Circuits Syst Signal Process (2008) 27: 627–643
DOI 10.1007/s00034-008-9046-7

Robust H_∞ Control for Class of Discrete-Time Markovian Jump Systems with Time-Varying Delays Based on Delta Operator

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Received: 10 April 2007 / Revised: 5 August 2007 / Published online: 30 July 2008

Abstract In this paper, the problem of robust H_∞ state feedback control using a delta operator approach for a class of linear fractional uncertain jump systems with time-varying delays is investigated. Based on the Lyapunov–Krasovskii functional in the delta domain, a new delay-dependent H_∞ state feedback controller which requires both robust stability and a prescribed H_∞ performance is presented in terms of linear matrix inequalities. The sampling period T appears as an explicit parameter; therefore, it is easy to observe and analyze the effect of the results with different sampling periods. Furthermore, the proposed method can unify some previous related continuous and discrete systems into the framework of delta operator systems. Numerical examples are presented to illustrate the effectiveness of the developed techniques.

Keywords Markovian jump parameters · Linear fractional uncertainties · Time-varying delays · Discrete-time systems · Delta operators · H_∞ control · Linear matrix inequalities

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1 Introduction

It is well known that many discrete systems are the result of sampling continuous systems. However, when sampling is fast using the traditional shift operator, the poles are located in the stable boundary, and the discrete systems will lose stability in a finite word length computer. Goodwin constructed a delta operator instead of the traditional shift operator for sampling continuous systems in [7]. It is easy to see that the discrete delta operator approximates the Euler derivative, which can cause a discrete-time system to become a quasi-continuous-time system for fast sampling frequencies. The quasi-continuous-time system called the delta operator system, which describes some practical models, can obtain a better result than the discrete shift operator. The delta operator model also has better numerical properties at high sampling rates, which was proved in [21]. In contrast to the discrete shift operator approximations, the delta operator approach means that the Euler derivative can lead to a quasi-continuous-time s-domain model for high sampling frequencies; for example, see [5]. The robustness problem for some delta operator systems with parametric uncertainties also has been investigated. The papers [19] and [20] consider the stability of delta operator systems, and the paper [4] reports an H_∞ control approach for delta operator systems, demonstrating a discrete-time controller approach to continuous-time systems using a delta operator.

Because time delays may lead to system instability, systems with time delays have received considerable attention; see, for example, [1–3, 6, 12–14, 22, 25, 27]. The paper [24] extends some of the robust stability results of the class of linear discrete-time systems with time-varying delays. The class of Markovian jump systems is an important family of systems subject to abrupt variations. In particular, the class of continuous systems with Markovian jump parameters is considered; see, for example, [17, 18, 29]. There are also a few papers that consider discrete systems with time delays [10, 16, 26]. H_∞ control was used in [23] via a linear matrix inequality (LMI) approach. This kind of problem has been considered for discrete-time systems with delays in [15], and for jump systems in the continuous case in [8]. Robust stochastic stability for jumping linear continuous-time and discrete-time systems with perturbed transition rates and probabilities has been studied in [11]. The delta operator system may also have finite modes, and the modes may switch from one to another at different times. The jumping between different system modes can be governed by a stochastic Markov chain.

In this paper, we focus on both time-varying delayed delta operator systems and Markovian jump parameters. The problem is to design a state feedback controller such that for all admissible linear fractional uncertainties the closed-loop system is robust, asymptotically stable and satisfies a prescribed H_∞ performance level. Control design methods in the sense of delay-dependent stability are developed for solving the robust control problems, and all results are presented in terms of some new Lyapunov functions in the delta domain and LMI form. The sampling period T is an explicit parameter in our result, so that it is easy to observe and convenient to analyze the effect of the state feedback controller with different sampling periods. Note that we employ a fast sampling method in discrete systems, where the delays are actually time varying. It is assumed that both the lower delay bound and upper delay bound can be denoted to the summations of the same sampling periods, respectively. Numerical examples are given to illustrate the feasibility and effectiveness of the developed technique.

Notation Throughout this paper, we let $t = kT$ for convenience in the following analysis. $\Re^n$ denotes the n-dimensional Euclidean space; the notation $X > Y$ ($X \geq Y$) means that the matrix $X - Y$ is positive definite ($X - Y$ is semi-positive definite, respectively); $P > 0$ means that P is symmetric and positive definite; I is the identity matrix of appropriate dimension and O denotes a zero matrix of appropriate dimension. For any matrix A, A^T denotes the transpose of matrix A, and A^{-1} denotes the inverse of matrix A. The shorthand $\mathrm{diag}\{M_1\ M_2\ \ldots\ M_r\}$ denotes a block diagonal matrix where the diagonal blocks are the matrices $M_1, M_2, \ldots, M_r$; the symmetric terms in a symmetric matrix are denoted by $*$.

2 Problem Statement

We are given a probability space $(\Omega, \mathbf{F}, \mathbf{P})$, where Ω is the sample space, $\mathbf{F}$ is the algebra of events and $\mathbf{P}$ is the probability measure defined on $\mathbf{F}$. $\{\eta_t,\ t \geq 0\}$ is a homogeneous, finite-state Markovian process with right continuous trajectories that takes values in a finite set $S = \{1, 2, \ldots, s\}$ with generator $\Lambda = (\lambda_{ij})$. The transition probability from mode i at time t to mode j at time $t + T$, $i,\ j \in S$, is

$$\Pr(\eta_{t+T} = j \,|\, \eta_t = i) = \begin{cases} \lambda_{ij} T, & i \neq j, \\ 1 + \lambda_{ij} T, & i = j, \end{cases} \tag{2.1}$$

where the transition probability rates satisfy $\lambda_{ij} \geq 0$ for $i,\ j \in S$, $i \neq j$ and $\lambda_{ii} = -\sum_{j=1,\ j \neq i}^{s} \lambda_{ij}$. In this paper, the following time quasi-continuous uncertain delta operator system with time-varying delays is considered:

