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Temperature dependence of an elastic modulus in generalized linear micropolar thermoelasticity

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Abstract. The model of the equations of generalized linear micropolar thermoelasticity with two relaxation times in an isotropic medium with temperature-dependent mechanical properties is established. The modulus of elasticity is taken as a linear function of reference temperature. Laplace and exponential Fourier transform techniques are used to obtain the solution by a direct approach. The integral transforms have been inverted by using a numerical technique to obtain the temperature, displacement, force and couple stress in the physical domain. The results of these quantities are given and illustrated graphically. A comparison is made with results obtained in case of temperature-independent modulus of elasticity. The problem of generalized thermoelasticity has been reduced as a special case of our problem.

Keywords. Elastic modulus, micropolar thermoelastic medium.

1. Introduction

The elastic modulus is an important physical property of materials reflecting the elastic deformation capacity of the material when subjected to an applied external load. Most of investigations were done under the assumption of the temperature-independent material properties, which limit the applicability of the solutions obtained to certain ranges of temperature. At high temperature the material characteristics such as the modulus of elasticity, Poisson's ratio, the coefficient of thermal expansion and the thermal conductivity are no longer constants [1]. In recent years due to the progress in various fields in science and technology the necessity of taking into consideration the real behavior of the material characteristics became actual. Temperature dependent measurements of Young's modulus were performed for the first time on black and transparent bulk material of chemical vapor deposited diamond by a dynamic three point bending method in a temperature range from -150 to 850 deg C [2]. A lower Young's modulus of polycrystalline diamond is caused by crystal imperfections and impurities [2]. The temperature dependencies of shear elasticity of some liquids have been investigated in [3]. It was found that the shear modulus decreases with increasing temperature. This decrease may be explained by the increase of the fluctuation free volume [3]. The dynamic resonance method was used to determine the temperature dependence of the modulus of elasticity of some plasma-sprayed materials [4]. Rising in test temperature was found to cause a monotonic decrease in the modulus of elasticity. It was shown that the modulus of elasticity of coating materials decreases during the temperature rise to 1200 deg C by 25 to 60 percent [4].

Analyzing the problems of high-frequency short-wavelength vibrations and ultrasonic waves in elastic media, the classical theory of elasticity is inadequate to describe the real phenomena, whence the necessity of recurring to the micropolar theory. The micropolar elasticity theory takes into consideration the granular character of the medium, and is intended to be applied to materials for problems where the classical theory of elasticity fails owing to the microstructure of the material. The general theory of linear micropolar elasticity was given by Eringen [5–7] and Nowacki [8–9]. Under this theory, solids can undergo macro-deformations and micro-rotations, and support couple stresses in addition to force stresses.

The micropolar theory was extended to include thermal effects by Nowacki [10], Eringen [5, 11], Tauchert et al. [12], Tauchert [13] and Nowacki and Olszak [14]. One can refer to Dhaliwal and Singh [15] for a review on the micropolar thermoelasticity and a historical survey of the subject, as well as to Eringen and Kafadar [16] in "Continuum Physics" series in which the general theroy of micromorphic media has been summed up.

The classical theory of heat conduction predicts infinite speed of heat transportation, if a material conducting heat is subjected to thermal disturbances, which contradicts the physical facts. During the last three decades non-classical theories have been developed to remove this paradox. Lord and Shulman [17] incorporated a flux rate term into the Fourier's law of heat conduction and formulated a generalized theory admitting finite speed for thermal signals. Green and Lindsay [18] have developed a temperature rate dependent thermoelasticity by including temperature rate among the constitutive variables which does not violate the classical Fourier law's of heat conduction when the body under consideration has a center of symmetry and this theory also predicts a finite speed of heat propagation.

Recently some authors discussed different type of problems in generalized micropolar thermoelasticity medium with temperature-independent modulus of elasticity (Kumar and Singh [19, 20], Singh and Kumar [21, 22], and Kumar [23]). Motivated by the recent experimental studies [1–4] showing the necessity of taking into consideration the real behavior of the material characteristics, this paper presents an attempt to examine the temperature dependency of elastic modulus on the behavior of two-dimensional solutions in a micropolar thermoelastic medium. This article is a continuation of the work [24, 25] to include the effect of reference temperature on thermal stress distribution.

2. Formulation of the problem

We shall consider an isotropic micropolar thermoelastic medium with temperature-dependent mechanical properties. Following Eringen [5-7], the components of the force stress $\sigma_{ij}$and couple stress m_{ij} tensors are given, respectively, by:

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + k(u_{j,i} - \varepsilon_{ijr}\phi_r) - \gamma \left(T - T_0 + \nu \frac{\partial T}{\partial t} \right) \delta_{ij}, \quad (2.1)$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \delta\phi_{j,i}. \quad (2.2)$$

The usual summation convention for repeated indices is used throughout and the comma denotes differentiation with respect to spatial coordinates. u_i is the displacement, ε_{ijr} is the permutation symbol and ϕ_i is the microrotation. T is the absolute temperature of the medium, ν is a constant with dimension of time, called a relaxation time, γ is a material constant given by $\gamma = (3\lambda + 2\mu + k)\alpha_t$, and α_t being the coefficient of linear thermal expansion. T_0 is a reference temperature chosen such that $|(T - T_0)/T_0| \ll 1$.

In three dimensions, the isotropic micropolar elastic solid requires six elastic constants λ, μ, α, β, γ and k for its description. The engineering material constants for micropolar elasticity are as follows:

$$\begin{aligned} &\text{i) Characteristic Length:} && L^2 = \frac{\delta}{4\mu + 2k}, \\ &\text{ii) Coupling factor:} && N^2 = \frac{k}{2\mu + 2k}, \\ &\text{iii) Poisson's ratio:} && \nu^* = \frac{\lambda}{2\mu + 2\lambda + k}, \\ &\text{iv) Young's Modulus:} && E = \frac{(2\mu + 3\lambda + k)(2\mu + k)}{2\mu + 2\lambda + k}. \end{aligned} \quad (2.3)$$

Classical elasticity corresponds to the special case of micropolar elasticity in which $L \to 0$.

