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A SNR Enhancement Method for Desert Seismic Data: Simplified Low-Rank Selection in Time–Frequency Decomposition Domain

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Abstract—In seismic data processing, low-frequency random noise with non-Gaussian and non-stationary characteristics heavily contaminates the reflected signals in Tarim area, which brings great difficulties in interpretation of seismic records in northwest China. To achieve more satisfied resolution, more greater fidelity, together with much higher increased signal-to-noise ratio (SNR), this paper proposes a SNR enhancement method based on the combination of variational mode decomposition (VMD) and Semi-soft Go Decomposition (Semi-Soft GoDec), named VMD-SSGoDec, which can realize the simplification of low-rank extraction in time–frequency representation (TFR) domain. Firstly, each trace of the rough seismic record is decomposed into several modes to reconstruct a component matrix by VMD. Due to the semi-low rank or approximate low-rank character of the desert low-frequency noise component matrix in TFR domain, secondly, we apply the Semi-soft GoDec, a low-rank matrix estimation to extract the low-frequency random noise components from the VMD results obtained in the first step. Repeating the above single-trace procedure to each trace rather than decomposing the entire record but use low-rank estimation once can lead to a more reduced dimension of the component matrix, and thus simplify the low-rank selection in Semi-soft GoDec. Finally, with the extracted random noise results in the second step, we can obtain the denoised record by making a difference with the original input. The proposed algorithm is tested by both synthetic record and field desert seismic data. Experimental results show outstanding advantages in low-frequency noise attenuation comparing with those of *f-x* deconvolution and SSWT-OptShrink. Both low-frequency random noise and surface waves are almost thoroughly attenuated by the proposed method, while the reflected signals are left nearly intact, revealing a significant enhancement in SNR.

Keywords: Variational mode decomposition (VMD), Semi-soft GoDec, SNR enhancement, Low-rank matrix approximation, Time–frequency representation.

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1. Introduction

Seismic data acquisition is always interfered by various noises in petroleum exploration. Especially in the desert area in northwest China, due to the particularity and complexity of geological structure (Wang et al., 2015; Xu et al., 2020), the seismic signals collected by the arrays are typically and seriously polluted by some non-stationary, non-gaussian, nonlinear, but low-frequency noises. These noises submerge the reflected signals and thus result in low SNR, low resolution and low fidelity of the seismic records. In view of these unclearly detected characteristics of the desert seismic noise, extracting the reflected information, improving the SNR of seismic data, but maintaining the characteristics of the reflected signal becomes one of the main problems in high-quality desert seismic data processing.

For non-stationary signals, time–frequency transform (TFR) is more informative than representing only in frequency or time domain (one-dimensional), and thus is often used to localize individual oscillatory components of seismic signals. So far, many different time–frequency representations have been proposed and improved to suppress seismic noise, such as *f-x* deconvolution (Spitz & Deschizeaux, 1994), prediction error filtering in *t-x* and *f-x* domains (Abma and Claerbout, 1995) and *f-x* singular spectrum analysis (Oropeza and Sacchi, 2011; Majumder et al. 2019). Between 2016 and 2017, Mostafa Mousavi and Rasoul Anvari applied Synchrosqueezed Wavelet Transform (SSWT) to noise reduction of seismic signals and achieved good results (Mousavi et al., 2016; Mousavi & Langston, 2017; Anvari et al., 2017). Variational Mode Decomposition (VMD) is also a time–frequency transform based technique and provides sharpened time–frequency representation with higher time and frequency resolution. Unlike traditional reassignment methods in SSWT (Daubechies et al., 2011), VMD is adaptive to different types of data and has rich visual information. Together with its simple and effective reconstruction formula, VMD became a powerful and popular tool for precise decomposition and analysis in recent years (Dragomiretskiy & Zosso 2014). However, under the circumstance of the complicated characteristics of the noise, VMD itself can cause spectral aliasing and could not achieve satisfied results in desert seismic denoising. In our research, we utilize the characteristic that the desert seismic noise preserved its low-rank property (Siahsar et al., 2016) when transformed with some sparse TFR, and propose a new method to enhance the SNR of desert seismic data. To extract the low rank parameter, many typical low-rank matrix approximation algorithms have been proposed and improved by scholars, including RPCA (Wright et al., 2009), WNNM (Gu et al., 2009), GoDec (Siahsar et al. 2016; Zhou & Tao, 2011), and so on. Semi-Soft GoDec has been greatly improved than the traditional ones, as it introduces soft thresholds to speed up the calculation (Zhou & Tao, 2013).

Hence, based on the above, the proposed method begins by transforming desert seismic signals with VMD. Then in each mode, since the low rank of the seismic noise is lower than that of the reflected signal, Semi-soft GoDec (Zhou & Tao, 2013) is selected to extract the low-rank components to estimate the seismic noise. Finally, a noise free seismic data could be obtained by subtracting the noise obtained by mode superposition from the original seismic signal. Within this method, a record is processed trace by trace to simplify the low rank extraction. By the way, no component is discarded at will.

This paper is organized as follows. The second part briefly elaborate the principle of the Semi-soft GoDec algorithm and the SNR enhancement method based on that. In the third part, both synthetic and field data experiments are processed and discussed in detail. The last two parts give further discussion and conclusion.

