



Calculation of Confidence Intervals for the Maximum Magnitude of Earthquakes in Different Seismotectonic Zones of Iran

MONA SALAMAT,¹ MEHDI ZARE,¹ MATTHIAS HOLSCHNEIDER,² and GERT ZÖLLER²

Abstract—The problem of estimating the maximum possible earthquake magnitude m_{\max} has attracted growing attention in recent years. Due to sparse data, the role of uncertainties becomes crucial. In this work, we determine the uncertainties related to the maximum magnitude in terms of confidence intervals. Using an earthquake catalog of Iran, m_{\max} is estimated for different predefined levels of confidence in six seismotectonic zones. Assuming the doubly truncated Gutenberg–Richter distribution as a statistical model for earthquake magnitudes, confidence intervals for the maximum possible magnitude of earthquakes are calculated in each zone. While the lower limit of the confidence interval is the magnitude of the maximum observed event, the upper limit is calculated from the catalog and the statistical model. For this aim, we use the original catalog which no declustering methods applied on as well as a declustered version of the catalog. Based on the study by Holschneider et al. (Bull Seismol Soc Am 101(4):1649–1659, 2011), the confidence interval for m_{\max} is frequently unbounded, especially if high levels of confidence are required. In this case, no information is gained from the data. Therefore, we elaborate for which settings finite confidence levels are obtained. In this work, Iran is divided into six seismotectonic zones, namely Alborz, Azerbaijan, Zagros, Makran, Kopet Dagh, Central Iran. Although calculations of the confidence interval in Central Iran and Zagros seismotectonic zones are relatively acceptable for meaningful levels of confidence, results in Kopet Dagh, Alborz, Azerbaijan and Makran are not that much promising. The results indicate that estimating m_{\max} from an earthquake catalog for reasonable levels of confidence alone is almost impossible.

Key words: Maximum magnitude of earthquake, Level of confidence, Confidence interval.

1. Introduction

Despite of the essential role of precise estimation of the maximum possible earthquake magnitude m_{\max} in

earthquake engineering and seismic hazard assessment, only a few studies with sufficient accuracy are available. The knowledge of m_{\max} is, however, crucial for purposes of seismic hazard assessment. Methods for evaluating m_{\max} can be divided into probabilistic approaches (Kijko and Singh 2011) and deterministic approaches as by Wells and Coppersmith (1994), Wheeler (2009) and Muller (2010). In a simplest way, the magnitude of the largest historic event maybe in combination with a small increment accounting for uncertainties is assumed to be the maximum possible magnitude (Reiter 1991). In other empirical methods, m_{\max} is roughly estimated from geological and paleoseismic data (Wheeler 2009). Many statistical procedures based on probabilistic models and earthquake catalogs are used to evaluate the parameter m_{\max} (Kagan and Schoenberg 2001; Kagan and Jackson 2013; Pisarenko 1991; Pisarenko et al. 1996). Kagan and Jackson (2013) assume the Pareto distribution and use the moment conservation principle in combination with tectonic data. Different point estimators of m_{\max} are explained by Kijko and Singh (2011) for parametric and non-parametric procedures. For example, Kijko and Singh (2011) present various point estimators for m_{\max} for the Gutenberg–Richter distribution as well as for an unspecified distribution. The best point estimators with low variance can be constructed using theoretical results from statistics (Pisarenko 1991). Pisarenko et al. (1996) derived the unbiased point estimator of the maximum magnitude m_{\max} with the lowest possible variance. Considering the pros and cons of different point estimators, a proper treatment of the corresponding uncertainties is, however, missing. A meaningful way to quantify uncertainties is the calculation of confidence intervals of m_{\max} from the frequentist and the Bayesian point of view. As

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discussed by Holschneider et al. (2011), the frequentist confidence interval diverges in many settings leading to the trivial result that m_{\max} lies between the maximum observed earthquake and infinity. For the Bayesian approach (Cornell 1994), it is found that the posterior distribution depends predominantly on the arbitrarily chosen prior distribution.

In the current work, we focus on the calculation of the confidence interval of m_{\max} based on a predefined confidence level in different seismotectonic zones of Iran from the frequentist point of view. The goal is to identify settings where information on m_{\max} from the catalog is gained. For a high confidence level, or low error probability, there will be no information gain, because the amount of data is too small. Therefore, we will specify the limit of the confidence level that leads to an information gain for the available data.

