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ON CURRENT FEEDBACK OPERATIONAL AMPLIFIER-BASED REALIZATIONS OF CHUA'S CIRCUIT*

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Abstract. Because they offer some significant advantages over conventional operational amplifiers, current feedback operational amplifiers (CFOAs) have been used instead of voltage operational amplifiers (VOAs) in new implementations of the chaotic Chua's circuit. In this study, after giving a comparative investigation of CFOA-based realizations of Chua's circuit, we will present an alternative CFOA-based realization of Chua's circuit combining the CFOA-based circuit topologies proposed for nonlinear resistor and inductor elements in the literature. This realization offers an alternative solution for readers studying Chua's circuit and high-frequency chaotic oscillations. The PSpice simulation experiments performed in both the time and frequency domains confirm that the proposed circuit can generate the original chaotic oscillations of Chua's circuit and that it exhibits excellent performance at high frequencies.
Key words: Chaos, Chua's circuit, high-frequency chaotic oscillations, current feedback operational amplifier.

1. Introduction

Among the chaotic circuits designed to date, Chua's chaotic circuit [1] has a special position. Because Chua's circuit is an extremely simple system and yet exhibits a rich variety of bifurcations and chaos, it has been studied extensively and accepted as a paradigm for studying chaos and chaotic signals. Chua's circuit, shown in Figure 1a, contains three energy storage elements, a linear resistor, and a nonlinear resistor N_R, namely Chua's diode. Chua's circuit is defined by the

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following state equations:

$$L\frac{di_L}{dt} = -V_{C2} - i_L \cdot R_S \tag{1}$$

$$C_2\frac{dV_{C2}}{dt} = i_L - \frac{1}{R}(V_{C2} - V_R) \tag{2}$$

$$C_1\frac{dV_R}{dt} = \frac{1}{R}(V_{C2} - V_R) - f(V_R), \tag{3}$$

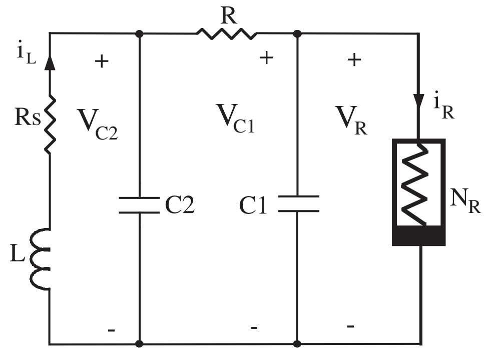
where $f(V_R)$ is a piecewise linear function, shown graphically in Figure 1b, defined by

$$i_R = f(V_R) = G_b V_R + \tfrac{1}{2} \cdot (G_a - G_b) \times (|V_R + B_p| - |V_R - B_p|), \tag{4}$$

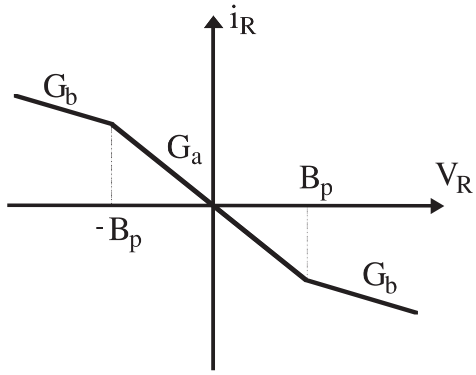
where G_a and G_b are the slopes in the inner and outer regions, respectively, and $\pm B_p$ denote the breakpoints.

Several realizations of Chua's circuit have been proposed in the literature. The methodologies used in these realizations can be divided into two basic categories. In the first approach, a variety of circuit topologies have been considered for realizing the nonlinear resistor N_R in Chua's circuit [2], [4], [7], [8], [11], [14]. The main idea in the second approach related to the implementation of Chua's circuit is inductorless realization of Chua's circuit. For this purpose, various circuit topologies providing a synthetic inductor, i.e., by inductance simulation, and RC configurations are used instead of the inductor element and LC resonator in Chua's circuit [3], [5], [6], [9], [10], [11], [13]. In early implementations of Chua's circuit, conventional voltage operational amplifiers (VOAs) and operational transconductance amplifiers have been used as the active elements. In general, VOA-based nonlinear resistor structures, VOA-based inductance simulators, and VOA-based RC configurations have been proposed for realizing Chua's circuit. Because of the frequency limitations and nonidealities of VOAs, the early VOA-based implementations of Chua's circuit impose an upper limit on the operating frequency. Therefore, in the literature, new design ideas are considered to improve the realization of Chua's circuit. Central to these new design ideas is using current feedback operational amplifiers (CFOAs), which are recognized for their excellent performance high-speed and high slew-rate analog signal processing, instead of VOAs. Recently, three CFOA-based realizations of Chua's circuit have been proposed in the literature. Two of these CFOA-based realizations have been reported by Elwakil and Kennedy [4], [5]; the other one has been presented by Senani and Gupta [11]. In these implementations, the authors aim to use the voltage-current capabilities of a CFOA, which offers several advantages over a classic VOA.