$$\begin{aligned} \delta x(t) &= \big(A(\eta_t) + \Delta A(\eta_t, t)\big)x(t) + \big(A_d(\eta_t) + \Delta A_d(\eta_t, t)\big)x\big(t - d(k)\big) \\ &\quad + B_d(\eta_t)\omega(t) + \big(B(\eta_t) + \Delta B(\eta_t, t)\big)u(t), \qquad (2.2) \\ z(t) &= \big(C(\eta_t) + \Delta C(\eta_t, t)\big)x(t) + \big(C_d(\eta_t) + \Delta C_d(\eta_t, t)\big)x\big(t - d(k)\big) \\ &\quad + D_d(\eta_t)\omega(t) + \big(D(\eta_t) + \Delta D(\eta_t, t)\big)u(t), \qquad (2.3) \\ x(t) &= \phi(t), \quad t \in [-d_{\mathrm{M}}, 0], \end{aligned}$$

where the delta operator $\delta x(t)$ is defined by

$$\delta x(t) = \begin{cases} \frac{\mathrm{d}x(t)}{\mathrm{d}t}, & T = 0, \\ \frac{x(t+T) - x(t)}{T}, & T \neq 0, \end{cases}$$

where T is a sampling period. $x(t) \in \Re^n$ is the state variable; $u(t) \in \Re^p$ is the control input; $z(t) \in \Re^p$ is the control output; $\omega(t) \in \Re^l$ is the disturbance input which belongs to $L_2[0, \infty)$. The time delay $d(k)$ is a time-varying function that satisfies

$0 \le d_{\mathrm{m}} \le d(k) \le d_{\mathrm{M}}$, with $d_{\mathrm{m}} = n_{\mathrm{m}}T$ and $d_{\mathrm{M}} = n_{\mathrm{M}}T$, where n_{m} and n_{M} are two known positive and finite integers, from which we let $n_{\mathrm{m}} \le n \le n_{\mathrm{M}}$. For notational simplicity, in the sequel, for $\eta_k = i \in S$, we denote $A(\eta_t)$ by A_i, $\Delta A(\eta_t,\ t)$ by $\Delta A_i(t)$, and so on. The linear fractional parametric uncertainties $\Delta A_i(t)$, $\Delta A_{di}(t)$, $\Delta B_i(t)$, $\Delta C_i(t)$, $\Delta C_{di}(t)$ and $\Delta D_i(t)$ are time-varying matrices with appropriate dimensions, which are defined as follows:

$$\begin{bmatrix} \Delta A_i(t) & \Delta A_{di}(t) & \Delta B_i(t) \\ \Delta C_i(t) & \Delta C_{di}(t) & \Delta D_i(t) \end{bmatrix} = \begin{bmatrix} H_{1i} \\ H_{2i} \end{bmatrix} \hat{F}_i(t)[\,E_{1i} \quad E_{2i} \quad E_{3i}\,], \tag{2.4}$$

where H_{1i}, H_{2i}, E_{1i}, E_{2i} and E_{3i} are known constant real matrices with appropriate dimensions, $\hat{F}_i(t) = F_i(t)[I - G_iF_i(t)]^{-1}$, and $F_i(t)$ is an unknown time-varying matrix satisfying $F_i^T(t)F_i(t) \le I$, $\forall t \ge 0$. It is assumed that the matrix $[I - G_iF_i(t)]^{-1}$ is invertible for any $F_i(t)$ and $I - G_i^TG_i > 0$.

Remark 2.1 The model (2.4) describes a wider class of parameter uncertainties than norm-bounded parameter uncertainties. It is easy to see that the linear fractional parameter uncertainties can be reduced to norm-bounded parameter uncertainties when $G_i = 0$.

Definition 2.1 Consider the uncertain system (2.1)–(2.2) and the state feedback $u(t) = K_ix(t)$. If the following conditions are satisfied:

(i) With $\omega(t) = 0$, the closed-loop system (2.1)–(2.2) and $u(t) = K_ix(t)$ is asymptotically stable.
(ii) With zero initial condition (i.e. $\phi = 0$), the following condition is satisfied:

$$J = \sum_{t=0}^{\infty}\left[\gamma^{-1}z^T(t)z(t) - \gamma\omega^T(t)\omega(t)\right] \le 0,$$

for some $\gamma > 0$, then the control $u(t) = K_ix(t)$ is said to be the H_∞ control of system (2.1)–(2.2) with disturbance attenuation γ. The parameter γ is called the H_∞ norm bound of the control.

Definition 2.2 A delta operator system is stochastically asymptotically stable in the delta domain if the following conditions hold:

(i) $V(x(t)) \ge 0$, with equality if and only if $x(t) = 0$
(ii) $\sigma V(x(t)) = [\mathbf{E}[V(x(t+T))] - V(x(t))]/T < 0$

where $V(x(t))$ is a Lyapunov function in the delta domain.

Before ending this section, the following lemmas will be used to prove our main results in this paper.

Lemma 2.1 [9] *For any constant positive semi-definite symmetric matrix W, and two positive integers r and r_0 satisfying $r \geq r_0 \geq 1$, the following inequality holds*:

$$\left(\sum_{i=r_0}^{r} x(i)\right)^T W \left(\sum_{i=r_0}^{r} x(i)\right) \leq (r - r_0 + 1) \sum_{i=r_0}^{r} x^T(i) W x(i).$$

Lemma 2.2 [28] *For some given matrices Υ, D and E of appropriate dimension and with Υ symmetric, we have*

$$\Upsilon + D\hat{F}(t)E + E^T \hat{F}^T(t) D^T \leq 0,$$

with $\hat{F}(t)$ as in (2.4), if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Upsilon + \begin{bmatrix} \varepsilon^{-1} E^T & \varepsilon D \end{bmatrix} \begin{bmatrix} I & -G \\ -G^T & I \end{bmatrix} \begin{bmatrix} \varepsilon^{-1} E \\ \varepsilon D^T \end{bmatrix} \leq 0.$$

