathrm{T}},\\
\sigma_{12}&=M_1\big(\Delta A_\tau(t)+\Delta B_\tau(t)K\big)+\big(\Delta A(t)+\Delta B(t)K\big)^{\mathrm{T}}M_2^{\mathrm{T}},\\
\sigma_{13}&=0,\qquad \sigma_{14}=\big(\Delta A(t)+\Delta B(t)K\big)^{\mathrm{T}}M_3^{\mathrm{T}},\\
\sigma_{22}&=M_2\big(\Delta A_\tau(t)+\Delta B_\tau(t)K\big)+\big(\Delta A_\tau(t)+\Delta B_\tau(t)K\big)^{\mathrm{T}}M_2^{\mathrm{T}},\\
\sigma_{23}&=0,\qquad \sigma_{24}=\big(\Delta A_\tau(t)+\Delta B_\tau(t)K\big)^{\mathrm{T}}M_3^{\mathrm{T}},\\
\sigma_{1i}&=\sigma_{2i}=\sigma_{sl}=0\quad (i=5,6,7,8,\ 3\le s\le l\le 8),\\
c_{11}&=S_1\Delta E(t)+\Delta E^{\mathrm{T}}(t)S_1^{\mathrm{T}},\qquad c_{12}=S_1\Delta E_\tau(t)+\Delta E^{\mathrm{T}}(t)S_2^{\mathrm{T}},\\
c_{13}&=c_{14}=0,\qquad c_{15}=\Delta E^{\mathrm{T}}(t)S_3^{\mathrm{T}},\qquad c_{22}=S_2\Delta E_\tau(t)+\Delta E_\tau^{\mathrm{T}}(t)S_2^{\mathrm{T}},\\
c_{23}&=c_{24}=0,\qquad c_{25}=\Delta E_\tau^{\mathrm{T}}(t)S_3^{\mathrm{T}},\\
c_{1\alpha}&=c_{2\alpha}=c_{sl}=0\quad (\alpha=6,7,8,\ 3\le s\le l\le 8).
\end{aligned}$$

Now, we set

$$J(t)=\mathcal{E}\left\{\int_0^t\big[z^{\mathrm{T}}(s)z(s)-\gamma^2v^{\mathrm{T}}(s)v(s)\big]\,\mathrm{d}s\right\},$$

where $t>0$. Because $V(\phi(t),0)=0$ under zero initial conditions, that is, $\phi(t)=0$ for $t\in[-\sigma,0]$, then, by Itô's formula, we derive

$$\begin{aligned}
J(t)&=\mathcal{E}\left\{\int_0^t\big[z^{\mathrm{T}}(s)z(s)-\gamma^2v^{\mathrm{T}}(s)v(s)\big]\,\mathrm{d}s\right\}\\
&=\mathcal{E}\left\{\int_0^t\big[z^{\mathrm{T}}(s)z(s)-\gamma^2v^{\mathrm{T}}(s)v(s)+\mathcal{L}V(x_s,s)\big]\,\mathrm{d}s\right\}-\mathcal{E}V(x_t,t)\\
&\le\mathcal{E}\left\{\int_0^t\big[z^{\mathrm{T}}(s)z(s)-\gamma^2v^{\mathrm{T}}(s)v(s)+\mathcal{L}V(x_s,s)\big]\,\mathrm{d}s\right\}\\
&\le\mathcal{E}\left\{\int_0^t\xi^{\mathrm{T}}(s)\varXi\xi(s)\,\mathrm{d}s\right\},
\end{aligned}$$

