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Simultaneous Inversion of Hypocentral Parameters and Structure Velocity of the Central Region of Madagascar as a Premise for the Mitigation of Seismic Hazard in Antananarivo

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Abstract—A layered velocity is obtained using arrival-time data of *P* and *S* waves from local earthquakes for the Central Region of Madagascar. A damped least-squares method is applied in the inversion of the data. The data used are 770 *P*-wave arrival times for 154 events which have epicenters in the region inside the Malagasy network operated by the Institut et Observatoire de Géophysique d'Antananarivo (IOGA). These data are jointly used in the inversion for the earthquake hypocenters and *P*- and *S*-wave velocity models. *S* waves are not used in the first step of the inversion, since their use leads to large location errors. If the error on the phase reading for the *P* wave is about 0.1 s, for the *S* wave it is considerably bigger. The reference average model used here is a variant of the model given by RAKOTONDRAINIBE (1977).

Key words: Seismic velocity model, hypocenter location, Itasy, Ankaratra, Madagascar.

1. Introduction

The island of Madagascar is mainly formed by cratonic basement. Sedimentary basins with Karoo to quaternary deposits are only located along the western coast (Morondava and Mahajanga) (BESAIRIE, 1971; FURON, 1968).

The basement consists of folded, metamorphosed, migmatized and reworked Precambrian rocks (BESAIRIE, 1971, 1973a; HOTTIN, 1976; CAEN *et al.*, 1984). However, two components have been distinguished (HOTTIN, 1976) and are approximately separated by the Bongolava-Ranotsara N20°W-trending line (see Fig. 1) which corresponds to an important tectonic trend in Madagascar. Metamorphosed and migmatized Archean rocks, eventually reworked and dated about 3000–2600 Ma are found to the North of the Bongolava-Ranotsara tectonic line (CAEN-VACHETTE, 1979; CAEN *et al.*, 1984). Localized near the east coast at Masoara and Antongil and in the central part of the island, cores of 'Katarchean'

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terrains older then 3000 Ma are found. Folded and metamorphosed Proterozoic rocks (2000–1800 Ma) are the principal components of the southwestern part of the basement. They consist of the 'Série Schisto-Quartzo-Calcaire' (BESAIRIE, 1971) and the Amborompotsy and Ikalamavony formations. And finally, the cratonic character of the Malagasy basement has been definitively linked with the tectono-thermal Panafrican event of about 550 Ma (CAEN-VACHETTE, 1979).

Volcanism has been active in Madagascar. It is characterized by Cretaceous, Tertiary and Quaternary episodes (BESAIRIE, 1973b). Cretaceous volcanism is characterized by dominant tholeitic basalts which may be related to the Gondwanan dislocation history. Such volcanism is concentrated around the perimeter of the island. On the western border, it corresponds to basaltics lavas which are typical of passive continental margin volcanism (NICOLET, 1982). Tertiary (Paleocene) and Quaternary volcanism is mainly concentrated in the axial zone. In the central region of Madagascar, Antananarivo, these volcanic rocks constitute the Ankaratra and Itasy massifs.

The main tectonic features and faults inherited from tectono-metamorphic history are shown in Figure 2. Two main tectonic trends occur N20°W (Bongolava-Ranotsara, Fig. 2) and N20°E (east coast trend) which have evidently shaped the coasts of the island. The N20°W may be related to the separation of Madagascar from Africa with the subsequent opening of the Mozambic channel and the development of the Mahajanga and Morondava basins. The N20°E trend (east coast) corresponds to the northward motion of separation of India from Madagascar during the Cretaceous times, which resulted in a very straight and steep coast on the eastern margin of Madagascar (SEGOUFIN and PATRIAT, 1981; SCHLICH, 1982).

In addition to these two main tectonic trends, a N8°W trend is characterized by alignment of magmatic rocks of various age. This trend is considered to be a main axis of magmatic reactivation (KUTINA, 1972) which could correspond to a deep fracture zone or a zone of crustal weakness (ANKARATRA and Itasy regions). Finally, the N10°E trend locally observed in the 'Hautes Terres' (Andaingo, Alaotra; Fig. 3) is associated with limited Neogene grabens such as the Alaotra graben (BESAIRIE, 1971, 1973a) to the northeast of Antananarivo.

The most seismically active areas in Madagascar are the Mahajanga Karoo basin and, in the Precambrian basement, the Alaotra, Ankaratra, Itasy and Famoizankova regions (Fig. 3). The central region of Madagascar, studied in this paper, is the Precambrian basement near the seismic stations of the Malagasy network (Fig. 4): Ankaratra and Itasy regions (between latitude 18°.4S, 19°.9S and longitude 46°.1E, 47°.8E). The seismicity of this area seems to be concentrated along several zones following the N8°W trend, characterized by alignment of magmatic rocks for the Ankaratra and Itasy regions (RAKOTONDRAINIBE, 1977; RAMBOLAMANANA, 1989; RAMBOLAMANANA and RATSIMBAZAFY, 1991).

The first Malagasy seismic station was installed by the Jesuits' Society in 1898 at the site of the Observatory of Antananarivo. It was a vertical short-period seismograph. In 1967, Rakotondrainibe installed a 3 short-period stations network. In 1988, this network was extended to 5 stations with a digital recording, covering the area around Antananarivo and the northeast of the Ankaratra region. The network records around 1700 earthquakes ($M_L > 2.9$) originating in the central region of Madagascar every year. The most important events are the 1985 October

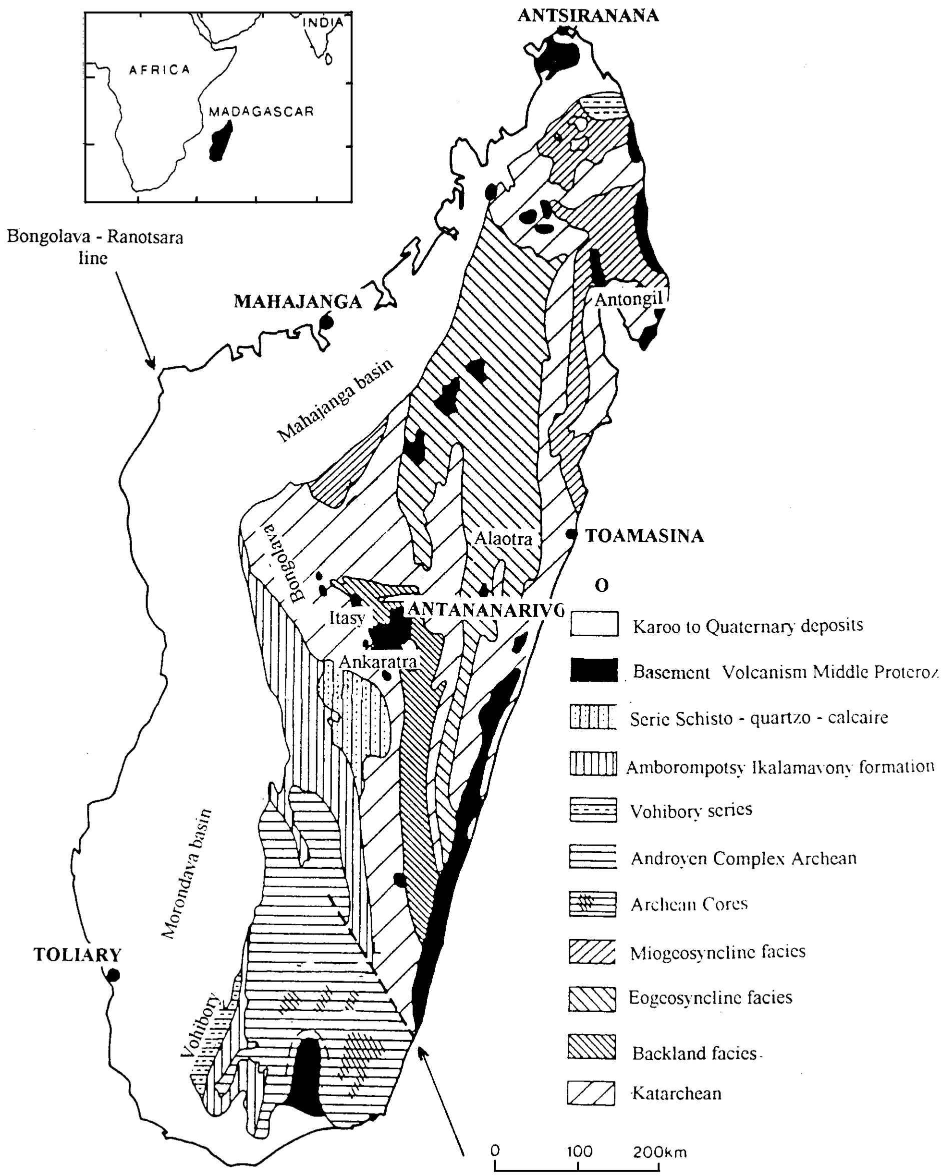


Figure 1
Map of the Malagasy basement (after JOURDE, 1971; HOTTIN, 1976; and CAEN-VACHETTE, 1979).

