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A microstretch continuum approach to model dielectric elastomers

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Abstract. A continuum model for dielectric elastomers is proposed on the basis of a micromorphic theory of electroelasticity. A biaxial microstretch deformation is considered to describe macrostretch and electric polarization due to applied mechanical loads and electric fields. A statistical isotropic condition is exploited to express the dependence of strain tensors on microstretch, and the equilibrium balance laws are given for micro- and macrodeformation and the electric potential. A one-dimensional problem is formulated to model a layer of dielectric elastomer subject to electric potential and mechanical traction. Some numerical results are obtained, which show consistence with the expected electroelastic physical behavior of such structures.

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1. Introduction

Electroelasticity of soft materials is characterized by large deformations connected to a coupled action of mechanical loads and electromagnetic fields. In particular, dielectric elastomers consist of polymer chains of molecules that, under the application of an external electric field, are equipped with an electric polarization and a deformation which includes stretches and chain's spatial reorientation. These phenomena are typically nonlinear and imply a dependence of electric permittivity from stretch and the actual electric field.

The most common approaches to model such electroelastic behaviors are continuum electromagneto-elastic theories based on nonlinear constitutive assumptions, allowing to give a phenomenological basis to describe various material performances and stability [1–3]. Statistical mechanics approaches have been also considered to account for the polymeric structure and its polarization by suitable statistical averages [4–6]. In the present work, we consider an alternative continuum mechanical approach which relies on a micromorphic electromagneto-elastic model. The general mechanical theory [7] accounts for a microstructure of the continuum particle by introducing internal degrees of freedom and the corresponding strain measures. An extension of this theory to electromagneto-elastic interactions has been proposed in previous works to obtain explicit expressions of polarization and magnetization which connect the electromagnetic field with the microdeformation [8,9]. Here, we consider a dielectric, nonmagnetic model and assume that the continuum particle is subject to a biaxial microstretch, in order to describe the polymer chain deformation of the dielectric elastomer. This model introduces two microstretch variables beside one orientational unit vector. The kinematics is developed in Sect. 2 where both microstrain tensors and polarization are derived as functions of the previous variables. In particular, evaluating polarization we account for first-order (dipole) and second-order (quadrupole) charge moments. Section 3 is devoted to a statistical averaging by the requirement of isotropy in order to obtain dipole and quadrupole densities and the microstretch tensors which describe the macrocontinuum model. With the aim of writing an explicit form of the set of balance equations for a boundary value problem, in Sect. 4 we write the constitutive equations for mechanical and electric stress and couple stress. Although we simply choose linear mechanical constitutive laws, our formulation yields a nonlinear problem due to the dependence on the stretch variables. The incompressibility assumption is then introduced, and a one-dimensional problem for an elastomer layer structure is formulated to discuss the effectiveness of our model. As expected, a nonlinear dependence of the stretch on the applied electric potential appears, together with a clear dependence of polarization on the stretch.

2. Kinematics for microstretch continua

The following analysis is based on the classical micromorphic theory of continuum media [7], extended to electroelastic coupling modeled by charge microdensity [8,9]. As usual, we denote by $\mathbf{x} = \mathfrak{f}(\mathbf{X}, t)$ the center of mass of the continuum particle $\mathcal{P}_t$ in the actual configuration, where $\mathbf{X}$ is the corresponding vector in the reference configuration $\mathcal{P}$. $\mathfrak{f}$ is a smooth function with gradient (deformation tensor) $\mathbf{F} = (\nabla \mathfrak{f})^T$. The microdeformation vector $\boldsymbol{\xi}$, within the particle in the actual configuration, is represented by $\boldsymbol{\xi} = \boldsymbol{\chi}(\mathbf{X}, t)\boldsymbol{\Xi}$ where $\boldsymbol{\Xi}$ is the microdeformation vector in the reference configuration. The microdeformation tensor $\boldsymbol{\chi}$ characterizes the kinematics of deformation within $\mathcal{P}_t$. In particular, we describe the elastomer element by its polymeric end-to-end direction $\mathbf{n}$ assuming a biaxial stretch given by the real positive stretches $\bar{\lambda}$ and λ, respectively, along and orthogonal to $\mathbf{n}$. All these quantities are supposed to be sufficiently smooth functions of $\mathbf{X}$ and t, and we have

$$\boldsymbol{\chi} = \lambda \mathbf{I} + (\bar{\lambda} - \lambda)\mathbf{n} \otimes \mathbf{n},$$

where $\mathbf{I}$ is the identity tensor. In the following, we shall denote components in the reference and actual configurations, respectively, by capital and lowercase indices so that the previous equation reads

$$\chi_{hH} = \lambda \delta_{hH} + (\bar{\lambda} - \lambda) n_H n_K \delta_{Kh}, \tag{2.1}$$

where the symbol δ_{hH} is the so-called *shifter*. We observe that the unit vector $\mathbf{n}$ is expressed in the reference configuration in order to discuss the arbitrariness of its orientation for the not deformed state, as explained in the next section.

The inverse microstretch tensor is given by

$$\mathfrak{X}_{Kl} = \frac{1}{\lambda}\delta_{Kl} + \frac{\lambda - \bar{\lambda}}{\lambda \bar{\lambda}} n_K n_H \delta_{Hl} \tag{2.2}$$

so that $\mathfrak{X}_{Kl}\chi_{lH} = \delta_{KH}$ and $\chi_{hK}\mathfrak{X}_{Kl} = \delta_{hl}$.

According to the general micromorphic theory, we introduce the strain tensors $\boldsymbol{\epsilon}$, $\mathbf{e}$, $\boldsymbol{\gamma}$, known as relative deformation, microdeformation and wryness tensors

$$\epsilon_{kl} = \delta_{kl} - F^{-1}_{Hk}\chi_{lH}, \qquad e_{kl} = \frac{1}{2}(\delta_{kl} - \mathfrak{X}_{Hk}\mathfrak{X}_{Hl}), \qquad \gamma_{klm} = \chi_{kH,M}\mathfrak{X}_{Hl}F^{-1}_{Mm} \tag{2.3}$$

In view of equations (2.1)–(2.3), we get

$$\begin{aligned}
\epsilon_{kl} &= \delta_{kl} - F^{-1}_{Hk}[\lambda \delta_{lH} + (\bar{\lambda} - \lambda) n_H n_K \delta_{Kl}] \\
e_{kl} &= \frac{1}{2}\left(1 - \frac{1}{\lambda^2}\right)\delta_{kl} + \frac{1}{2}\left(\frac{1}{\lambda^2} - \frac{1}{\bar{\lambda}^2}\right) n_H n_K \delta_{Hk}\delta_{Kl} \\
\gamma_{klm} &= \left\{\frac{\lambda_{,M}}{\lambda}\delta_{kl} + \left[\left(\frac{\bar{\lambda}_{,M}}{\bar{\lambda}} - \frac{\lambda_{,M}}{\lambda}\right) n_H n_J + (\bar{\lambda} - \lambda)\left(\frac{1}{\lambda} n_{H,M} n_J + \frac{1}{\bar{\lambda}} n_H n_{J,M}\right)\right]\delta_{kJ}\delta_{Hl}\right\} F^{-1}_{Mm}
\end{aligned} \tag{2.4}$$