2. *SNR Enhancement Theory*

A. Semi-soft GoDec

GoDec is one of the typical low-rank matrix approximation algorithms aiming at recovering potential low-rank matrices from the given degraded observations. It imposes hard constraints on the rank of the low-rank matrix L and the cardinality of the sparse matrix S. In addition, by using the low-order approximation based on bilateral random projection (BRP) and the controllable rank of L (Zhou & Tao, 2011, 2013), the noise decomposition could be accelerated. Given the input matrix X, the formula of GoDec is shown as follow:

$$\begin{gathered}\min_{L,S} X - L - S_F^2 \\ s.t.\, rank(L) \le r, card(S) \le k\end{gathered} \tag{1}$$

If introduce the regularization method with a soft threshold λ, the formula becomes:

$$\begin{gathered}\min_{L,S} X - L - S_F^2 + \lambda S_1 \\ s.t.\, rank(L) \le r\end{gathered} \tag{2}$$

Semi-soft GoDec solves this problem by alternatively optimizing the following two sub-problems until convergence:

$$\begin{cases} L^t = \arg \min\limits_{rank(L) \le r} X - L - S^{t-1}{}_F^2 \\ S^t = \arg \min\limits_{S} X - L^t - S_F^2 + \lambda S_1. \end{cases} \tag{3}$$

The two sub-problems can be solved by alternatively updating L^t via singular value hard thresholding (svd) of $X - S^{t-1}$ and S^t via soft thresholding (P_λ) of $X - L^t$ (Zhou & Tao, 2011), resctively:

$$\begin{cases} \overline{L^t \sum_{i=1}^{r} \lambda_i U_i V_i^T, \quad svd(X - S^{t-1}) = U \Lambda V^T} \\ S^t = P_\lambda(X - L^t), \quad P_\lambda(x) = sign(x) \max(|x| - \lambda, 0) \end{cases} \tag{4}$$

B. Simplified Low-rank Selection in VMD

In this paper, we propose a method with Semi-Soft GoDec in the TFR domain for enhancing the SNR of desert seismic data. As a mature time–frequency decomposition algorithm, VMD has been wely used so that the specific principle is no longer described in detail here. Hence, we describe the two main steps of the proposed algorithm as follows:

Part 1. Time–Frequency Decomposition

Assume the desert seismic record as a $m \times n$ matrix, where m is the number of samples and n is the number of traces. Choose a single trace $x(t)$ along the m (time) dimensional and decompose it into N modes with VMD algorithm to obtain a noised component matrix of order $m \times N$. The column number N constrains the range for the following simplified rank selection to $(1, N)$.

Part 2. Simplified Low-rank Selection and Denoising

Since the numerical value of the mode N is relatively small, it is reasonable and operational to implement Semi-Soft GoDec algorithm by trying every rank i, where $i \in (1, N)$, to extract different levels of noise components from the $m \times N$ matrix in part 1, resulting in several (up to N) filtered $m \times N$ matrices. For each of these matrices, with the superposition of every mode, we can obtain one low-rank component, denoted $n_i(t)$. Here the filtered components are not corresponding to the denoised signal but the noise component, because only the noise rather than the effective signal has the low-rank characteristic. Hence, we can observe the SNR enhanced signal $\hat{x}_i(t)$ by subtracting the estimated noise $n_i(t)$ from the original trace $x(t)$. The optimal result is among the set $\{\hat{x}_i(t) | i \in (1, N)\}$, and how to choose the optimal parameters is discussed in the fourth part.

Repeat the above two steps to all the traces within the $m \times n$ seismic matrix and obtain a SNR enhanced record in the end.

In fact, there is an alternative to extract the low-rank components for noise separation, where the whole seismic record would be considered as an 2D matrix and decomposed with VMD for only once, thus also be estimated with Semi-Soft GoDec for only once. That idea comes from the recently published algorithm SSWT-OptShrink (Anvari et al., 2017), but the parameter of the low rank extraction is hard to select from a wide range of $(1, nN)$(Ma et al., 2019). However, in our simplified low rank selection strategy, the value range of the low rank parameter could be reduced to $(1, N)$ by increasing the number of VMD procedure by n times. In most experiments, the mode number of N is less than 10 so that it is possible to try all the rank values by enumeration and choose an optimal result from $\{\hat{x}_i(t) | i \in (1, N)\}$. In terms of computational cost, the simplified low rank extraction in this paper is much more time saving than the 2D-VMD implementation. For synthetic seismic data experiments, it usually takes a few minutes to accomplish, and for more complex field seismic records, the denoising process can be done within 20 min.

3. Examples and Results

A. Synthetic Desert Seismic Record

We constructed a 1400 × 50 synthetic record in Fig. 1a, where the dominant frequency of the reflected wavelets varies from 30 to 35 Hz and the sampling frequency is 500 Hz. The synthetic record contains several seismic reflection events with complex characteristics, including crossovers, linearity (flat events and steep events), curve, and discontinuity events. Then we add some synthetic random noise arises from the desert seismic model (G. Li et al., 2016), shown in Fig. 1(b), and make the SNR fall down to –8.52 dB in Fig. 1(c). In order to verify the performance of both reflected signal enhancement and noise reduction objectively, we make quantitative analyses of each record afterward with SNR and mean square error (MSE) as follows:

$$SNR = 10 \log \frac{\sum_i \sum_t |s(t,i)|^2}{\sum_i \sum_t |x(t,i) - s(t,i)|^2} \tag{5}$$