After giving a comparative investigation of these CFOA-based realizations of Chua's circuit, we will also present an alternative CFOA-based realization of Chua's circuit combining the CFOA-based circuit topologies proposed for the nonlinear resistor and inductor elements in Chua's circuit. Our CFOA-based realization of Chua's circuit, which has an inductorless circuit structure, is able to



(a)



(b)

Figure 1. (a) Chua's circuit, (b) the three-segment piecewise linear characteristic of the nonlinear resistor in Chua's circuit.

generate the original chaotic oscillations of Chua's circuit, and it exhibits excellent performance at high frequencies. So, we hope that this realization provides an alternative solution to the readers studying Chua's circuit and high-frequency chaotic oscillations. The paper is organized as follows. A comparative investigation of CFOA-based realizations of Chua's circuit in the literature is given in Section 2. In Section 3, an alternative CFOA-based realization of Chua's circuit is described, and simulation results are presented. Some concluding remarks are discussed in Section 4.

2. CFOA-based realizations of Chua's circuit

Because they offer some significant advantages over the conventional VOAs, the CFOAs are receiving considerable attention recently. The CFOA has the advantages of a constant bandwidth (up to 100 MHz) which is independent of the closed loop gain and its high slew rate, which is typically around 2000 V/μs. It is a cascade of second generation current conveyor (CCII) and a voltage buffer [12].

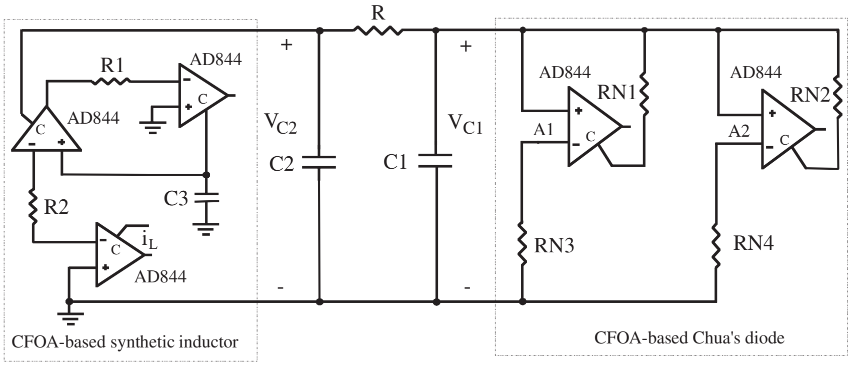
Up to now, three CFOA-based realizations of Chua's circuit have been reported in the literature, two by Elwakil and Kennedy [4], [5] and one by Senani and Gupta [11]. In this section, we will investigate these realizations.

A realization of Chua's circuit proposed by Senani and Gupta [11] is shown in Figure 2a. As shown in the figure, the authors have used the AD844-type CFOA as an active circuit element for realizing both a nonlinear resistor, namely Chua's diode, and a synthetic inductor. In Senani and Gupta's CFOA-based nonlinear resistor circuit structure, each of the CFOAs is configured as a negative impedance converter, with resistors R_{N1} and R_{N2} shorted. In this case, the circuit is basically a parallel connection of two negative resistors $-R_{\text{N3}}$ and $-R_{\text{N4}}$. Adding resistors R_{N1} and R_{N2} and using different power supply voltages for the two CFOAs, the authors offer the circuit realization for Chua's diode with the following parameters: $R_{\text{N1}} = 9.558$ kΩ, $R_{\text{N2}} = 542$ Ω, $R_{\text{N3}} = 5.482$ kΩ, $R_{\text{N4}} = 1.606$ kΩ; and the power supplies $\pm V_{\text{C1}} = \pm 4.05$ V, $\pm V_{\text{C2}} = \pm 11.23$ V for A1 and A2, respectively. The use of these parameters yields $G_a \cong -0.8$ mA/V, $G_b \cong -0.5$ mA/V, and $\pm B_p = \pm 1$ V. The simulated v-i characteristic of Chua's diode of Figure 2a is shown in Figure 2b. In the CFOA-based synthetic inductor, shown in Figure 2a, the equivalent inductance value can be computed as follows:

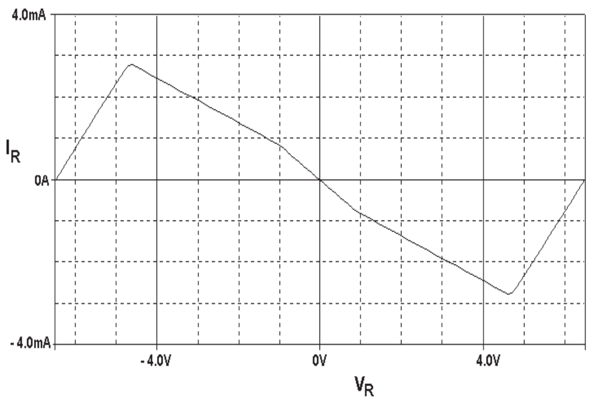
$$L_{eq} = C_3 R_1 R_2. \tag{5}$$

Considering the parasitic impedance of the CFOAs, the authors determined the circuit parameter values as $R_1 = 3.744$ kΩ, $R_2 = 1.459$ kΩ, and $C_3 = 5$ nF, giving an inductance value of $L_{eq} = 28.53$ mH. The other circuit parameters of Chua's circuit were determined as $R = 1.503$ kΩ, $C_1 = 5$ nF, and $C_2 = 50$ nF such that the circuit exhibits a double-scroll chaotic attractor.