By introducing the microdensity of bound charges, $\sigma'(\mathbf{x} + \boldsymbol{\xi}, t)$, in the actual configuration, we can write electric multipoles of different order in $\boldsymbol{\xi}$. For dipole and quadrupole densities, we have

$$\mathbf{p}(\mathbf{x}, t) = \frac{1}{\Delta v'}\int_{\mathcal{P}_t} \sigma'(\mathbf{x} + \boldsymbol{\xi}, t)\boldsymbol{\xi}\, \mathrm{d}v', \qquad \mathbf{Q}(\mathbf{x}, t) = \frac{1}{\Delta v'}\int_{\mathcal{P}_t} \sigma'(\mathbf{x} + \boldsymbol{\xi}, t)\boldsymbol{\xi} \otimes \boldsymbol{\xi}\, \mathrm{d}v' \tag{2.5}$$

The corresponding quantities in the reference configuration are given by

$$\mathbb{P}(\mathbf{X}) = \frac{1}{\Delta V'} \int_{\mathcal{P}} \sigma_0'(\mathbf{X}+\boldsymbol{\Xi})\boldsymbol{\Xi}\,\mathrm{d}V', \qquad \mathbb{Q}(\mathbf{x}) = \frac{1}{\Delta V'} \int_{\mathcal{P}} \sigma_0'(\mathbf{X}+\boldsymbol{\Xi})\boldsymbol{\Xi}\otimes\boldsymbol{\Xi}\,\mathrm{d}V' \tag{2.6}$$

We assume the conservation of charge microdensity so that $\sigma_0' = j\sigma'$ where $j = \det\boldsymbol{\chi}$. This allows to obtain the following relation between multipoles in actual and reference configuration. In components

$$p_i = \frac{1}{j}\chi_{iH}\mathbb{P}, \qquad Q_{ij} = \frac{1}{j}\chi_{iH}\chi_{jK}\mathbb{Q}_{HK}. \tag{2.7}$$

Owing to Eq. (2.1), we have $j = \lambda^2\bar{\lambda}$. Then, we pose

$$\mathbb{P}_H = \delta_{Hk}\pi_k, \qquad \mathbb{Q}_{HK} = \delta_{Hh}\delta_{Kk}\mathcal{Q}_{hk}$$

and from Eqs. (2.7) and (2.1) we obtain the following representation of electric dipole and quadrupole densities in the actual deformed configuration

$$p_i = \frac{1}{\lambda^2\bar{\lambda}}[\lambda\pi_i + (\bar{\lambda}-\lambda)n_H n_K \delta_{Hh}\delta_{Ki}\pi_h]$$

$$Q_{ij} = \frac{1}{\lambda^2\bar{\lambda}}\left\{\lambda^2\mathcal{Q}_{ij} + (\bar{\lambda}-\lambda)n_J\left[\lambda(n_H\delta_{jK}\delta_{Ji} + n_K\delta_{iH}\delta_{Jj}) + (\bar{\lambda}-\lambda)n_H n_K n_M \delta_{Ji}\delta_{Mj}\right]\mathbb{Q}_{HK}\right\} \tag{2.8}$$

In the following, these result will be exploited to express the polarization of the continuum according to the result

$$\mathbf{P} = \mathbf{p} - \frac{1}{2}\nabla\cdot\mathbf{Q}, \tag{2.9}$$

up to the second-order multipoles [10].

3. Statistical averages

In the following, we shall consider a spatial random orientation of the elastomer element in the reference configuration assuming an isotropic distribution. The orientational vector $\mathbf{n}$ of the continuum particle can be written as

$$\mathbf{n} = (\cos\theta, \sin\theta\cos\psi, \sin\theta\sin\psi), \tag{3.1}$$

where θ and ψ are spherical angles referred to a fixed frame in the reference configuration. The statistical average of a quantity h for a probability distribution $g(\mathbf{X},\mathbf{n})$ is

$$\langle h\rangle = \int_0^{2\pi}\int_0^{\pi} h(\mathbf{X},\mathbf{n})g(\mathbf{X},\mathbf{n})\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\psi$$

Under the isotropic assumption, $g(\mathbf{X},\mathbf{n}) = 1/4\pi$ and we get the following results

$$\langle n_H n_K\rangle = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} n_H n_K \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\psi = \frac{1}{3}\delta_{HK} \tag{3.2}$$

$$\langle n_H n_K n_J n_M\rangle = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} n_H n_K n_J n_M \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\psi$$

$$= \frac{1}{15}(\delta_{HK}\delta_{JM} + \delta_{HJ}\delta_{KM} + \delta_{HM}\delta_{KJ}) \tag{3.3}$$

Moreover, assuming constant gradients $\nabla\theta$ and $\nabla\psi$, for the random distribution in the reference configuration, we get

$$\langle n_{H,M} n_K \rangle = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} n_{H,M} n_K \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\psi = \frac{1}{3} \varepsilon_{HMK} \psi_{,M}, \quad (M \text{ not summed}) \tag{3.4}$$

and on the basis of the required isotropy we pose $\psi_{,M} = k$, $(M = 1, 2, 3)$ to obtain

$$\langle n_{H,M} n_K \rangle = \frac{k}{3} \varepsilon_{HMK}. \tag{3.5}$$

The meaning of the quantity k concerns an intrinsic measure of randomness in the reference distribution of orientation. As shown in the next section, it turns to be a factor in the mechanical parameters of the constitutive equation of the wryness tensor.

We apply these results to the statistical averages of the strain measures for the macroscopic continuum description. In view of the next developments, we introduce the macrodisplacement $\mathbf{u}$ such that

$$u_h = f_h - X_H \delta_{Hh}, \qquad u_{h,j} = \delta_{hj} - F^{-1}_{Hj} \delta_{Hh}. \tag{3.6}$$

We obtain the following results

$$\langle \epsilon_{kl} \rangle = \left(1 - \frac{2}{3}\lambda - \frac{1}{3}\bar{\lambda}\right) \delta_{kl} + \left(\frac{2}{3}\lambda + \frac{1}{3}\bar{\lambda}\right) u_{l,k}, \tag{3.7}$$

$$\langle e_{kl} \rangle = \left(\frac{1}{2} - \frac{1}{3\lambda^2} - \frac{1}{6\bar{\lambda}^2}\right) \delta_{kl} \tag{3.8}$$

$$\langle \gamma_{klm} \rangle = \frac{1}{3}\left(2\frac{\lambda_{,m}}{\lambda} + \frac{\bar{\lambda}_{,m}}{\bar{\lambda}}\right) \delta_{kl} + \frac{1}{3} k \frac{(\bar{\lambda} - \lambda)^2}{\lambda\bar{\lambda}} (\varepsilon_{klm} - \varepsilon_{klh} u_{h,m}) \tag{3.9}$$

Analogously, for the electric multipole densities,

$$\langle p_i \rangle = \frac{1}{\lambda^2 \bar{\lambda}} \left(\frac{2}{3}\lambda + \frac{1}{3}\bar{\lambda}\right) \pi_i, \tag{3.10}$$

$$\langle Q_{ij} \rangle = \frac{1}{15\lambda^2\bar{\lambda}} \left[(\bar{\lambda} - \lambda)^2 \mathcal{Q}_{hh} \delta_{ij} + (7\lambda^2 + 2\bar{\lambda}^2 + 6\lambda\bar{\lambda}) \mathcal{Q}_{ij}\right] \tag{3.11}$$

In the absence of microdeformation, i.e., for $\lambda = \bar{\lambda} = 1$, the previous strain measure ϵ_{kl} reduces to the corresponding macrocontinuum classical strain $u_{l,k}$ in the absence of microdeformation. In this case, e_{kl} and γ_{klm} vanish and the electric multipole densities reduce to the corresponding reference values π_i and $\mathcal{Q}_{ij}$.