$$MSE = \frac{\sum_i \sum_t [s(t,i) - \hat{x}(t,i)]^2}{n \times m} \tag{6}$$

where $s(t,i)$, $x(t,i)$, $\hat{x}(t,i)$ are clean, noisy, and

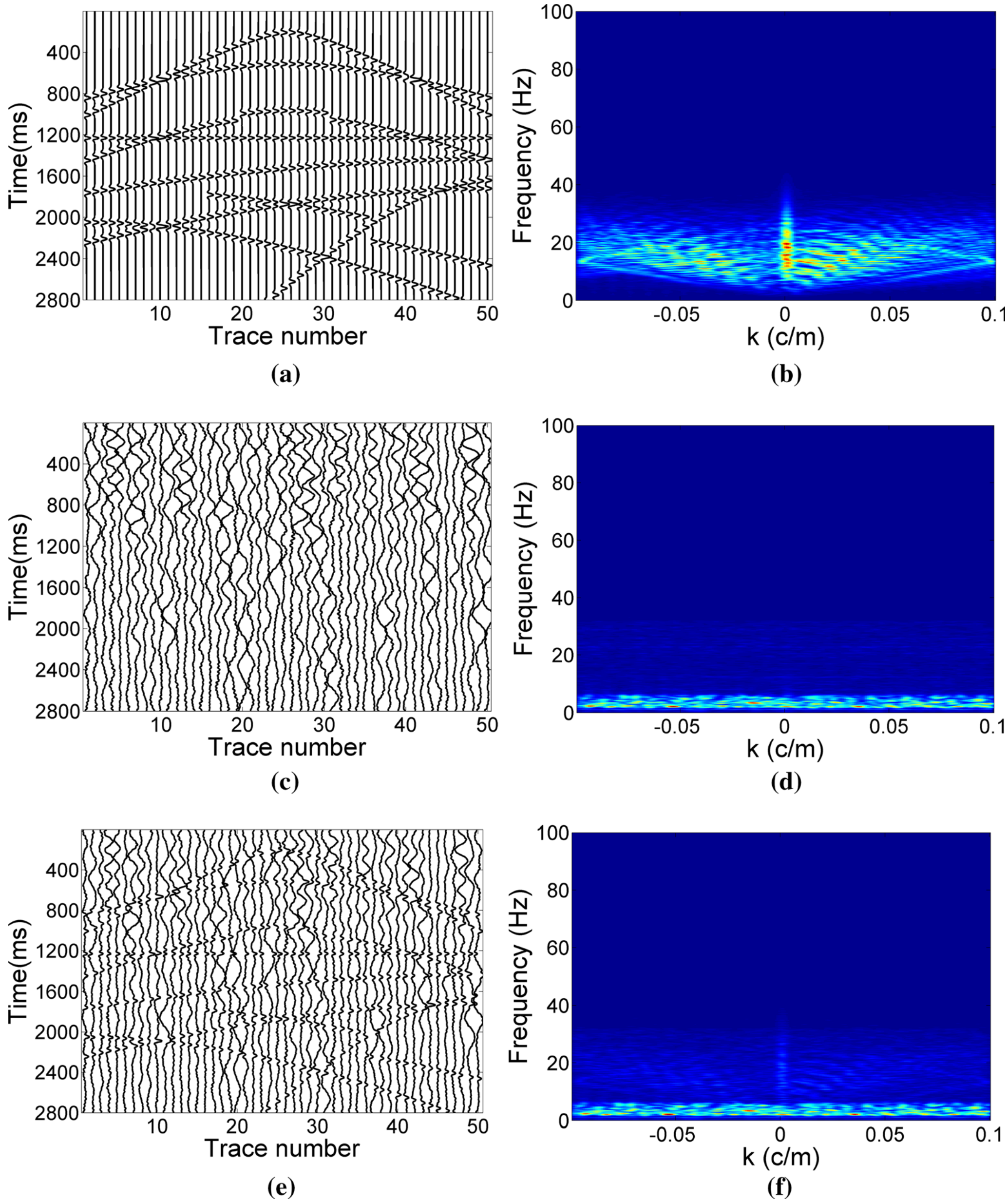


Figure 1
Synthetic desert seismic records and their FK spectra. **a**, **b** Synthetic pure desert seismic signals. **c**, **d** Synthetic desert seismic noise. **e**, **f** Synthetic noisy record with the additive noise (SNR = -8.52 dB)

denoised synthetic desert seismic data, respectively, t denotes the sample point, $t \in (1, m)$, and i denotes the trace number, $i \in (1, n)$.

f-x deconvolution together with the SSWT-OptShrink are selected as comparative experimental methods to the proposed VMD-SSGoDec, and the results are exhibited both in Fig. 2; Table 1. Here, the processing frequency band of *f-x* deconvolution is tuned to 10 Hz–50 Hz with the operator length of 25 sample points. The rank value of the SSWT-OptShrink is set to 12 when extracting the low-rank component. In the proposed algorithm, we first decompose the noisy