where

$$\Xi = \bar{\Lambda}_1 + \Delta\Lambda_1 + \Delta\Lambda_2$$

with

$$\bar{\Lambda}_1 = \begin{bmatrix} \bar{\phi}_{11} & \bar{\phi}_{12} & -G_1 & \phi_{14} & \phi_{15} & -N_1 & -G_1 & \phi_{18} \\ * & \bar{\phi}_{22} & -G_2 & \phi_{24} & \phi_{25} & -N_2 & -G_2 & \phi_{28} \\ * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 & \phi_{48} \\ * & * & * & * & \phi_{55} & 0 & 0 & \phi_{58} \\ * & * & * & * & * & -\frac{1}{\sigma}R & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\sigma}R & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\bar{\phi}_{11} = \tilde{\phi}_{11} + (C+DK)^{\mathrm{T}}(C+DK),$$
$$\bar{\phi}_{12} = \tilde{\phi}_{12} + (C+DK)^{\mathrm{T}}(C_\tau+D_\tau K),$$
$$\bar{\phi}_{22} = \tilde{\phi}_{22} + (C_\tau+D_\tau K)^{\mathrm{T}}(C_\tau+D_\tau K).$$

According to (4) and Lemma 2, $\Delta\Lambda_1$ and $\Delta\Lambda_2$ can be expressed as follows:

$$\Delta\Lambda_1 = \Sigma_1 HF(t)\Pi_1 + \Pi_1^{\mathrm{T}}F^{\mathrm{T}}(t)H^{\mathrm{T}}\Sigma_1^{\mathrm{T}} \le \varepsilon_1^{-1}\Sigma_1 HH^{\mathrm{T}}\Sigma_1^{\mathrm{T}} + \varepsilon_1\Pi_1^{\mathrm{T}}\Pi_1, \quad (22)$$

$$\Delta\Lambda_2 = \Sigma_2 HF(t)\Pi_2 + \Pi_2^{\mathrm{T}}F^{\mathrm{T}}(t)H^{\mathrm{T}}\Sigma_2^{\mathrm{T}} \le \varepsilon_2^{-1}\Sigma_2 HH^{\mathrm{T}}\Sigma_2^{\mathrm{T}} + \varepsilon_2\Pi_2^{\mathrm{T}}\Pi_2, \quad (23)$$

where

$$\Sigma_1^{\mathrm{T}} = \begin{bmatrix} M_1^{\mathrm{T}} & M_2^{\mathrm{T}} & 0 & M_3^{\mathrm{T}} & 0_{1\times 4} \end{bmatrix},$$
$$\Sigma_2^{\mathrm{T}} = \begin{bmatrix} S_1^{\mathrm{T}} & S_2^{\mathrm{T}} & 0 & 0 & S_3^{\mathrm{T}} & 0_{1\times 3} \end{bmatrix},$$
$$\Pi_1 = [\, N_a + N_b K \quad N_{a\tau} + N_{b\tau}K \quad 0_{1\times 6}\,],$$
$$\Pi_2 = [\, N_e \quad N_{e\tau} \quad 0_{1\times 6}\,].$$

Applying Schur's complement to (10), we know that $\Xi < 0$. Moreover, $J(t) < 0$ for all $t > 0$. Consequently, $\|z(t)\|_{E_2} < \gamma\|v(t)\|_2$ holds for any nonzero $v(t) \in L_2[0,\infty)$. Furthermore, when $v(t) = 0$, the following inequality is true:

$$\mathcal{E}\mathcal{L}V(x_t,t) \le \zeta^{\mathrm{T}}(t)[\Theta + \Delta\Theta_1 + \Delta\Theta_2]\zeta(t),$$

where

$$\Theta = \begin{bmatrix} \phi_{11} & \phi_{12} & -G_1 & \phi_{14} & \phi_{15} & -N_1 & -G_1 \\ * & \phi_{22} & -G_2 & \phi_{24} & \phi_{25} & -N_2 & -G_2 \\ * & * & -Q_2 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 \\ * & * & * & * & \phi_{55} & 0 & 0 \\ * & * & * & * & * & -\frac{1}{\sigma}R & 0 \\ * & * & * & * & * & * & -\frac{1}{\sigma}R \end{bmatrix},$$

$$\Delta\Theta_1 = (\sigma_{ij})_{7\times 7}, \qquad \Delta\Theta_2 = (c_{ij})_{7\times 7},$$
$$\zeta^{\mathrm{T}}(t) = \big[x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-\tau(t)) \quad x^{\mathrm{T}}(t-\sigma) \quad y^{\mathrm{T}}(t) \quad g^{\mathrm{T}}(t)$$
$$\int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)\,\mathrm{d}s \quad \int_{t-\sigma}^{t-\tau(t)} y^{\mathrm{T}}(s)\,\mathrm{d}s\big].$$