For incompressible materials, we have isochoric deformations. As shown in a previous work [11], isochoric microdeformations imply isochoric macrodeformations so that we describe the incompressibility condition by posing $j = \bar{\lambda}\lambda^2 = 1$. Then, we get $\bar{\lambda} = 1/\lambda^2$, $\bar{\lambda}_{,m} = -2\lambda^{-3}\lambda_{,m}$ and we obtain

$$\begin{aligned}
\langle \epsilon_{kl} \rangle &= \delta_{kl} + \frac{1}{3\lambda^2}(2\lambda^3 + 1)(u_{l,k} - \delta_{kl}) \\
\langle e_{kl} \rangle &= \left(\frac{1}{2} - \frac{1}{3\lambda^2} - \frac{\lambda^4}{6}\right) \delta_{kl} \\
\langle \gamma_{klm} \rangle &= \frac{1}{3} k \left(\frac{1}{\lambda^2} - \lambda\right) (\varepsilon_{klm} - \varepsilon_{klh} u_{h,m}) \qquad (3.12) \\
\langle p_i \rangle &= \left(\frac{2}{3}\lambda + \frac{1}{3\lambda^2}\right) \pi_i
\end{aligned}$$

$$\langle Q_{ij} \rangle = \frac{1}{15}\left(\frac{1}{\lambda^2} - \lambda\right)^2 \mathcal{Q}_{hh} \delta_{ij} + \frac{1}{15}\left(7\lambda^2 + \frac{2}{\lambda^4} + \frac{6}{\lambda}\right) \mathcal{Q}_{ij}. \tag{3.13}$$

In the following, we shall restrict our analysis to incompressible materials. To simplify writing, angular brackets in the previous averaged quantities will be omitted.

4. Constitutive equations

Now we proceed in writing the constitutive equations according to the linear isotropic micromorphic mechanical theory. We observe that, despite this assumption, the balance equations obtained by this approach are not linear, due to the previous expressions of the strain tensors. From the general micromorphic model [7], the Cauchy stress $\mathbf{T}$ and the couple stress $\mathbf{m}$ are given by

$$T_{ij} = A_{ijhk}\epsilon_{hk} + E_{ijhk}e_{hk} + T^E_{ij}, \qquad m_{ijk} = C_{jkihlp}\gamma_{hlp} + m^E_{ijk} \tag{4.1}$$

where $\mathbf{T}^E$ and $\mathbf{m}^E$ are purely electric contributions which are introduced to complete the effects of electroelastic coupling considered in the next balance equations. The isotropy condition implies

$$\begin{aligned} A_{ijhk} &= \alpha\delta_{ij}\delta_{hk} + (\mu+\kappa)\delta_{ih}\delta_{jk} + \mu\delta_{ik}\delta_{jh} \\ E_{ijhk} &= (\alpha+\nu)\delta_{ij}\delta_{hk} + (\mu+\sigma)(\delta_{ih}\delta_{jk} + \delta_{ik}\delta_{jh}) \\ C_{jkihlp} &= \tau_1(\delta_{jk}\delta_{ih}\delta_{lp} + \delta_{jp}\delta_{ki}\delta_{hl}) + \tau_2(\delta_{jk}\delta_{il}\delta_{hp} + \delta_{ji}\delta_{kp}\delta_{hl}) + \tau_3\delta_{jk}\delta_{ip}\delta_{hl} \\ &\quad + \tau_4\delta_{jh}\delta_{ki}\delta_{lp} + \tau_5(\delta_{ji}\delta_{kh}\delta_{lp} + \delta_{jl}\delta_{ki}\delta_{hp}) + \tau_6\delta_{ji}\delta_{kl}\delta_{hp} + \tau_7\delta_{jh}\delta_{kl}\delta_{ip} \\ &\quad + \tau_8(\delta_{jl}\delta_{kp}\delta_{ih} + \delta_{jp}\delta_{kh}\delta_{il}) + \tau_9\delta_{jh}\delta_{kp}\delta_{il} + \tau_{10}\delta_{jl}\delta_{kh}\delta_{ip} + \tau_{11}\delta_{jp}\delta_{kl}\delta_{ih} \end{aligned} \tag{4.2}$$

with real parameters $\alpha, \mu, \kappa, \nu, \sigma, \tau_i (i = 1, \ldots, 11)$ subjected to restrictions imposed by thermodynamics [7]. Also we have

$$T^E_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2}\|\mathbf{E}\|^2 \delta_{ij} \right) \tag{4.3}$$

$$m^E_{ijk} = \frac{1}{2}\left(\mathcal{Q}_{ik}E_j - \mathcal{Q}_{ij}E_k - \varepsilon_{hjk}\mathcal{Q}_i E_h\right) \tag{4.4}$$

where $\mathbf{E}$ is the electric field and where we posed $\mathcal{Q}_i = \sum_{h=1}^{3} \mathcal{Q}_{ih}$. Equation (4.3) represents a Maxwell stress, while Eq. (4.4) arises from a dimensional analysis, accounting for linearity and for the skew-symmetric requirement due to the fact that, as in the micropolar theory, we shall write the balance of moment of momentum in its dual vectorial form.
In view of Eq. (3.12), in the isochoric case we obtain the following explicit expression for $\mathbf{T}$,

$$\begin{aligned} T_{ij} = &\left(\alpha_0 - \alpha_1\lambda - \alpha_2\frac{1}{\lambda^2} - \alpha_3\lambda^4 - p \right)\delta_{ij} \\ &+ \frac{1}{3}\left(2\lambda + \frac{1}{\lambda^2}\right)[\alpha u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \kappa u_{j,i}] + T^E_{ij} \end{aligned} \tag{4.5}$$

where p is the pressure variable introduced by the incompressibility condition and

$$\alpha_0 = \frac{9}{2}\alpha + 3\mu + \kappa + \frac{3}{2}\nu + \sigma, \qquad \alpha_1 = 2\alpha + \frac{4}{3}\mu + \frac{2}{3}\kappa,$$
$$\alpha_2 = 2\alpha + \frac{4}{3}\mu + \frac{1}{3}\kappa + \nu + \frac{2}{3}\sigma, \qquad \alpha_3 = \frac{1}{2}\alpha + \frac{1}{3}\mu + \frac{1}{2}\nu + \frac{1}{3}\sigma,$$