The generalized equation of heat conduction has the form

$$KT_{,ii} = \rho c_E \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T + T_0 \gamma \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2} \right) u_{i,i}, \quad (2.4)$$

where K is the coefficient of the thermal conductivity of the medium, c_E is the specific heat at constant strain, τ_0 is an other relaxation time and n_0 is a non-dimensional constant. For the generalized theory of thermoelasticity with two relaxation times (Green Lindsay theory), the thermal relaxations τ_0 and ν satisfy the inequality $\nu > \tau_0$ and $n_0 = 0$, whereas for coupled theory $\nu = \tau_0 = 0$. The Lord Shulman model can be deduced from Green Lindsay theory by taking $\nu = 0$ and $n_0 = 1$.

Our goal is to investigate the effect of temperature dependency of modulus of elasticity keeping the other elastic and thermal parameters constants, therefore we assume

$$E = E_0 f(T), \quad \lambda = E_0\lambda_0 f(T), \quad \mu = E_0\mu_0 f(T), \quad k = E_0 k_0 f(T),$$
$$\alpha = E_0\alpha_0 f(T), \quad \beta = E_0\beta_0 f(T), \quad \delta = E_0\delta_0 f(T), \quad \gamma = E_0\gamma_0 f(T),$$

where E_0, ν^* and α_t are considered constants, $f(T)$ is a given non-dimensional function of temperature, in case of temperature-independent modulus of elasticity $f(T) \equiv 1$, and $E = E_0$.

The system of governing equations of a micropolar thermoelastic solid, without body forces and body couples, consists of [5-7]:

$$\sigma_{ji,j} = \rho\frac{\partial^2 u_i}{\partial t^2}, \tag{2.5}$$

$$\varepsilon_{ijr}\sigma_{jr} + m_{ji,j} = j\rho\frac{\partial^2 \phi_i}{\partial t^2}, \tag{2.6}$$

where ρ is the density and j is the microinertia. Substituting Eqs. (2.1)–(2.2) into equations of motion (2.5) and (2.6) we get

$$\begin{aligned}
\rho\frac{\partial^2 u_i}{\partial t^2} &= E_0 f\left[(\lambda_0+\mu_0)u_{j,ji} + (\mu_0+k_0)u_{i,jj} + k_0\varepsilon_{ijr}\phi_{r,j} - \gamma_0\left(1+\nu\frac{\partial}{\partial t}\right)T_{,i}\right]\\
+E_0 f_{,j} &\left[\lambda_0 u_{r,r}\delta_{ij} + \mu_0(u_{i,j}+u_{j,i}) + k_0(u_{i,j}+\varepsilon_{ijr}\phi_r) - \gamma_0\left(T - T_0 + \nu\frac{\partial T}{\partial t}\right)\delta_{ij}\right]\\
j\rho\frac{\partial^2 \phi_i}{\partial t^2} &= E_0 f[(\alpha_0+\beta_0)\phi_{j,ji}\delta_{ij} + \delta_0\phi_{i,jj} + k_0\varepsilon_{ijr}u_{r,j} - 2k_0\phi_i]\\
&\quad + E_0 f_{,j}[\alpha_0\phi_{r,r}\delta_{ij} + \delta_0\phi_{i,j} + \beta_0\phi_{j,i}].
\end{aligned} \tag{2.7}$$

Now we introduce the following non-dimensional variables:

$$\begin{aligned}
x_i^* &= \frac{\eta_0}{c_0}x_i; \quad u_i^* = \frac{\rho\eta_0 c_0}{\gamma_0 T_0}u_i; \quad t^* = \eta_0 t; \quad \tau_0^* = \eta_0\tau_0; \; \nu^* = \eta_0\nu;\\
\theta &= E_0\frac{T-T_0}{T_0}; \quad \sigma_{ij}^* = \frac{\sigma_{ij}}{\gamma_0 T_0}; \quad m_{ij}^* = \frac{\eta_0}{c_0\gamma_0 T_0}m_{ij}; \; \phi_i^* = \frac{\rho c_0^2}{\gamma_0 T_0}\phi_i
\end{aligned}$$

where $\eta_0 = \rho c_E c_0^2/K$ and $c_0^2\rho = \mu_0 E_0$. Eqs. (2.1), (2.2), (2.4) and (2.7) take the following form (dropping the asterisks for convenience):

$$\begin{aligned}
\mu_0\sigma_{ij} &= \left[\lambda_0 u_{r,r}\delta_{ij} + \mu_0(u_{i,j}+u_{j,i}) + k_0(u_{j,i} - \varepsilon_{ijr}\phi_r)\right.\\
&\qquad \left. - \mu_0\left(1+\nu\frac{\partial}{\partial t}\right)\theta\delta_{ij}\right]f(\theta)\\
\mu_0 m_{ij} &= [\alpha_0\phi_{r,r}\delta_{ij} + \beta_0\phi_{i,j} + \delta_0\phi_{j,i}]f(\theta)\\
\mu_0\frac{\partial^2 u_i}{\partial t^2} &= \left[(\lambda_0+\mu_0)u_{j,ji} + (k_0+\mu_0)u_{i,jj} + k_0\varepsilon_{ijr}\phi_{r,j}\right.
\end{aligned}$$