By following a similar line as (22)–(23), and applying Schur's complement to (10), we can obtain that $\mathcal{E}\mathcal{L}V(x_t,t)<0$, and consequently, by using Definition 1 and [15], we can conclude that the trivial solution of the resulting closed-loop system (8) is robustly stochastically stable. The proof is completed. □

Remark 1 In the proof of Theorem 1, some free-weighting matrices such as N_p, G_p, M_q and S_q $(p=1,2,\ q=1,2,3)$ are introduced, which may lead to a less conservative condition.

Remark 2 When the differential of $\tau(t)$ is unknown, and the delay $\tau(t)$ satisfies $0<\tau(t)\le\sigma$, by setting $Q_1=0$ in (14), we know that the system (8) and (9) is delay-dependent and rate-independent robustly stochastically stable with disturbance attenuations γ for any delay $\tau(t)$ satisfying $0<\tau(t)\le\sigma$.

4 Robust H_∞ Control

This section is devoted to the design of the feedback gain matrix K such that the resulting closed-loop system (8) and (9) is robustly stochastically stable with disturbance attenuation γ. For this problem, under the assumption of zero initial conditions, we have the following result.

Theorem 2 *Consider the closed-loop system* (8) *and* (9). *For given scalars* $\sigma>0$, h, $\gamma>0$, a_q *and* b_q $(q=1,2,3)$, *the stochastic system* (8) *and* (9) *is robustly stochastically stabilizable with disturbance attenuation* γ *for any* $\tau(t)$ *satisfying* $0<\tau(t)\le\sigma$, $\dot\tau(t)\le h<\infty$, *if there exist matrices* X, L, $\bar P>0$, $\bar Q_p>0$, $\bar R>0$, $\bar G_p$ *and* $\bar N_p$ $(p=1,2)$, *as well as positive scalars* ϵ_1 *and* ϵ_2, *satisfying the following LMI*:

$$\begin{bmatrix}
\varphi_{11} & \varphi_{12} & -\bar G_1 & \varphi_{14} & \varphi_{15} & -\bar N_1 & -\bar G_1 & \varphi_{18} & \varphi_{19} & \varphi_{110} & \varphi_{111}\\
* & \varphi_{22} & -\bar G_2 & \varphi_{24} & \varphi_{25} & -\bar N_2 & -\bar G_2 & \varphi_{28} & \varphi_{29} & \varphi_{210} & \varphi_{211}\\
* & * & -\bar Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & \varphi_{44} & 0 & 0 & 0 & \varphi_{48} & 0 & 0 & 0\\
* & * & * & * & \varphi_{55} & 0 & 0 & \varphi_{58} & 0 & 0 & 0\\
* & * & * & * & * & -\frac{1}{\sigma}\bar R & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & -\frac{1}{\sigma}\bar R & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & -I & 0 & 0\\
* & * & * & * & * & * & * & * & * & -\epsilon_1 I & 0\\
* & * & * & * & * & * & * & * & * & * & -\epsilon_2 I
\end{bmatrix} < 0, \tag{24}$$