and the following expression for $\mathbf{m}$,

$$\begin{aligned} m_{ijk} = &\frac{1}{3}k\lambda\left(\frac{1}{\lambda^2} - \lambda\right)^2 \{\varepsilon_{ijk}(\tau_7 + 2\tau_8 - \tau_9 - \tau_{10} - \tau_{11}) \\ &+ [(\tau_2 - \tau_1)\delta_{jk}\varepsilon_{ipq} + (\tau_5 - \tau_4)\delta_{ki}\varepsilon_{jpq} + (\tau_6 - \tau_5)\delta_{ij}\varepsilon_{kpq}]u_{q,p} \\ &+ (\tau_{10} - \tau_7)\varepsilon_{jkq}u_{q,i} + (\tau_9 - \tau_8)\varepsilon_{ijq}u_{q,k} + (\tau_{11} - \tau_8)\varepsilon_{kiq}u_{q,j}\} + m^E_{ijk} \end{aligned} \tag{4.6}$$

As expected, in the absence of microstretch ($\lambda = 1$) and electric field, the microstress tensor m_{ijk} vanishes and the Cauchy stress tensor reduces to the classical form

$$T_{ij} = (\alpha u_{k,k} - p)\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \kappa u_{j,i}.$$

The explicit expression for polarization, from Eq. (2.9) in the isochoric case, becomes

$$\mathbf{P} = \frac{1}{3}\left(2\lambda + \frac{1}{\lambda^2}\right)\boldsymbol{\pi} + \frac{1}{15}\left(\frac{2}{\lambda^5} - \frac{1}{\lambda^2} - \lambda\right)(\mathrm{tr}\,\boldsymbol{\mathcal{Q}})\nabla\lambda - \frac{1}{15}\left(7\lambda - \frac{4}{\lambda^5} - \frac{3}{\lambda^2}\right)\boldsymbol{\mathcal{Q}}\,\nabla\lambda. \tag{4.7}$$

We note that, in the absence of an intrinsic electric dipole $\boldsymbol{\pi}$ of the elastomeric element, the contribution to polarization arises from the gradient of microstretch, connected to the bound charge distribution characterizing the quadrupole density $\boldsymbol{\mathcal{Q}}$.

Looking to the next vectorial form of the balance equation for moment of momentum, we write the following microstress tensor

$$\begin{aligned} \mu_{il} &= \varepsilon_{jkl} m_{ijk} \\ &= \frac{1}{3}\left(\lambda^3 + \frac{1}{\lambda^3} - 2\right)\{[(2\beta_1 + \beta_2) - \beta_1 u_{k,k}]\,\delta_{il} + \beta(u_{i,l} - u_{l,i}) + (\beta_1 - \beta_2)u_{l,i}\} \\ &\quad + \frac{1}{2}\varepsilon_{hkl}(\mathcal{Q}_{ki}E_h - \mathcal{Q}_{hi}E_k) - \mathcal{Q}_i E_l \end{aligned} \tag{4.8}$$

where

$$\beta = k(\tau_4 - 2\tau_5 + \tau_6), \quad \beta_1 = k(2\tau_8 - \tau_9 - \tau_{11}), \quad \beta_2 = 2k(\tau_7 - \tau_{10}).$$

5. Equilibrium balance equations and boundary conditions

We consider the balance laws for equilibrium of the dielectric elastomers according to the general micromorphic theory of electroelasticity for not magnetizable dielectric continua [8]. In the absence of external mechanical body forces, and accounting for the absence of net charge density, we can write the balance laws for momentum, moment of momentum and the Gauss' law in the following form

$$\nabla \cdot \mathbf{T} + \mathbf{p} \cdot \nabla\mathbf{E} + \frac{1}{2}(\mathbf{Q}\nabla)\nabla\mathbf{E} = \mathbf{0} \tag{5.1}$$

$$\nabla \cdot \boldsymbol{\mu} + \mathbf{t} + \mathbf{E} \times \mathbf{p} + \mathbf{H} = \mathbf{0} \tag{5.2}$$

$$\nabla \cdot (\epsilon_0 \mathbf{E} + \mathbf{P}) = 0 \tag{5.3}$$

where the vectors $\mathbf{t}$ and $\mathbf{H}$ are given by

$$t_i = \varepsilon_{ihk} t_{kh}, \quad H_i = \varepsilon_{ikh} Q_{kj} E_{h,j},$$

and where ϵ_0 is the dielectric permittivity in vacuum. Equation (5.2) has been obtained from the reduction of the tensorial balance of moment of momentum to its dual vectorial form after skew-symmetrization (multiplication by the permutation symbol) as occurs in the micropolar reduction [12]. Within the quasi-static model of dielectrics, we introduce the electric potential φ and assume $\mathbf{E} = \mathbf{E}^{(0)} - \nabla\varphi$, where $\mathbf{E}^{(0)}$ is a possible constant external applied field. Then, from the constitutive model of the previous section in the isochoric case we obtain the following explicit balance laws,

$$\begin{aligned} &-\nabla p + \left[\frac{2}{3}\alpha\left(1 - \frac{1}{\lambda^3}\right)\nabla\cdot\mathbf{u} - \alpha_1 + 2\alpha_2\frac{1}{\lambda^3} - 4\alpha_3\lambda^3\right]\nabla\lambda \\ &\quad + \frac{2}{3}\left(1 - \frac{1}{\lambda^3}\right)\left[\mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right) + \kappa(\nabla\mathbf{u})^T\right]\nabla\lambda + \frac{1}{3}\left(2\lambda + \frac{1}{\lambda^2}\right)[(\alpha + \mu)\nabla(\nabla\cdot\mathbf{u}) + (\mu + \kappa)\Delta\mathbf{u}] \end{aligned}$$

$$-\frac{1}{3}\left(2\lambda+\frac{1}{\lambda^2}\right)(\boldsymbol{\pi}\cdot\nabla)\nabla\varphi-\frac{1}{30}\left[\left(\frac{1}{\lambda^2}-\lambda\right)^2(\mathrm{tr}\,\boldsymbol{\mathcal{Q}})\mathbf{I}+\left(7\lambda^2+\frac{2}{\lambda^4}+\frac{6}{\lambda}\right)\boldsymbol{\mathcal{Q}}\right]\nabla[\nabla(\nabla\varphi)]$$
$$+\epsilon_0(\nabla\varphi-\mathbf{E}^{(0)})\Delta\varphi=\mathbf{0} \tag{5.4}$$