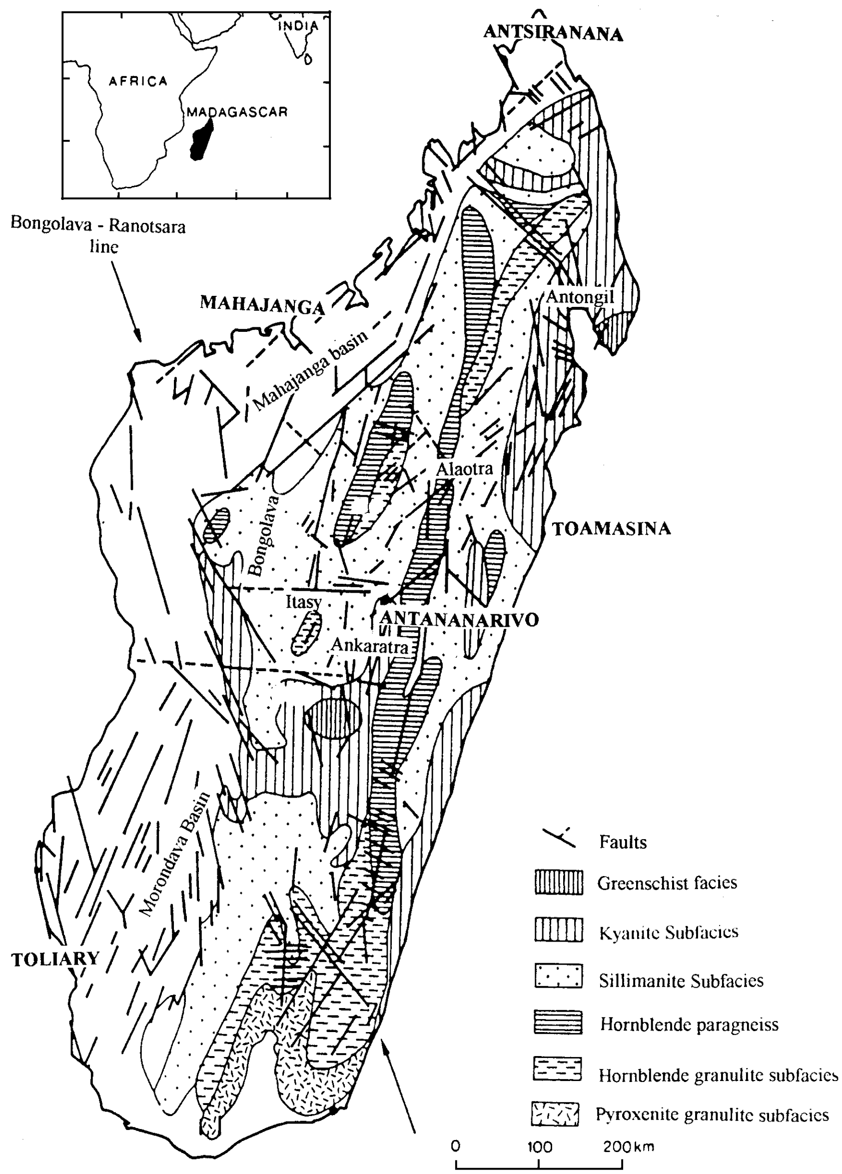


Figure 2
Simplified map of tectonic and metamorphic features of the Precambrian basement (after BESAIRIE, 1971; BAZOT *et al.*, 1971 and KUTINA, 1972).

5 ($M_L = 5.2$), in the Andaingo region and the 1991 April 21 ($M_L = 5.5$), in the Famoizankova region.

The method used to determine the location was developed by RAKOTONDRAINIBE (1977) and it is based on least-squares calculations that give the

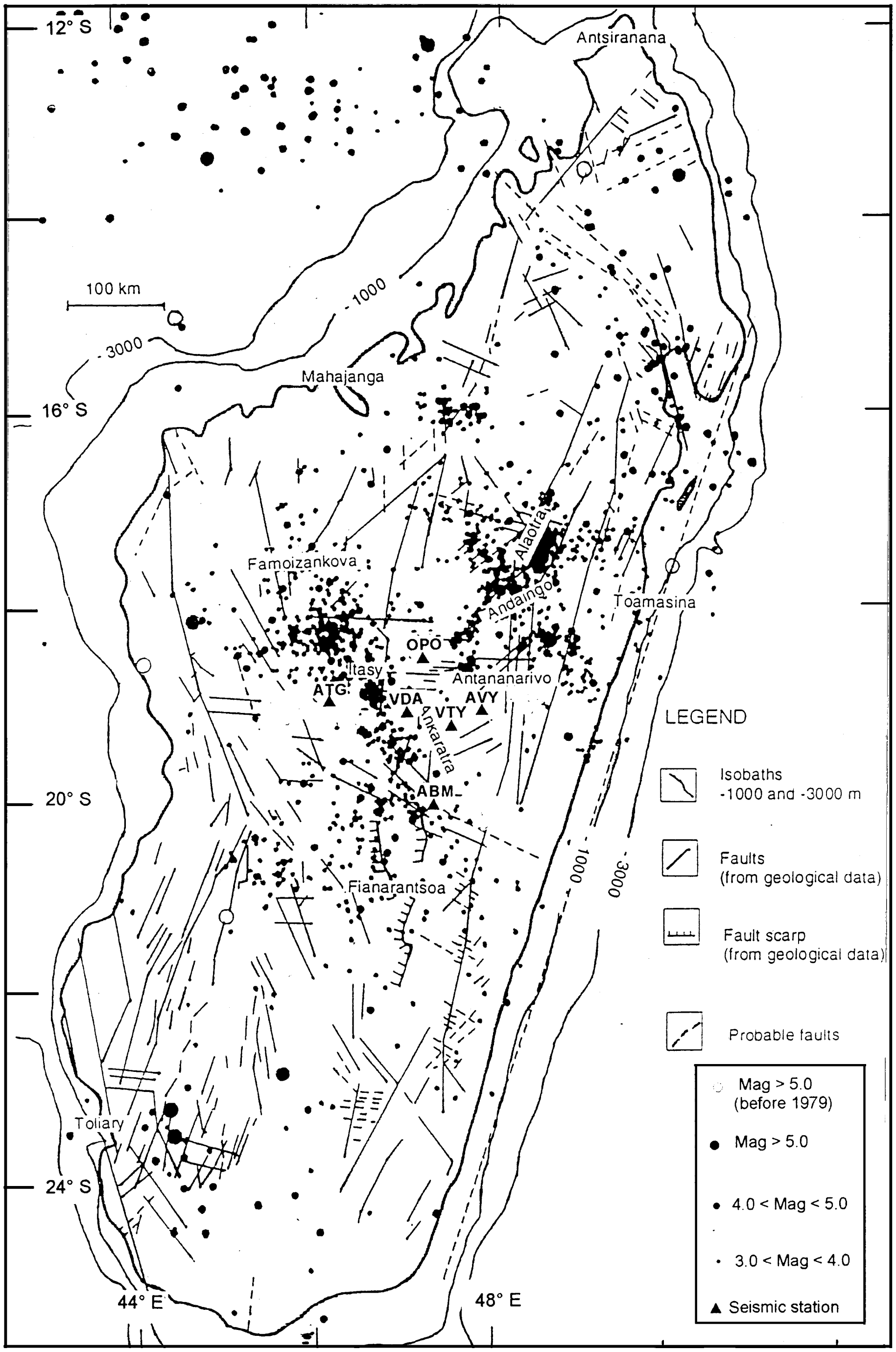


Figure 3
Seismicity of Madagascar region recorded by the Malagasy network from 1979 to 1994.