Elwakil and Kennedy [4], [5] have also proposed two CFOA-based realizations of Chua's circuit. In the first realization [4], the main design idea was to develop a CFOA-based nonlinear resistor for realizing Chua's circuit. In Elwakil and Kennedy's [4] implementation of Chua's diode, shown in Figure 3a, the design methodology is similar to that of Kennedy's VOA-based nonlinear resistor [7]. Whereas the authors used a CFOA connected as a CCII with resistor R_{N4} to operate as a linear negative impedance converter (NIC) operating primarily in



(a)



(b)

Figure 2. (a) The CFOA-based inductorless realization of Chua's circuit proposed by Senani and Gupta [11], (b) simulated v-i characteristic of the nonlinear resistor of (a).

its linear region in Kennedy's circuit, they configured resistors R_{N1}, R_{N2}, R_{N3} and the associated CFOA to operate as a nonlinear voltage-controlled negative impedance converter (VNIC) instead of the VOA and its three resistors in [7]. Although the authors used the same values of R_{N1}, R_{N2}, and R_{N4} as in [7], they determined R_{N3} by adjusting its value to observe chaos. Two AD844-type CFOAs biased with ± 9 V are used in this implementation. With these parameters, the desired dc characteristics $G_a \cong -0.8$ mA/V and $G_b \cong -0.5$ mA/V have been achieved. The simulated dc characteristic of Chua's diode in Figure 3a with the circuit parameters $R_{\text{N1}} = R_{\text{N2}} = 22$ kΩ, $R_{\text{N3}} = 500$ Ω, and $R_{\text{N4}} = 2.2$ kΩ is shown in Figure 3b. In the second realization [5], the main design idea of the authors was to implement an inductorless Chua's circuit using a CFOA-based RC configuration. As shown in Figure 4, the authors used a second-order Wien-bridge oscillator instead of the LC resonator of Chua's circuit. Here, the Wien-bridge oscillator is implemented using a CFOA configured as a noninverting voltage-controlled voltage source with gain $K = R_B/R_A$ [5]. This Wien-bridge based realization of Chua's circuit is defined by the following state equations:

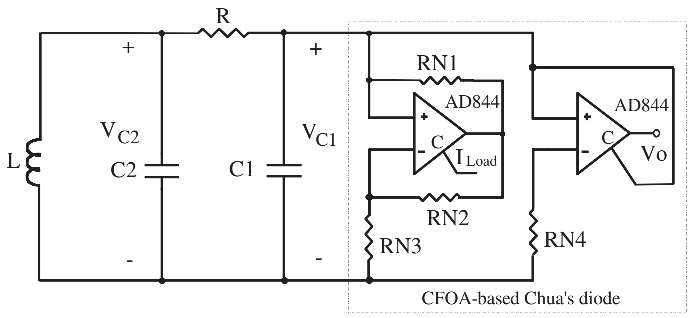
$$C_2\frac{dV_{C2}}{dt} = \frac{V_{C1} - V_{C2} - KV_{C3}}{R} - \frac{V_{C2} + (K-1)V_{C3}}{R_1}, \tag{6}$$

$$C_3\frac{dV_{C3}}{dt} = \frac{V_{C2} + (K-1)V_{C3}}{R_1} - \frac{V_{C3}}{R_2}, \tag{7}$$

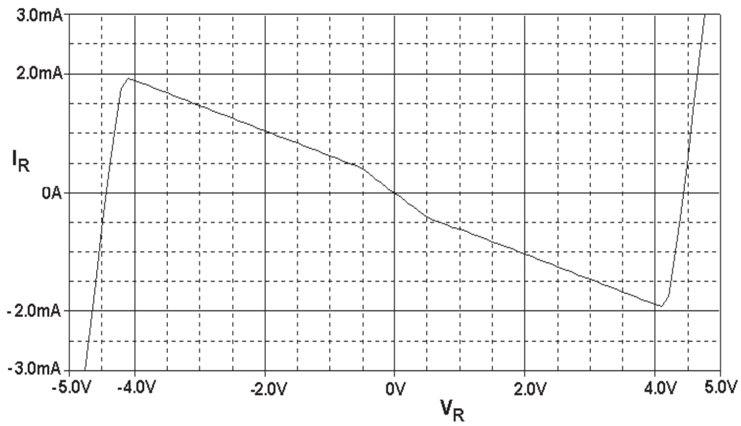
$$C_1\frac{dV_{C1}}{dt} = -\frac{V_{C1} - V_{C2} - KV_{C3}}{R} - f(V_R), \tag{8}$$

where $f(V_R)$ is a piecewise linear function defined by equation (4). In the realization of Figure 4, the authors used Kennedy's VOA-based nonlinear resistor structure [7] with the same parameter values, and they determined the other circuit parameters as $C_2 = C_3 = 50$ nF, $R_1 = R_2 = 200\ \Omega$, $R_A = 1\ k\Omega$, $R_B = 3.4$ kΩ, $C_1 = 1.5$ nF, and $R = 1.5$ kΩ, such that the circuit exhibits the double-scroll attractor.