3 Main Results

The following control law is employed to deal with the problem of stabilization via state feedback:

$$u(t) = K(\eta_t) x(t), \tag{3.1}$$

where $K(\eta_t) = K_i$ is the state feedback controller gain to be determined such that the closed-loop system is asymptotically stable for any $d_{\mathrm{m}} \leq d(k) \leq d_{\mathrm{M}}$. Consider system (2.1)–(2.2) associated with the control law (3.1). The resulting closed-loop system can be expressed as follows:

$$\delta x(t) = \big(A_{ki} + \Delta A_{ki}(t)\big)x(t) + \big(A_{di} + \Delta A_{di}(t)\big)x\big(t - d(k)\big) + B_{di}\omega(t), \tag{3.2}$$

$$z(t) = \big(C_{ki} + \Delta C_{ki}(t)\big)x(t) + \big(C_{di} + \Delta C_{di}(t)\big)x\big(t - d(k)\big) + D_{di}\omega(t), \tag{3.3}$$

where $A_{ki} = A_i + B_i K_i$, $\Delta A_{ki}(t) = \Delta A_i(t) + \Delta B_i(t) K_i$, $C_{ki} = C_i + D_i K_i$, $\Delta C_{ki}(t) = \Delta C_i(t) + \Delta D_i(t) K_i$. We let $A_{ki}(t) = A_{ki} + \Delta A_{ki}(t)$, $C_{ki}(t) = C_{ki} + \Delta C_{ki}(t)$ for convenience.

In this section, an LMI approach for designing the desired H_∞ state feedback controller is developed. The following theorem presents a delay-range-dependent result in terms of LMIs.

Theorem 3.1 *Given a scalar $\gamma > 0$, the uncertain jump delta operator system in (3.2)–(3.3) is robustly stable with disturbance attenuation γ, if there exist matrices $\tilde{P}_i > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$ and N_i, as well as positive scalars α_i such that the following*

LMI holds:

$$\Sigma=\begin{bmatrix}\Omega_{11} & \Omega_{12} & A_{di}\tilde{P}_i & B_{di}\tilde{P}_i & 0 & 0 & \alpha_i H_{1i}\\ * & \Omega_{22} & \frac{1}{d_{\mathrm{M}}}\tilde{R}+A_{di}\tilde{P}_i & B_{di}\tilde{P}_i & \tilde{P}_i C_i^T+N_i^T D_i^T & \tilde{P}_i E_{1i}^T+N_i^T E_{3i}^T & \alpha_i H_{1i}\\ * & * & -\tilde{Q}-\frac{1}{d_{\mathrm{M}}}\tilde{R} & 0 & \tilde{P}_i C_{di}^T & \tilde{P}_i E_{2i}^T & 0\\ * & * & * & -\gamma I & D_{di}^T & 0 & 0\\ * & * & * & * & -\gamma I & 0 & \alpha_i H_{2i}\\ * & * & * & * & * & -\alpha_i I & \alpha_i G_i\\ * & * & * & * & * & * & -\alpha_i I\end{bmatrix}<0, \tag{3.4}$$

with

$$\Omega_{11}=T^2\sum_{j=1}^{s}\lambda_{ij}\tilde{P}_j-2\tilde{P}_i+T\tilde{P}_i+d_{\mathrm{M}}\tilde{R},\qquad \Omega_{12}=T\sum_{j=1}^{s}\lambda_{ij}\tilde{P}_j+A_i\tilde{P}_i+B_iN_i^T,$$

$$\Omega_{22}=\sum_{j=1}^{s}\lambda_{ij}\tilde{P}_j+A_i\tilde{P}_i+B_iN_i^T+\tilde{P}_iA_i^T+N_iB_i^T+(d_{\mathrm{M}}-d_{\mathrm{m}}+1)\tilde{Q}-\frac{1}{d_{\mathrm{M}}}\tilde{R}.$$

Then the system (3.2)–(3.3) *is asymptotically stable. Moreover, a suitable stabilizing* H_∞ *jump state feedback controller can be chosen by* $u(t)=N_i\tilde{P}_i^{-1}x(t)$.

Proof Construct a Lyapunov–Krasovskii functional in the delta domain as follows:

$$V(x,t,\eta_t)=V_1(x,t,\eta_t)+V_2(x,t,\eta_t)+V_3(x,t,\eta_t)+V_4(x,t,\eta_t),$$

with

$$V_1(x,t,\eta_t)=x^T(t)P(\eta_t)x(t),\qquad V_2(x,t,\eta_t)=T\sum_{i=1}^{n}x^T(t-iT)Qx(t-iT),$$

$$V_3(x,t,\eta_t)=T^2\sum_{i=n_{\mathrm{m}}+1}^{n_{\mathrm{M}}}\sum_{j=1}^{i}x^T(t-jT)Qx(t-jT),$$

$$V_4(x,t,\eta_t)=\sum_{i=1}^{n}\sum_{j=1}^{i}e^T(t-jT)Re(t-jT),$$

where $e(j)=x(j)-x(j+T)$, so there exist $\delta x(j)=-e(j)/T$ and $e(t-iT)=x(t-iT)-x(t-(i-1)T)$. The delta operator of the stochastic process $\{x(t),\ \eta_t,\ t\geq 0\}$, acting on $V(x,\ t,\ \eta_t)$ at the point $\{x,\ t,\ \eta_t=i\}$, can be expressed as follows:

$$\sigma\big(V(x,t,\eta_t)\big)=\frac{\mathbf{E}[V(x,t+T,\eta_{t+T})]-V(x,t,\eta_t)}{T}. \tag{3.5}$$

Let the mode at time t be i, that is, $\eta_t=i$. Recall that, at time $t+T$, the system may jump to any mode $\eta_{t+T}=j$. By letting $\mathbf{E}(P_j)=T\sum_{j=1}^{s}\lambda_{ij}P_j+P_i$ and taking the stochastic delta operator manipulations along the trajectory of system (3.2), we can obtain

$$\begin{aligned}\sigma V_1(x,t,\eta_t) &= \frac{1}{T}\bigl\{\mathbf{E}\bigl[x^T(t+T)P(\eta_{t+T})x(t+T)\bigr] - x^T(t)P\bigl(\eta(t)\bigr)x(t)\bigr\}\\ &= \sum_{j=1}^{s}\lambda_{ij}x^T(t+T)P_jx(t+T)\\ &\quad + \frac{1}{T}\bigl[x^T(t+T)P_ix(t+T) - x^T(t)P_ix(t)\bigr]\\ &= T^2\sum_{j=1}^{s}\lambda_{ij}\delta^T\bigl(x(t)\bigr)P_j\delta\bigl(x(t)\bigr) + T\sum_{j=1}^{s}\lambda_{ij}\delta^T\bigl(x(t)\bigr)P_jx(t)\\ &\quad + T\sum_{j=1}^{s}\lambda_{ij}x^T(t)P_j\delta\bigl(x(t)\bigr)\\ &\quad + \sum_{j=1}^{s}\lambda_{ij}x^T(t)P_jx(t) + T\delta^T\bigl(x(t)\bigr)P_i\delta\bigl(x(t)\bigr) + \delta^T\bigl(x(t)\bigr)P_ix(t)\\ &\quad + x^T(t)P_i\delta\bigl(x(t)\bigr),\end{aligned} \tag{3.6}$$