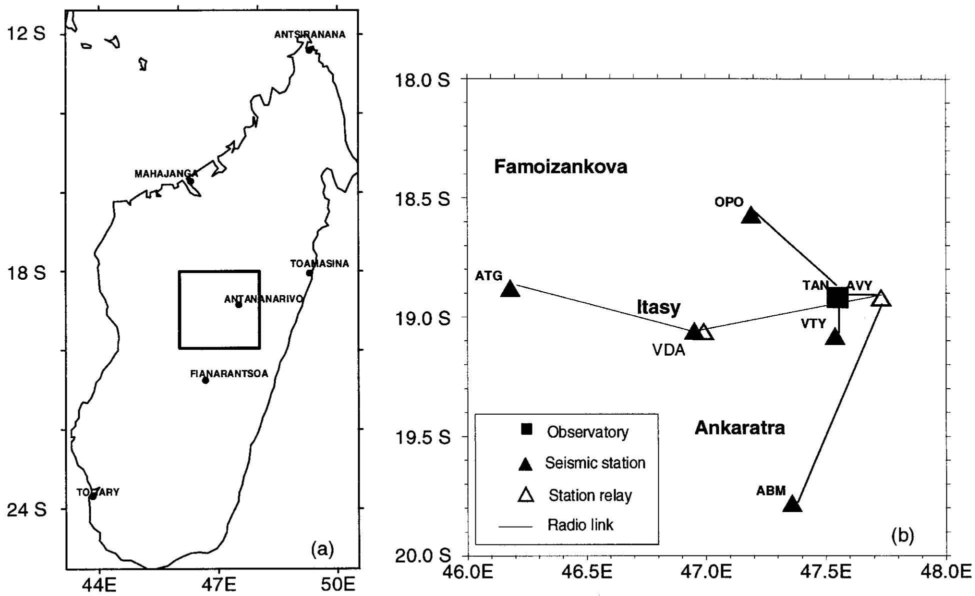


Figure 4
a) Map of Madagascar. Inset shows the studied area. b) The Malagasy seismic network in the studied area.

apparent velocities of P and S waves, the receiver-source azimuth and the arrival-times of the seismic wave front. The distance to the epicenter (D) is given by the formula

$$D = (T_S - T_P - E)\,\frac{V_P V_S}{(V_P - V_S)}$$

where T_P and T_S are arrival times, V_P and V_S the velocities of P and S waves respectively, and E is a parameter which depends on the crustal model and the focal depth (RAKOTONDRAINIBE, 1977). The epicenter is calculated by varying E in steps of 0.1 s for all combinations of three stations and for direct and refracted waves. From 1988, another method is used, based on least squares too, but the configuration of the network does not allow an accurate hypocenter determination. For the location, the program chooses the depth which corresponds to the smallest residual. Since most of the events occur outside the network, the first attempt of a simultaneous inversion of hypocentral parameters and velocity structures was undertaken in 1992 (RAMBOLAMANANA, unpublished), but the results were not very satisfactory.

In July 1994, the configuration of the network was changed. Station ATG was installed to include within the network the Ankaratra and Itasy regions (Fig. 4). The seismicity of these regions is very important, since it is high and relatively near the capital. In 1897 an event occurred in the Itasy region with an epicentral intensity of VIII on the Mercalli's scale. In 1945 and 1955, two other events with an epicentral intensity of approximately VII on the Mercalli's scale occurred in the same region. If a major event occurs in these regions, it will certainly produce severe damage in the capital and in nearby cities. The Ankaratra region is about 40 km southwest of the capital and the Itasy region about 80 km to the west.

To reduce seismic hazard it is necessary, as a first step, to have a more reliable velocity structure for these regions, based on the recorded data. The zonation of a territory in terms of seismic hazard is an essential preventive countermeasure. The maximum expected peak ground acceleration (PGA), at different frequencies, is a very important parameter considered by civil engineers when designing, reinforcing or retrofitting the built environment. A very useful tool to estimate PGA, starting from the available information such as the regional velocity structure, seismic sources and level of seismicity of the investigated area, is the deterministic procedure developed by COSTA *et al.* (1993), in which theoretical accelerations are computed by the modal summation technique (PANZA, 1985; FLORSCH *et al.*, 1991). The use of synthetic seismograms allows us to estimate the seismic hazard even if historical or instrumental information of those areas is not available. Following the procedure of COSTA *et al.* (1993), it is also possible to quite easily simulate different types of source mechanisms, to consider different structural models, and to compare the relative results in order to evaluate the influence on ground motion of each parameter. The second step is to study the effects of important events, occurring in the surroundings, in the cities and the capital, and for this purpose microzoning studies may be conducted using the realistic modelling of seismic input (e.g., PANZA *et al.*, 1996). In this paper we address the first step of the seismic hazard mitigation.

In fact, the arrival times of microearthquakes originating near and inside the network area provide us with the possibility to determine the velocity structure inside the regional network being used.

In geophysical observations, the inversion of data is essentially based on the iterative procedure of refining an *a priori* established earth model by introducing small perturbations (BACKUS and GILBERT, 1967; 1968; 1970; WIGGINS, 1972).

CROSSON (1976a,b), as well as AKI and LEE (1976) developed a nonlinear least-squares modelling procedure to simultaneously estimate hypocenter parameters, station corrections and a velocity model, by using arrival times from local earthquakes. PAVLIS and BOOKER (1980), SPENCER and GUBBINS (1980) improved their method by mathematically decoupling the hypocenter parameters from the velocity parameters in the inversion procedure. For the definition of velocity structure and hypocenter parameters, we apply in this study the method by MAO and SUHADOLC (1992), which also determines the model parameters perturbation, resolution and covariance.

2. *Inversion Theory*

The usual data obtained with local seismic networks are the arrival times of body waves radiated from seismic sources within the network. Such data contain information about the location of these sources and the seismic velocity structure. We can write the arrival time of the ith event recorded at the jth station as

$$T_{ij} = T_{ij}(t_{0i}, x_i, y_i, z_i, \alpha_k), \tag{1}$$

where t_{0i}, x_i, y_i, z_i are the hypocenter parameters and α_k $(k = 1, \ldots, Q)$ the parameters which describe the structure (velocities, thicknesses, densities, . . .).

A small change in travel time is given by

$$\Delta T_{ij} = \frac{\partial T_{ij}}{\partial t_{0i}} dt_{0i} + \frac{\partial T_{ij}}{\partial x_i} dx_i + \frac{\partial T_{ij}}{\partial y_i} dy_i + \frac{\partial T_{ij}}{\partial z_i} dz_i + \sum_{k=1}^{Q} \frac{\partial T_{ij}}{\partial \alpha_k} d\alpha_k, \tag{2}$$

that can be put in the form

$$\Delta T_{ij} = \sum_{p=1}^{4} \frac{\partial T_{ij}}{\partial h_{ip}} dh_{ip} + \sum_{k=1}^{Q} \frac{\partial T_{ij}}{\partial \alpha_k} d\alpha_k. \tag{3}$$

Following MAO and SUHADOLC (1992), we can write equation (3) in the form

$$\mathbf{r} = \mathbf{B}\delta\mathbf{h} + \mathbf{A}\delta\alpha, \tag{4}$$

where $\mathbf{r}$ is an L-dimensional $(L = NS \times NE)$ vector of travel-time residuals, NS is the number of stations and NE is the number of events used, $\delta\mathbf{h}$ is a 4 NE-dimensional vector of hypocenter parameters adjustment, $\delta\alpha$ is a Q-dimensional vector of structure parameters adjustment, $\mathbf{B}$ is an $L \times 4NE$ matrix of derivatives of travel times with respect to the hypocenter parameters and $\mathbf{A}$ is an $L \times Q$ matrix of the derivatives of travel times with respect to the structure parameters. For this study α represents the slowness of the structural model.