where

$$
\begin{aligned}
\varphi_{11} &= \bar{Q}_1 + \bar{Q}_2 + \bar{N}_1 + \bar{N}_1^{\mathrm{T}} + a_1 A X^{\mathrm{T}} + a_1 X A^{\mathrm{T}} + a_1 B L + a_1 L^{\mathrm{T}} B^{\mathrm{T}} \\
&\quad + b_1 E X^{\mathrm{T}} + b_1 X E^{\mathrm{T}} + \epsilon_1 a_1^2 H H^{\mathrm{T}} + \epsilon_2 b_1^2 H H^{\mathrm{T}}, \\
\varphi_{12} &= -\bar{N}_1 + \bar{N}_2^{\mathrm{T}} + \bar{G}_1 + a_1 A_\tau X^{\mathrm{T}} + a_1 B_\tau L + a_2 X A^{\mathrm{T}} + a_2 L^{\mathrm{T}} B^{\mathrm{T}} \\
&\quad + b_1 E_\tau X^{\mathrm{T}} + b_2 X E^{\mathrm{T}} + \epsilon_1 a_1 a_2 H H^{\mathrm{T}} + \epsilon_2 b_1 b_2 H H^{\mathrm{T}}, \\
\varphi_{14} &= \bar{P} - a_1 X^{\mathrm{T}} + a_3 X A^{\mathrm{T}} + a_3 L^{\mathrm{T}} B^{\mathrm{T}} + \epsilon_1 a_1 a_3 H H^{\mathrm{T}}, \\
\varphi_{15} &= -b_1 X^{\mathrm{T}} + b_3 X E^{\mathrm{T}} + \epsilon_2 b_1 b_3 H H^{\mathrm{T}}, \qquad \varphi_{18} = a_1 B_v + b_1 E_{v,} \\
\varphi_{19} &= X C^{\mathrm{T}} + L^{\mathrm{T}} D^{\mathrm{T}}, \qquad \varphi_{110} = X N_a^{\mathrm{T}} + L^{\mathrm{T}} N_b^{\mathrm{T}}, \qquad \varphi_{111} = X N_e^{\mathrm{T}} \\
\varphi_{22} &= -(1-h)\bar{Q}_1 - \bar{N}_2 - \bar{N}_2^{\mathrm{T}} + \bar{G}_2 + \bar{G}_2^{\mathrm{T}} + a_2 A_\tau X^{\mathrm{T}} + a_2 X A_\tau^{\mathrm{T}} + a_2 B_\tau L \\
&\quad + a_2 L^{\mathrm{T}} B_\tau^{\mathrm{T}} + b_2 E_\tau X^{\mathrm{T}} + b_2 X E_\tau^{\mathrm{T}} + \epsilon_1 a_2^2 H H^{\mathrm{T}} + \epsilon_2 b_2^2 H H^{\mathrm{T}}, \\
\varphi_{24} &= -a_2 X^{\mathrm{T}} + a_3 X A_\tau^{\mathrm{T}} + a_3 L^{\mathrm{T}} B_\tau^{\mathrm{T}} + \epsilon_1 a_2 a_3 H H^{\mathrm{T}}, \\
\varphi_{25} &= -b_2 X^{\mathrm{T}} + b_3 X E_\tau^{\mathrm{T}} + + \epsilon_2 b_2 b_3 H H^{\mathrm{T}}, \qquad \varphi_{28} = a_2 B_v + b_2 E_{v,} \\
\varphi_{29} &= X C_\tau^{\mathrm{T}} + L^{\mathrm{T}} D_\tau^{\mathrm{T}}, \qquad \varphi_{210} = X N_{a\tau}^{\mathrm{T}} + L^{\mathrm{T}} N_{b\tau}^{\mathrm{T}}, \qquad \varphi_{211} = X N_{e\tau}^{\mathrm{T}}, \\
\varphi_{44} &= \sigma \bar{R} - a_3 X - a_3 X^{\mathrm{T}} + \epsilon_1 a_3^2 H H^{\mathrm{T}}, \qquad \varphi_{48} = a_3 B_v, \qquad \varphi_{58} = b_3 E, \\
\varphi_{55} &= \bar{P} - b_4 X - b_4 X^{\mathrm{T}} + \epsilon_2 b_3^2 H H^{\mathrm{T}}.
\end{aligned}
$$