where we can let $x^T(t)P_i\delta(x(t)) = x^T(t)P_i[A_{ki}(t)x(t) + A_{di}(t)x(t-d(k)) + B_{di}\omega(t)]$. Taking the stochastic delta operator manipulations of $V_2(x,t,\eta_t)$ and $V_3(x,t,\eta_t)$, we can obtain

$$\begin{aligned}\sigma V_2(x,t,\eta_t) &= \frac{1}{T}\Biggl[T\sum_{i=1}^{n}x^T\bigl(t-(i-1)T\bigr)Qx\bigl(t-(i-1)T\bigr)\\ &\quad - T\sum_{i=1}^{n}x^T(t-iT)Qx(t-iT)\Biggr]\\ &\le x^T(t)Qx(t) - x^T\bigl(t-d(k)\bigr)Qx\bigl(t-d(k)\bigr)\\ &\quad + T\sum_{i=n_{\mathrm{m}}+1}^{n_{\mathrm{M}}}x^T(t-iT)Qx(t-iT).\end{aligned} \tag{3.7}$$

$$\begin{aligned}\sigma V_3(x,t,\eta_t) &= T\sum_{i=n_{\mathrm{m}}+1}^{n_{\mathrm{M}}}\Biggl(\sum_{j=1}^{i}x^T\bigl(t-(j-1)T\bigr)Qx\bigl(t-(j-1)T\bigr)\\ &\quad - \sum_{j=1}^{i}x^T(t-jT)Qx(t-jT)\Biggr)\\ &= (d_{\mathrm{M}}-d_{\mathrm{m}})x^T(t)Qx(t) - T\sum_{i=n_{\mathrm{m}}+1}^{n_{\mathrm{M}}}x^T(t-iT)Qx(t-iT).\end{aligned} \tag{3.8}$$

Using Lemma 2.1 and taking the stochastic delta operator manipulations of $V_4(x,t,\eta_t)$, there exists

$$\begin{aligned}
\sigma V_4(x,t,\eta_t) &= \frac{1}{T}\Bigg[\sum_{i=1}^{n}\sum_{j=1}^{i} e^T\big(t-(i-1)T\big)Re\big(t-(i-1)T\big) \\
&\quad - \sum_{i=1}^{n}\sum_{j=1}^{i} e^T(t-iT)Re(t-iT)\Bigg] \\
&= \frac{1}{T}\Bigg[\sum_{i=1}^{n} e^T(t)Re(t) - \sum_{i=1}^{n} e^T(t-iT)Re(t-iT)\Bigg] \\
&\le \frac{n}{T}e^T(t)Re(t) - \frac{1}{nT}\Bigg[\sum_{i=1}^{n} e(t-iT)\Bigg]^T R\Bigg[\sum_{i=1}^{n} e(t-iT)\Bigg] \\
&= d_{\mathrm{M}}\delta^T\big(x(t)\big)R\delta\big(x(t)\big) - \frac{1}{d_{\mathrm{M}}}\big[x\big(t-d(k)\big)-x(t)\big]^T \\
&\quad \times R\big[x\big(t-d(k)\big)-x(t)\big].
\end{aligned} \tag{3.9}$$

For any real matrix P_i, one has that

$$\begin{aligned}
0 &= -2\delta^T\big(x(t)\big)P_i\big[\delta\big(x(t)\big) - A_{ki}(t)x(t) - A_{di}(t)x\big(t-d(k)\big) - B_{di}\omega(t)\big] \\
&= -2\delta^T\big(x(t)\big)P_i\delta\big(x(t)\big) + 2\delta^T\big(x(t)\big)P_iA_{ki}(t)x(t) \\
&\quad + 2\delta^T\big(x(t)\big)P_iA_{di}(t)x\big(t-d(k)\big) + 2\delta^T\big(x(t)\big)P_iB_{di}\omega(t).
\end{aligned} \tag{3.10}$$

And there exists

$$\begin{aligned}
&\gamma^{-1}z^T(t)z(t) - \gamma\omega^T(t)\omega(t) \\
&\quad = \gamma^{-1}\big[x^T(t)C_{ki}^T(t)C_{ki}(t)x(t) + x^T(t)C_{ki}^T(t)C_{di}(t)x\big(t-d(k)\big) \\
&\qquad + x^T(t)C_{ki}^T(t)D_{di}\omega(t) + x^T\big(x-d(k)\big)C_{di}^T(t)C_{ki}(t)x(t) \\
&\qquad + x^T\big(x-d(k)\big)C_{di}^T(t)C_{di}(t)x\big(t-d(k)\big) + x^T\big(x-d(k)\big)C_{di}^T(t)D_{di}\omega(t) \\
&\qquad + \omega^T(t)D_{di}^TC_{ki}(t)x(t) + \omega^T(t)D_{di}^TC_{di}(t)x\big(t-d(k)\big) + \omega^T(t)D_{di}^TD_{di}\omega(t)\big] \\
&\qquad - \gamma\omega^T(t)\omega(t).
\end{aligned} \tag{3.11}$$