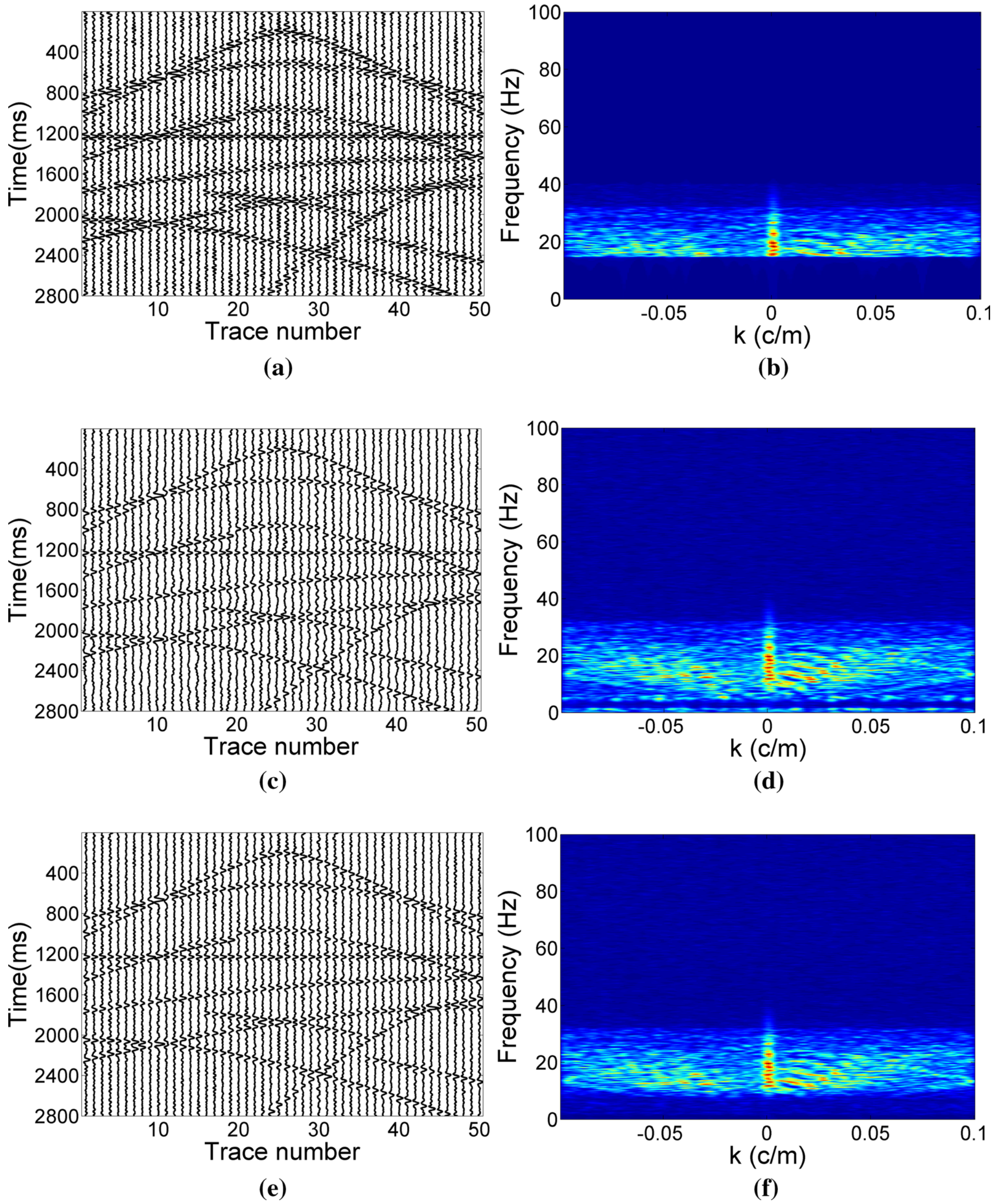


Figure 2
Noise attenuation results of synthetic desert seismic records and their FK spectra by utilizing three methods. **a**, **b** *f-x* deconvolution, **c**, **d** SSWT-OptShrink, and **e**, **f** the proposed VMD-SSGoDec method

record into 4 modes with VMD, and then extract the low-rank component with the rank of 1 in Semi-Soft GoDec procedure to obtain an optimal denoising performance. From the denoised figures and statistical data in Fig. 2; Table 1, we can see that all these methods can achieve noise attenuation, but to a different degree. From the frequency–wave number spectra (FK spectra), the denoising effect of *f-x* deconvolution is similar to a band-pass filter, where the energy loss of low frequency reflected signal is serious but the noise remains strong within other frequency bands. In SSWT-OptShrink results, the

Table 1

Comparison of denoising performance

Denoising Methods	*f-x* deconvolution	SSWT-OptShrink	VMD-SSGoDec
Output SNR (dB)	2.13	3.42	5.19
Output MSE	0.0359	0.0267	0.0178

overall residual noise seems less in Fig. 2(c) but it is clear that some low-frequency noise still retains in the FK spectrum, so that the SNR enhancement is secondary. In Fig. 2(e), much more noise is removed than the former two methods. The continuity and intactness of the reflection event are well recovered by the VMD-SSGoDec. The SNR is increased with a great improvement by 13 dB and the resolution is enhanced as well.

Figure 3 shows the differences and corresponding FK spectra of the above three denoising algorithms. Obviously, some reflection events remain in the residual noise after the *f-x* deconvolution in Fig. 3(a), and both the SSWT-OptShrink denoising method and the proposed VMD-SSGoDec can separate the low frequency noise from the noisy record, seen in Fig. 3(b, c), respectively.