The matrices $\mathbf{B}$ and $\mathbf{A}$ can be written as

$$\mathbf{B} = \begin{pmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_{NE} \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_{NE} \end{pmatrix}$$

where

B_i are $NS \times 4$ submatrices
A_i are $NS \times Q$ submatrices
$i = 1, \ldots, NE$

The basic problem is to find $\delta\mathbf{h}$ and $\delta\alpha$ that minimize

$$\|\mathbf{r} - \mathbf{B}\delta\mathbf{h} - \mathbf{A}\delta\alpha\|^2. \tag{5}$$

Following CROSSON's (1976a) method, the solution of equation (5) requires large quantities of computer time and storage due to the large number of equations, a few thousand, and several hundred unknowns.

Therefore, in the inversion, the hypocentral parameters should be separated from the structural parameters (SPENCER and GUBBINS, 1980),

$$\delta\mathbf{h} = \mathbf{B}^+(\mathbf{r} - \mathbf{A}\delta\alpha), \tag{6}$$

$$\delta\alpha = (\mathbf{OA})^+\mathbf{Or}, \tag{7}$$

where "+" denotes the Moore-Penrose generalized inverse of a matrix, and $\mathbf{O}$ is given by

$$\mathbf{O} = \mathbf{A}^T - \mathbf{A}^T\mathbf{B}\mathbf{B}^+. \tag{8}$$

Equation (7) indicates that the determination of $\delta\alpha$ has no direct relation with $\delta\mathbf{h}$; it depends only on the current model parameters. $\delta\mathbf{h}$ then can be obtained from the structural parameter adjustments $\delta\alpha$.

Equations (6) and (7) show that one must find the inverse of two matrices: the $Q \times Q$ matrix $\mathbf{OA}$ and the $L \times 4NE$ matrix $\mathbf{B}$. $\mathbf{B}$ is a large matrix, but since it is a block diagonal matrix, its inverse can be obtained from the generalized inverse submatrices $\mathbf{B_i}$ (MAO and SUHADOLC, 1992).

According to the singular value decomposition theorem, the generalized inverse of matrix $\mathbf{B}$ is written as

$$\mathbf{B}^+ = \mathbf{W}\mathbf{S}^{-1}\mathbf{U}^T, \tag{9}$$

where $\mathbf{U}$ is an $(L \times L)$ orthogonal matrix, $\mathbf{W}$ is an $(4\,NE \times 4\,NE)$ orthogonal matrix, $\mathbf{S}$ is an $(L \times 4NE)$ diagonal matrix with diagonal elements consisting of the values s_i $(i = 1, 2, \ldots, 4NE)$. s_i are the nonnegative square roots of the eigenvalues of $\mathbf{B}^T\mathbf{B}$. The matrix $\mathbf{S}^{-1}$ is defined by $s_i^{-1} = 1/s_i$ if $s_i > 0$ and $s_i^{-1} = 0$ if $s_i = 0$. It means that even if some $s_i = 0$, the inverse exists.

In practice, due to numerical errors, very small singular values s_i of the coefficient matrix are usually encountered.

LEVENBERG (1944) and MARQUARDT (1963) developed the so-called "damped least-squares" method. According to this method, the matrix $\mathbf{B}^+$ is defined as

$$\mathbf{B}^+ = \mathbf{W}\{(s^2 + \theta^2\mathbf{I})^{-1}s\}\mathbf{U}^T, \tag{10}$$

where θ is an adjustable parameter. θ is usually much smaller than the largest singular value s_1.

The resolution matrices corresponding to the solution vectors (6) and (7) are

$$\mathbf{R}_h = \mathbf{B}^+\mathbf{B} - \mathbf{B}^+\mathbf{A}(\mathbf{OA})^+\mathbf{OB}, \tag{11}$$

$$\mathbf{R}_\alpha = (\mathbf{OA})^+\mathbf{OA}, \tag{12}$$

where $\mathbf{R}_h$ and $\mathbf{R}_\alpha$ relate the true solution to the estimated solution. WIGGINS (1972), AKI and RICHARDS (1980) and MENKE (1984) give the meaning and the qualitative definition of good resolution. If the resolution is an identity matrix, each component of the model parameter vector is perfectly resolved and our solution is unique. However, because of inadequacies in the parameterization of the model and in the data, a unique solution may never be obtained. The resolution of model parameters is a measure of the inversion results, but not a unique standard, since it cannot estimate the accuracy of the inversion results. For this purpose, the covariance matrices are introduced by

$$\mathbf{C}_h = \sigma^2(\mathbf{B}^+ - \mathbf{B}^+\mathbf{A}(\mathbf{OA})^+\mathbf{O})(\mathbf{B}^+ - \mathbf{B}^+\mathbf{A}(\mathbf{OA})^+\mathbf{O})^T, \tag{13}$$

$$\mathbf{C}_\alpha = \sigma^2((\mathbf{OA})^+\mathbf{O})((\mathbf{OA})^+\mathbf{O})^T, \tag{14}$$

where σ^2 is the standard deviation of the observed data (MAO and SUHADOLC, 1992). It is well known that there is a trade-off between resolution and covariance. An increase in the density of model parameters is accompanied by a decrease in the resolution of model parameters. Thus, the resolution and the covariance of model parameters depend on the model parameterization. The adjustable parameter θ controls the trade-off between resolution and covariance. In general, it is desirable to choose θ as small as possible to get a maximum resolution however in practice small values of θ result in an instability in the inversion.

3. Data Selection

We used the arrival-time data from the Malagasy seismological network of the Institut et Observatoire de Géophysique d'Antananarivo (IOGA). One-second vertical component seismographs are in operation at 5 stations in a region of about 190 km by 140 km (Fig. 4). 770 *P*- and *S*-wave arrival times from 154 earthquakes were chosen among the events occurring in the time interval September 1994 to May 1995.

We used the program HYPO71PC to refine the location and especially the depth from the routine package ISIS (Integrated Software In Seismology) developed by the Laboratoire de Détéction et de Géophysique (LDG) in France. The results of the location given by IOGA are used as *a priori* input data for

HYPO71PC. The locations obtained by the two methods do not show a great difference but the focal depths obtained from HYPO71PC do not present artificial concentrations at certain depths. In fact, the ISIS located depths show concentrations at 16 km, 18 km, 27 km and 37 km (Fig. 5a).