Finally, it follows from (3.6)–(3.11) that

$$\sigma V(x,t,\eta_t) \le \varphi^T(t)\Sigma_1\varphi(t) < 0, \tag{3.12}$$

where

$$\varphi^T(t) = \begin{bmatrix}\delta^T(x(t)) & x^T(t) & x^T(t-d(k)) & \omega^T(t)\end{bmatrix},$$

$$\Sigma_1=\begin{bmatrix}(1,1)&(1,2)&P_iA_{di}(t)&P_iB_{di}\\ *&(2,2)&(2,3)&\gamma^{-1}C_{di}^T(t)D_{di}+P_iB_{di}\\ *&*&(3,3)&\gamma^{-1}C_{di}^T(t)D_{di}\\ *&*&*&-\gamma I+\gamma^{-1}D_{di}^TD_{di}\end{bmatrix},$$

with

$$(1,1)=T^2\sum_{j=1}^{s}\lambda_{ij}P_j-2P_i+TP_i+d_{\mathrm{M}}R,\qquad (1,2)=T\sum_{j=1}^{s}\lambda_{ij}P_j+P_iA_{ki}(t),$$

$$(2,2)=\sum_{j=1}^{s}\lambda_{ij}P_j+P_iA_{ki}(t)+A_{ki}^T(t)P_i+(d_{\mathrm{M}}-d_{\mathrm{m}}+1)Q$$
$$-\frac{1}{d_{\mathrm{M}}}R+\gamma^{-1}C_{ki}^T(t)C_{ki}(t),$$

$$(2,3)=\gamma^{-1}C_{ki}^T(t)C_{di}(t)+\frac{1}{d_{\mathrm{M}}}R+P_iA_{di}(t),$$

$$(3,3)=-Q+\gamma^{-1}C_{di}^T(t)C_{di}(t)-\frac{1}{d_{\mathrm{M}}}R.$$

Using Schur's complement, $\Sigma_1<0$ can be changed to

$$\Sigma_2=\begin{bmatrix}(1,1)&(1,2)&P_iA_{di}(t)&P_iB_{di}&0\\ *&(2,2)&\frac{1}{d_{\mathrm{M}}}R+P_iA_{di}(t)&P_iB_{di}&C_{ki}^T(t)\\ *&*&-Q-\frac{1}{d_{\mathrm{M}}}R&0&C_{di}^T(t)\\ *&*&*&-\gamma I&D_{di}^T\\ *&*&*&*&-\gamma I\end{bmatrix}<0, \tag{3.13}$$

$$(1,1)=T^2\sum_{j=1}^{s}\lambda_{ij}P_j-2P_i+TP_i+d_{\mathrm{M}}R,\qquad (1,2)=T\sum_{j=1}^{s}\lambda_{ij}P_j+P_iA_{ki}(t),$$

$$(2,2)=\sum_{j=1}^{s}\lambda_{ij}P_j+P_iA_{ki}(t)+A_{ki}^T(t)P_i+(d_{\mathrm{M}}-d_{\mathrm{m}}+1)Q-\frac{1}{d_{\mathrm{M}}}R.$$

From (2.4) and (3.13) we can get

$$\Sigma_2=\Upsilon+\xi_i\hat{F}(t)\zeta_i+\zeta_i^T\hat{F}^T(t)\xi_i^T<0, \tag{3.14}$$

with

$$\Upsilon=\begin{bmatrix}(1,1)&(1,2)&P_iA_{di}&P_iB_{di}&0\\ *&(2,2)&\frac{1}{d_{\mathrm{M}}}R+P_iA_{di}&P_iB_{di}&C_{ki}^T\\ *&*&-Q-\frac{1}{d_{\mathrm{M}}}R&0&C_{di}^T\\ *&*&*&-\gamma I&D_{di}^T\\ *&*&*&*&-\gamma I\end{bmatrix},$$

$$\xi_i = \begin{bmatrix} P_i H_{1i} \\ P_i H_{1i} \\ 0 \\ 0 \\ H_{2i} \end{bmatrix}, \qquad \zeta_i = \begin{bmatrix} 0 \\ E_{1i}^T + E_{3i}^T \\ E_{2i}^T \\ 0 \\ 0 \end{bmatrix}^T,$$

with

$$(1,1) = T^2 \sum_{j=1}^{s} \lambda_{ij} P_j - 2P_i + T P_i + d_{\mathrm{M}} R, \qquad (1,2) = T \sum_{j=1}^{s} \lambda_{ij} P_j + P_i A_{ki},$$

$$(2,2) = \sum_{j=1}^{s} \lambda_{ij} P_j + P_i A_{ki} + A_{ki}^T P_i + (d_{\mathrm{M}} - d_{\mathrm{m}} + 1) Q - \frac{1}{d_{\mathrm{M}}} R.$$

By Lemma 2.2, there exists a scalar $\varepsilon_i > 0$ such that (3.14) is equal to the following inequality:

$$\Sigma_3 = \Upsilon + \begin{bmatrix} \varepsilon_i \zeta_i^T & \varepsilon_i^{-1} \xi_i \end{bmatrix} \begin{bmatrix} -I & G \\ G^T & -I \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_i \zeta_i \\ \varepsilon_i^{-1} \xi_i^T \end{bmatrix} < 0. \tag{3.15}$$

Using Schur's complement, $\Sigma_3 < 0$ can be changed to $\Sigma_4 < 0$, where

$$\Sigma_4 = \begin{bmatrix} (1,1) & (1,2) & P_i A_{di} & P_i B_{di} & 0 & 0 & \varepsilon_i^{-1} P_i H_{1i} \\ * & (2,2) & \frac{1}{d_{\mathrm{M}}} R + P_i A_{di} & P_i B_{di} & C_{ki}^T & \varepsilon_i E_{1i}^T + \varepsilon_i K_i^T E_{3i}^T & \varepsilon_i^{-1} P_i H_{1i} \\ * & * & -Q - \frac{1}{d_{\mathrm{M}}} R & 0 & C_{di}^T & \varepsilon_i E_{2i}^T & 0 \\ * & * & * & -\gamma I & D_{di}^T & 0 & 0 \\ * & * & * & * & -\gamma I & 0 & \varepsilon_i^{-1} H_{2i} \\ * & * & * & * & * & -I & G_i \\ * & * & * & * & * & * & -I \end{bmatrix},$$

with

$$(1,1) = T^2 \sum_{j=1}^{s} \lambda_{ij} P_j - 2P_i + T P_i + d_{\mathrm{M}} R, \qquad (1,2) = T \sum_{j=1}^{s} \lambda_{ij} P_j + P_i A_{ki},$$

$$(2,2) = \sum_{j=1}^{s} \lambda_{ij} P_j + P_i A_{ki} + A_{ki}^T P_i + (d_{\mathrm{M}} - d_{\mathrm{m}} + 1) Q - \frac{1}{d_{\mathrm{M}}} R.$$