In this section, the CFOA-based circuit topologies proposed for Chua's diode and inductor element in Chua's circuit in the literature will be compared. A comparison between Elwakil and Kennedy's CFOA-based nonlinear resistor [4] and Senani and Gupta's CFOA-based nonlinear resistor [11] is now in order. The CFOA-based designs in both [11] and [4] for Chua's diode employ fewer resistors than the six resistors in Kennedy's VOA-based design [7]. Elwakil and Kennedy's CFOA-based nonlinear resistor has a buffered output voltage directly representing a state variable. One of the two output voltages can be used as the carrier signal in chaotic communication systems; thus, the feature of a buffered and isolated voltage output directly representing a state variable in the Chua's diode design of Elwakil and Kennedy constitutes an advantage over Senani and Gupta's design. Both Elwakil and Kennedy's and Senani and Gupta's circuit topologies can be configured so that the chaotic spectrum is extended to higher frequencies than Kennedy's VOA-based original implementation. Whereas Elwakil and Kennedy's CFOA-based nonlinear resistor structure constitutes some advantage over Senani and Gupta's CFOA-based nonlinear resistor structure, the CFOA-based synthetic inductor proposed by Senani and Gupta for the inductor element in Chua's circuit has some advantages over the other inductance simulators in the literature. Although this inductance simulator employs three CFOAs, it uses the minimum number of passive components. In most of the designs for simulating the inductor in Chua's circuit, the third state variable (the inductor current i_L in Chua's circuit) cannot be determined in a direct manner; however, in Senani and Gupta's CFOA-based design [11], the inductor current i_L is available directly as a current output.



(a)



(b)

Figure 3. (a) The CFOA-based Chua's circuit proposed by Elwakil and Kennedy [4], (b) simulated v-i characteristic of the nonlinear resistor of (a).

3. An Alternative CFOA-based realization of chua's circuit

After the preceding comparison, we can conclude that Elwakil and Kennedy's CFOA-based nonlinear resistor structure [4] and Senani and Gupta's CFOA-based synthetic inductor structure [11] are superior to the other designs. But these CFOA-based structures have not been used together in the same design for realizing Chua's circuit in the literature. If these two structures are used together for realizing Chua's circuit, the realized circuit will be very advantageous and useful among the other realizations of Chua's circuit. Along this idea, we

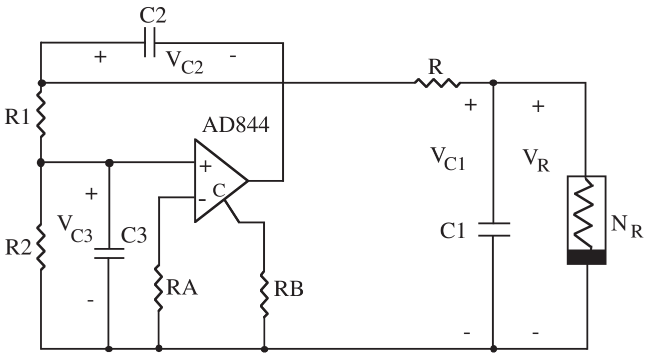


Figure 4. Wien-bridge based inductorless realization of Chua's circuit proposed by Elwakil and Kennedy [5].