To fix the detail, we also select a single trace (the 48th trace) of each seismic record and compare their amplitudes in Fig. 4. *f-x* deconvolution method results in serious energy loss (20–50%) of the Ricker wavelet in Fig. 4(a); SSWT-OptShrink performs better in preservation of the reflected signals when eliminating the low frequency noise; the proposed VMD-SSGoDec is superior in low frequency noise attenuation, especially in the interval of 0–700 ms. Also, as we mentioned before, since the strategy used in VMD-SSGoDec can more significantly decrease the difficulty in rank selection than in SSWT-OptShrink, the proposed method is undoubtedly a good choose in low frequency seismic noise attenuation.

We also repeat the synthetic experiments with various input SNR to test their performances in SNR enhancement and MSE descend, shown in Fig. 5. We can see that the denoising framework proposed in this paper is superior to the other two methods no matter whether the input SNR is down to −12 dB or up to 0 dB. When the input SNR is around 0 dB, the output SNR of VMD-SSGoDec is 7 dB, that is 2–4 dB higher than the other results; when the input SNR is as low as −11 dB, the output SNR of VMD-SSGoDec is increased by about 13 dB. Among the statistical results of MSE, the proposed VMD-SSGoDec also gives out the minimum value and shows its accuracy in reflected signal preservation. In general, the framework introduced in this paper is more effective in low frequency seismic noise reduction.

B. Field Desert Seismic Data

Figure 6 shows a field seismic record collected in desert area in northwest China, which has 210 traces, 2000 samples points in each trace with a sampling interval of 2 ms. The denoising results and residual noises are shown in Fig. 7.

The processing frequency band of the comparison method *f-x* deconvolution is still tuned to 10 Hz–50 Hz with the operator length of 25 sample points. The rank value of the SSWT-OptShrink algorithm is set to 60. When using the proposed VMD-SSGoDec to remove the noise, we still decompose the noisy record into 4 modes with VMD, and then extract the low-rank component with the rank of 1 in Semi-Soft GoDec procedure to obtain an optimal denoising performance. We can see that the proposed algorithm can almost cleanly attenuate the low-frequency seismic noise, including the surface waves in the field data. Unlike *f-x* deconvolution, the reflection event can be preserved with the proposed VMD-SSGoDec, seen in the red rectangle. Compared with the SSWT-OptShrink method, VMD-SSGoDec can remove more noise with low frequency characteristics. Therefore, more reflected signals are clearly revealed and both the continuity and the coherence of the seismic reflection events are improved in Fig. 7(e).

To summarize, the SNR enhancement method proposed in this paper is effective and efficient in attenuating the low-frequency seismic noise, and can achieve the requirements for improving the resolution of the seismic record.