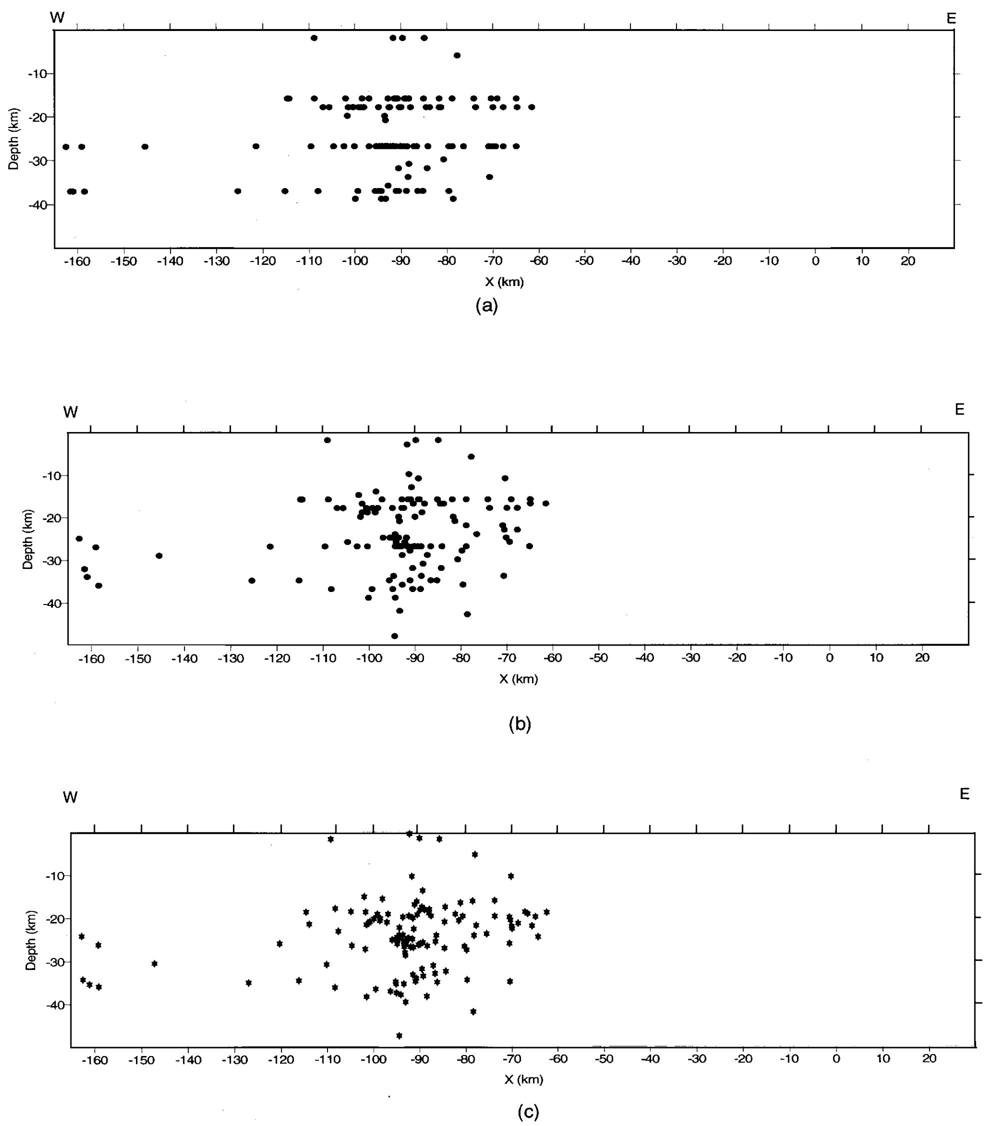


Figure 5
Focal depth distribution in the $x-z$ plane (east-west cross section). a) Focal depths from ISIS (see text) location; b) focal depths determined by HYPO71PC; c) focal depths obtained from the solution of the inversion.

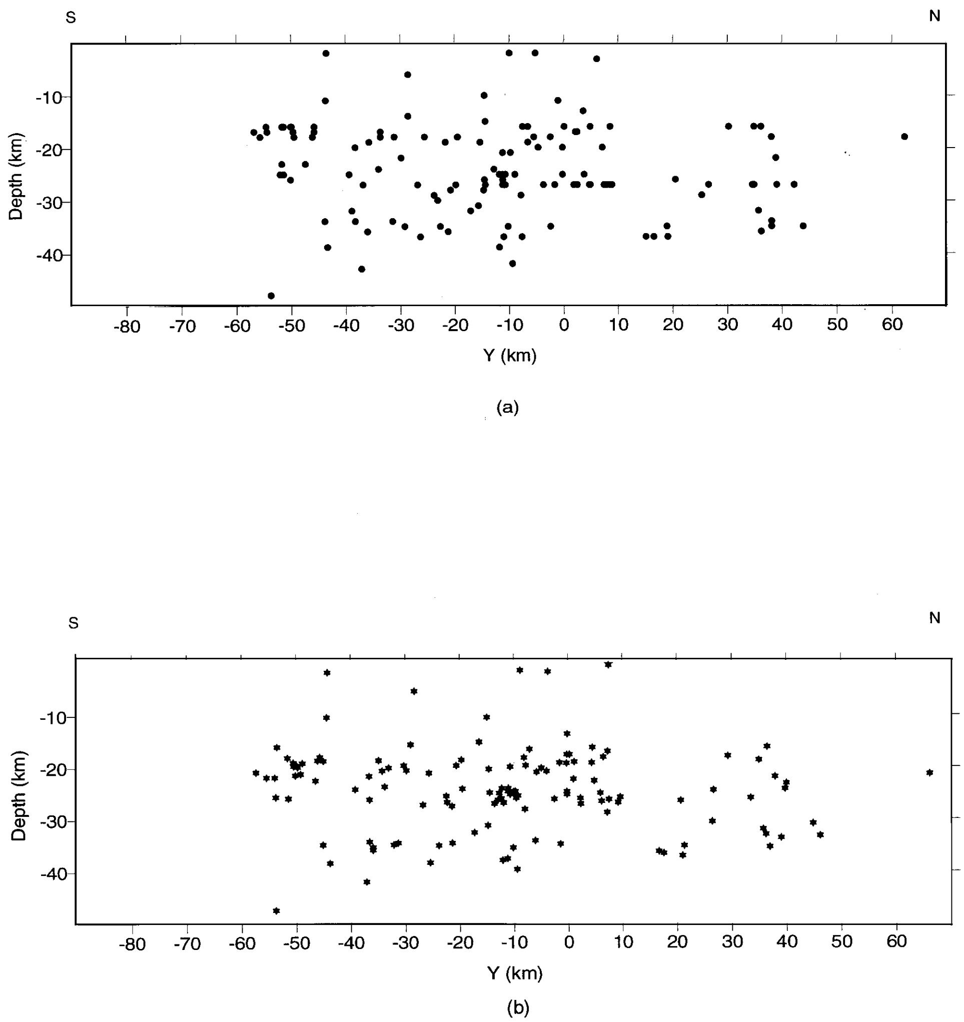


Figure 6
Focal depth distribution in the y–z plane (north-south cross section). a) Initial distribution of focal depths from HYPO71PC; b) focal depths distribution obtained from the inversion.

The first velocity model of Madagascar was determined by RAKOTONDRAINIBE (1977), using a simplified approach based on the difference $T_S - T_P$ and a geometrical method. IOGA used this model and method for the location of earthquakes up to 1988. From 1988 to 1994, the location package ISIS developed by the LDG (France) was used to locate earthquakes. Since 1994, an update of this routine is being utilized. The velocity model used in this routine is an average of the 1977 model. This average model and the hypocenter locations obtained with HYPO71PC, were used as the *a priori* model input for the simultaneous inversion.

Figures 5b, 6a, and 7 show the distribution of the epicenters and the hypocenters in the (x, y, z) space. All of the events are located inside the network, and are concentrated in the western region of Ankaratra and in the volcanic region of Itasy. These events are recorded at five stations linked via radio with a common time system.

We used for the inversion only the events in the magnitude range from $M_L = 2.0$ to $M_L = 4.7$. This means that we used in the inversion only well-determined first arrival times. The accuracy of the reading-time for P phases is 0.1 s. In the first step of the inversion, we used only P-arrival times to obtain the velocity structure for P waves. The location obtained from this first step is subsequently used for the determination of the S-waves velocity structure. Therefore, the use of the location obtained from P phases minimizes the instability in the S-waves velocity structure inversion. Both direct and refracted waves are considered in the inversion.

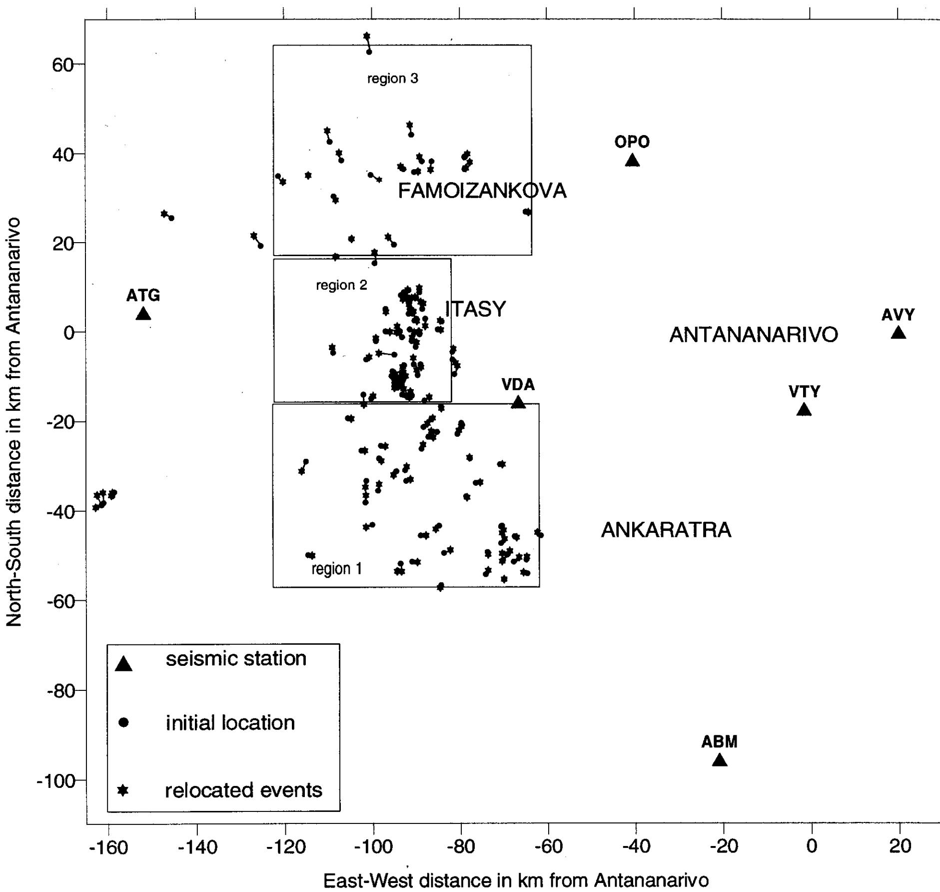


Figure 7
Map of the initial (dots) and relocated (stars) epicenters. Each relocated epicenter is connected with a segment to the corresponding initial one.