Based on geological and tectonic settings, Iran is divided into six seismotectonic zones, namely Alborz, Azerbaijan, Zagros, Central Iran, Makran and Kopeh Dagh (Karimiparidari et al. 2013). Seismic events in each zone are gathered based on the national and international combined earthquake catalogs extended from 1900 to 2015. The upper bound of confidence intervals for predefined levels of confidence is calculated in the original and declustered sub-catalogs of each seismotectonic zone. This allows to evaluate the influence of a possible violation of the assumption of the independence of events.

The remainder of the paper is structured as follows: in the next section, we describe the seismotectonic features of the studied region. Then we explain the methodology to determine the uncertainty of the maximum magnitude in terms of confidence intervals. Finally, we present results for different levels of completeness in each zone and summarize our findings.

2. Seismotectonic Features of the Studied Region

The existence of numerous active faults and the history of earthquake casualties (Ambraseys and Melville 1982) present Iran as one of the most hazardous countries in the world. The distribution of active faults and earthquake occurrences in Iranian seismotectonic zones are shown in Figs. 1 and 2, respectively.

Considering the geological and tectonic features of the region and former developed maps by Mirzaei et al. (1998) and Tavakoli (1996), Iran is divided into six seismotectonic zones (Karimiparidari et al. 2013) which are shown in Fig. 1. The Alborz seismotectonic zone which is extended through the Caspian sea is an active region of complex crustal deformation with the high level of seismicity which experienced lots of destructive historic earthquakes. Besides several major faults such as Rudbar and Ipak which trend east–west generally, this zone includes some significant minor faults.

One of the seismically most active intracontinental ranges is the Zagros fold and thrust belt which is the significant element in the active tectonic of the middle east (Talebian and Jackson 2004; Nissen et al. 2011). The Iranian central seismic zone as one of the most tectonic region consists of Tabas, Lut, Posht-Badam and Yazd blocks, is restricted by the Zagros in southwest, the Alborz in north, the Kopeh Dagh in northeast and the Makran in southeastern part. The Doruneh fault system with about 600 Km length is known as one of the notable active fault in this zone (Karimiparidari et al. 2013). The Azerbaijan seismic zone as one of the most active and seismic regions which is located in the north western part of Iran experienced lots of damaging earthquakes. The 2012 Ahar-Varzaghan double earthquakes with the magnitude of M_w 6.5 and M_w 6.2 is one of the recent examples which caused lots of casualties in this region (Donner et al. 2013). The Kopeh Dagh seismic region located in the northeastern part shows 6.5 ± 2 mm/year of the shortening (Vernant et al. 2004). The only subduction zone which is the result of subducting the oceanic portion of the Arabian beneath the Eurasia is the Makran subduction zone which seismicity is low at present. The 1945 earthquake with the magnitude M_s 8.0 (Ambraseys and Melville 1982) which was reported in this region is the strongest earthquake in the Iranian catalog.

To investigate m_{\max} , a homogenized catalog is collected from 1900 to 2015. For this purpose, national and international data are used (Karimiparidari et al. 2013; Shahvar et al. 2013). The Institute of the Iranian Geophysics (IGUT 2015) and the International Institute of Earthquake Engineering and Seismology (IIEES 2015) are used as national data banks and the International Seismological Center UK (ISC 2015b),