$$
\begin{aligned}
&- \mu_0 \left(1 + \nu \frac{\partial}{\partial t}\right) \theta_{,i} \Big] f(\theta) \\
&+ \left[\lambda_0 u_{r,r} - \mu_0 \left(1 + \nu \frac{\partial}{\partial t}\right) \theta\right] f_{,i} \\
&+ \left[\mu_0 (u_{i,j} + u_{j,i}) + k_0 (u_{i,j} + \varepsilon_{ijr} \phi_r)\right] f_{,j} \\
\mu_0 j \frac{\partial^2 \phi_i}{\partial t^2} &= \left[(\alpha_0 + \beta_0) \phi_{j,ji} + \delta_0 \phi_{i,jj} + \frac{k_0 c_0^2}{\eta_0^2} \varepsilon_{ijr} u_{r,j} - 2 \frac{k_0 c_0^2}{\eta_0^2} \phi_i\right] f(\theta) \\
&+ \alpha_0 \phi_{r,r} f_{,i}(\theta) + [\delta_0 \phi_{i,j} + \beta_0 \phi_{j,i}] f_{,j}, \\
\nabla^2 \theta &= \left(\frac{\partial}{\partial t} - \tau_0 \frac{\partial^2}{\partial t^2}\right) \theta - \varepsilon f(\theta) \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}\right) \nabla^2 u_{i,i},
\end{aligned}
$$

where $\varepsilon = \frac{\gamma_0^2 E_0^2 T_0}{K \rho \eta_0}$. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface $z = 0$ with z axis vertical into the medium is introduced. If we restrict our analysis parallel to xz-plane with displacement vector $\mathbf{u} = (u, 0, w)$ and microrotation vector $\phi = (0, \phi_2, 0)$, the set of the system of equations (2.8) reduces to

$$
\begin{aligned}
\mu_0 \frac{\partial^2 u}{\partial t^2} &= \left[(\lambda_0 + \mu_0) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right) + (\lambda_0 + k_0) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right)\right. \\
&- \left. k_0 \frac{\partial \phi_2}{\partial z} - \mu_0 \left(1 + \nu \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial x}\right] f(\theta) \\
&+ \left[(\lambda_0 + 2\mu_0 + k_0) \frac{\partial u}{\partial x} + \lambda_0 \frac{\partial w}{\partial z} - \mu_0 (1 + \nu \frac{\partial}{\partial t}) \theta\right] \frac{\partial f}{\partial x} \\
&+ \left[\mu_0 \frac{\partial w}{\partial x} + (\mu_0 + k_0) \frac{\partial u}{\partial z} - k_0 \phi_2\right] \frac{\partial f}{\partial z} \\
\mu_0 \frac{\partial^2 w}{\partial t^2} &= \left[(\lambda_0 + \mu_0) \left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 u}{\partial x \partial z}\right) + (\lambda_0 + k_0) \left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2}\right)\right. \\
&+ \left. k_0 \frac{\partial \phi_2}{\partial x} - \mu_0 \left(1 + \nu \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial z}\right] f(\theta) \\
&+ \left[(\lambda_0 + 2\mu_0 + k_0) \frac{\partial w}{\partial z} + \lambda_0 \frac{\partial u}{\partial x} - \mu_0 \left(1 + \nu \frac{\partial}{\partial t}\right) \theta\right] \frac{\partial f}{\partial z} \\
&+ \left[\mu_0 \frac{\partial u}{\partial z} + (\mu_0 + k_0) \frac{\partial w}{\partial x} + k_0 \phi_2\right] \frac{\partial f}{\partial x} \\
\mu_0 j \frac{\partial^2 \phi_2}{\partial t^2} &= \left[\delta_0 (\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial z^2}) + \frac{k_0 c_0^2}{\eta_0^2} (\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) - 2 \frac{k_0 c_0^2}{\eta_0^2} \phi_2\right] f(\theta) \\
&+ \delta_0 \left(\frac{\partial \phi_2}{\partial x} \frac{\partial f}{\partial x} + \frac{\partial \phi_2}{\partial z} \frac{\partial f}{\partial z}\right), \\
\nabla^2 \theta &= \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) \theta + \varepsilon f(\theta) \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}\right) \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right).
\end{aligned}
$$

In generalized thermoelasticity, as well as in the coupled theory only the infinitesimal temperature deviations from reference temperature are considered. Therefore $f(\theta)$ can be taken in the form $f(\theta) = 1 - \xi^* T_0$, where ξ^* is an empirical material constant $(1/K)$. The last system of equations is linearized and reduces to the linear system:

$$\xi \frac{\partial^2 u}{\partial t^2} = \frac{1}{\varepsilon_1}\nabla^2 u + \frac{1}{\varepsilon_2}\frac{\partial}{\partial z}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) - \frac{\varepsilon_3}{\varepsilon_2}\frac{\partial \phi_2}{\partial z} - \left(1 + \nu\frac{\partial}{\partial t}\right)\frac{\partial \theta}{\partial x}, \quad (2.8)$$

$$\xi \frac{\partial^2 w}{\partial t^2} = \frac{1}{\varepsilon_1}\nabla^2 w + \frac{1}{\varepsilon_2}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \frac{\varepsilon_3}{\varepsilon_2}\frac{\partial \phi_2}{\partial x} - \left(1 + \nu\frac{\partial}{\partial t}\right)\frac{\partial \theta}{\partial z}, \quad (2.9)$$

$$\xi\varepsilon_5 \frac{\partial^2 \phi_2}{\partial t^2} = \nabla^2 \phi_2 - 2\varepsilon_4\phi_2 + \varepsilon_4\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right), \quad (2.10)$$

$$\nabla^2\theta = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\theta + \varepsilon_0\left(\frac{\partial}{\partial t} + n_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right),$$

where

$$\xi = \frac{1}{1-\xi^* T_0}, \quad \varepsilon_0 = \frac{\varepsilon}{\xi}, \quad \varepsilon_1 = \frac{\mu_0}{\lambda_0 + 2\mu_0 + k_0}, \quad \varepsilon_2 = \frac{\mu_0}{\mu_0 + k_0},$$
$$\varepsilon_3 = \frac{k_0}{\mu_0 + k_0}, \quad \varepsilon_4 = \frac{k_0 c_0^2}{\delta_0 \eta_0^2}, \quad \varepsilon_5 = \frac{j\mu_0}{\delta_0}.$$