Table 1

Structure model for Madagascar from RAKOTONDRAINIBE (1977). a) Western part. b) Eastern part

(a)		
$V_P = 5.9$ km/s	$V_S = 3.4$ km/s	$h = 16$ km
$V_P = 6.7$ km/s	$V_S = 3.8$ km/s	$h = 21$ km
$V_P = 8.0$ km/s	$V_S = 4.6$ km/s	$h = \infty$
(b)		
$V_P = 5.9$ km/s	$V_S = 3.4$ km/s	$h = 16$ km
$V_P = 6.7$ km/s	$V_S = 3.8$ km/s	$h = 18$ km
$V_P = 8.0$ km/s	$V_S = 4.6$ km/s	$h = \infty$

Table 2

First starting P-wave velocity model

P velocity (km s^{-1})	Thickness (km)
5.90	9.0
5.90	9.0
6.70	10.5
6.70	10.5
8.00	30.0

4. Velocity Structure

The maximum epicentral depth of the events chosen for the inversion computation is about 48 km; the velocity structure will, therefore, be defined above this depth. As we can see from Figure 7, the area inside the network may be divided into three regions: region 1 (west Ankaratra) is between $X = -60, -120$ km; $Y = -20, -60$ km, region 2 (Itasy) is between $X = -80, -120$ km; $Y = +15, -20$ km, and region 3 (Famoizankova) is $X = -60, -120$ km; $Y = +15, +60$ km. Due to the small number of events, we cannot separate these regions in the inversion. RAKOTONDRAINIBE (1977) formulated two structural models for Madagascar, one for its western and one for its eastern part (Table 1). Thus we used as *a priori* model a variant (Table 2) of the model given by RAKOTONDRAINIBE (1977) for Western Madagascar. The formalism for ray-tracing in one-dimensional layered models is given by LEE and STEWART (1981). The trial take-off angle is given by MAO and SUHADOLC (1992)

$$\alpha = tg^{-1} \frac{v_j \Delta}{\xi v_j + \sum_{i=1}^{j-1} h_i v_i}, \tag{15}$$

where v_j and h_j are the velocity and thickness of the jth layer, Δ is the epicentral distance and ξ the depth from the top of the jth layer. We used both direct and head waves in the inversion. The calculation of the derivatives of travel time with respect to hypocenter parameters and velocities is given by MAO and SUHADOLC (1992). The control of the inversion accuracy is given by LEONARD and JOHNSON (1987)

$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \frac{r_i^2}{\sigma_i^2}, \tag{16}$$

where N is the number of travel times used in the inversion, r_i is the ith travel-time residual and σ_i is the observational error of the ith travel time. An acceptable model is obtained when χ^2 is approximately equal to 1.0. The validity of this method tested with synthetic data is shown by MAO and SUHADOLC (1992).

The selection of layer thickness plays an important role in arrival-time inversion results and their resolution. The layer thickness is assumed *a priori* on the basis of independent information from geophysical and geological studies.

In the central region of Madagascar, little geophysical information is known. From the results obtained by RECHENMANN (1982), FOURNO (1994), RAKOTONDRAOMPIANA *et al.* (1995), it appears that the thickness of the crust under the Ankaratra region and the Itasy region is more than 37 km, but the uncertainty is substantial. We plan to study the depth of the Mohorovičić's discontinuity in a future paper.

In general, small thickness, compatible with the error in the phase readings and therefore with the attainable resolution, should be sought for the parameterization of the velocity model. In this case the limited ray coverage, due to the focal depth distribution and the number of the events used in the inversion, prevents the use of small layer thickness.

5. Results

We started with the *a priori* model shown in Table 2. This model is a variant of the model given by RAKOTONDRAINIBE (1977). We divided the first and second layer by two. After 7 iterations, we obtained the results for the velocities and the resolution matrix shown in Table 3.

We have to calculate the structure parameter adjustment $\delta\alpha$ and thereafter estimate the hypocenter adjustment δh. Thus we have to use values of the adjustable parameter θ. One for calculating the generalized inverse of the submatrices $\mathbf{B_i^+}$, and one for the submatrices $\mathbf{OA_i^+}$. We choose $\theta_B = 0.1\%\ s_1$, where s_1 is the maximum value of the singular values of the submatrices $\mathbf{B_i^+}$ and $\theta_{OA} = 0.1\%$ s_2, where s_2 is the maximum value of the singular values of the submatrices $\mathbf{OA_i^+}$. Values smaller than these do not give stable results in the inversion, while values greater than these do not lead to a good resolution of the model. The resolution matrix shown in Table 3b indicates that only the velocity of the first and third layers is sufficiently well resolved. Thus, we decided to use several different starting models, presenting variations with respect to the model given by RAKOTONDRAINIBE (1977). For each starting model we compared the obtained results (resolution matrix and covariance matrix). The starting model (Table 4) which led to the best resolution and covariance matrix has slightly higher velocities in the middle crust and a deeper Moho. Only 2 iterations were necessary to obtain the results presented in Table 5, and the related resolution matrix (Table 5b) is very close to the identity matrix.

Figure 7 displays both the initial (HYPO71PC) earthquake locations and the final ones determined by the inversion process. The initial and final epicenters are connected with a line. The dashed lines in Figure 8 illustrate the starting model velocities for *P* and *S* waves and the solid lines show the results obtained by the inversion. The distribution of the *P*- and *S*-wave arrival time residuals is given in Figure 9. The *P*-wave residuals (Fig. 9a) obtained from the initial model of Table 4 (dashed line) are mostly negative. The *P*-wave residuals (solid line) obtained from the result of the inversion, the model in Table 5a, show a normal distribution, and, 71.6% of the *P*-wave residuals are within the range of $|t_{\mathrm{obs}} - t_{\mathrm{cal}}| < 0.075$ s and 84.0% of them within the range of $|t_{\mathrm{obs}} - T_{\mathrm{cal}}| < 0.125$ s. This indicates that the inversion results represent an acceptable 1-D model of the crustal structure in the central region of Madagascar.

In order to provide general outline of the variations of the hypocentral location, histograms of the number of earthquakes versus the variation of the origin time and

Table 3

a) Results of the inversion (unit in km s^{-1}) for the P wave using the starting model of Table 2. b) Resolution matrix obtained from the inversion

V_P	Standard errors
(a)	
5.85	0.044
6.08	0.007
6.88	0.822
6.59	0.230
7.98	0.001

(b)

+0.993	+0.015	−0.000	+0.001	−0.002
+0.015	+0.804	+0.020	+0.004	+0.000
−0.000	+0.020	+0.997	−0.006	−0.006
+0.001	+0.004	−0.006	+0.891	+0.086
−0.002	+0.000	−0.006	+0.086	+0.563

Table 4

Second starting P-wave velocity model

P velocity (km s^{-1})	Thickness (km)
5.90	10
6.10	10
6.70	10
6.80	12
8.10	26

Table 5

a) Results of the inversion (unit in km s^{-1}) for the P wave using the starting velocity model of Table 4. b) Resolution matrix obtained from the inversion

V_P	Standard errors
(a)	
5.84	0.07
6.37	0.03
6.94	0.02
6.64	0.05
8.09	0.14

(b)

0.997	0.001	0.000	0.000	0.001
0.001	1.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.998	0.002
0.001	0.000	0.000	0.002	0.924

the space location are shown in Figure 10. The maximum depth variation for some events in the seismic areas of Ankaratra and Itasy is 5.5 km. for 99% of the events the variation in the origin time and in the epicenter is respectively less than 0.4 s and 2.5 km; for 71% of the events, the variation of the focal depth is less than 1.5 km and for 90% of them, the variation is less than 3.5 km.