Pre-multiplying $\Sigma_4 < 0$ by the diagonal matrix $\mathrm{diag}\{P_i^{-1}\ P_i^{-1}\ P_i^{-1}\ I\ I\ \varepsilon_i^{-1} I\ \varepsilon_i^{-1} I\}$ and post-multiplying by the diagonal matrix $\mathrm{diag}\{P_i^{-T}\ P_i^{-T}\ P_i^{-T}\ I\ I\ \varepsilon_i^{-1} I\ \varepsilon_i^{-1} I\}$, respectively, and then setting $\varepsilon_i^{-2} = \alpha_i$, $\tilde{P}_i = P_i^{-1}$, $N_i = K_i P_i^{-1}$, $\tilde{P}_j = P_i^{-1} P_j P_i^{-T}$, $\tilde{Q} = P_i^{-1} Q P_i^{-T}$, $\tilde{R} = P_i^{-1} R P_i^{-T}$, the inequality $\Sigma_4 < 0$ is equivalent to $\Sigma < 0$, from which we can get

$$\Sigma_0=\begin{bmatrix}(1,1) & (1,2) & A_{di}\tilde{P}_i & 0 & \alpha_i H_{1i}\\ * & (2,2) & \frac{1}{d_{\mathrm{M}}}\tilde{R}+A_{di}\tilde{P}_i & \tilde{P}_i E_{1i}^T+N_i^T E_{3i}^T & \alpha_i H_{1i}\\ * & * & -\tilde{Q}-\frac{1}{d_{\mathrm{M}}}\tilde{R} & \tilde{P}_i E_{2i}^T & 0\\ * & * & * & -\alpha_i I & \alpha_i G_i\\ * & * & * & * & -\alpha_i I\end{bmatrix}<0, \quad (3.16)$$

with

$$(1,1)=T^2\sum_{j=1}^{s}\lambda_{ij}\tilde{P}_j-2\tilde{P}_i+T\tilde{P}_i+d_{\mathrm{M}}\tilde{R},$$

$$(1,2)=T\sum_{j=1}^{s}\lambda_{ij}\tilde{P}_j+A_i\tilde{P}_i+B_iN_i^T,$$

$$(2,2)=\sum_{j=1}^{s}\lambda_{ij}\tilde{P}_j+A_i\tilde{P}_i+B_iN_i^T+\tilde{P}_iA_i^T+N_iB_i^T+(d_{\mathrm{M}}-d_{\mathrm{m}}+1)\tilde{Q}-\frac{1}{d_{\mathrm{M}}}\tilde{R}.$$

This means that the closed-loop system (2.1) with $\omega(t)=0$ and $u(t)=K_ix(t)$ is asymptotically stable. The solution to the robust H_∞ state feedback controller design problem is presented. Considering the H_∞ performance index in Definition 2.1,

$$J=\sum_{t=0}^{\infty}\left[\gamma^{-1}z^T(t)z(t)-\gamma\omega^T(t)\omega(t)\right],$$

and assuming, without loss of generality, that system (3.2) is stable with zero initial condition, implies that $V(\cdot)|_{t=0}=0$ and $V(\cdot)|_{t\to\infty}\to\epsilon$ with $\epsilon\to 0$, if $\omega(t)=0$ or $\epsilon<\infty$ if $\omega(t)\neq 0$. In this way, the above index can be rewritten as

$$J=\sum_{t=0}^{\infty}\left[\gamma^{-1}z^T(t)z(t)-\gamma\omega^T(t)\omega(t)+\delta V(x,t,\eta_t)\right]. \quad (3.17)$$

And it can be obtained that

$$J=\sum_{t=0}^{\infty}\left[\gamma^{-1}z^T(t)z(t)-\gamma\omega^T(t)\omega(t)\right]\le\sum_{t=0}^{\infty}\left[\zeta^T(t)\Sigma\zeta(t)\right]\le 0, \quad (3.18)$$

with

$$\zeta^T(t)=\begin{bmatrix}\delta^T\big(x(t)\big) & x^T(t) & x^T\big(t-d(k)\big) & \omega^T(t) & y_0^T(t) & y_1^T(t) & y_2^T(t)\end{bmatrix}.$$

where $y_0(t)$, $y_1(t)$ and $y_2(t)$ are arbitrary vectors with appropriate dimensions. From the delta domain Lyapunov–Krasovskii stability theorem, it is easily concluded that the jump delta operator system (2.2)–(2.3) with $u(t)=K_ix(t)$ is asymptotically stable if (3.4) holds. Furthermore, the explicit expression for the H_∞ state feedback controller is given by $u(t)=N_iP_i^{-1}x(t)$.

From the proof of Theorem 3.1, it is easy to see that if there are no uncertainties in the delta operator system (2.2)–(2.3), i.e. $\hat{F}_i(t) \equiv 0$ $(i = 1, 2, \dots, s)$, then

$$\delta x(t) = A(\eta_t)x(t) + A_d(\eta_t)x\big(t - d(k)\big) + B(\eta_t)u(t) + B_d(\eta_t)\omega(t), \tag{3.19}$$

$$z(t) = C(\eta_t)x(t) + C_d(\eta_t)x\big(t - d(k)\big) + D(\eta_t)u(t) + D_d(\eta_t)\omega(t), \tag{3.20}$$

$$x(t) = \phi(t), \quad t \in [-d_{\mathrm{M}},\ 0].$$

And then we have the following corollary.