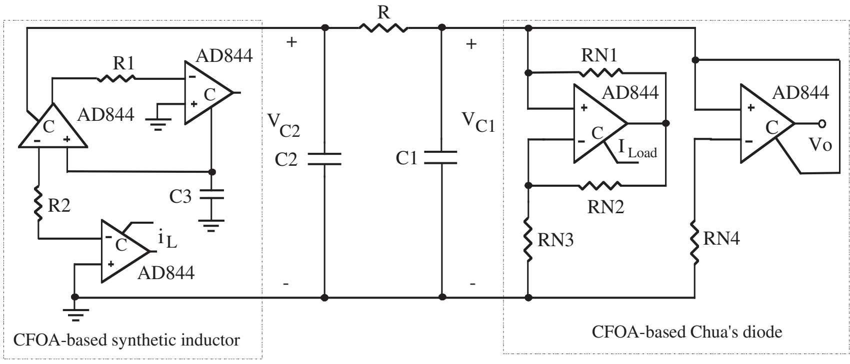
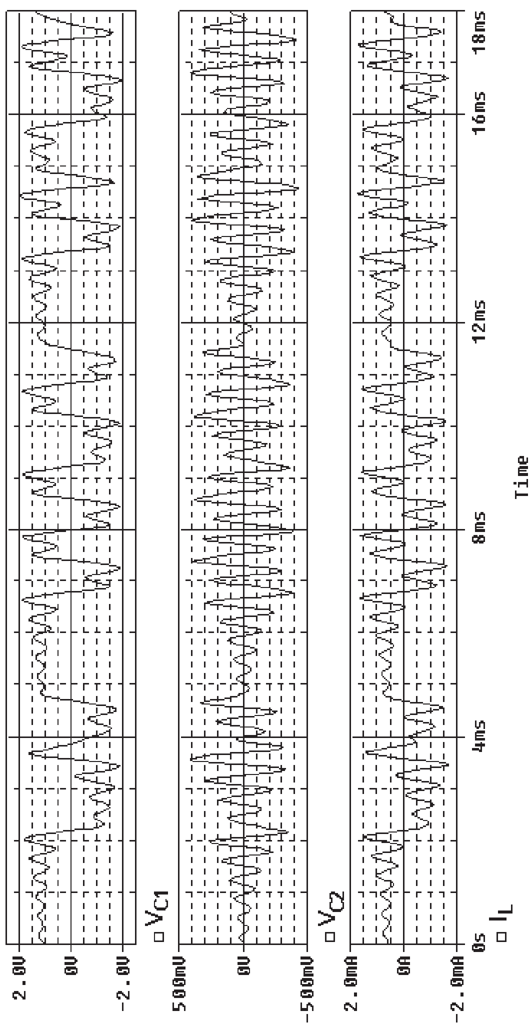


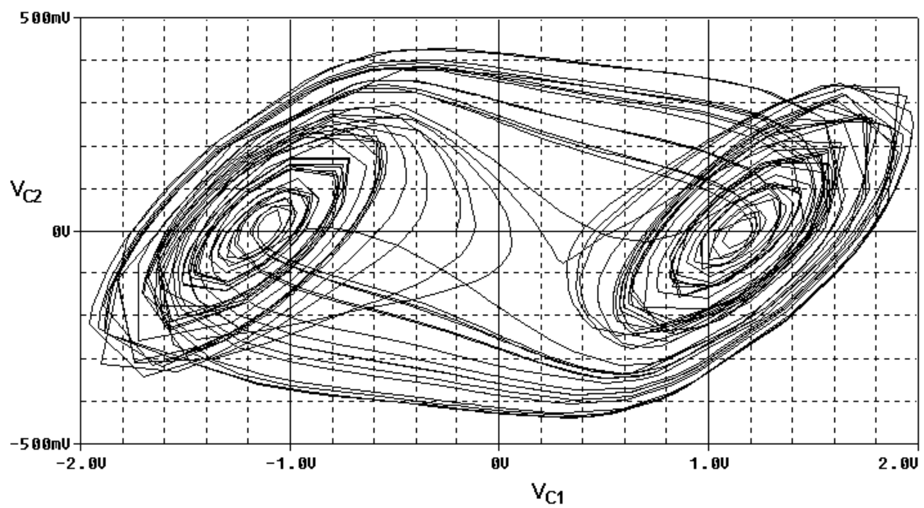
Figure 5. An alternative CFOA-based realization of Chua's circuit.

constructed an alternative CFOA-based realization of Chua's circuit combining Elwakil and Kennedy's CFOA-based nonlinear structure and Senani and Gupta's CFOA-based synthetic inductor structure so that the novel circuit exhibits the original chaotic behavior of Chua's circuit and high-frequency chaotic oscillations. Figure 5 shows the novel realization of Chua's circuit using two AD844-type CFOAs biased with ±9 V and four resistors to implement the nonlinear resistor, and using three AD844-type CFOAs biased with ±9 V, two resistors, and a capacitor to implement the synthetic inductor. In this realization, we fixed the parameter values of the nonlinear resistor as $R_{N1} = R_{N2} = 22$ kΩ, $R_{N3} = 500$ Ω, and $R_{N4} = 2.2$ kΩ as in [4]. Then, to illustrate the circuit's operation for different frequency ranges, especially at high frequencies in which the CFOA has excellent performance, we configured the rest of the circuit parameters as three groups. In the first parameter group, to show that the proposed



(a)

Figure 6. (a) Simulations of chaotic circuit dynamics $V_{C1}(t)$, $V_{C2}(t)$, $i_L(t)$. (cont)



(b)

Figure 6. (b) The double-scroll attractor, projection in the (V_{C2}–V_{C1}) plane, of the CFOA-based realization of Chua's circuit configured with the first parameter group.