In Figure 11, we show the histogram of the number of earthquakes versus the focal depth for the relocated events, and the result of the P-wave velocity inversion. Most of the events are concentrated in the depth range between 15 km and 27.5 km.

As we previously mentioned, the location obtained from the determination of P-wave velocity structure is subsequently used to obtain the S-wave velocity structure. The error in phase readings for S wave is larger than the one affecting P wave, and we assume an error of 0.2 s for S-wave phases readings. The starting model for the S-wave velocity structure is shown in Table 6. After two iterations, we obtain the result shown in Table 7. The resolution matrix given in Table 7b for

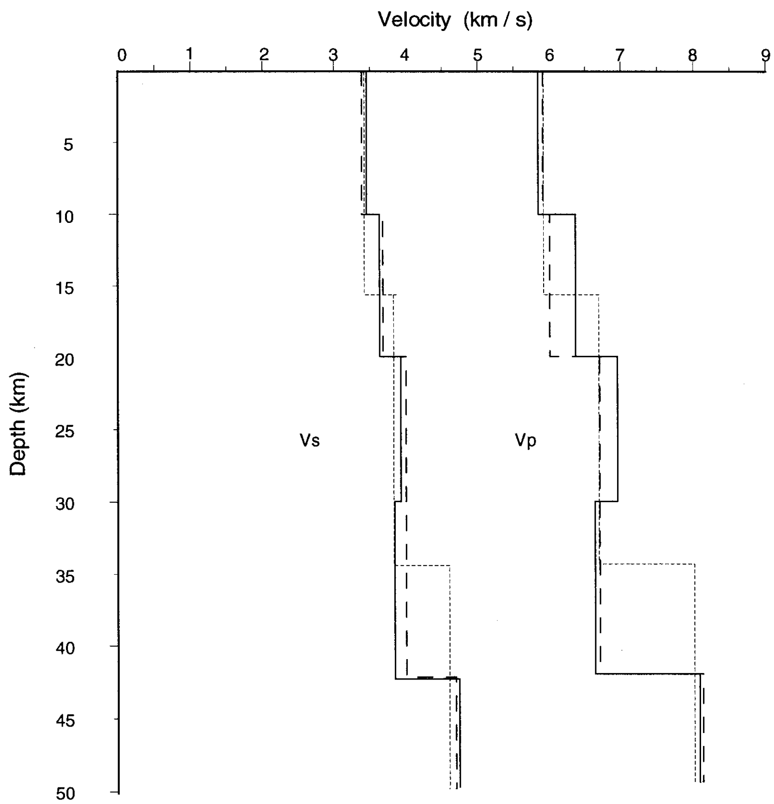


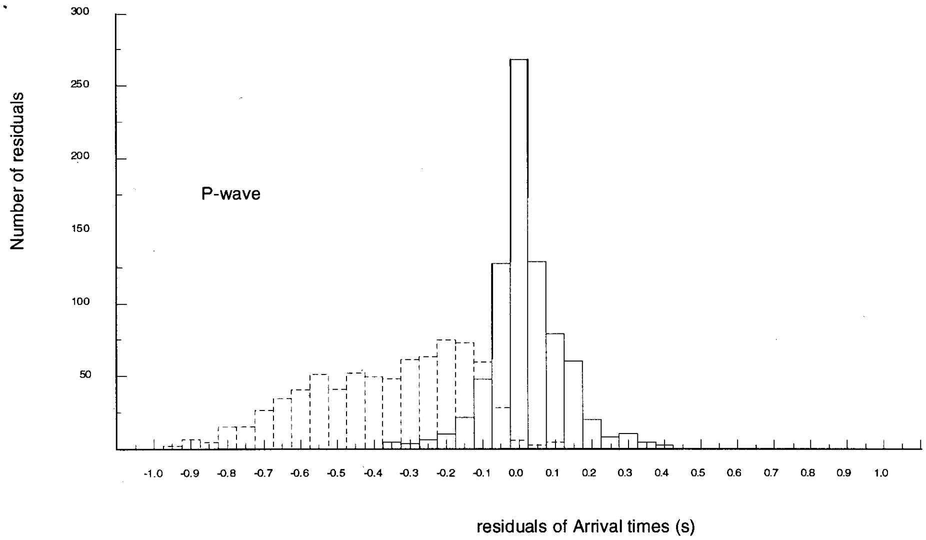
Figure 8
The dotted lines indicate the RAKOTONDRAINIBE (1977) model, the dashed lines our starting model (Table 4), and the solid lines denote the final results of the inversion (this paper) for the P- and S-wave velocity structures.

the determination of S-wave velocity structure is a quasi-identity matrix. The convergence to the solution is fast and the inversion is stable.

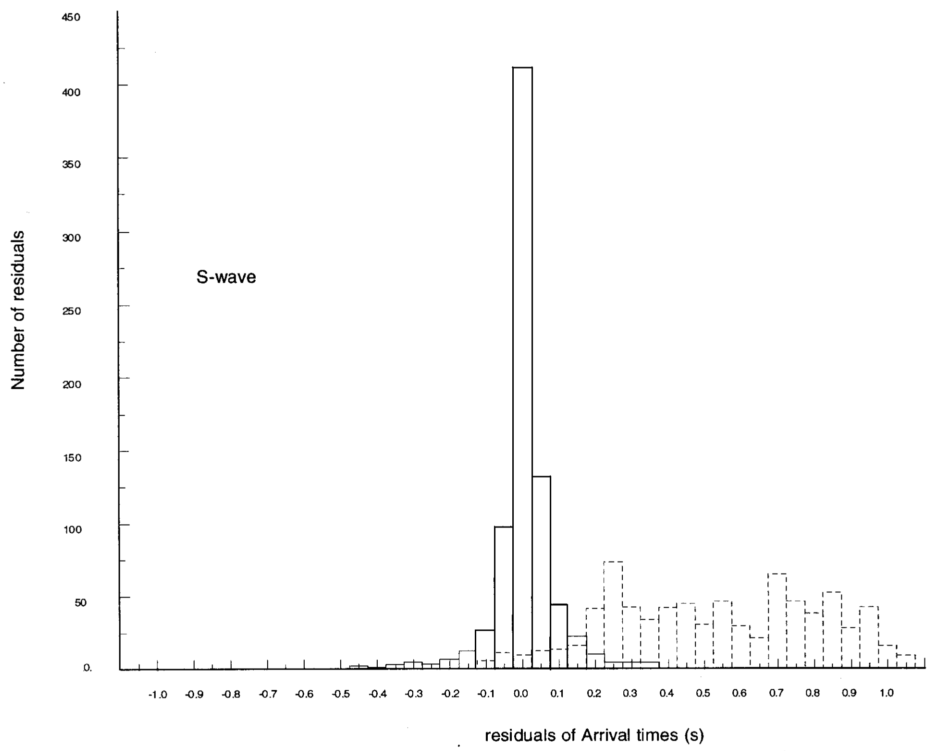
The S-wave residuals in Figure 9b (dashed line), derived from the starting model of Table 7, are mostly positive. The S-wave residuals obtained from the results of the inversion are, as expected, normally distributed (solid line).

From these results, a V_P/V_S ratio can be calculated for each layer, since the velocity structures are inverted independently for the P-wave and the S-wave arrival times (see Table 8).

Figure 9
Histograms of the arrival-time residuals. a) Arrival-time residuals of P waves; b) arrival-time residuals for S waves. The dashed lines denote the residuals of the starting models and the solid lines denote the residuals of the inversion.