Moreover, the feedback gain matrix K is given by

$$K = L X^{-\mathrm{T}}.$$

Proof According to Theorem 1, if we pre- and post-multiply (10) by

$$\Omega = \operatorname{diag}(X \quad X \quad X \quad X \quad X \quad X \quad X \quad I \quad I \quad I \quad I)$$

and its transpose, we have that $\Omega(10)\Omega^{\mathrm{T}} < 0$. Now, we define new variables in $\Omega(10)\Omega^{\mathrm{T}} < 0$ as follows:

$$
\begin{aligned}
& M_q = a_q X^{-1}, \qquad S_q = b_q X^{-1}, \qquad \bar{P} = X P X^{\mathrm{T}}, \qquad \bar{Q}_p = X Q_p X^{\mathrm{T}}, \\
& \bar{R} = X R X^{\mathrm{T}}, \\
& \bar{N} = X N X^{\mathrm{T}}, \qquad \bar{G} = X G X^{\mathrm{T}}, \qquad \epsilon_1 = \frac{1}{\varepsilon_1}, \qquad \epsilon_2 = \frac{1}{\varepsilon_2}, \qquad L = K X^{\mathrm{T}}, \\
& \quad p = 1, 2, \quad q = 1, 2, 3,
\end{aligned}
$$

with X being an invertible matrix. Then (24) follows immediately by applying Schur's complement to $\Omega(10)\Omega^{\mathrm{T}} < 0$. The proof is completed. □

Remark 3 For uncertain stochastic systems with both state and input delays, Theorem 2 presents a delay-dependent robust H_∞ control scheme.Under the assumption that the time delays are time varying, delay-independent and delay-dependent conditions have been proposed respectively in [5] and [27] in which h must satisfy $h < 1$. However, in this paper, $\dot{\tau}(t) \le h < \infty$.

Remark 4 In order to determine the feedback gain matrices, we have to set $M_q = a_q X^{-1}$, $S_q = b_q X^{-1}$ $(q = 1, 2, 3)$ to obtain the LMI condition. The parameters a_q and b_q should be given prior to solve LMI (24). How to choose these design parameters to optimize is still an open problem.

For the case when $v(t) = 0$ we have the following corollary.

Corollary 1 *Consider the closed-loop system* (8). *For given scalars* $\sigma > 0, h$, a_q *and* b_q $(q = 1, 2, 3)$, *the stochastic system* (8) *is robustly stochastically stabilizable for any* $\tau(t)$ *satisfying* $0 < \tau(t) \le \sigma$, $\dot{\tau}(t) \le h < \infty$, *if there exist matrices* X, L, $\bar{P} > 0$, $\bar{Q}_p > 0$, $\bar{R} > 0$, $\bar{G}_p$ *and* $\bar{N}_p$ $(p = 1, 2)$, *as well as constant* $\epsilon_1 > 0$ *and* $\epsilon_2 > 0$, *satisfying the following LMI*:

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & -\bar{G}_1 & \varphi_{14} & \varphi_{15} & -\bar{N}_1 & -\bar{G}_1 & \varphi_{110} & \varphi_{111} \\ * & \varphi_{22} & -\bar{G}_2 & \varphi_{24} & \varphi_{25} & -\bar{N}_2 & -\bar{G}_2 & \varphi_{210} & \varphi_{211} \\ * & * & -\bar{Q}_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{1}{\sigma}\bar{R} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\sigma}\bar{R} & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon_1 I & 0 \\ * & * & * & * & * & * & * & * & -\epsilon_2 I \end{bmatrix} < 0.$$

Moreover, the feedback gain matrix K *is given by*

$$K = LX^{-\mathrm{T}}.$$

5 Numerical Examples

In this section, the following two examples are used to demonstrate the effectiveness of the proposed method.