$$\frac{1}{3}\left(\lambda^3+\frac{1}{\lambda^3}-2\right)[(\beta-\beta_1)\nabla(\nabla\cdot\mathbf{u})-(\beta-\beta_1+\beta_2)\Delta\mathbf{u}]$$
$$+\left(\lambda^2-\frac{1}{\lambda^4}\right)[(2\beta_1+\beta_2-\beta_1\nabla\cdot\mathbf{u})\nabla\lambda+(\beta_1-\beta_2-\beta)(\nabla\lambda)\cdot\nabla\mathbf{u}+\beta(\nabla\mathbf{u})^T\cdot\nabla\lambda]$$
$$+\kappa\nabla\times\mathbf{u}+\frac{1}{3}\left(2\lambda+\frac{1}{\lambda^2}\right)(\mathbf{E}^{(0)}-\nabla\varphi)\times\boldsymbol{\pi}-\frac{1}{15}\left(7\lambda^2+\frac{2}{\lambda^4}+\frac{6}{\lambda}\right)\boldsymbol{\mathcal{H}}+\boldsymbol{\mathcal{M}}=\mathbf{0} \tag{5.5}$$

$$-\varepsilon_0\Delta\varphi+\frac{2}{3}\left(1-\frac{1}{\lambda^3}\right)\boldsymbol{\pi}\cdot\nabla\lambda$$
$$+\frac{2}{15}\left(\lambda+\frac{1}{\lambda^2}-\frac{2}{\lambda^5}\right)(\mathrm{tr}\,\boldsymbol{\mathcal{Q}})\Delta\lambda+\frac{2}{15}\left(7\lambda-\frac{3}{\lambda^2}-\frac{4}{\lambda^5}\right)\boldsymbol{\mathcal{Q}}:\nabla(\nabla\lambda)$$
$$+\frac{2}{15}\left(1-\frac{2}{\lambda^3}+\frac{10}{\lambda^6}\right)(\mathrm{tr}\,\boldsymbol{\mathcal{Q}})\|\nabla\lambda\|^2+\frac{2}{15}\left(7+\frac{6}{\lambda^3}+\frac{20}{\lambda^6}\right)(\nabla\lambda)\cdot\boldsymbol{\mathcal{Q}}(\nabla\lambda)=0 \tag{5.6}$$

where

$$\mathcal{H}_i=\varepsilon_{ikh}\mathcal{Q}_{kj}\varphi_{,hj} \qquad \mathcal{M}_i=\mathcal{Q}_j\varphi_{,ij}+\frac{1}{2}(\mathcal{Q}_{hj}\varphi_{,kj}-\mathcal{Q}_{kj}\varphi_{,hj}).$$

If $\mathbf{N}$ is the external normal to the boundary of the continuum domain, we have the following boundary conditions [12]

$$\mathbf{NT}=\boldsymbol{\tau}, \qquad \mathbf{N}\boldsymbol{\mu}=\boldsymbol{\sigma}, \qquad [[\mathbf{D}]]\cdot\mathbf{N}=0, \tag{5.7}$$

where $\boldsymbol{\tau}$, $\boldsymbol{\sigma}$ are, respectively, the force traction and couple traction on the boundary and double square brackets denote the difference $[[\mathbf{D}]]=\mathbf{D}^- - \mathbf{D}^+$ across the surface.

6. One-dimensional problem

In order to discuss the effectiveness of the present model in describing electroelastic coupling for elastomers, here we formulate a one-dimensional isochoric equilibrium problem for fields with spatial dependence on $x_1 \equiv x$, assuming

$$\mathbf{u}(x), \quad \lambda(x), \quad \varphi(x), \quad p(x).$$

We consider a layer of thickness d in the reference configuration, delimited by two parallel planes orthogonal to the $\mathbf{e}_1$ direction along x. No bounds are considered along the axes $\mathbf{e}_2$ and $\mathbf{e}_3$, thus describing an unbounded material region between planes $X=0$ and $X=d$. The surface $X=0$ is fixed $(x=0)$ while at $X=d$, a displacement along $\mathbf{e}_1$ is described by the local value of u_1, due to the coupling of the electric and mechanical boundary conditions (see Fig. 1). This means that the variable x is bounded as $x\in[0,\hat{d}]$ where $\hat{d}=d+\hat{u}_1$ and $\hat{u}_1$ is implicitly defined by

$$\hat{u}_1=u_1(d+\hat{u}_1). \tag{6.1}$$

Denoting differentiation with respect to x by a prime, the balance laws (5.4)–(5.6) reduces, in components, to the following set of equations

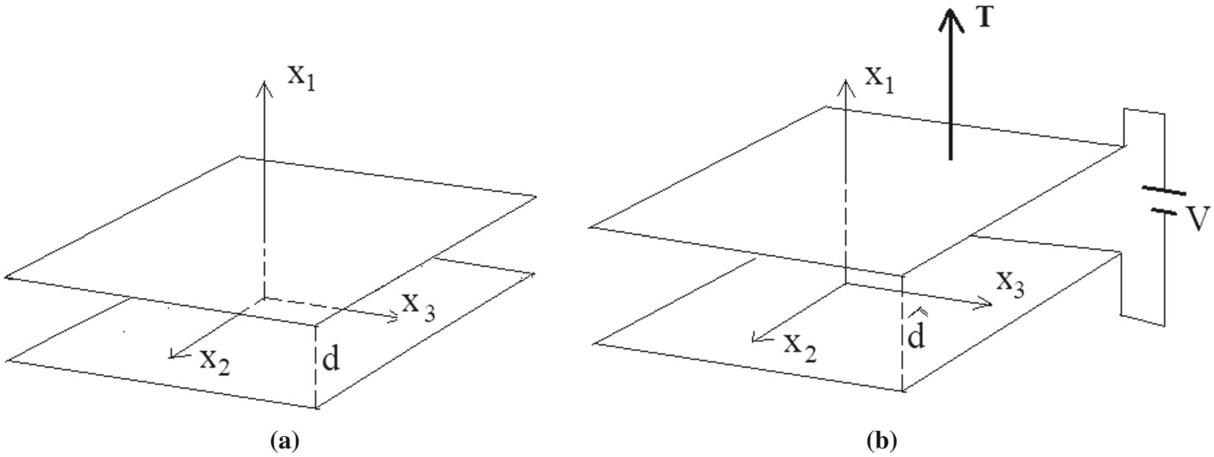

FIG. 1. Layer of dielectric elastomer. **a** Reference configuration, **b** deformed configuration due to applied electric potential V and traction $\mathbf{T}$

$$-p' + \frac{2}{3}(\alpha + 2\mu + \kappa)\left(1 - \frac{1}{\lambda^3}\right)u_1'\lambda' - \left(\alpha_1 - \alpha_2\frac{2}{\lambda^3} + 4\alpha_3\lambda^3\right)\lambda' + \frac{1}{3}\left(2\lambda + \frac{1}{\lambda^2}\right)(\alpha + 2\mu + \kappa)u_1'' - \frac{1}{3}\left(2\lambda + \frac{1}{\lambda^2}\right)\pi_1\varphi'' - \frac{1}{30}\left[\left(\frac{1}{\lambda^2} - \lambda\right)^2\mathcal{Q}_{hh} + \left(7\lambda^2 + \frac{2}{\lambda^4} + \frac{6}{\lambda}\right)\mathcal{Q}_{11}\right]\varphi''' + \epsilon_0(\varphi' - E_1^{(0)})\varphi'' = 0, \tag{6.2}$$

