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Fault Reconstruction for Lipschitz Nonlinear Descriptor Systems via Linear Matrix Inequality Approach

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Abstract By using the linear matrix inequality for a nonlinear descriptor system with a Lipschitz constraint, a state-space observer is designed to simultaneously reconstruct the descriptor system states and the fault signals. For a nonlinear descriptor system with bounded input disturbances, a robust state-space observer is proposed to simultaneously reconstruct the descriptor system states and the faults and attenuate the input bounded disturbances. The fault considered can be unbounded (provided that the qth derivative of the fault is zero piecewise or norm bounded); thus, the present fault reconstruction approach can handle a large class of fault signals including time-invariant and time-varying signals. A numerical example is given to illustrate the proposed design approach.

Keywords Descriptor systems · Fault reconstruction · Nonlinearity · Observers · Robust estimation

1 Introduction

Fault detection and isolation (FDI) for dynamic systems has long been recognized as one of the important aspects in improving the reliability of practical control systems.

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During the past several decades, numerous results have been reported, particularly for state-space dynamical systems; for instance, see [2, 4, 6, 7] and the references therein. However, very few efforts have been made to investigate FDI for nonlinear descriptor systems. A nonlinear descriptor system is a class of rather complex systems, which not only possesses nonlinearities, but also has algebraic constraints of a singular nature. Therefore, investigations on this class of systems are more difficult and challenging [3, 11, 16]. So far, only a limited amount of work on FDI for nonlinear descriptor systems can be found in the literature [5, 14, 15]. Many practical systems, such as constrained robots, chemical processes, electronic electric circuits and power systems, can be modeled as nonlinear descriptor systems. Therefore, it is of significance to design and implement FDI schemes for this class of dynamic systems from the viewpoint of reliability and safety.

Via FDI, an alarm can be originated when a fault occurs. However, the magnitude of the fault cannot be provided by FDI. The process to estimate the magnitude of the fault is called fault reconstruction. If the fault can be reconstructed, one can readily make a fault-tolerant design [8]. However, so far very few results have been obtained on fault reconstruction for nonlinear descriptor systems. Very recently, interesting simultaneous state and fault reconstruction approaches have been proposed in [9, 10] for Lipschitz nonlinear descriptor systems. However, the work in [10] focuses on sensor fault signals, while the result in [9] is for actuator faults only. This has motivated us to develop a novel fault reconstruction approach for systems with both actuator and sensor faults.

In this study, a novel fault reconstruction approach is presented for nonlinear Lipschitz descriptor systems using the linear matrix inequality (LMI) technique. Only original coefficient matrices are utilized in the design. This design technique is thus reliable in computations and convenient in implementation. Moreover, the proposed fault reconstruction technique can handle actuator fault and sensor fault signals at the same time. Furthermore, a robustness analysis of the present fault/state estimator is presented.

In this paper, the notation used is rather standard. $\mathcal{R}$ denotes the set of real numbers, A^{-1} denotes the inverse of A, I_m denotes an identity matrix with the dimension $m \times m$, $0_{n\times p}$ denotes an $n \times p$ matrix with zero entries, 0 denotes a scalar zero or a zero matrix with appropriate dimension, $P > 0$ (or $P < 0$) indicates that the symmetric matrix P is positive (or negative) definite, $\forall$ means "for all", and $\mathcal{L}_2[0, T_f]$ represents the set of all signals which are square integrable satisfying $\int_0^{T_f} \mathrm{d}^T(\tau)\,\mathrm{d}(\tau)\,\mathrm{d}\tau < \infty$ and $\|d\|_{T_f} := (\int_0^{T_f} \mathrm{d}^T(\tau)\,\mathrm{d}(\tau)\mathrm{d}\tau)^{\frac{1}{2}}$.

2 Fault Reconstruction

Consider the following nonlinear descriptor systems with faults:

$$\begin{cases} E\dot{x} = Ax + Bu + B_f f + \Phi(t, x, u), \\ y = Cx + Du + D_f f \end{cases} \tag{1}$$

where $x \in \mathcal{R}^n$ is the descriptor state vector, $u \in \mathcal{R}^m$ and $y \in \mathcal{R}^p$ are respectively the control input and measurement output vectors, $f \in \mathcal{R}^k$ is the unknown fault vector,