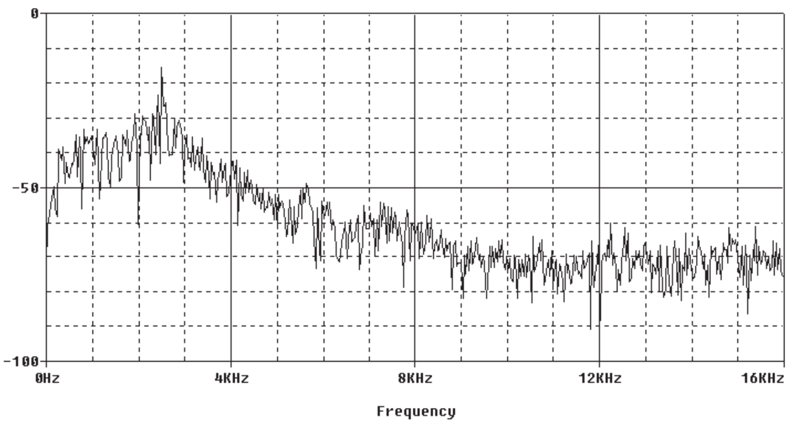
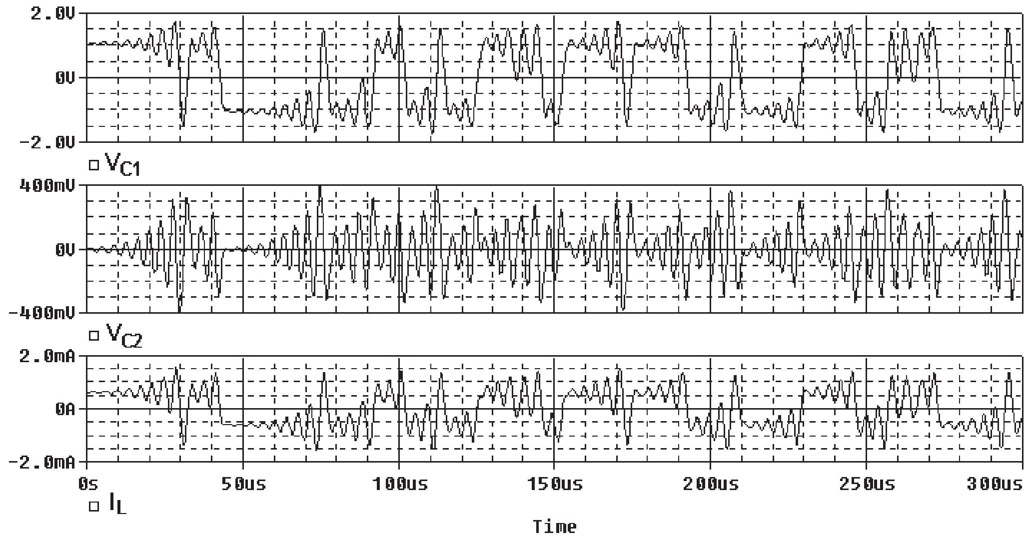


Figure 7. The chaotic spectrum for simulated time waveform V_{C2}(t) from the CFOA-based Chua's circuit configured with the first parameter group.

circuit can exhibit the original chaotic behavior of Chua's circuit with the most studied parameters ($L = 18$ mH, $C_1 = 10$ nF, $C_2 = 100$ nF, and $R = 1.7$ kΩ) in the literature, we determined the circuit parameters in Figure 5 as $C_1 = 10$ nF, $C_2 = 100$ nF, $R = 1.7$ kΩ, and the parameters of the CFOA-based synthetic inductor as $R_1 = R_2 = 1$ kΩ, $C_3 = 18$ nF to obtain $L = 18$ mH according to equation (5). The circuit dynamics V_{C1}, V_{C2}, i_L, and the double-scroll attractor are shown in Figure 6, and the chaotic frequency spectrum is shown in Figure 7. These graphs confirm that the proposed circuit is able to exhibit the original chaotic behavior of Chua's circuit with the most used parameters. As shown in Figure 7, the chaotic frequency spectrum is centered approximately around 3 kHz.

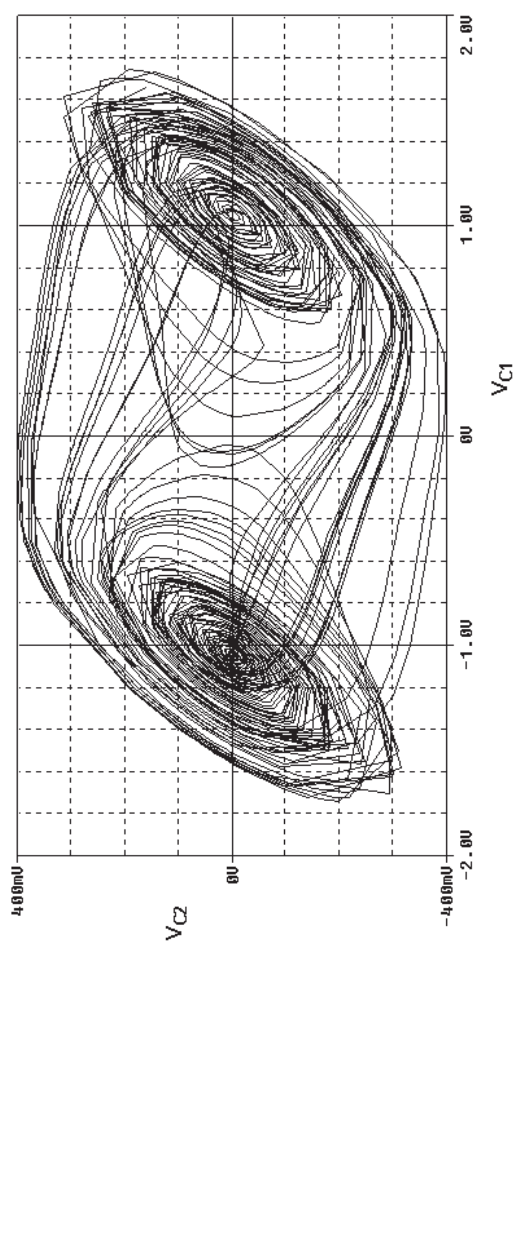
After investigating the proposed circuit with the first parameter group, to show that the proposed CFOA-based circuit exhibits excellent performance at high frequencies, we examined the circuit by using the second and third parameter groups, in which the synthetic inductor and capacitors are scaled down to extend the operating frequency. In the second parameter group, by scaling down values of the synthetic inductor and capacitors by a factor of 100, we determined the circuit parameters in Figure 5 as $C_1 = 100$ pF, $C_2 = 1$ nF, $R = 1.7$ kΩ, and the parameters of the CFOA-based synthetic inductor as $R_1 = R_2 = 1$ kΩ, $C_3 = 0.18$ nF to obtain $L = 0.18$ mH according to equation (5). For this operation mode, the circuit dynamics V_{C1}, V_{C2}, i_L, and the double-scroll attractor are shown in Figure 8, and the chaotic frequency spectrum is shown in Figure 9. As shown in Figure 9, the operating frequency is extended, and the chaotic frequency spectrum is centered approximately around 250 kHz.