Example 1 Consider the uncertain stochastic system (1),

$$A = \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}, \qquad A_\tau = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 \\ -1 & -4 \end{bmatrix},$$

$$E = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.1 \end{bmatrix}, \qquad E_\tau = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.1 \end{bmatrix}, \qquad M = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$B_\tau = N_a = N_{a\tau} = N_b = N_{b\tau} = N_e = N_{e\tau} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then, by using Corollary 1 with $a_1 = a_2 = 0.04$, $a_3 = 0.08$, $b_1 = b_2 = 0.04$ and $b_3 = 0.3$, it is found that the maximal allowable delay is $\sigma = 1.9296$, and the feedback gain matrix is computed as

$$K = \begin{bmatrix} 0.1640 & -0.6430 \\ 0.3078 & 1.6630 \end{bmatrix}.$$

Specifically, when $B_\tau = \Delta B_\tau = 0$ in this example, the system reduces to the one in [5]. By Theorem 1 in [5], it is found that the maximal allowable delay is $\sigma = 1.2499$ and the corresponding stabilizing control gain matrix is given by

$$K = \begin{bmatrix} -1.0518 & 0.4515 \\ 2.4766 & -0.5100 \end{bmatrix}.$$

However, by Corollary 1 in our paper, we can obtain that the maximal allowable delay is $\sigma = 1.9255$ and the feedback gain matrix is given as follows:

$$K = \begin{bmatrix} -1.1200 & -1.2845 \\ 0.7985 & 2.5232 \end{bmatrix}.$$

Therefore, from this example we can see that our delay-dependent condition for robust stochastic stabilization gives a less conservative result than that obtained by the methods in [5].

Example 2 Consider the uncertain stochastic system (1)–(2),

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \qquad A_\tau = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}, \qquad E = \begin{bmatrix} 0.01 & 0 \\ 0 & -0.01 \end{bmatrix},$$
$$E_\tau = \begin{bmatrix} 0.01 & 0.01 \\ 0 & -0.01 \end{bmatrix}, \qquad B = [0 \quad 1]^{\mathrm{T}}, \qquad B_\tau = [-0.2 \quad -0.1]^{\mathrm{T}},$$
$$B_v = [1 \quad 1]^{\mathrm{T}}, \qquad E_v = [0.1 \quad 0.2]^{\mathrm{T}}, \qquad C = [0 \quad 1],$$
$$C_\tau = [-1 \quad 1], \qquad D = 0.1, \qquad D_\tau = 1.2, \qquad M = [0.2 \quad 0.2]^{\mathrm{T}},$$
$$N_b = [1 \quad 1]^{\mathrm{T}}, \qquad N_{b\tau} = [1 \quad 1]^{\mathrm{T}}, \qquad N_a = N_{a\tau} = N_e = N_{e\tau} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then, for $\gamma = 1$, by using Theorem 2 with $a_1 = a_2 = 0.04$, $a_3 = 0.08$, $b_1 = b_2 = 0.04$ and $b_3 = 0.3$ and $h = 0$, it has been found that the maximal allowable delay is $\sigma = 0.3444$, and the feedback gain matrix is computed as

$$K = [0.4075 \quad -1.3654].$$

Specifically, when $B_\tau = \Delta B_\tau = E = E_\tau = C_\tau = D_\tau = 0$, the system becomes the one in [5]. As stated in [5], for $\gamma = 4.86$ Theorem 2 in [5] provides the maximal allowable delay $\sigma = 0.39$, and

$$K = [0.4742 \quad -2.7005],$$

while the method in [27] does not work for this system. However, for $\gamma = 1$, by Theorem 2 in our paper, the maximal allowable delay is $\sigma = 0.5520$, and the feedback gain matrix is

$$K = [0.0576 \quad -3.5430].$$

6 Conclusions

In this paper, the problem of delay-dependent robust H_∞ control has been addressed for uncertain stochastic systems with state and input delays. The free-weighting matrix technique has been used to develop the delay-dependent stability conditions, and a robust H_∞ controller has been constructed. Some numerical examples have been given to illustrate the effectiveness of our results, and the simulations show that our results are less conservative than the existing ones.

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