Number of residuals
300
250
200
150
100
50
P-wave
-1.0 -0.9 -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
residuals of Arrival times (s)

(a)


Number of residuals
450
400
350
300
250
200
150
100
50
0.
S-wave
-1.0 -0.9 -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
residuals of Arrival times (s)

(b)

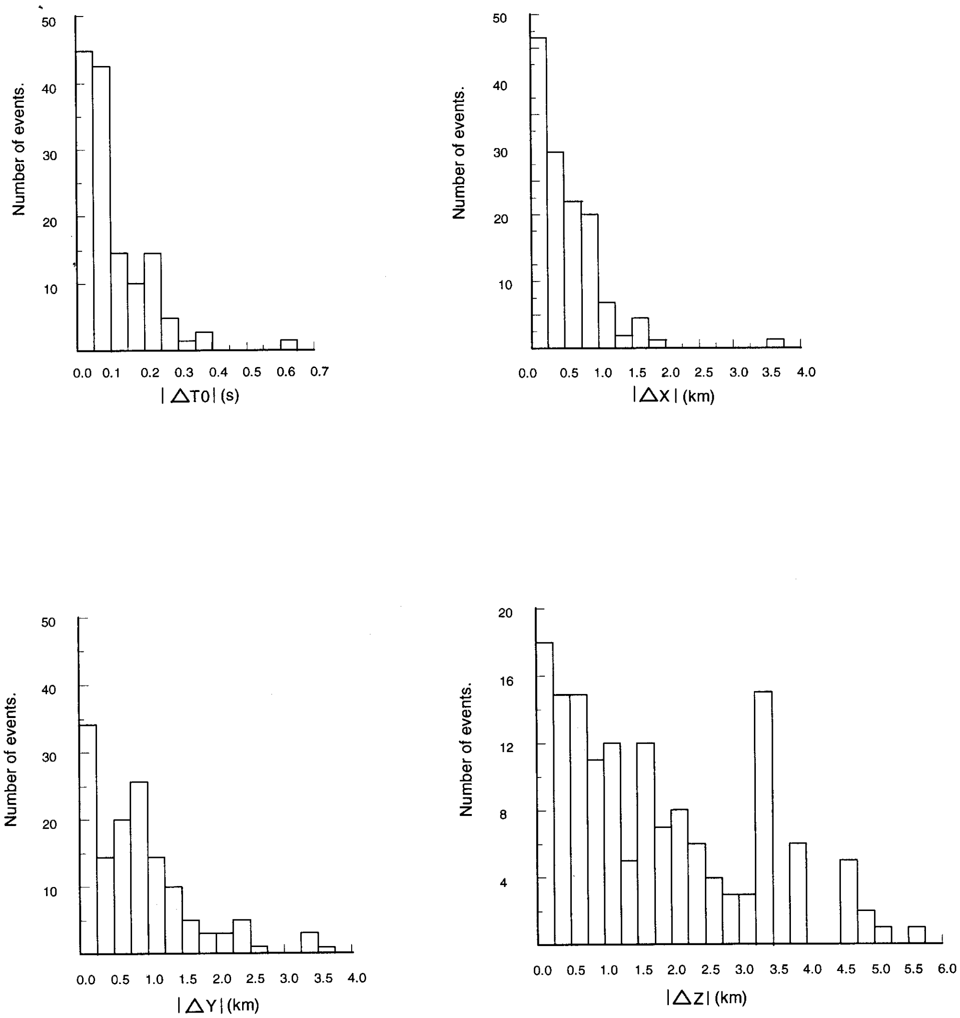


Figure 10
Histograms of number of events versus variations between hypocentral parameters obtained by HYPO71PC and those by this study (the solid line model in Fig. 8). ΔT_0 is error in origin time, ΔX, ΔY, ΔZ are errors in E-W, N-S locations and depth of hypocentral location, respectively.

6. Conclusions

We have obtained both P- and S-wave velocity models by applying the MAO and SUHADOLC (1992) inversion method to arrival times of the Central Madagascar earthquakes. The determination of the crustal velocity within a seismic network, using local travel-time data, requires accurate hypocentral locations. As regards the seismic area studied in this paper, the network is not very dense and it is not

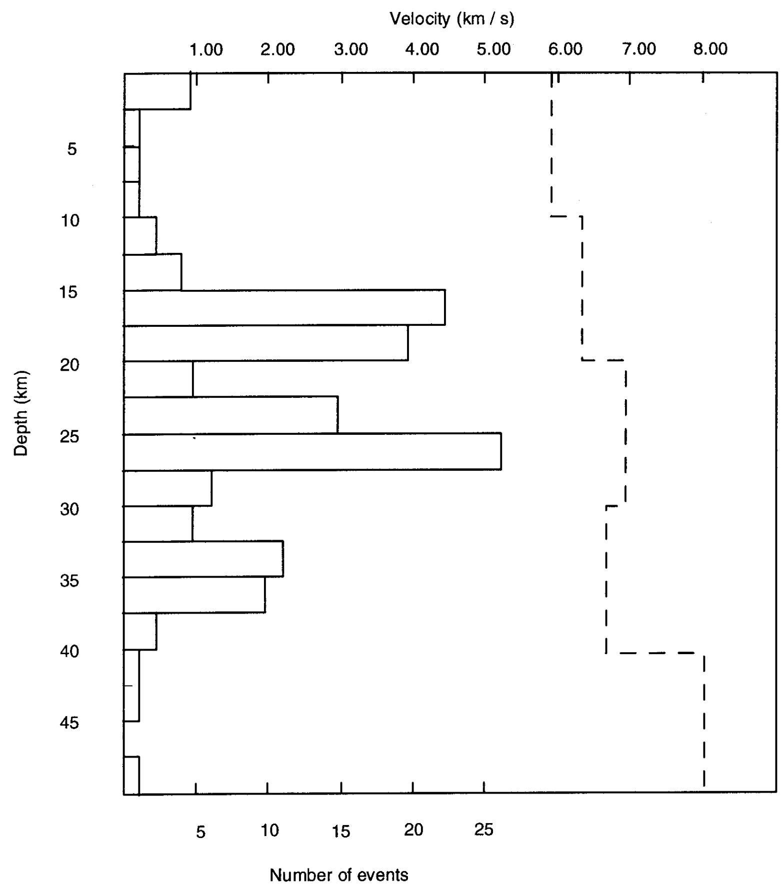


Figure 11
Histogram of the number of earthquakes versus focal depth. The results of the inversion for the P-wave velocity structure is also shown in the figure (dashed line).

impossible to perform separate inversions for the various seismogenetic areas of Central Madagascar (Fig. 7).

The results obtained with the inversion of P-wave arrival times indicate that the velocity in the first layer is about the same as that assumed by IOGA. The P-wave velocities in the depth range 10–30 km are found to be substantially larger than those assumed in the model proposed by RAKOTONDRAINIBE (1977) and a smaller difference is found for $S =$ waves velocity. The inversion leads to a well constrained low-velocity channel in the lower crust between 30 km and 42 km of depth. The 42 km discontinuity is obviously associated with the Moho. This value is slightly more than the average, 37 km, proposed by RAKOTONDRAINIBE (1977). FOURNO (1994), in his study of the Moho discontinuity from the inversion of gravimetric data, proposes a Moho discontinuity at 40 km in the southeastern Ankaratra. The

Table 6

Starting S-wave velocity model

S velocity (km s^{-1})	Thickness (km)
3.30	10
3.50	10
3.90	10
3.90	12
4.70	26

Table 7

a) Results of the inversion (unit in km s^{-1}) for the S wave. b) Resolution matrix obtained from the inversion

V_S	Standard errors
(a)	
3.45	0.00
3.61	0.01
3.91	0.01
3.83	0.04
4.70	0.06

(b)

1.000	0.000	0.000	0.000	0.000
0.000	1.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.993	0.002
0.000	0.000	0.000	0.002	0.939

Table 8

V_P/V_S ratios derived from the inversion

Layer in km	Ratio V_P/V_S
10.0	1.68
10.0	1.76
10.0	1.77
12.0	1.73
26.0	1.72

relocated hypocenters do not reveal any concentration at some depths, and the travel-time residuals are minimized and have a normal distribution.

The obtained velocity structure will permit better localizations of central Madagascar events. It also represents the basis both for earthquake source studies in the region and for seismic hazard estimates based on deterministic approaches.

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