$$\frac{1}{3}(\mu + \kappa)\left[2\left(1 - \frac{1}{\lambda^3}\right)u_i'\lambda' + \left(2\lambda + \frac{1}{\lambda^2}\right)u_i''\right] - \epsilon_0 E_i^{(0)}\varphi'' = 0, \quad i = 2, 3. \tag{6.3}$$

$$-\frac{1}{3}\beta_2\left(\lambda^3 + \frac{1}{\lambda^3} - 2\right)u_1'' + \left(\lambda^2 - \frac{1}{\lambda^4}\right)[2\beta_1 + \beta_2 - \beta_2 u_1']\lambda' + \frac{1}{3}\left(2\lambda + \frac{1}{\lambda^2}\right)(\pi_3 E_2^{(0)} - \pi_2 E_3^{(0)}) + \mathcal{Q}_1\varphi'' = 0, \tag{6.4}$$

$$\frac{1}{3}(\beta_1 - \beta_2 - \beta)\left[\left(\lambda^3 + \frac{1}{\lambda^3} - 2\right)u_2'' + 3\left(\lambda^2 - \frac{1}{\lambda^4}\right)u_2'\lambda'\right] - \kappa u_3' + \frac{1}{3}\left(2\lambda + \frac{1}{\lambda^2}\right)[\pi_1 E_3^{(0)} - \pi_3(E_1^{(0)} - \varphi')] - \frac{1}{15}\left(7\lambda^2 + \frac{2}{\lambda^4} + \frac{6}{\lambda} - 15\right)\mathcal{Q}_{31}\varphi'' = 0, \tag{6.5}$$

$$\frac{1}{3}(\beta_1 - \beta_2 - \beta)\left[\left(\lambda^3 + \frac{1}{\lambda^3} - 2\right)u_3'' + 3\left(\lambda^2 - \frac{1}{\lambda^4}\right)u_3'\lambda'\right] - \kappa u_2' + \frac{1}{3}\left(2\lambda + \frac{1}{\lambda^2}\right)[\pi_2(E_1^{(0)} - \varphi') - \pi_1 E_2^{(0)}] + \frac{1}{15}\left(7\lambda^2 + \frac{2}{\lambda^4} + \frac{6}{\lambda} - 15\right)\mathcal{Q}_{21}\varphi'' = 0. \tag{6.6}$$

$$-\epsilon_0\varphi'' + \frac{2}{3}\left(1 - \frac{1}{\lambda^3}\right)\pi_1\lambda' + \frac{1}{15}\left[\left(\frac{2}{\lambda^5} - \frac{1}{\lambda^2} - \lambda\right)\mathcal{Q}_{hh} - \left(7\lambda - \frac{4}{\lambda^5} - \frac{3}{\lambda^2}\right)\mathcal{Q}_{11}\right]\lambda'' - \frac{1}{15}\left[\left(\frac{10}{\lambda^6} - \frac{2}{\lambda^3} + 1\right)\mathcal{Q}_{hh} + \left(7 + \frac{20}{\lambda^6} + \frac{6}{\lambda^3}\right)\mathcal{Q}_{11}\right](\lambda')^2 = 0. \tag{6.7}$$

As to the boundary conditions, we assume that an electric potential is applied on the layer assuming $\varphi(0) = 0$, $\varphi(\hat{d}) = V$ with $\mathbf{E}^{(0)} = \mathbf{0}$. In addition, a normal traction is applied at $x = \hat{d}$, expressed as

$\mathbf{e}_1\mathbf{T}(\hat{d}) = \tau\mathbf{e}_1$. For the additional condition on $\boldsymbol{\mu}$, we pose $\mathbf{e}_1\boldsymbol{\mu}(\hat{d}) = \boldsymbol{\sigma}$ assuming no couple traction along axis $\mathbf{e}_1$ ($\boldsymbol{\sigma}\cdot\mathbf{e}_1 = 0$).

In view of a description of effects due to the coupling between microdeformation and electric field, we suppose that the dielectric material has no intrinsic polarization in the undeformed configuration. This means that $\boldsymbol{\pi} = \mathbf{0}$, while $\boldsymbol{\mathcal{Q}} \neq \mathbf{0}$ in order to describe polarization due to deformation, according to the general expression of $\mathbf{P}$. As follows, this assumption, together with the absence of external transverse electric fields, notably simplifies the problem.

From Eq. (6.2), accounting for the boundary conditions $T_{12}(\hat{d}) = T_{13}(\hat{d}) = 0$, we get $u_2'(x) = u_3'(x) = 0$, $x \in [0, \hat{d}]$. This implies that both Eqs. (6.4) and (6.5) reduce to

$$\left(7\lambda^2 + \frac{2}{\lambda^4} + \frac{6}{\lambda} - 15\right)\varphi'' = 0,$$

and excluding the trivial solution $\lambda = 1$, we get $\varphi'' = 0$ which implies φ' to be constant within the layer. Due to the boundary conditions on φ, we simply obtain

$$\varphi(x) = \frac{V}{\hat{d}}x, \qquad x \in [0, \hat{d}]. \tag{6.8}$$

The remaining boundary conditions on the stress $T_{11}(\hat{d}) = \tau$ and on the couple stress $\boldsymbol{\mu}$ give

$$\alpha_0 - \alpha_1\hat{\lambda} - \alpha_2\frac{1}{\hat{\lambda}^2} - \alpha_3\hat{\lambda}^4 + \frac{1}{3}(\alpha + 2\mu + \kappa)\left(2\hat{\lambda} + \frac{1}{\hat{\lambda}^2}\right)\hat{u}_1' - \hat{p} + \frac{1}{2}\epsilon_0\left(\frac{V}{\hat{d}}\right)^2 = \tau \tag{6.9}$$

$$\frac{1}{3}\left(\hat{\lambda}^3 + \frac{1}{\hat{\lambda}^3} - 2\right)(2\beta_1 + \beta_2 - \beta_2\hat{u}_1') + \mathcal{Q}_1\frac{V}{\hat{d}} = 0,$$

$$\sigma_2 = \mathcal{Q}_{13}\frac{V}{\hat{d}}, \qquad \sigma_3 = \mathcal{Q}_{12}\frac{V}{\hat{d}}, \tag{6.10}$$

where

$$\hat{\lambda} = \lambda(\hat{d}), \qquad \hat{u}_1' = u_1'(\hat{d}), \qquad \hat{p} = p(\hat{d}).$$

The integration of Eqs. (6.1) and (6.3), exploiting conditions (6.8) and (6.9) gives the following results

$$p = \frac{1}{3}(\alpha + 2\mu + \kappa)\left(2\lambda + \frac{1}{\lambda^2}\right)u_1' + \alpha_1\lambda + \alpha_2\frac{1}{\lambda^2} + \alpha_3\lambda^4 - \hat{\tau}, \tag{6.11}$$

$$\frac{1}{3}\left(\lambda^3 + \frac{1}{\lambda^3} - 2\right)(2\beta_1 + \beta_2 - \beta_2 u_1') = -\mathcal{Q}_1\frac{V}{\hat{d}} \tag{6.12}$$

where

$$\hat{\tau} = \tau - \frac{1}{2}\epsilon_0\left(\frac{V}{\hat{d}}\right)^2 - \alpha_0 + 2\left(\alpha_1\hat{\lambda} + \alpha_2\frac{1}{\hat{\lambda}^2} + \alpha_3\hat{\lambda}^4\right).$$