3. Formulation and solution in the transformed domain.

Introducing potentials $\Phi(x, z, t)$ and $\Psi(x, z, t)$ which are related to displacement components, we get

$$u = \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial z}, \qquad w = \frac{\partial \Phi}{\partial z} - \frac{\partial \Psi}{\partial x} \quad (3.1)$$

Applying the Laplace transform $\bar{f}(x, z, s)$ of a function $f(x, z, t)$ defined by the relation:

$$\bar{f}(x, z, s) = \int_0^\infty f(x, z, t)e^{-st}dt, \quad (3.2)$$

and then the exponential Fourier transform with respect to the variable z (denoted by an asterisk) and defined by the relation

$$\bar{f}^*(x, q, s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\mathbf{i}qz}\bar{f}(x, z, s)dz, \quad (3.3)$$

on both sides of Eq. (2.9), we get:

$$(D^2 - q^2 - \varepsilon_1\xi s^2)\bar{\Phi}^* - \varepsilon_1(1 + \nu s)\bar{\theta}^* = 0, \quad (3.4)$$

$$(D^2 - q^2 - \varepsilon_2\xi s^2)\bar{\Psi}^* - \varepsilon_3\bar{\phi}_2^* = 0, \quad (3.5)$$

$$(D^2 - q^2 - 2\varepsilon_4 - \varepsilon_5\xi s^2)\bar{\phi}_2^* + \varepsilon_4(D^2 - q^2)\bar{\Psi}^* = 0, \quad (3.6)$$

$$(D^2 - q^2 - s - \tau_0 s^2)\bar{\theta}^* - \varepsilon_0 s(1 + n_0\tau_0 s)(D^2 - q^2)\bar{\Phi}^* = 0, \quad (3.7)$$

where $D = \partial/\partial x$. Substituting Eq. (3.7) into Eq. (3.4) and Eq. (3.6) into Eq. (3.5), we get:

$$(D^4 - AD^2 + B)(\bar{\Phi}^*, \bar{\theta}^*) = 0, \tag{3.8}$$

$$(D^4 - CD^2 + E)(\bar{\Psi}^*, \bar{\phi}_2^*) = 0, \tag{3.9}$$

where

$$\begin{gathered}A = 2q^2 + s(1+\tau_0 s) + \varepsilon_1 \xi s^2 + \varepsilon_0 \varepsilon_1 s(1+\nu s)(1+n_0\tau_0 s),\\ B = (q^2 + s + \tau_0 s^2)(q^2 + \varepsilon_1 \xi s^2) + \varepsilon_0\varepsilon_1 q^2 s(1+\nu s)(1+n_0\tau_0 s),\\ C = 2q^2 + (\varepsilon_2 + \varepsilon_5)\xi s^2 + \varepsilon_4(2-\varepsilon_3),\\ E = (q^2 + \varepsilon_2\xi s^2)(q^2 + 2\varepsilon_4 + \varepsilon_5 \xi s^2) - \varepsilon_3\varepsilon_4 q^2.\end{gathered} \tag{3.10}$$

The solution of Eq. (3.8), which is bounded for $x > 0$, is given by

$$\bar{\Phi}^* = \sum_{n=1}^{2} A_n e^{-k_n x}, \quad \bar{\theta}^* = \sum_{n=1}^{2} \frac{k_n^2 - q^2 - \varepsilon_1 \xi s^2}{\varepsilon_1(1+\nu s)} A_n e^{-k_n x}, \tag{3.11}$$

where k_1^2 and k_2^2 are the roots of the characteristic equation

$$k^4 - Ak^2 + B = 0 \tag{3.12}$$

In a similar manner, the solution of Eq. (3.9), which is bounded for $x > 0$, is given by

$$\bar{\Psi}^* = \sum_{n=1}^{2} A_{n+2} e^{-k_{n+2} x}, \quad \bar{\phi}_2^* = \frac{1}{\varepsilon_3} \sum_{n=1}^{2} (k_{n+2}^2 - q^2 - \varepsilon_2 \xi s^2 A_{n+2} e^{-k_{n+2} x}) \tag{3.13}$$

where k_3^2 and k_4^2 are the roots of the characteristic equation

$$k^4 - Ck^2 + E = 0 \tag{3.14}$$

4. Application

We consider a thermal boundary condition that the surface of the half-space is known

$$\theta(0, z, t) = g(z, t) = \theta_0 H(t) H(b - |z|) \tag{4.1}$$

where $H(.)$ is the Heaviside unit step function and θ_0 is a constant. This means that heat is applied on the surface of the half-space on a narrow band of width $2b$ surrounding the $z-axis$ to keep it at temperature θ_0, while the rest of the surface is kept at zero temperature.

The surface of the half-space is traction free

$$\sigma_{zz}(0, z, t) = \sigma_{zx}(0, z, t) = m_{zy}(0, z, t) = 0. \tag{4.2}$$

Making use of Eqs. (3.11) and (3.13) in the boundary conditions (4.1)–(4.2) after applying the transforms defined by Eqs. (3.2)–(3.3), we obtain the expressions for displacement components, and stresses fields as

$$\begin{aligned}
\bar{u}^* &= \sum_{n=1}^{2}\Big[-k_nA_ne^{-k_nx}+\mathbf{i}qA_{n+2}e^{-k_{n+2}x}\Big],\\
\bar{w}^* &= \sum_{n=1}^{2}\Big[\mathbf{i}qA_ne^{-k_nx}+k_{n+2}A_{n+2}e^{-k_{n+2}x}\Big],\\
\xi\bar{m}^*_{zy} &= \frac{\mathbf{i}q\varepsilon_3\varepsilon_4}{\varepsilon_2}\sum_{n=1}^{2}\Big[k^2_{n+2}-q^2-\varepsilon_2\xi s^2\Big]A_{n+2}e^{-k_{n+2}x},\\
\xi\bar{\sigma}^*_{zz} &= \sum_{n=1}^{2}\Big[(\xi s^2-\varepsilon_7k_n^2)A_ne^{-k_nx}+\mathbf{i}q\varepsilon_7k_{n+2}A_{n+2}e^{-k_{n+2}x}\Big],\\
\xi\sigma^*_{zx} &= \sum_{n=1}^{2}\Big[-\mathbf{i}q\varepsilon_7k_nA_ne^{-k_nx}+(\xi s^2-\varepsilon_7k^2_{n+2})A_{n+2}e^{-k_{n+2}x}\Big],
\end{aligned}\tag{4.3}$$