E and $A \in \mathcal{R}^{n\times n}$, E may be singular, B, B_f, C, D and D_f are constant real matrices of appropriate dimensions, and $\Phi(t,x,u) \in \mathcal{R}^n$ is a real nonlinear vector function with the Lipschitz constant θ, namely,

$$\begin{aligned} &\left\|\Phi(t,\hat{x},u) - \Phi(t,x,u)\right\| \le \theta\|\hat{x} - x\|, \\ &\forall(t,\hat{x},u),\ (t,x,u) \in \mathcal{R}\times\mathcal{R}^n\times\mathcal{R}^m. \end{aligned} \tag{2}$$

Remark 1 As pointed out in [1, 12, 13], the Lipschitz system plays a very important role in nonlinear system analysis and design. Any nonlinear system $E\dot{x} = g(x,u)$ can be expressed in the form of the dynamic equation (1) (with f being null), at least locally, if $E\dot{x} = g(x,u)$ is continuously differentiable with respect to x. In addition, when $\Phi(t,x,u)$ is locally Lipschitz, all the results given in this paper are valid in a neighborhood of a nominal point.

The fault vector $f(t)$ considered in this study is assumed to have the following form:

$$A_0 + A_1 t + A_2 t^2 + \cdots + A_{q-1}t^{q-1} \tag{3}$$

with A_i $(i = 0,1,2,\ldots,q-1)$ being unknown constant matrices with appropriate dimensions, and t being the time interval. Clearly, the qth derivative of this fault is zero, i.e., $f^{(q)} = 0$. In Sect. 3, we will relax the condition, that is, $f^{(q)}$ need not necessarily be zero piecewise provided that it is norm bounded.

Remark 2 With the proposed design, it is feasible to reconstruct the fault signal, whose qth derivative of the dominant component is bounded. Compared with the previous results [5, 14, 15], the fault considered here covers a larger class of fault signals.

In this section, $f^{(q)}$ is assumed to be zero piecewise.

Let

$$\xi_i = f^{(q-i)} \quad (i = 1,2,\ldots,q), \tag{4}$$

and the following relationships hold:

$$\begin{cases} \dot{\xi}_1 = f^{(q)} = 0, \\ \dot{\xi}_2 = \xi_1, \\ \dot{\xi}_3 = \xi_2, \\ \vdots \\ \dot{\xi}_{q-1} = \xi_{q-2}, \\ \dot{\xi}_q = \xi_{q-1}. \end{cases} \tag{5}$$

Using (1) and (5), we can construct an augmented nonlinear descriptor plant as follows

$$\begin{cases} \bar{E}\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{\Phi}(t,x,u), \\ y = \bar{C}\bar{x} + Du \end{cases} \tag{6}$$

where

$$\bar{n} = n + kq,$$
$$\bar{x} = \begin{bmatrix} x^T, & \xi_1^T, & \xi_2^T, & \dots, & \xi_q^T \end{bmatrix}^T \in \mathcal{R}^{\bar{n}},$$
$$\bar{\Phi}(t,x,u) = \begin{bmatrix} \Phi^T(t,x,u), & 0, & \dots, & 0 \end{bmatrix}^T \in \mathcal{R}^{\bar{n}},$$
$$\bar{E} = \begin{bmatrix} E & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \\ 0 & 0 & \cdots & 0 & I \end{bmatrix} \in \mathcal{R}^{\bar{n}\times\bar{n}},$$
$$\bar{A} = \begin{bmatrix} A & 0 & \cdots & 0 & B_f \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \in \mathcal{R}^{\bar{n}\times\bar{n}},$$
$$\bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \mathcal{R}^{\bar{n}\times m},$$
$$\bar{C} = \begin{bmatrix} C & 0 & 0 & \cdots & 0 & D_f \end{bmatrix} \in \mathcal{R}^{p\times\bar{n}}. \tag{7}$$

Consider the nonlinear state-space dynamical system

$$\begin{cases} \dot{\bar{z}} = (\bar{A} - \bar{L}_P\bar{C})\hat{\bar{x}} + (\bar{B} - \bar{L}_P D)u + \bar{L}_P y + \bar{\Phi}(t,\hat{x},u), \\ \hat{\bar{x}} = (\bar{E} + \bar{L}_D\bar{C})^{-1}(\bar{z} + \bar{L}_D y - \bar{L}_D Du) \end{cases} \tag{8}$$

where $\hat{\bar{x}} \in \mathcal{R}^{\bar{n}}$ is the estimate of the augmented descriptor state $\bar{x} \in \mathcal{R}^{\bar{n}}$, $\hat{x} = [I_n \ 0_{n\times kq}]\hat{\bar{x}} \in \mathcal{R}^n$ is the estimate of the original system state $x \in \mathcal{R}^n$, and $\bar{L}_P$ and $\bar{L}_D$ are the design parameters in the form

$$\bar{L}_D = \begin{bmatrix} L_D \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad \bar{L}_P = \begin{bmatrix} L_P \\ L_I^1 \\ L_I^2 \\ \vdots \\ L_I^q \end{bmatrix} \in \mathcal{R}^{\bar{n}\times p}. \tag{9}$$