In the third parameter group, by scaling down values of the synthetic inductor and capacitors by a factor of 1000, we determined the circuit parameters in



(a)

Figure 8. (a) Simulations of chaotic circuit dynamics $V_{C1}(t)$, $V_{C2}(t)$, $i_L(t)$. (cont)



(b)

Figure 8. (b) The double-scroll attractor, projection in the (V_{C2}–V_{C1}) plane, of the CFOA-based realization of Chua's circuit configured with the second parameter group for high-frequency operating mode.

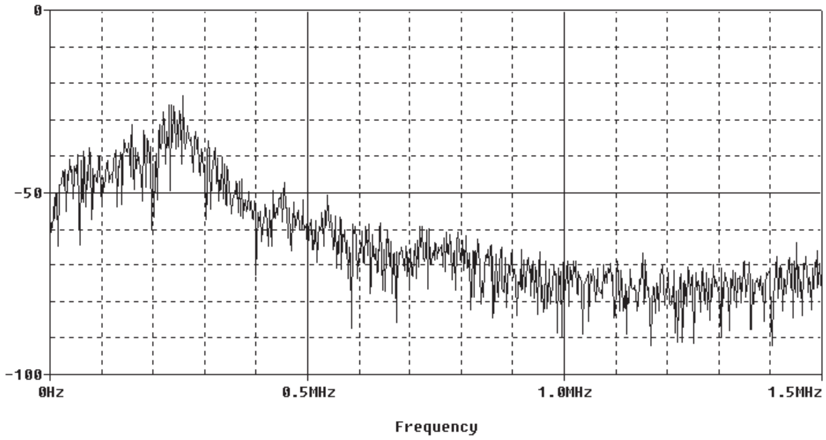
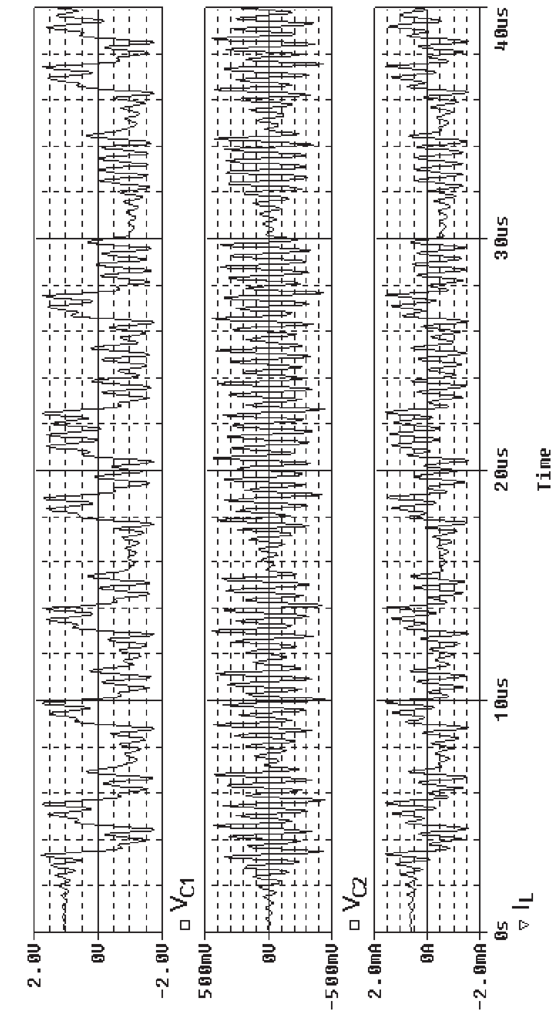


Figure 9. The chaotic spectrum for simulated time waveform V_{C2}(t) from CFOA-based Chua's circuit configured with the second parameter group.