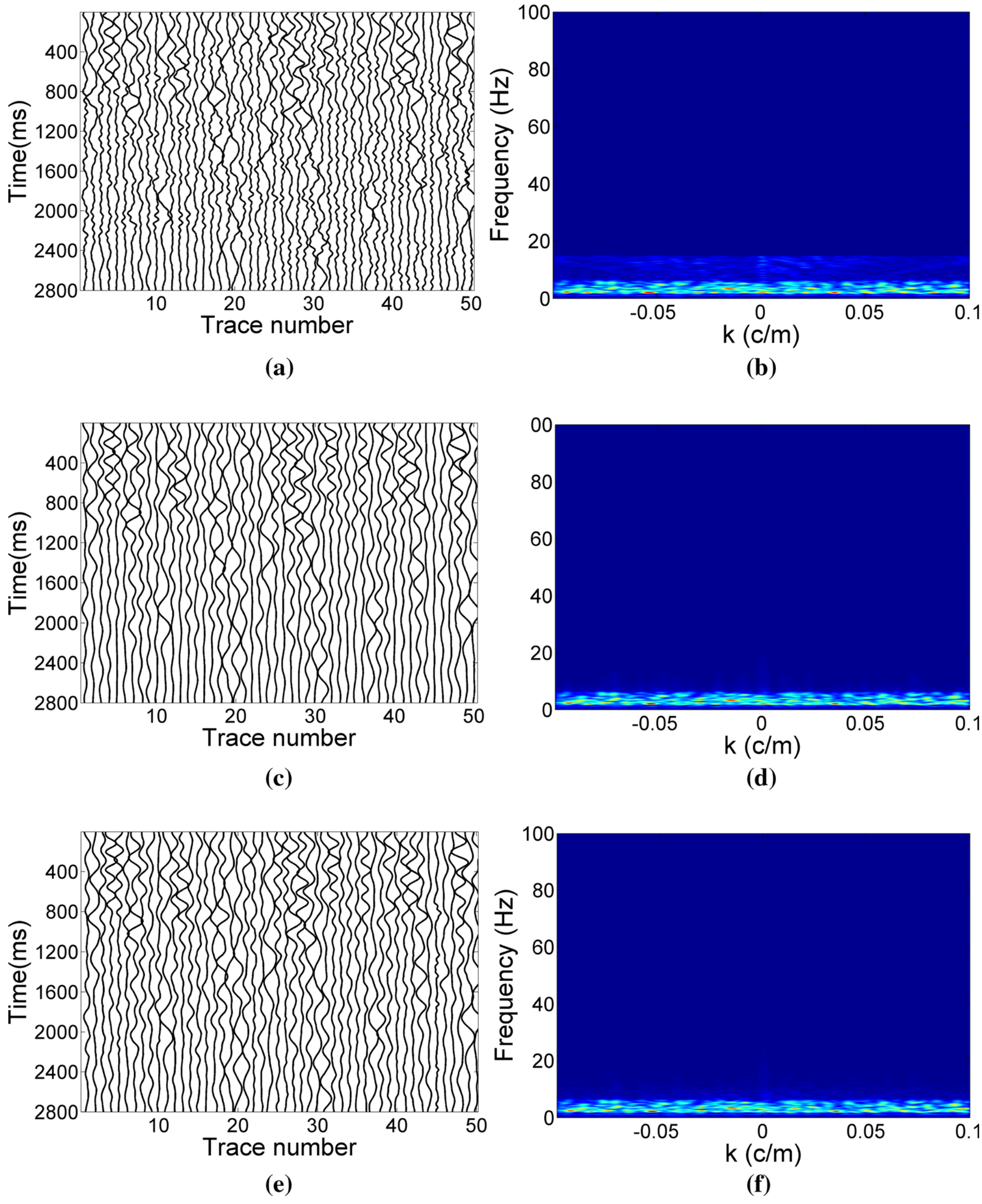


Figure 3
Comparison of residual noise in synthetic desert seismic records and the FK spectra. **a**, **b** *f-x* deconvolution, (**c**, **d**) SSWT-OptShrink, and (**e**, **f**) the proposed VMD-SSGoDec method

4. Discussion

According to the foregoing, the SNR enhancement method for desert seismic data denoising in this paper mainly