Finally, by integrating Eq. (6.6) under the boundary condition $(5.7)_3$ we obtain

$$\psi(\lambda) - \psi(\hat{\lambda}) = 15\epsilon_0\frac{V}{\hat{d}}(x - \hat{d}), \tag{6.13}$$

where

$$\psi(\lambda) = \left(\frac{1}{\lambda} - \frac{1}{2\lambda^4} - \frac{1}{2}\lambda^2\right)\mathcal{Q}_{hh} - \left(\frac{7}{2}\lambda^2 + \frac{1}{\lambda^4} + \frac{3}{\lambda}\right)\mathcal{Q}_{11}.$$

Now, the solution to our problem can be achieved in the following way. As a first step, we evaluate parameters $\hat{\lambda}$ and $\hat{u}_1'$ from Eqs. (6.9) and $(6.10)_1$. Then, substitution into Eqs. (6.11) and (6.12) and (6.13) allows us to obtain the functions $p(x)$, $\lambda(x)$ and $u_1'(x)$ on $[0, \hat{d}]$. The successive integration of u_1' allows to exploit the implicit formula (6.1) to obtain $\hat{u}_1$ and then $\hat{d}$. By this final result, we achieve explicit results for the problem variables p, λ, u_1.

Table 1. *Material parameters*

α_0 (10^6N/m^2)	$\bar\alpha_1$	$\bar\alpha_2$	$\bar\alpha$	β_2 (10^{-6}N/m)	b	$\mathcal{Q}_{11}$ $(10^{-9}C\text{m}^{-1})$	$\mathcal{Q}_{hh}$ $(10^{-9}C\text{m}^{-1})$	$\mathcal{Q}_1$ $(10^{-9}C\text{m}^{-1})$
1	0.2	0.5	2	−1	1.5	2	4	−1

7. A numerical simulation

In order to apply our analysis on a possible soft electroelastic material and looking for a test on the effectiveness of the present micromorphic model for the qualitative electroelastic behavior of elastomers, we present a numerical solution of the problem depicted in the previous section. To this aim, we rewrite Eqs. (6.9) and $(6.10)_1$ in the following dimensionless form

$$\begin{aligned} &\bar\tau = 1 - \bar\alpha_1\hat\lambda - \bar\alpha_2\frac{1}{\hat\lambda^2} - \hat\alpha_3\hat\lambda^4 + \frac{1}{3}\bar\alpha\left(2\hat\lambda + \frac{1}{\hat\lambda^2}\right)\hat u_1' + \frac{1}{2}\frac{\epsilon_0}{\alpha_0}\left(\frac{V}{\hat d}\right)^2, \\ &\frac{1}{3}(\hat\lambda^3 - 1)^2(b - \hat u_1') + \frac{\mathcal{Q}_1}{\beta_2}\frac{V}{\hat d}\hat\lambda^3 = 0, \end{aligned} \tag{7.1}$$

where we posed

$$\bar\tau = \frac{\tau + \hat p}{\alpha_0}, \quad \bar\alpha = \frac{\alpha + 2\mu + \kappa}{\alpha_0}, \quad b = \frac{2\beta_1 + \beta_2}{\beta_2}, \quad \bar\alpha_i = \frac{\alpha_i}{\alpha_0}, \quad i = 1,2,3.$$

We observe that, owing to the definitions of α_i $(i = 0,1,2,3)$, we have $\sum_{i=1}^3 \bar\alpha_i = 1$, so that we consider here six mechanical parameters and three parameters for the quadrupolar matrix $\boldsymbol{\mathcal{Q}}$. In Table 1, a set of values for these quantities are proposed after a comparison with some mechanical parameters of elastomers, micromorphic elastic coefficients and accounting for the order of magnitude of electric quadrupoles in monomeric molecules [6,13,14].
Then, we proceed along the description given at the end of the previous section and evaluate numerically the quantities $\lambda(x)$, $u_1(x)$ on $[0, \hat d]$ for a layer with a thickness $d = 0.005$m in the reference configuration. We introduce the macrostretch of the layer as

$$\Lambda = 1 + \frac{u_1}{d}.$$

In Fig. 2, we show the result on Λ as a function of the electric potential V(V) under three values of the dimensionless traction $\bar\tau$. A nonlinear increase of thickness appears in the present problem. In coherence with a physical analysis and experimental results on similar problems [2,15–17,19], this behavior can be connected to the occurrence, in the present model, of a dependence of polarization on the mechanical stretch. In fact, in addition to the contribution of the Maxwell stress $\mathbf{T}^E$, the electric forces and couples appearing in the balance Eqs. (5.1) and (5.2), depend on the microdeformation. To confirm this deduction we consider the results on polarization for the present problem. From Eq. (4.7), we have

$$\begin{aligned} &P_1 = \frac{1}{15}\left[\left(\frac{2}{\lambda^5} - \frac{1}{\lambda}^2 - \lambda\right)\text{tr}\,\boldsymbol{\mathcal{Q}} - \left(7\lambda - \frac{4}{\lambda^5} - \frac{3}{\lambda^2}\right)\mathcal{Q}_{11}\right]\lambda', \\ &P_2 = -\frac{1}{15}\left(7\lambda - \frac{4}{\lambda^5} - \frac{3}{\lambda^2}\right)\mathcal{Q}_{21}\lambda', \quad P_3 = -\frac{1}{15}\left(7\lambda - \frac{4}{\lambda^5} - \frac{3}{\lambda^2}\right)\mathcal{Q}_{31}\lambda'. \end{aligned} \tag{7.2}$$

We have computed the mean value of polarization within the layer in order to compare its value with the applied potential and traction. The results are shown in Figs. 3 and 4 where a longitudinal (P_1) polarization and a transverse (P_2) polarization are shown. For the transverse components of $\mathbf{P}$, we have chosen $\mathcal{Q}_{12} = \mathcal{Q}_{13}$ so that the present result on P_3 is the same as P_2. In Fig. 3, two values of traction are considered. An increasing traction implies a decreasing polarization for both longitudinal and transverse

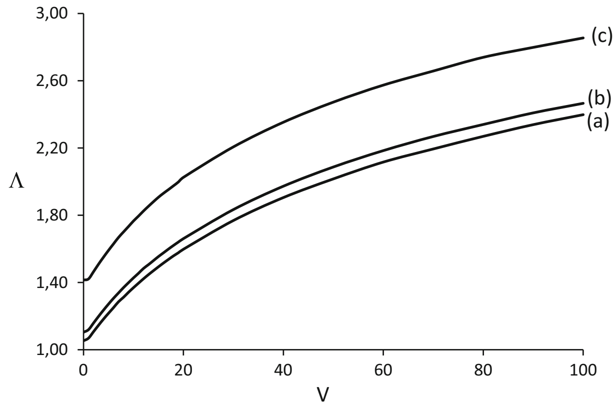

FIG. 2. Macrostretch versus V(V) for different values of dimensionless traction. (a) $\bar{\tau} = 0.01$, (b) $\bar{\tau} = 0.1$, (c) $\bar{\tau} = 0.5$