Clearly, $\hat{\bar{x}} = [\hat{x}^T, \ \hat{\xi}_1^T, \ \hat{\xi}_2^T, \ \dots, \ \hat{\xi}_q^T]^T$ includes components such as the estimated state $\hat{x}$, estimates $\hat{\xi}_i$ $(i = 1, 2, \dots, q-1)$ for the derivatives of the fault, and the fault estimate $\hat{\xi}_q = \hat{f}$.

Now we have the following theorem.

Theorem 1 *For the plant* (6), *there exists an asymptotic state-space estimator in the form of* (8) *if there exist a positive definite matrix* $\bar{P} \in \mathcal{R}^{\bar{n}\times\bar{n}}$ *and a matrix* $\bar{Y} \in \mathcal{R}^{\bar{n}\times p}$ *such that*

$$\begin{bmatrix} \Omega + \theta^2 I & \bar{P}\bar{S}^{-1} \\ \bar{S}^{-T}\bar{P} & -I \end{bmatrix} < 0 \tag{10}$$

where $\Omega = \bar{A}^T\bar{S}^{-T}\bar{P} + \bar{P}\bar{S}^{-1}\bar{A} - \bar{C}^T\bar{Y}^T - \bar{Y}\bar{C}$ *and* $\bar{S} = \bar{E} + \bar{L}_D\bar{C}$ *is made nonsingular by selecting an appropriate derivative gain* $\bar{L}_D$. *Furthermore, the proportional gain* $\bar{L}_P$ *can be computed as* $\bar{L}_P = \bar{S}\bar{P}^{-1}\bar{Y}$.

Proof The augmented plant (6) and the system (8) can be rewritten, respectively, as follows:

$$\begin{aligned} (\bar{E} + \bar{L}_D\bar{C})\dot{\bar{x}} &= (\bar{A} - \bar{L}_P\bar{C})\bar{x} + (\bar{B} - \bar{L}_P D)u \\ &\quad + \bar{L}_P y + \bar{L}_D\dot{y} - \bar{L}_D D\dot{u} + \bar{\Phi}(t,x,u), \end{aligned} \tag{11}$$

$$\begin{aligned} (\bar{E} + \bar{L}_D\bar{C})\dot{\hat{\bar{x}}} &= (\bar{A} - \bar{L}_P\bar{C})\hat{\bar{x}} + (\bar{B} - \bar{L}_P D)u \\ &\quad + \bar{L}_P y + \bar{L}_D\dot{y} - \bar{L}_D D\dot{u} + \bar{\Phi}(t,\hat{x},u). \end{aligned} \tag{12}$$

Letting $\bar{e} = \hat{\bar{x}} - \bar{x}$, $e_x = \hat{x} - x$, $\tilde{\Phi} = \bar{\Phi}(t,\hat{x},u) - \bar{\Phi}(t,x,u)$, and subtracting (11) from (12), the error dynamic equation can be characterized as

$$\dot{\bar{e}} = \bar{S}^{-1}(\bar{A} - \bar{L}_P\bar{C})\bar{e} + \bar{S}^{-1}\tilde{\Phi}. \tag{13}$$

Taking a Lyapunov candidate as

$$V_o(\bar{e}) = \bar{e}^T\bar{P}\bar{e}, \tag{14}$$

and using (2), one can derive that

$$\begin{aligned} \dot{V}_o(\bar{e}) &= \bar{e}^T\{[\bar{S}^{-1}(\bar{A} - \bar{L}_P\bar{C})]^T\bar{P} + \bar{P}[\bar{S}^{-1}(\bar{A} - \bar{L}_P\bar{C})]\}\bar{e} \\ &\quad + \tilde{\Phi}^T\bar{S}^{-T}\bar{P}\bar{e} + \bar{e}^T\bar{P}\bar{S}^{-1}\tilde{\Phi} \\ &\le \bar{e}^T\{[\bar{S}^{-1}(\bar{A} - \bar{L}_P\bar{C})]^T\bar{P} + \bar{P}[\bar{S}^{-1}(\bar{A} - \bar{L}_P\bar{C})]\}\bar{e} \\ &\quad + \tilde{\Phi}^T\tilde{\Phi} + \bar{e}^T\bar{P}\bar{S}^{-1}\bar{S}^{-T}\bar{P}\bar{e} \\ &\le \bar{e}^T[\bar{A}^T\bar{S}^{-T}\bar{P} + \bar{P}\bar{S}^{-1}\bar{A} - (\bar{S}^{-1}\bar{L}_P\bar{C})^T\bar{P} \\ &\quad - \bar{P}\bar{S}^{-1}\bar{L}_P\bar{C} + \theta^2 I + \bar{P}\bar{S}^{-1}\bar{S}^{-T}\bar{P}]\bar{e}. \end{aligned} \tag{15}$$