relies on Semi-Soft GoDec algorithm in extracting the low-frequency noise components from the VMD representation. In this algorithm, we treat the 2D seismic record trace by trace, rather than applying the 2D-VMD only once to simplify the rank selection in the following Semi-Soft GoDec and save

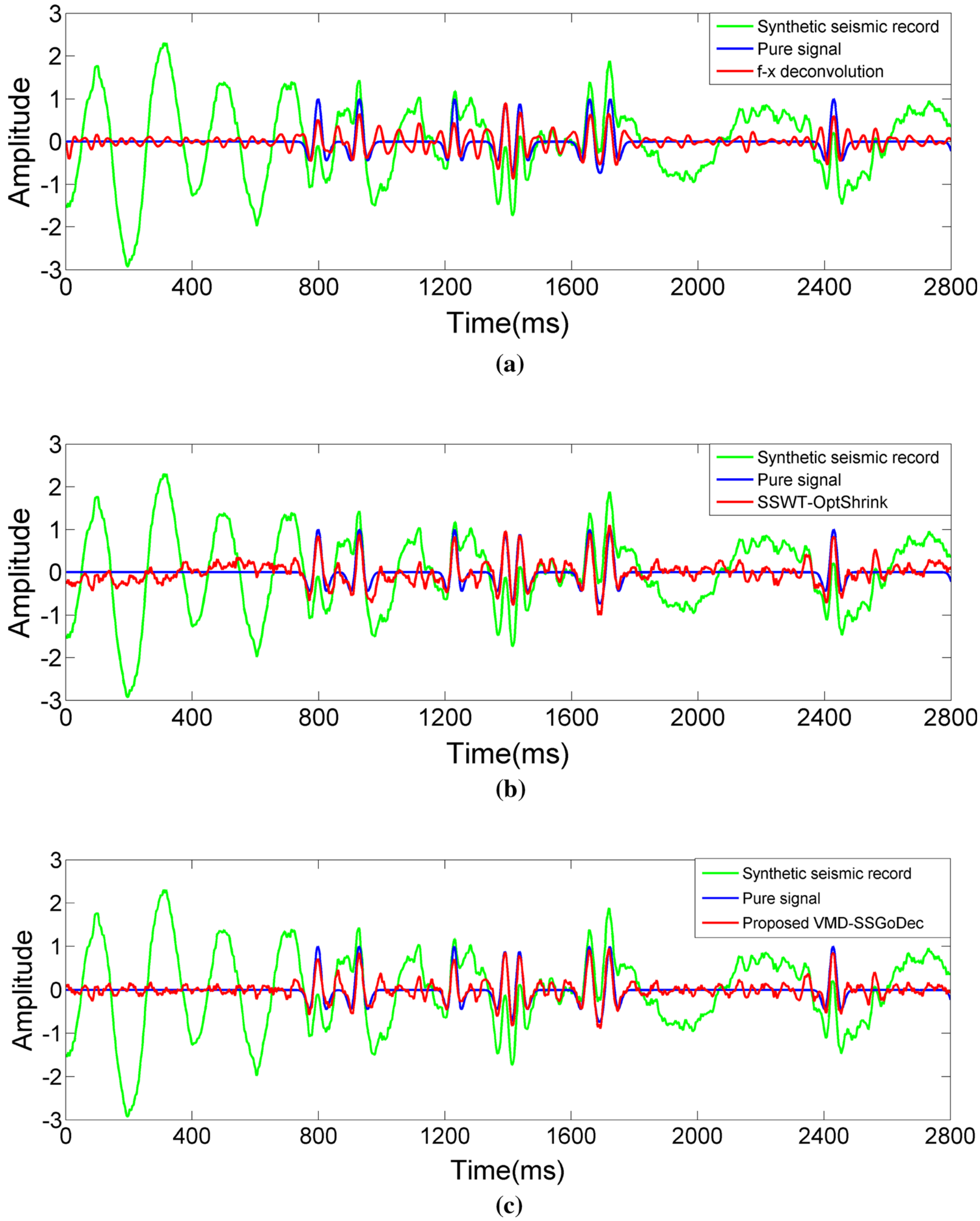
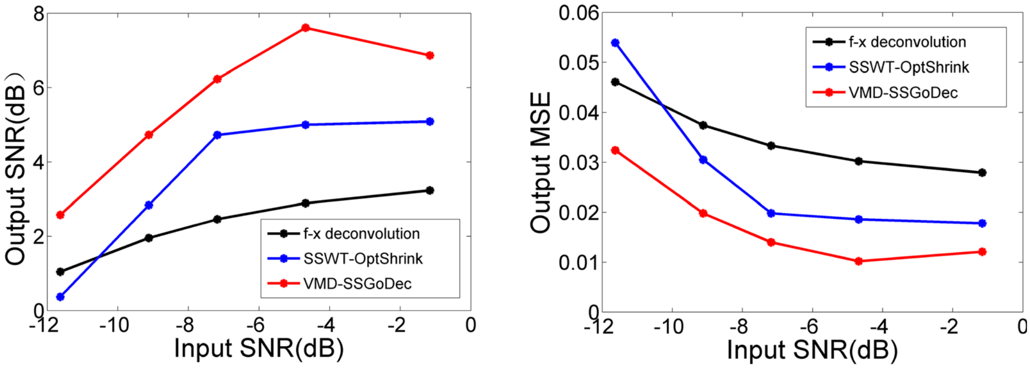