where

$$\begin{aligned}
A_1 &= (k_4-k_3)\Big[(\xi s^2-\varepsilon_7k_2^2)(\xi s^2-\varepsilon_7(q^2+\varepsilon_2\xi s^2))(k_4+k_3)\\
&\quad -q^2\varepsilon_7^2k_2(k_3k_4+q^2+\varepsilon_2\xi s^2)\Big]/\Delta,\\
A_2 &= -(k_4-k_3)\Big[(\xi s^2-\varepsilon_7k_1^2)(\xi s^2-\varepsilon_7(q^2+\varepsilon_2\xi s^2))(k_4+k_3)\\
&\quad -q^2\varepsilon_7^2k_1(k_3k_4+q^2+\varepsilon_2\xi s^2)\Big]/\Delta,\\
A_3 &= \mathbf{i}q\varepsilon_7(k_4^2-q^2-\varepsilon_2\xi s^2)(k_1-k_2)(\xi s^2+\varepsilon_7k_2k_1)/\Delta,\\
A_4 &= -\mathbf{i}q\varepsilon_7(k_3^2-q^2-\varepsilon_2\xi s^2)(k_1-k_2)(\xi s^2+\varepsilon_7k_2k_1)/\Delta,\\
\Delta &= \frac{(k_4-k_3)(k_1-k_2)}{\varepsilon_1(1+\nu s)\bar{g}^*(q,s)}\Big[(\xi s^2\\
&\quad -\varepsilon_7(q^2+\varepsilon_1\xi s^2))(\xi s^2-\varepsilon_7(q^2+\varepsilon_2\xi s^2))(k_4+k_3)(k_1+k_2)\\
&\quad -q^2\varepsilon_7^2(k_3k_4+q^2+\varepsilon_2\xi s^2)(k_1k_2+q^2+\varepsilon_1\xi s^2)\Big].
\end{aligned}$$

Particular case: If we neglect the microrotational effect by putting $\alpha_0=\beta_0=\delta_0=k_0=j=0$ in the system of equations (2.9), the expressions for displacement and stresses fields in a generalized thermoelastic medium are given by the system of equations (4.3) with A_i, $i=1,2,3,4$, defined as

$$\begin{aligned}
A_1 &= \Big[(\xi s^2-2k_2^2)(\xi s^2-2k_3^2)-4q^2k_2k_3\Big]/\Delta,\\
A_2 &= -\Big[(\xi s^2-2k_1^2)(\xi s^2-2k_3^2)-4q^2k_1k_3\Big]/\Delta,\\
A_3 &= 2\mathbf{i}q(k_1-k_2)(\xi s^2+2k_1k_2)/\Delta,\quad A_4=0,
\end{aligned}$$

$$\Delta = \frac{(k_1 - k_2)}{\varepsilon_1(1+\nu s)\bar{g}^*(q,s)}\Big[(k_1+k_2)(\xi s^2 - 2k_3^2)\Big(\xi s^2 - 2(q^2+\varepsilon_1\xi s^2)\Big)$$
$$-4q^2 k_3(k_1k_2 + q^2 + \varepsilon_1 \xi s^2)\Big].$$

5. Inversion of the double transforms

We shall now outline the numerical method used to find the solution in the physical domain.

Let $\bar{f}^*(x,q,s)$ be the double Fourier-Laplace transform of a function $f(x,z,t)$. First, we invert the Fourier transform using the inversion formula to obtain a Laplace transform expression $\bar{f}(x,z,s)$ of the form

$$\bar{f}(x,z,s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \bar{f}^*(x,q,s)e^{\mathbf{i}qz}dq$$
$$= \sqrt{\frac{2}{\pi}}\int_0^{+\infty}\Big(\cos(qz)\bar{f}_e^*(x,q,s) + \mathbf{i}\sin(qz)\bar{f}_o^*(x,q,s)\Big)dq,$$

where $\bar{f}_e^*$ and $\bar{f}_o^*$ denote the even and odd parts of the function $\bar{f}^*(q,z,s)$, respectively.

The inversion formula for the Laplace transforms can be written as

$$f(x,z,t) = \frac{1}{2\pi\mathbf{i}}\int_{c-\mathbf{i}\infty}^{c+\mathbf{i}\infty} e^{st}\bar{f}(x,z,s)\,ds,$$

where c is an arbitrary constant greater than all real parts of the singularities of $\bar{f}(x,z,s)$.

Taking $s = c + \mathbf{i}y$, we get

$$f(x,z,t) = \frac{e^{ct}}{2\pi}\int_{-\infty}^{\infty} e^{\mathbf{i}yt}\,\bar{f}(x,z,c+\mathbf{i}y)\,dy \quad .$$

Expanding the function $h(x,z,t) = e^{-ct}f(x,z,t)$ in a Fourier series in the interval $[0,2T]$, we obtain the approximate formula Honig and Hirdes [26]:

$$f(x,z,t) = f_\infty(x,z,t) + E_D$$

where

$$f_\infty(x,z,t) = \frac{1}{2}c_0 + \sum_{k=1}^{\infty} c_k, \quad \text{for} \quad 0 \le t \le 2T, \tag{5.1}$$

and

$$c_k = \frac{e^{ct}}{T}\Re e[e^{\mathbf{i}k\pi t/T}\bar{f}(x,z,c+\mathbf{i}k\pi/T)]$$

E_D, the discretization error, can be made arbitrarily small by choosing c large enough [26]. Since the infinite series in Eq. (5.1) can only be summed up to a finite number N of terms, the approximate value of $f(x,z,t)$ becomes

$$f_N(x,z,t) = \frac{1}{2}c_0 + \sum_{k=1}^{N} c_k, \quad \text{for} \quad 0 \le t \le 2T. \tag{5.2}$$

Using the above formula to evaluate $f(x,z,t)$, we introduce a truncation error E_T that must be added to the discretization error to produce the total approximation error.