Using the Schur complement, condition (10) is equivalent to

$$\begin{aligned} &\bar{A}^T\bar{S}^{-T}\bar{P} + \bar{P}\bar{S}^{-1}\bar{A} - \bar{C}^T\bar{Y}^T - \bar{Y}\bar{C} + \theta^2 I \\ &\quad + \bar{P}\bar{S}^{-1}\bar{S}^{-T}\bar{P} < 0. \end{aligned} \tag{16}$$

Defining

$$\bar{Y} = \bar{P}\bar{S}^{-1}\bar{L}_P, \tag{17}$$

and substituting (16) into (15), one has

$$\dot{V}_o(\bar{e}) < 0. \tag{18}$$

Thus, $\bar{e} \to 0$ as $t \to \infty$. This completes the proof. □

Remark 3 When $f^{(q)} = 0$, the observer proposed above can accurately estimate the descriptor system state x, the fault f and its finite time derivatives $f^{(q-i)}$ $(i = 1, 2, \ldots, q-1)$ at the same time for a class of nonlinear Lipschitz descriptor systems. Clearly, the derivative gain $\bar{L}_D$ exists for making $\bar{S} = \bar{E} + \bar{L}_D\bar{C}$ nonsingular if and only if rank$[E^T \ C^T]^T = n$, and an algorithm for seeking such $\bar{L}_D$ can be found in the work by [8]. Therefore, the derivative gain $\bar{L}_D$ and the proportional gain $\bar{L}_P$ can both be computed conveniently by using standard commercially available computing softwares.

3 Robust Fault Reconstruction

It is to be noted that $f^{(q)}$ is assumed to be zero piecewise in the last section. In this section, we discuss a nonlinear Lipschitz descriptor system with $f^{(q)} \neq 0$, and an unknown input disturbance d.

When the bounded input disturbance exists, the plant given by (1) becomes

$$\begin{cases} E\dot{x} = Ax + Bu + B_f f + \Phi(t, x, u) + B_d d, \\ y = Cx + Du + D_f f \end{cases} \tag{19}$$

where $d \in \mathcal{R}^l$ is the bounded vector, $B_d \in \mathcal{R}^{n\times l}$ is a known matrix and the other symbols are the same as defined in (1).

Letting

$$\xi_i = f^{(q-i)}, (i = 1, 2, \ldots, q), \tag{20}$$

$$\bar{x} = \begin{bmatrix} x^T, & \xi_1^T, & \xi_2^T, & \ldots, & \xi_q^T \end{bmatrix}^T \in \mathcal{R}^{\bar{n}}, \tag{21}$$

and assuming $f^{(q)} \neq 0$, but norm bounded, we can construct an augmented plant as follows:

$$\begin{cases} \bar{E}\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{\Phi}(t, x, u) + \bar{N}\bar{d}, \\ y = \bar{C}\bar{x} + Du \end{cases} \tag{22}$$

where

$$\bar{N} = \begin{bmatrix} B_d & 0 \\ 0 & I_k \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \qquad \bar{d} = \begin{bmatrix} d \\ f^{(q)} \end{bmatrix}, \tag{23}$$

and the other symbols are the same as before.

We now present the following result.

Theorem 2 *For the plant* (19), *there exists a robust state-space estimator in the form of* (8) *such that*

$$\|\bar{e}\|_{T_f} \le \gamma \|\bar{d}\|_{T_f}, \quad \forall T_f \ge 0 \tag{24}$$

if there exist a positive definite matrix $\bar{P} \in \mathcal{R}^{\bar{n}\times\bar{n}}$ *and a matrix* $\bar{Y} \in \mathcal{R}^{\bar{n}\times p}$ *such that*

$$\begin{bmatrix} \Omega + (\theta^2+1)I & \bar{P}\bar{S}^{-1} & \bar{P}\bar{S}^{-1}\bar{N} \\ \bar{S}^{-T}\bar{P} & -I & 0 \\ \bar{N}^T\bar{S}^{-T}\bar{P} & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{25}$$