Corollary 3.1 *With a given scalar $\gamma > 0$, consider the uncertain delta operator system (3.19)–(3.20), if there exist matrices $\tilde{P}_i > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$ and N_i, as well as a positive scalar α_i such that the following LMI holds*:

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & A_{di}\tilde{P}_i & B_{di}\tilde{P}_i & 0 \\ * & \Xi_{22} & \frac{1}{d_{\mathrm{M}}}\tilde{R} + A_{di}\tilde{P}_i & B_{di}\tilde{P}_i & \tilde{P}_i C_i^T + N_i^T D_i^T \\ * & * & -\tilde{Q} - \frac{1}{d_{\mathrm{M}}}\tilde{R} & 0 & \tilde{P}_i C_{di}^T \\ * & * & * & -\gamma I & D_{di}^T \\ * & * & * & * & -\gamma I \end{bmatrix} < 0, \tag{3.21}$$

with

$$\Xi_{11} = T^2 \sum_{j=1}^{s} \lambda_{ij}\tilde{P}_j - 2\tilde{P}_i + T\tilde{P}_i + d_{\mathrm{M}}\tilde{R}, \qquad \Xi_{12} = T\sum_{j=1}^{s} \lambda_{ij}\tilde{P}_j + A_i\tilde{P}_i + B_i N_i^T,$$

$$\Xi_{22} = \sum_{j=1}^{s} \lambda_{ij}\tilde{P}_j + A_i\tilde{P}_i + B_i N_i^T + \tilde{P}_i A_i^T + N_i B_i^T + (d_{\mathrm{M}} - d_{\mathrm{m}} + 1)\tilde{Q} - \frac{1}{d_{\mathrm{M}}}\tilde{R}.$$

Then the system (3.19) is asymptotically stable. Moreover, a suitable stabilizing H_∞ state feedback controller for system (3.19)–(3.20) can be chosen by $u(t) = N_i\tilde{P}_i^{-1}x(t)$.

If the system mode set $S = \{1\}$, the jump system (2.2)–(2.3) is simplified into a general linear system as follows:

$$\delta x(t) = \big(A + \Delta A(t)\big)x(t) + \big(A_d + \Delta A_d(t)\big)x\big(t - d(k)\big) + \big(B + \Delta B(t)\big)u(t) + B_d\omega(t), \tag{3.22}$$

$$z(t) = \big(C + \Delta C(t)\big)x(t) + \big(C_d + \Delta C_d(t)\big)x\big(t - d(k)\big) + \big(D + \Delta D(t)\big)u(t) + D_d\omega(t), \tag{3.23}$$

$$x(t) = \phi(t), \quad t \in [-d_{\mathrm{M}},\ 0].$$

Simplified results for H_∞ control can be given as follows.

Corollary 3.2 *Given a scalar $\gamma > 0$, the uncertain delta operator system in (3.22)–(3.23) is robustly stable with disturbance attenuation γ, if there exist matrices $\tilde{P} > 0$,*

$\tilde{Q} > 0$, $\tilde{R} > 0$ *and* N, *as well as a positive scalar* α *such that the following LMI holds*:

$$\begin{bmatrix} \Gamma_{11} & A\tilde{P} + BN^T & A_d\tilde{P} & B_d\tilde{P} & 0 & 0 & \alpha H_1 \\ * & \Gamma_{22} & \frac{1}{d_{\mathrm{M}}}\tilde{R} + A_d\tilde{P} & B_d\tilde{P} & \tilde{P}C^T + N^T D^T & \tilde{P}E_1^T + N^T E_3^T & \alpha H_1 \\ * & * & -\tilde{Q} - \frac{1}{d_{\mathrm{M}}}\tilde{R} & 0 & \tilde{P}C_d^T & \tilde{P}E_2^T & 0 \\ * & * & * & -\gamma I & D_d^T & 0 & 0 \\ * & * & * & * & -\gamma I & 0 & \alpha H_2 \\ * & * & * & * & * & -\alpha I & \alpha G \\ * & * & * & * & * & * & -\alpha I \end{bmatrix} < 0, \tag{3.24}$$

with

$$\Gamma_{11} = -2\tilde{P} + T\tilde{P} + d_{\mathrm{M}}\tilde{R},$$

$$\Gamma_{22} = A\tilde{P} + BN^T + \tilde{P}A^T + NB_i^T + (d_{\mathrm{M}} - d_{\mathrm{m}} + 1)\tilde{Q} - \frac{1}{d_{\mathrm{M}}}\tilde{R}.$$

Then the system (3.22) *is asymptotically stable. Moreover, a suitable stabilizing state feedback controller for system* (3.22)–(3.23) *can be chosen by* $u(t) = N\tilde{P}^{-1}x(t)$.

4 Numerical Examples

The first numerical example will show the characteristic of a discrete-time system and delta operator system in sampling a continuous-time system.

Example 4.1 Consider a continuous-time system in the s-domain:

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \tag{4.1}$$

$$y(t) = [\,1\ 1\,]x(t). \tag{4.2}$$

By using the shift operator and delta operator in sampling the continuous-time system, respectively, we get the relevant different discrete-time system in the z-domain and the δ-domain. When $T = 1$, there exist

$$x\big((k+1)T\big) = \begin{bmatrix} 0.3679 & 0 \\ 0.2325 & 0.1353 \end{bmatrix} x(kT) + \begin{bmatrix} 0 \\ 0.4323 \end{bmatrix} u(kT);$$

$$\delta x(kT) = \begin{bmatrix} -0.6321 & 0 \\ 0.2325 & -0.8647 \end{bmatrix} x(kT) + \begin{bmatrix} 0 \\ 0.4323 \end{bmatrix} u(kT).$$

When $T = 0.55$, there exist

$$x\big((k+1)T\big) = \begin{bmatrix} 0.5769 & 0 \\ 0.2441 & 0.3329 \end{bmatrix} x(kT) + \begin{bmatrix} 0 \\ 0.3336 \end{bmatrix} u(kT);$$

$$\delta x(kT) = \begin{bmatrix} -0.7692 & 0 \\ 0.4438 & -1.2130 \end{bmatrix} x(kT) + \begin{bmatrix} 0 \\ 0.6065 \end{bmatrix} u(kT).$$