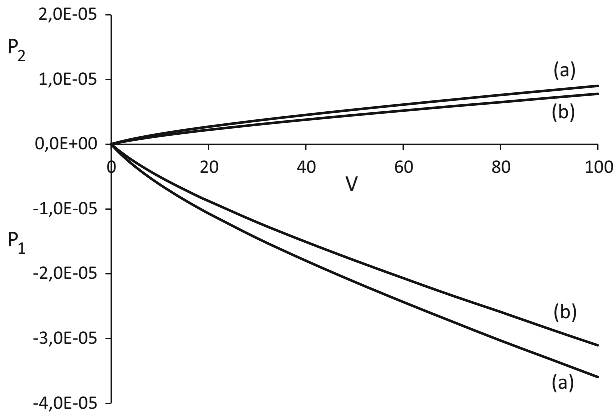

FIG. 3. Polarization components P_1 and P_2 (Cm^{-2}) versus V(V) for different values of dimensionless traction. (a) $\bar{\tau} = 0.1$, (b) $\bar{\tau} = 0.5$

components. To clarify this feature, the dependence on stretch is shown in Fig. 4 for $V = 50$V where the variation of stretch is due to the variation of the applied traction. Polarization decreases by an increasing stretch; this behavior has been observed in the past by experimental works [18,19].

8. Concluding remarks

The micromorphic approach to electroelasticity on soft materials described in this work relies on a biaxial microstretch of the continuum elementary particle. Dielectric properties are here related to such microdeformation on the basis of the representation of polarization by electric multipole densities. In particular, in the absence of intrinsic electric dipole of molecules, polarization arises from the quadrupole gradient. This approach is alternative to models based on constitutive assumptions for polarization where nonlinear phenomena are accounted by a possible dependence of permittivity on mechanical stretch. This electroelastic coupling is a basic feature of the present model where microstretch determines both electric polarization and macrostretch. Also, this allows to consider nonlinear electroelastic coupling assuming

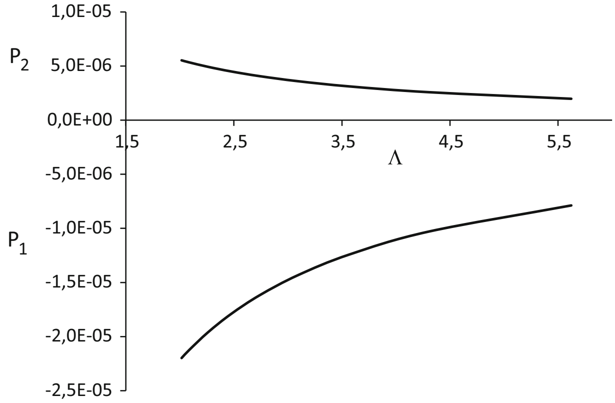

Fig. 4. Polarization components P_1 and P_2 (Cm^{-2}) as functions of the macrostretch due to applied mechanical traction, with $V = 50\mathrm{V}$

simply linear constitutive equations for mechanical stress and couple stress in the classical micromorphic theory.

The one-dimensional problem investigated in the previous sections can be considered as a simple application where the occurrence of the only nonzero displacement in the material layer yields a constant electric field. In this respect, we observe that this field, however, depends on the stretching layer thickness, thus suggesting that the hypothesis of independence of electric field on deformation and polarization, commonly adopted in the literature of phenomenological theories, can be restrictive in modeling nonlinear phenomena. We finally note that the condition of isotropy exploited in the statistical average of microstrain tensors and adopted in the mechanical constitutive laws, does not exclude the presence of a polarization not aligned with the electric field. This effect depends on the intrinsic dielectric molecular structure, and specifically, on the off-diagonal entries of the quadrupole matrix.

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References

[1] Dorfmann, L., Ogden, R.W.: Nonlinear electroelasticity. Acta Mech. **174**, 167–183 (2005)
[2] Jimenez, S.M.A., McMeeking, R.M.: Deformation dependent dielectric permittivity and its effects on actuator performance and stability. Int. J. Nonlinear Mech. **57**, 183–191 (2013)
[3] Dorfmann, L., Ogden, R.W.: Nonlinear electroelasticity: material properties, continuum theory and applications. Proc. R. Soc. A **473**, 20170311 (2017)
[4] Cohen, N., deBotton, G.: The electromechanical response of polymer networks with long-chain molecules. Math. Mech. Solids **20**, 721–728 (2014)
[5] Cohen, N., Dayal, K., deBotton, G.: Electroelasticity of polymer networks. J. Mech. Phys. Solids **92**, 105–126 (2016)
[6] Itskov, M., Khiêm, V., Waluyo, S.: Electroelasticity of dielectric elastomers based on molecular chain statistics. Math. Mech. Solids **24**, 862–873 (2019)
[7] Eringen, A.C.: Microcontinuum Field Theories I—Foundations and Solids. Springer, New York (1999)
[8] Romeo, M.: Micromorphic continuum model for electromagnetoelastic solids. Z.A.M.P. **62**, 513–527 (2011)
[9] Romeo, M.: A microstructure continuum approach to electromagneto-elastic conductors. Contin. Mech. Thermodyn. **28**, 1807–1820 (2016)
[10] Romeo, M.: Polarization in dielectrics modeled as micromorphic continua. Z.A.M.P. **66**, 1233–1247 (2015)
[11] Romeo, M.: A microstretch description of electroelastic solids with application to plane waves. Math. Mech. Solids **24**, 2181–2196 (2019)
[12] Romeo, M.: A variational formulation for electroelasticity of microcontinua. Math. Mech. Solids **20**, 1234–1250 (2015)

[13] Chen, Y., Lee, J.D.: Determining material constants in micromorphic theory through phonon dispersion relations. Int. J. Eng. Sci. **41**, 871–886 (2003)
[14] Villanueva-García, M., Robles, J., Martínez-Richa, A.: Quadrupolar moment calculations and mesomorphic character of model dimeric liquid crystals. Comput. Mater. Sci. **22**, 300–308 (2001)
[15] Schlögl, T., Leyendecker, S.: A polarization based approach to model the strain dependent permittivity of dielectric elastomers. Sens. Actuators A **267**, 156–163 (2017)
[16] Zhao, X., Hong, W., Suo, Z.: Electromechanical hysteresis and coexistent states in dielectric elastomers. Phys. Rev. B **76**, 134113 (2007)
[17] Zhao, X., Suo, Z.: Electrostriction in elastic dielectrics undergoing large deformations. J. Appl. Phys. **104**, 123530 (2008)
[18] Wissler, M., Mazza, E.: Electromechanical coupling in dielectric elastomer actuators. Sens. Actuators A **138**, 384–393 (2007)
[19] Li, B., et al.: Effects of mechanical pre-stretch on the stabilization of dielectric elastomer actuation. J. Phys. D Appl. Phys. **44**, 155301 (2011)

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