where $\Omega = \bar{A}^T\bar{S}^{-T}\bar{P} + \bar{P}\bar{S}^{-1}\bar{A} - \bar{C}^T\bar{Y}^T - \bar{Y}\bar{C}$ *and* $\bar{S} = \bar{E} + \bar{L}_D\bar{C}$ *is made nonsingular by selecting an appropriate derivative gain* $\bar{L}_D$. *Furthermore, the proportional gain* $\bar{L}_P$ *can be computed as* $\bar{L}_P = \bar{S}\bar{P}^{-1}\bar{Y}$.

Proof Using (12) and (22), one has

$$\dot{\bar{e}} = \bar{S}^{-1}(\bar{A} - \bar{L}_P\bar{C})\bar{e} + \bar{S}^{-1}\tilde{\Phi} - \bar{S}^{-1}\bar{N}\bar{d}. \tag{26}$$

By using (14), (17) and (26), one can derive that

$$\begin{aligned} \dot{V}_o(\bar{e}) \le \bar{e}^T\big[&\bar{A}^T\bar{S}^{-T}\bar{P} + \bar{P}\bar{S}^{-1}\bar{A} - \bar{C}^T\bar{Y}^T - \bar{Y}\bar{C} \\ &+ \theta^2 I + \bar{P}\bar{S}^{-1}\bar{S}^{-T}\bar{P}\big]\bar{e} + 2\bar{e}^T\bar{P}\bar{S}^{-1}\bar{N}\bar{d}. \end{aligned} \tag{27}$$

Letting

$$H_0 = \dot{V}_0(\bar{e}) + \bar{e}^T\bar{e} - \gamma^2\bar{d}^T\bar{d}, \tag{28}$$

one has

$$\begin{aligned} H_0 &\le \bar{e}^T\big[\bar{A}^T\bar{S}^{-T}\bar{P} + \bar{P}\bar{S}^{-1}\bar{A} - \bar{C}^T\bar{Y}^T - \bar{Y}\bar{C} \\ &\quad + (\theta^2+1)I + \bar{P}\bar{S}^{-1}\bar{S}^{-T}\bar{P}\big]\bar{e} + 2\bar{e}^T\bar{P}\bar{S}^{-1}\bar{N}\bar{d} - \gamma^2\bar{d}^T\bar{d} \\ &= \begin{bmatrix}\bar{e}^T & \bar{d}^T\end{bmatrix} \begin{bmatrix} \Omega + (\theta^2+1)I + \bar{P}\bar{S}^{-1}\bar{S}^{-T}\bar{P} & \bar{P}\bar{S}^{-1}\bar{N} \\ \bar{N}^T\bar{S}^{-T}\bar{P} & -\gamma^2 I \end{bmatrix} \begin{bmatrix}\bar{e} \\ \bar{d}\end{bmatrix}. \end{aligned} \tag{29}$$

According to the Schur complement, condition (25) indicates that H_0 in (29) is negative definite.

As a result, under zero initial conditions, we have

$$\begin{aligned} \int_0^{T_f} \big(\bar{e}^T\bar{e} - \gamma^2\bar{d}^T\bar{d}\big)\,\mathrm{d}\tau &\le \int_0^{T_f} \big(\bar{e}^T\bar{e} - \gamma^2\bar{d}^T\bar{d}\big)\,\mathrm{d}\tau + V_0(\bar{e}) \\ &= \int_0^{T_f} H_0\,\mathrm{d}\tau \le 0 \end{aligned} \tag{30}$$

which means

$$\int_0^{T_f} \bar{e}^T \bar{e}\,\mathrm{d}\tau \leq \gamma^2 \int_0^{T_f} \bar{d}^T \bar{d}\,\mathrm{d}\tau. \tag{31}$$

This completes the proof. □

Remark 4 By using the LMI technique, the above robust observer allows us to attenuate the effect coming from the bounded d and $f^{(q)}$, and estimate the system state, the fault and its derivatives from 1 to $q-1$ at the same time.

If we let $\bar{e}_w = \bar{W}\bar{e}$, where $\bar{W}$ is the pre-specified weight matrix, we can formulate the following corollary.