Two methods are used to reduce the total error. First, the Korrektur method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and hence to accelerate convergence. The Korrektur method uses the following formula to evaluate the function $f(x,z,t)$:

$$f(x,z,t) = f_\infty(x,z,t) - e^{-2cT} f_\infty(x,z,2T+t) + E'_D,$$

where the discretization errors $|E'_D| << |E_D|$. Thus, the approximate value of $f(x,z,t)$ becomes

$$f_{N_k}(x,z,t) = f_N(x,z,t) - e^{-2cT} f_{N'}(x,z,2T+t), \tag{5.3}$$

where N' is an integer less N.

We shall now describe the ε- algorithm that is used to accelerate the convergence of the series in Eq. (5.1). Let $N = 2q+1$ where q is a natural number, and let

$$s_m = \sum_{k=1}^{m} c_k$$

be the sequence of partial sums of Eq. (5.2), we define the ε-sequence by

$$\varepsilon_{0,m} = 0, \quad \varepsilon_{1,m} = s_m, \; m = 1,2,3,\cdots$$

and

$$\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n,m}}, \; n = m = 1,2,3,\cdots$$

It can be shown that [26] the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \cdots, \varepsilon_{N,1}$ converges to $f(x,z,t) + E_D - \frac{1}{2}c_0$ faster than the sequence of partial sums s_m, $m = 1,2,3,\cdots$ The actual procedure used to invert the Laplace transforms consists of using Eq.(5.2) together with the ε- algorithm. The values of c and T are chosen according the criteria outlined in [26].

The last step is to calculate the integral in Eq.(5.3). This method for evaluating this integral is described by Press et al. in [27], which involves the use of Romberg's integration adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

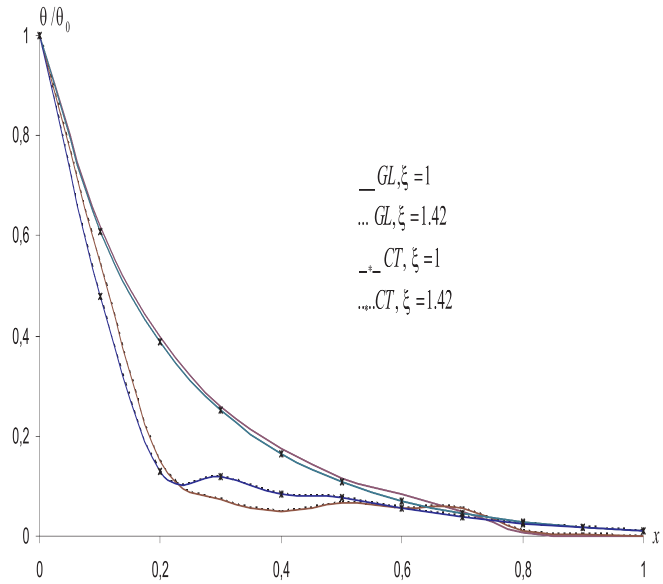

Figure 1. a) Effect of reference temperature T_0 on temperature in GMTE medium.

6. Discussion

For numerical computations, following Eringen [28], the values of relevant parameters for the case of magnesium crystal are taken:

Table 1. Values of the constants

$\lambda_0 = 9.4 \times 10^{11}$ dyne/cm^2	$\mu_0 = 4.0 \times 10^{11}$ dyne/cm^2	$k_0 = 1.0 \times 10^{11}$ dyne/cm^2
$\rho = 1.74$ gm/cm^3	$\delta_0 = 0.779 \times 10^{-4}$ dyne	$j = 0.2 \times 10^{-15}$ cm^2
$c_E = 0.23$ cal/gm °C	$\gamma_0 = 1.78 \times 10^{-5}$ /°C	$K = 0.6 \times 10^{-2}$ cal/cm s °C
$T_0 = 296$ K	$\xi^* = 0.001$ /K	$\varepsilon = 10.588$

The computations were carried out for a value of time $t = 0.1$ and on the surface of plane $z = 0$. The numerical values for the temperature, the horizontal displacement components u, the normal force stress component σ_{zz} and the couple stress m_{zy} on the surface of plane $z = 0$ are shown in figures 1a–3a and Fig. 4 in case of generalized micropolar thermoelasticity medium (GMTE) and in figures 1b–3b in case of generalized thermoelasticity medium (GTE). In these figures the dotted lines either without center symbol or with center symbol represent the solution obtained when the modulus of elasticity is taken as a linear function of reference temperature ($\xi = 1.42$), while the solid lines with or without

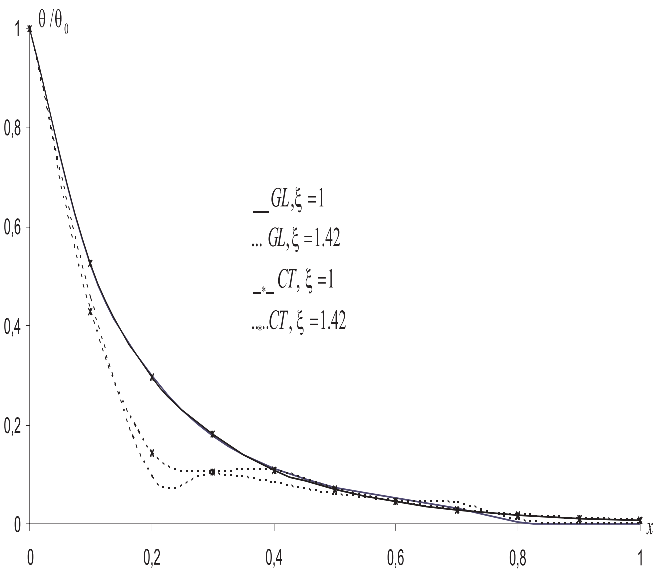

Figure 1. b) Effect of reference temperature T_0 on temperature in GTE medium.