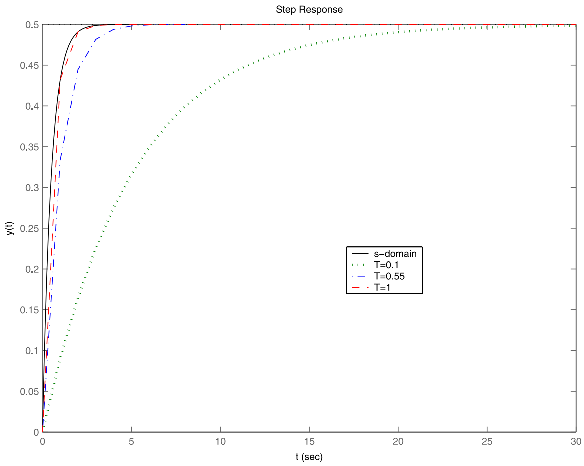

Fig. 1 The trajectories of the output in z-domain

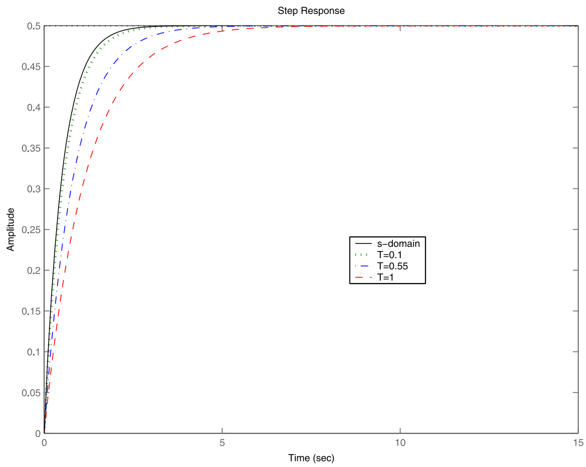

Fig. 2 The trajectories of the output in δ-domain

When $T = 0.1$, there exist

$$x\big((k+1)T\big) = \begin{bmatrix} 0.9048 & 0 \\ 0.0861 & 0.8187 \end{bmatrix} x(kT) + \begin{bmatrix} 0 \\ 0.0906 \end{bmatrix} u(kT);$$

$$\delta x(kT) = \begin{bmatrix} -0.9516 & 0 \\ 0.8611 & -1.8127 \end{bmatrix} x(kT) + \begin{bmatrix} 0 \\ 0.9063 \end{bmatrix} u(kT).$$

From the above results, it is easy to see that the virtue of the delta operator systems in sampling continuous-time systems will appear when the sampling is fast. Two output curve graphs of the above systems with different sampling periods are shown in Fig. 1 and Fig. 2.

Example 4.1 now demonstrates the results obtained in previous sections.

Example 4.2 The following Markovian jump delta operator system (2.2)–(2.3) with $i = 1, 2$ is considered:

$$A_1 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, \qquad W_{01} = \begin{bmatrix} -0.3 & 0.3 \\ 0.1 & -0.1 \end{bmatrix},$$

$$W_{11} = \begin{bmatrix} 0.1 & 0.1 \\ 0.3 & 0.3 \end{bmatrix}, \qquad G_1 = \begin{bmatrix} 0.03 & 0.03 \\ 0.03 & 0.03 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.12 & 0.12 \\ 0.12 & 0.12 \end{bmatrix}, \qquad E_{11} = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix},$$

$$E_{21} = \begin{bmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{bmatrix}, \qquad E_{31} = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.8 \end{bmatrix}, \qquad W_{02} = \begin{bmatrix} -0.2 & 0.2 \\ 0.3 & -0.3 \end{bmatrix},$$

$$W_{12} = \begin{bmatrix} 0.2 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.23 & 0.23 \\ 0.23 & 0.23 \end{bmatrix}, \qquad E_{12} = \begin{bmatrix} 0.03 & 0.03 \\ 0.03 & 0.03 \end{bmatrix},$$

$$E_{22} = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix}, \qquad E_{32} = \begin{bmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{bmatrix}.$$

The generator matrix of the stochastic process η_t is

$$\lambda = \begin{bmatrix} -\lambda_1 & \lambda_1 \\ \lambda_2 & -\lambda_2 \end{bmatrix}, \quad \text{where } \lambda_1 = 7,\ \lambda_2 = 6.$$

The neuron function is $f(x) = [|x+1| - |x-1|]/2$, and the time-varying delay function is $1 \le d(k) \le 2$. Using Theorem 3.1 and the LMI Control Toolbox in MATLAB, we find that the jump systems (2.2)–(2.3) is asymptotically stable and the solution of LMI (3.4) is given as follows:

$$P_1 = \begin{bmatrix} 84.8749 & 0.8572 \\ 0.8572 & 86.2738 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 85.4704 & 0.1115 \\ 0.1115 & 86.9122 \end{bmatrix},$$

$$M = \begin{bmatrix} 3.7018 & 3.5370 \\ 3.5370 & 4.9583 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 48.4294 & -2.3273 \\ -1.0523 & 44.0836 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 47.8911 & -0.2969 \\ -2.4274 & 44.1331 \end{bmatrix},$$

$$Q = \begin{bmatrix} 6.4566 & 7.3848 \\ 7.3848 & 12.2908 \end{bmatrix},$$

$$R = \begin{bmatrix} 17.9268 & -1.5895 \\ -1.5895 & 15.5675 \end{bmatrix}, \qquad S_1 = \begin{bmatrix} -38.5973 & 0 \\ 0 & -38.5973 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} -43.0586 & 0 \\ 0 & -43.0586 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} -0.4674 & 0 \\ 0 & -0.4674 \end{bmatrix}, \qquad N_2 = \begin{bmatrix} -2.7614 & 0 \\ 0 & -2.7614 \end{bmatrix},$$

$$\alpha_1 = 69.4574, \qquad \alpha_2 = 75.2159.$$

5 Conclusions

In this paper, the problem of H_∞ state feedback controller design for jump delta operator systems with time-varying delays has been investigated, involving both linear fractional uncertainties and Markovian jump parameters. The class of jump delta operator systems is characteristic of fast sampling periods and delay-range-dependent stability criteria. Using some new Lyapunov–Krasovskii functionals in the delta domain, sufficient conditions are given in terms of LMIs. The proposed method can unify some previous related results of continuous and discrete systems into the delta operator systems framework. Numerical examples were given to illustrate the effectiveness of the theoretic results obtained.

Acknowledgements The authors would like to thank the reviewers for their very helpful comments and suggestions which have improved the presentation of the paper. The work of Yuanqing Xia was supported by the National Natural Science Foundation of China under Grant 60504020 and the Excellent Young Scholars Research Fund of Beijing Institute of Technology 2006y0103, respectively.

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