Corollary 1 *For the plant* (19), *there exists a robust state-space estimator in the form of* (8) *such that*

$$\|\bar{e}_w\|_{T_f} \leq \gamma \left\|\bar{d}\right\|_{T_f}, \quad \forall T_f \geq 0 \tag{32}$$

if there exist a positive definite matrix $\bar{P} \in \mathcal{R}^{\bar{n}\times\bar{n}}$ *and a matrix* $\bar{Y} \in \mathcal{R}^{\bar{n}\times p}$ *such that*

$$\begin{bmatrix} \Omega + \theta^2 I + \bar{W}^T\bar{W} & \bar{P}\bar{S}^{-1} & \bar{P}\bar{S}^{-1}\bar{N} \\ \bar{S}^{-T}\bar{P} & -I & 0 \\ \bar{N}^T\bar{S}^{-T}\bar{P} & 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{33}$$

where $\Omega = \bar{A}^T\bar{S}^{-T}\bar{P} + \bar{P}\bar{S}^{-1}\bar{A} - \bar{C}^T\bar{Y}^T - \bar{Y}\bar{C}$ *and* $\bar{S} = \bar{E} + \bar{L}_D\bar{C}$ *is made nonsingular by selecting an appropriate derivative gain* $\bar{L}_D$. *Furthermore, the proportional gain* $\bar{L}_P$ *can be computed as* $\bar{L}_P = \bar{S}\bar{P}^{-1}\bar{Y}$.

Proof The proof is similar to that for Theorem 2, and is thus omitted. □

Remark 5 $\bar{W}$ can be selected according to the design goal. For example, if we only considered the fault reconstruction, we would choose $\bar{W}$ as

$$\bar{W} = \begin{bmatrix} 0_{k\times(n+qk-k)} & I_k \end{bmatrix}. \tag{34}$$

4 Numerical Example

Let us consider the plant in the form of

$$\begin{cases} E\dot{x} = Ax + Bu + \Phi(x) + B_d d + B_a f_a, \\ y = Cx + D_s f_s \end{cases} \tag{35}$$

where

$$E = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \qquad A = \begin{bmatrix} -1 & 0 \\ 1 & -3 \end{bmatrix},$$

$$B = B_a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad B_d = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}, \qquad D_s = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\Phi(x) = \begin{bmatrix} 0 \\ 0.1347 \sin(x_2) \end{bmatrix},$$

$$d = 0.2 \sin(20t), \qquad u = 4.$$

Case 1 Assume the actuator fault f_a occurs at 6 seconds with offset 20% and disappears at 7 seconds, and the sensor fault f_s is represented as

$$f_s = \begin{cases} 0.2(t-3) + 0.2 \sin[10(t-3)] + 0.5, & t \geq 3, \\ 0, & t < 3. \end{cases} \tag{36}$$

Letting

$$f = \begin{bmatrix} f_a \\ f_s \end{bmatrix},$$

we have

$$B_f = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad D_f = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Construct the augmented descriptor plant in the form of (22) with $q = 2$, where

$$\bar{E} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\bar{A} = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \bar{N} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0.2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Choosing

$$\bar{L}_D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \bar{W} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.85 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

and solving the LMI (33), one can obtain $\theta = 0.1347$ and $\gamma = 0.4132$. Then, by using $\bar{L}_P = \bar{S}\bar{P}^{-1}\bar{Y}$, one can compute