center symbol represent the solution obtained in case of temperature independent modulus of elasticity ($\xi = 1$). For the coupled theory (CT theory), the relaxation times are: $\tau_0 = \nu = 0$, while for the generalized theory with two relaxation times (GL theory): $\tau_0 = 0.02,\ \nu = 0.03$.

Some comparisons of both thermal theories and of the thermal (GTE) and micropolar (GMTE) theories are made.

Behavior of temperature for both thermal theories in GMTE and GTE media, as shown in Fig. 1*a* and Fig. 1*b* respectively, is similar. The temperature starts with its maximum value at the origin (due to the presence of the thermal shock) and decreases until attaining zero beyond the thermal wavefront for the generalized theory, whereas it is continuous everywhere else for the coupled theory. It should be noted that, in all cases the values of θ/θ_0 are less for $\xi = 1.42$ compared to those for $\xi = 1$, and they are large for GL theory in comparison with those for CT theory.

Figures 2a,b show the variation of displacement u/θ_0 in GMTE and GTE media, respectively. Initially, u/θ_0 starts with a negative value, and then increases until attaining zero beyond the thermal wavefront. Due to microrotation effect, values of u/θ_0 increase to zero following oscillatory pattern in GMTE medium, whereas they go towards zero in GTE medium. We note that values of u/θ_0 are less when $\xi = 1.42$ compared to those when $\xi = 1$ for GMTE medium, whereas the reverse happens in GTE media. It is also observed for both media, that values

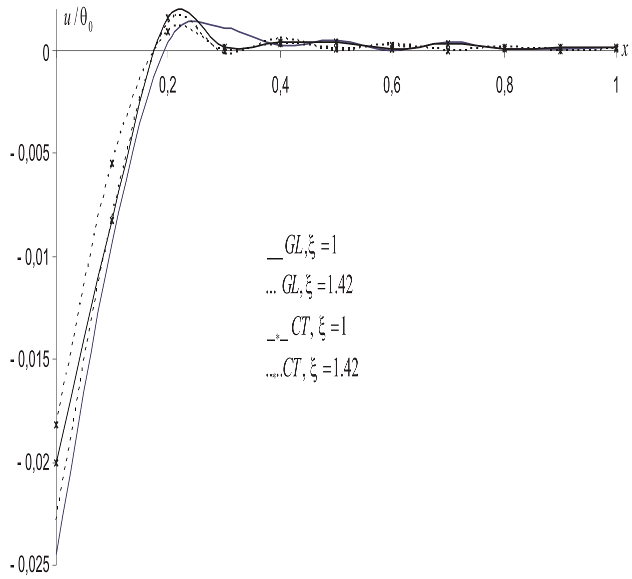

Figure 2. a) Effect of reference temperature T_0 on displacement in GMTE medium.

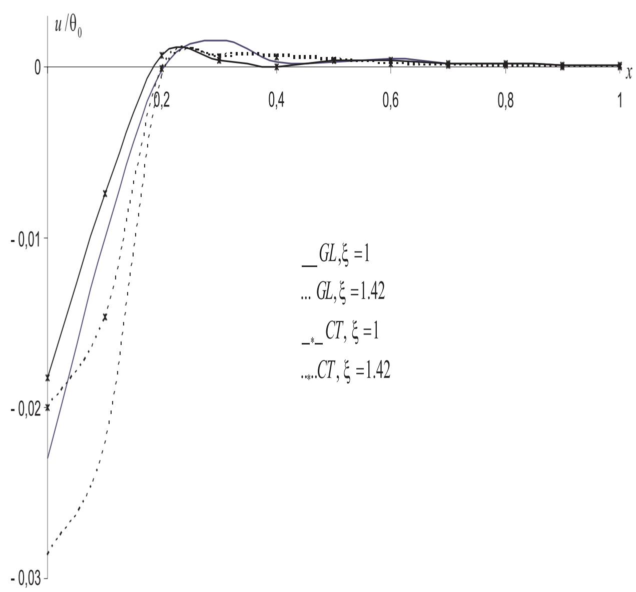

Figure 2. b) Effect of reference temperature T_0 on displacement in GTE medium.

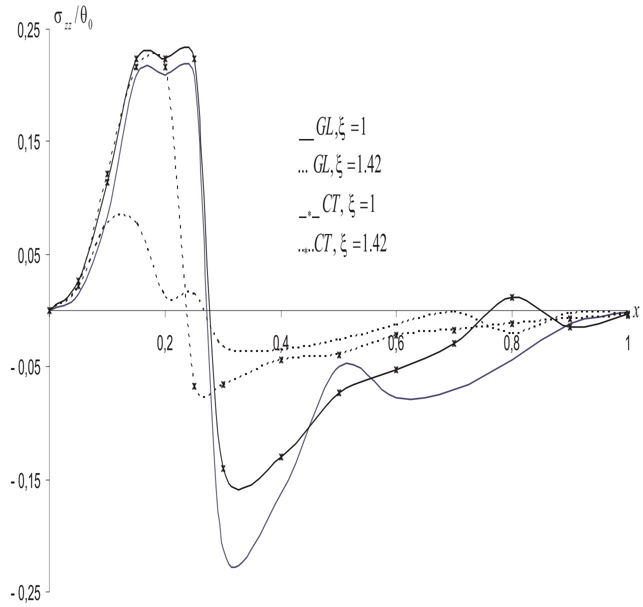

Figure 3. a) Effect of reference temperature T_0 on force stress in GMTE medium.