Figure 4

Amplitude comparison of reflected signals of the 48th trace after denoising. **a** *f-x* deconvolution, (**b**) SSWT-OptShrink, and (**c**) the proposed VMD-SSGoDec method

the overall computation cycles. In the above synthetic and field data experiments, we only demonstrate one result with relatively fixed parameters. Whereas here we want to discuss the two main parameters in the proposed method: the decomposition mode number in VMD and the rank value in Semi-Soft GoDec.

We repeat the synthetic experiment on Fig. 1(e), where the SNR is equal to -8.52 dB, with different mode numbers and rank values, and then exhibit the



(a) (b)

Figure 5
Denoising performance comparison of selected method in (**a**) output SNR and (**b**) output MSE under various input SNR

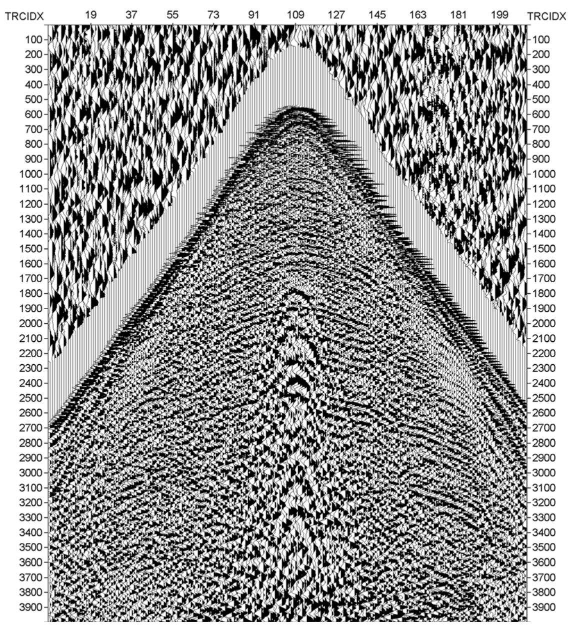


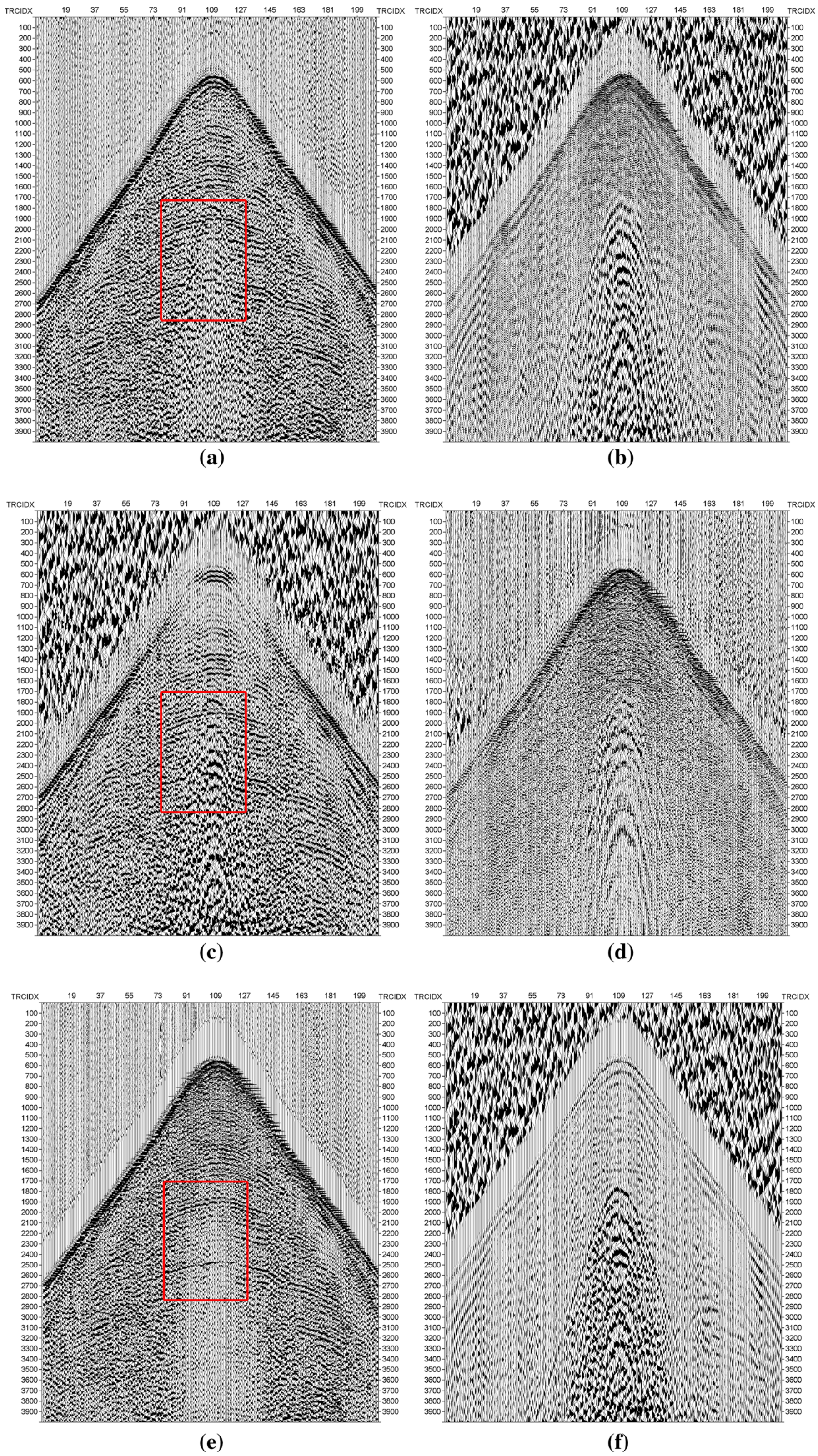
Figure 6
Field desert seismic record

output SNR and MSE in Tables 2, 3, respectively. In Table 2, no matter how much the mode number is, the output SNR would rise to the maximum value when the selected rank equals 1. In Table 3, no matter how much the mode number is, the output SNR would rise to the minimum value when the selected rank equals 1. When the rank value is set to 1, different mode numbers would not result in much fluctuations in the SNR or MSE. However, when the mode number in VMD is down to 3, though the result is acceptable somehow, the decomposition is incomplete and appears spectrum aliasing. On the contrary, if the mode number is too big, such as 7, it will cause the problem of over decomposition. Therefore, we select the medium mode number of 4 (5 is also fine) and the smallest rank value of 1 in the previous experiments to raise the SNR as much as possible and prevent the spectrum aliasing or over decomposition.

Another discussion is about the frequency band of the attenuated noise. This method only removes the noise that has the low-frequency (less than 10 Hz) characteristic in desert seismic signals. However, in the complex desert random noise, there are not only low-frequency noise, but also other types of high-frequency noise. This kind of high-frequency noise cannot be removed well by this method. In theory, the low-rank characteristics of high-frequency noise and reflected signal in VMD time–frequency representation domain still need to be explored. Perhaps the algorithm in this paper is imperfect and cannot achieve good implementation in high-frequency noise removal. More effort will be paid in the future to generalize this algorithm to other types of noises.

5. *Conclusion*

In this paper, we propose a SNR enhancement method VMD-SSGoDec, which utilizes VMD and Semi-Soft GoDec to extract the low-rank component and realize desert seismic data denoising in northwest


TRCIDX
(a)
(b)
(c)
(d)
(e)
(f)

◀Figure 7
Denoising results of the real desert seismic record and their residual noise by using (**a**, **b**) *f-x* deconvolution, (**c**, **d**) SSWT-OptShrink, and (**e**, **f**) the proposed VMD-SSGoDec method

Table 2

Relationship between parameter selection and the output SNR

Output SNR(dB)		Mode number				
		3	4	5	6	7
Selected rank	1	4.81	5.18	5.19	4.95	4.69
	2	0.02	1.12	1.27	1.35	1.93
	3	–	0.17	0.02	0.39	0.33
	4	–	–	− 0.07	− 0.10	− 0.17
	5	–	–	–	− 0.07	− 0.10
	6	–	–	–	–	− 0.05
	7	–	–	–	–	–

Table 3

Relationship between parameter selection and the output MSE

Output MSE		Mode number				
		3	4	5	6	7
Selected rank	1	0.0194	0.0178	0.0178	0.0188	0.0199
	2	0.0584	0.0454	0.0438	0.0430	0.0376
	3	–	0.0564	0.0584	0.0537	0.0543
	4	–	–	0.0597	0.0601	0.0611
	5	–	–	–	0.0597	0.0600
	6	–	–	–	–	0.0593
	7	–	–	–	–	–

China. Firstly, every trace of a seismic record is decomposed into several modes by VMD to restructure a signal matrix. Secondly, use Semi-Soft GoDec to extract the noise components of the matrix, and obtain the denoised signal by subtracting the estimated noise from the original trace. Repeat the above two steps until the final trace is finished. An advantage of this method is to simplify the parameter selection problem by using the VMD-SSGoDec trace by trace, which makes a comparatively small range of the rank selection in Semi-soft GoDec. Experiments show that VMD-SSGoDec can save much time while maintaining the performance of denoising comparing with another low rank extraction method SSWT-OptShrink. Experiment of synthetic and field desert seismic data also demonstrate the superiority in both low frequency noise reduction and reflected signal preservation than some conventional and state-of-the-art algorithms, especially in SNR enhancement.

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