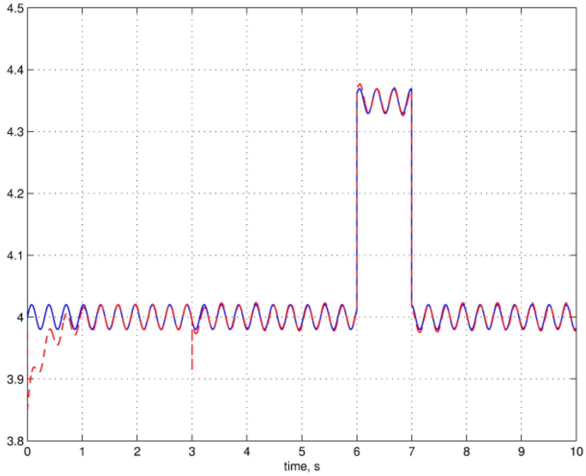

Fig. 1 State x_1 and its estimate $\hat{x}_1$: Case 1

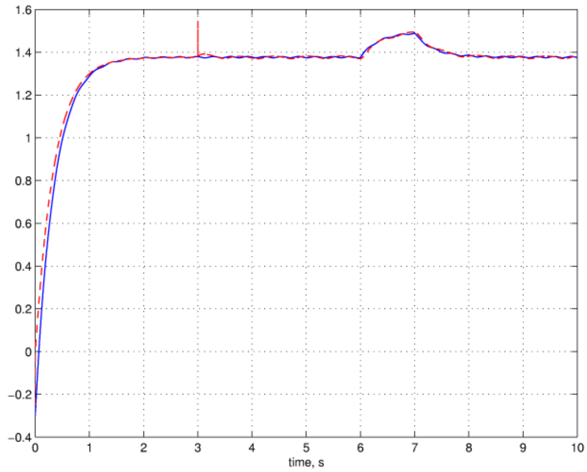

Fig. 2 State x_2 and its estimate $\hat{x}_2$: Case 1

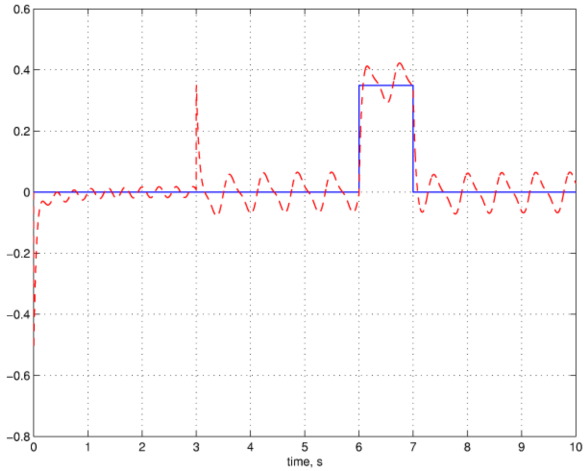

Fig. 3 Actuator fault f_a and its estimate $\hat{f}_a$: Case 1

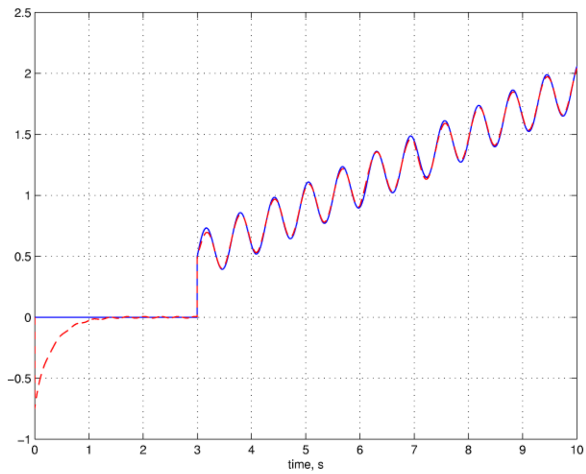

Fig. 4 Sensor fault f_s and its estimate $\hat{f}_s$: Case 1

$$\bar{L}_P = \begin{bmatrix} 0.1038 & -0.0022 \\ -0.1108 & 0.0024 \\ 1.8751 & 0.1307 \\ 0.0213 & 0.4492 \\ 2.3709 & 0.1390 \\ 0.3130 & 0.2643 \end{bmatrix} \times 10^5.$$

Figures 1–4 exhibit the states, the actuator fault and the sensor fault (the solid lines) and their estimated trajectories (the dashed lines). One can see that the estimation properties are as desired.

Remark 6 Specifically, the estimated trajectories of the states x_1, x_2 and the sensor fault f_s are excellent. However, the estimated property of the actuator fault f_a exhibited by Fig. 3 is not as good as those shown by Figs. 1, 2 and 4. It is not unusual, since the weight of $\hat{f}_a - f_a$ is smaller than the weights of $\hat{x}_1 - x_1$, $\hat{x}_2 - x_2$ and $\hat{f}_s - f_s$. If the weight of $\hat{f}_a - f_a$ were increased and the LMI (33) were also solvable, the tracking performance with respect to the actuator fault reconstruction could be improved.

Case 2 Assume that the sensor fault is null and the actuator fault is

$$f_a = \begin{cases} 0.2(t-3) + 0.1\sin[30(t-3)] + 0.5e^{-t}, & t \geq 3 \\ 0, & t < 3. \end{cases} \tag{37}$$

Clearly, the actuator signal is a combination of a slope signal, a negative exponent signal and a high-frequency signal. The simulated curves are exhibited in Figs. 5, 6, 7, and 8. One can see that the state and fault tracking behaviors are as desired.