Figure 5 as $C_1 = 10$ pF, $C_2 = 100$ pF, $R = 1.7$ kΩ, and the parameters of the CFOA-based synthetic inductor as $R_1 = R_2 = 1$ kΩ, $C_3 = 0.018$ nF to obtain $L = 0.018$ mH according to equation (5). For this very high-frequency operation mode, the circuit dynamics V_{C1}, V_{C2}, i_L, and the double-scroll attractor are shown in Figure 10, and the chaotic frequency spectrum is shown in Figure 11. As shown in Figure 11, the operating frequency is extended to a higher frequency range, and the chaotic frequency spectrum is centered approximately around 2.25 MHz.

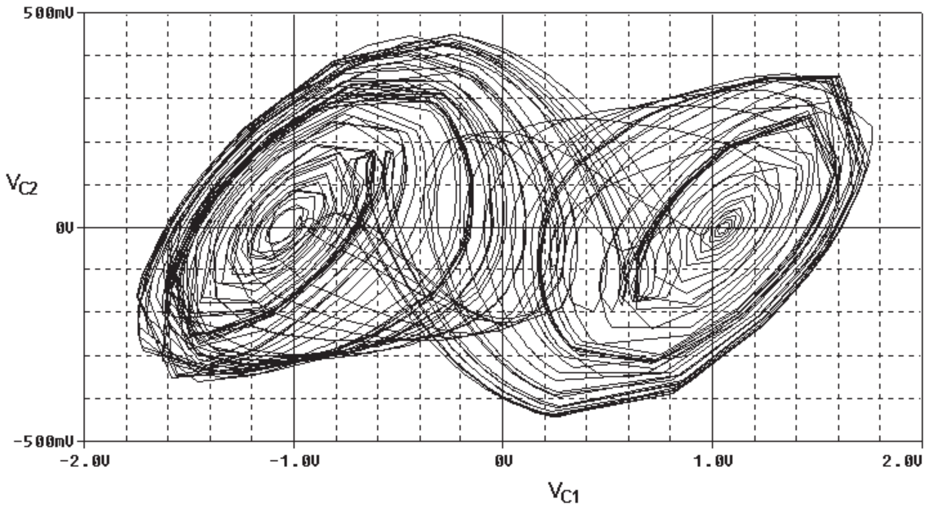
4. Conclusions

After comparing the CFOA-based realizations of Chua's circuit in the literature, we have presented an alternative CFOA-based realization of Chua's circuit. The aim of these implementations is to use the voltage-current capabilities of a CFOA, which offers several advantages over a classic VOA, in new realizations of Chua's circuit. Because our CFOA-based realization, which has an inductorless circuit structure, combines the advantageous features of CFOA-based circuit topologies for nonlinear resistor and inductor elements in the literature, it offers a very attractive circuit structure for realizing Chua's circuit operating at high frequencies. The PSpice simulation results, shown in both the time and frequency domains, verify that this realization is able to generate the original chaotic oscillations of Chua's circuit and exhibits excellent performance at high frequencies. Thus, we hope that this study will provide an alternative solution to the readers studying Chua's circuit and high-frequency chaotic oscillations.



(a)

Figure 10. (a) Simulations of chaotic circuit dynamics V_{C1}(t), V_{C2}(t), i_L(t). (cont)



(b)

Figure 10. (b) The double-scroll attractor, projection in the (V_{C2}–V_{C1}) plane, of the CFOA-based realization of Chua's circuit configured with the third parameter group for higher-frequency operating mode.

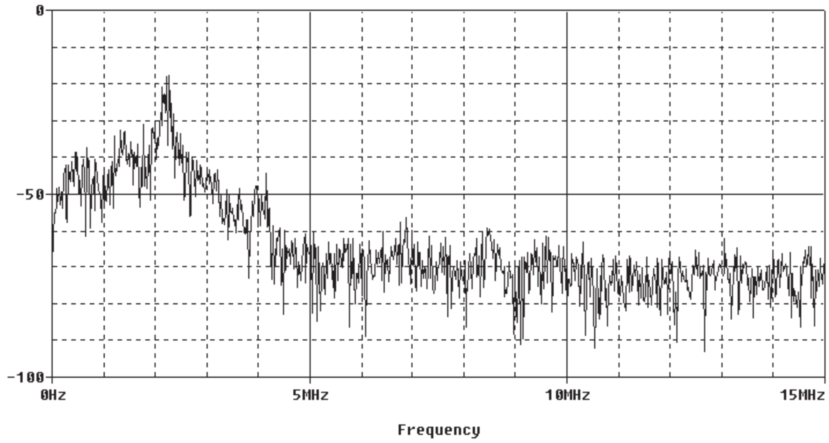


Figure 11. The chaotic spectrum for simulated time waveform V_{C2}(t) from CFOA-based Chua's circuit configured with the third parameter group.

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