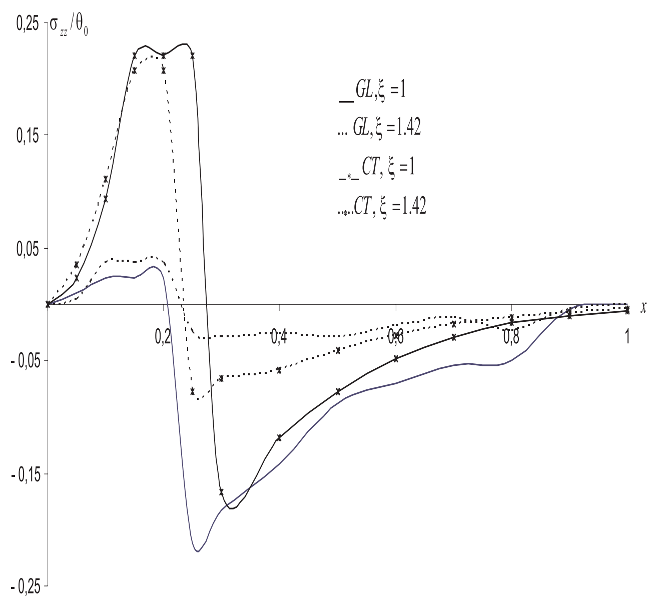

Figure 3. b) Effect of reference temperature T_0 on force stress in GTE medium.

of u/θ_0 for GL theory are large in comparison with those for CT theory.

Figures 3a,b show the variation of normal force stress σ_{zz}/θ_0 in GMTE and GTE media, respectively. In both media, normal force stress σ_{zz}/θ_0 starts with a zero value at the origin (according to the boundary condition given by Eq.(4.2)), and thereafter experiences a finite jump. Due to reference temperature effect, the discontinuity existing for $\xi = 1$ is eliminated for $\xi = 1.42$ in both media and for generalized theory only, while for coupled theory, the existing discontinuity persists to exist.

The variation of couple stress m_{zy}/θ_0 with x has been shown in Fig. 4 in GMTE medium. For both coupled and generalized theories, m_{zy}/θ_0 starts with a zero value at the origin (see Eq.(4.2)), and then oscillates in the range $0 < x \leq 0.3$ as x increases, whereas for $x > 0.3$ the values are very small compared to those in the first range. It is also observed that values of m_{zy}/θ_0 for GL theory are large in comparison with those for CT theory, and they are less for $\xi = 1.42$ compared to those for $\xi = 1$,

7. Concluding remarks

Based on the analysis presented here and the values of the parameters used, we state the following conclusions:

1. In all these figures, it is clear that the considered functions for the generalized theory are localized in a finite region of space surrounding the heating source and are identically zero outside this region. The edge of this region is the thermal wavefront which moves with a finite speed. It is clear from figures that the location of discontinuities changes with values of ξ, whereas the edge of thermal wavefront does not change. This edge is the same in both media, and is determined only by the values of time t and relaxation time τ_0. This is not the case for the coupled theory where an infinite speed of propagation is inherent and hence all the considered functions have non-zero (although may be very small) value for any point in the medium. In addition, the values of solutions for GL theory are large in comparison with those for CT theory. Under GL theory, the relaxation times are large ($\nu > \tau_0 > 0$), therefore the time available for the exchange of thermal energy with the domain is large and then the values of solutions are higher. All these remarks indicate that the generalized theory mechanism is completely different from the classic Fourier's in essence, and more realistic in dealing with practical problems involving very large heat fluxes and or short time intervals.

2. The normal force stress component σ_{zz} has a finite jump in both media for both theories in case of temperature independent modulus of elasticity ($\xi = 1$). The same situation arises in [29, 30, 31]. This jump which violates the requirement of continuity, is not physically realistic [32]. Thus, the dependence of the modulus of elasticity on reference temperature has a significant effect on the thermal and mechanical interaction by eliminating the existing discontinuities under generalized

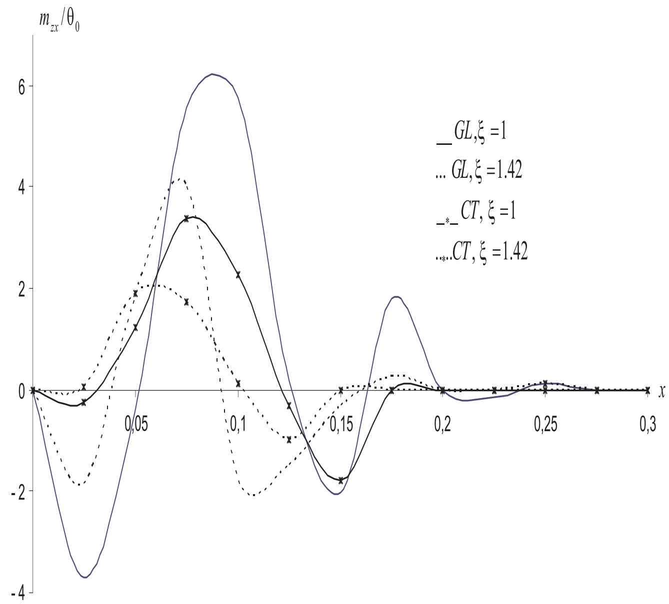

Figure 4. Effect of reference temperature T_0 on couple stress in GMTE medium.

theory only. Another important effect, is the decrease of the magnitude solution in GMTE medium, whereas this effect is not evident in GTE medium. This is due to the fact that the micropolar theory is more adequate to describe the real phenomena than the classical theory of elasticity, and results obtained with temperature dependency of modulus of elasticity are physically more acceptable than those obtained without.

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