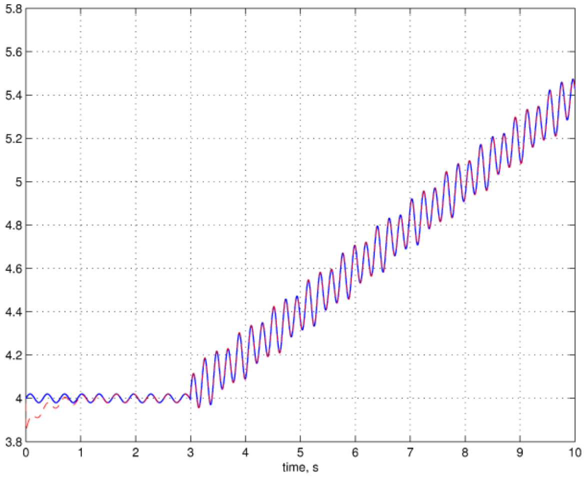

Fig. 5 State x_1 and its estimate $\hat{x}_1$: Case 2

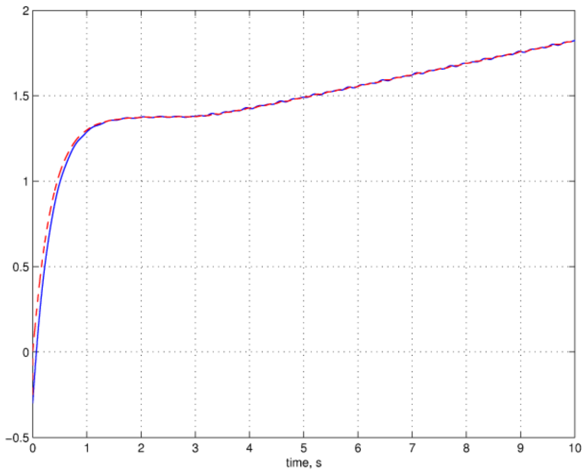

Fig. 6 State x_2 and its estimate $\hat{x}_2$: Case 2

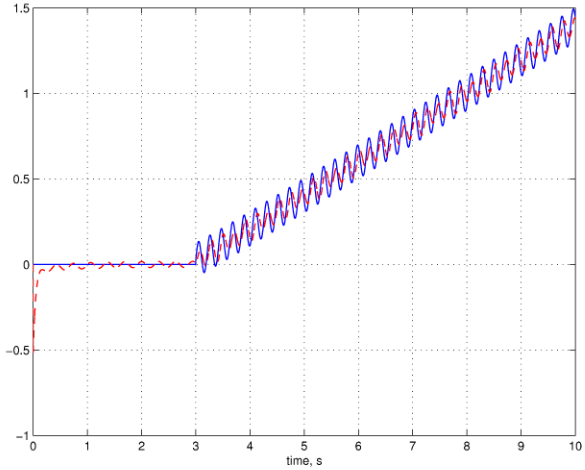

Fig. 7 Actuator fault f_a and its estimate $\hat{f}_a$: Case 2

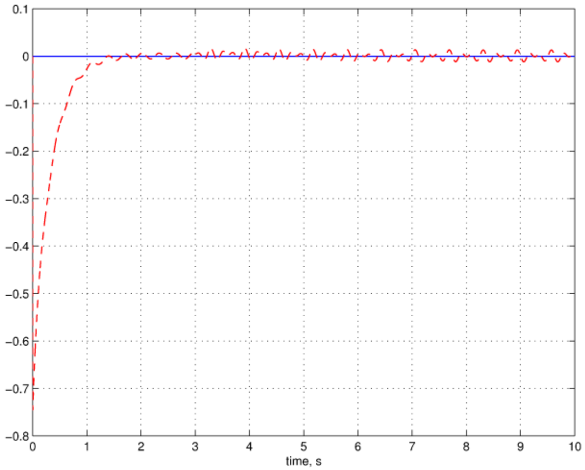

Fig. 8 Sensor fault f_s and its estimate $\hat{f}_s$: Case 2

Remark 7 Via the present state/fault simultaneous observer, one can reconstruct the state and fault successfully for a system subject to a large class of fault signals such as time-invariant and time-varying signals.

5 Conclusions

By using the LMI approach, a state/fault observer and a robust state/fault observer have both been presented to simultaneously reconstruct the descriptor states and the fault signals for Lipschitz nonlinear descriptor systems. The design procedure has been illustrated by a numerical example, and the efficiency has also been demonstrated by the simulations. The proposed design technique can handle time-invariant fault signals as well as time-varying signals. Moreover, for signals that are unbounded, but with bounded finite q-order derivatives, an observer with the order

$n + kq$ may be designed to reconstruct the unbounded fault signals using the proposed technique, n being the system order and k the vector dimension of the fault. Based on the reconstructed state and faults, one can obtain a fault-tolerant control design.

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