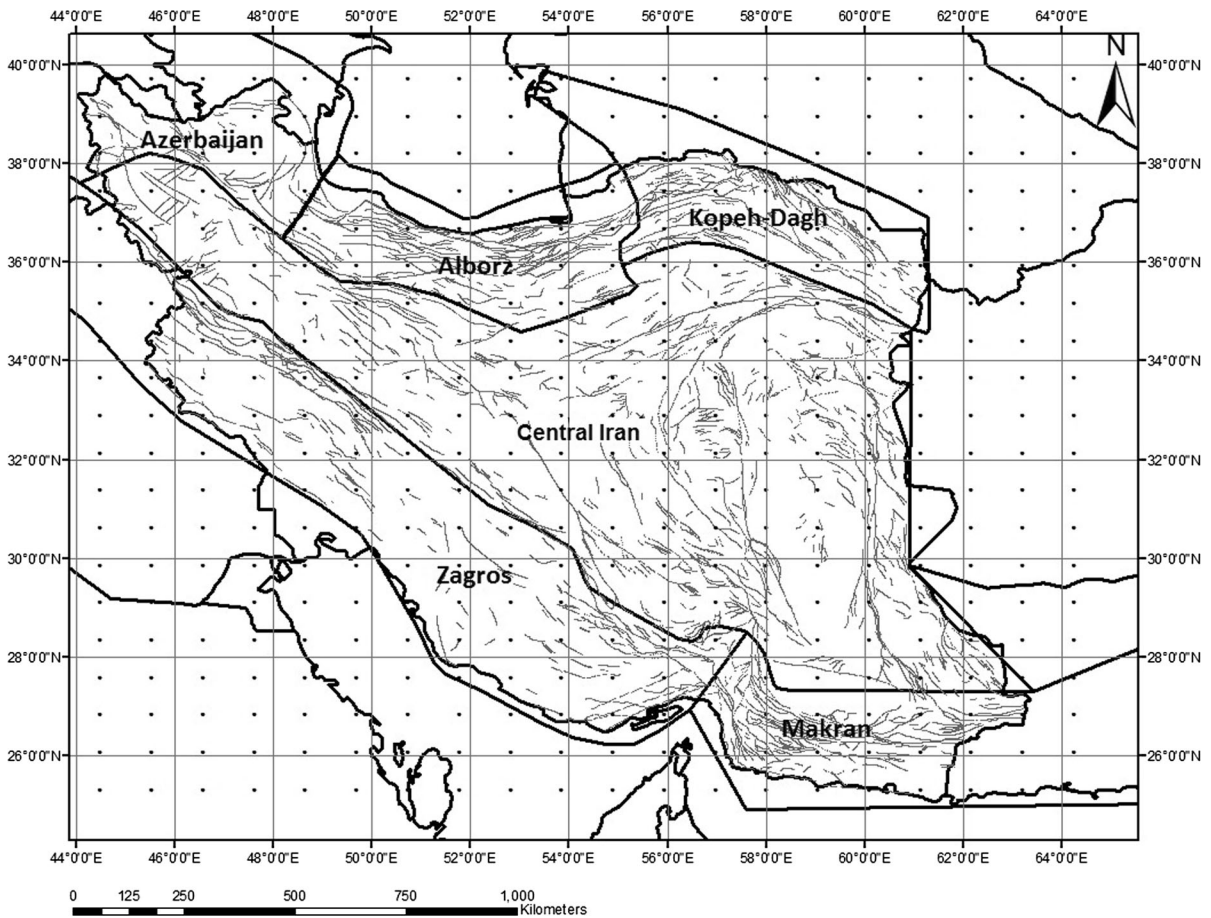


Figure 1
Distribution of active faults in six seismotectonic zones of Iran

The National Earthquake Information Center (NEIC 2015) and EHB (ISC 2015a) are used as international data banks. Table 1 shows earthquakes with $M_w \geq 7$ in different seismotectonic zones of Iran.

All magnitudes in the catalog are converted to the moment magnitude M_w (Shahvar et al. 2013). The final catalog covers events between $24^\circ\text{--}42^\circ\text{N}$ and $44^\circ\text{--}64^\circ\text{E}$, from 1900 to 2015 and consists of 28,979 events with a magnitude range between M_w 4.0–8.1. Figure 2 shows the distribution of earthquakes in six seismotectonic zones. To evaluate the influence of potential dependence of events, a declustered version of the catalog is studied in addition to the original catalog. For this purpose, the declustering method of Grünthal (1985) is used. The result of the declustering is shown in Table 2.

3. Methodology

In the current work, the doubly truncated Gutenberg–Richter model is assumed as the statistical model for the earthquake magnitude distribution.

The probability density function is defined by

$$f_{\beta m_{\max}}(m) = \begin{cases} 0 & m < m_0 \\ \frac{\beta \exp(-\beta m)}{\exp(-\beta m_0) - \exp(-\beta m_{\max})} & m_0 \leq m \leq m_{\max} \\ 0 & m > m_{\max} \end{cases} \quad (1)$$

with $\beta = \log(10) \times b$, and b is the Gutenberg–Richter b value (Gutenberg and Richter 1956) which typically varies between 0.6 and 1.5. Since the instrumental part of the catalog is used and the effect of the binning magnitude is considered negligible

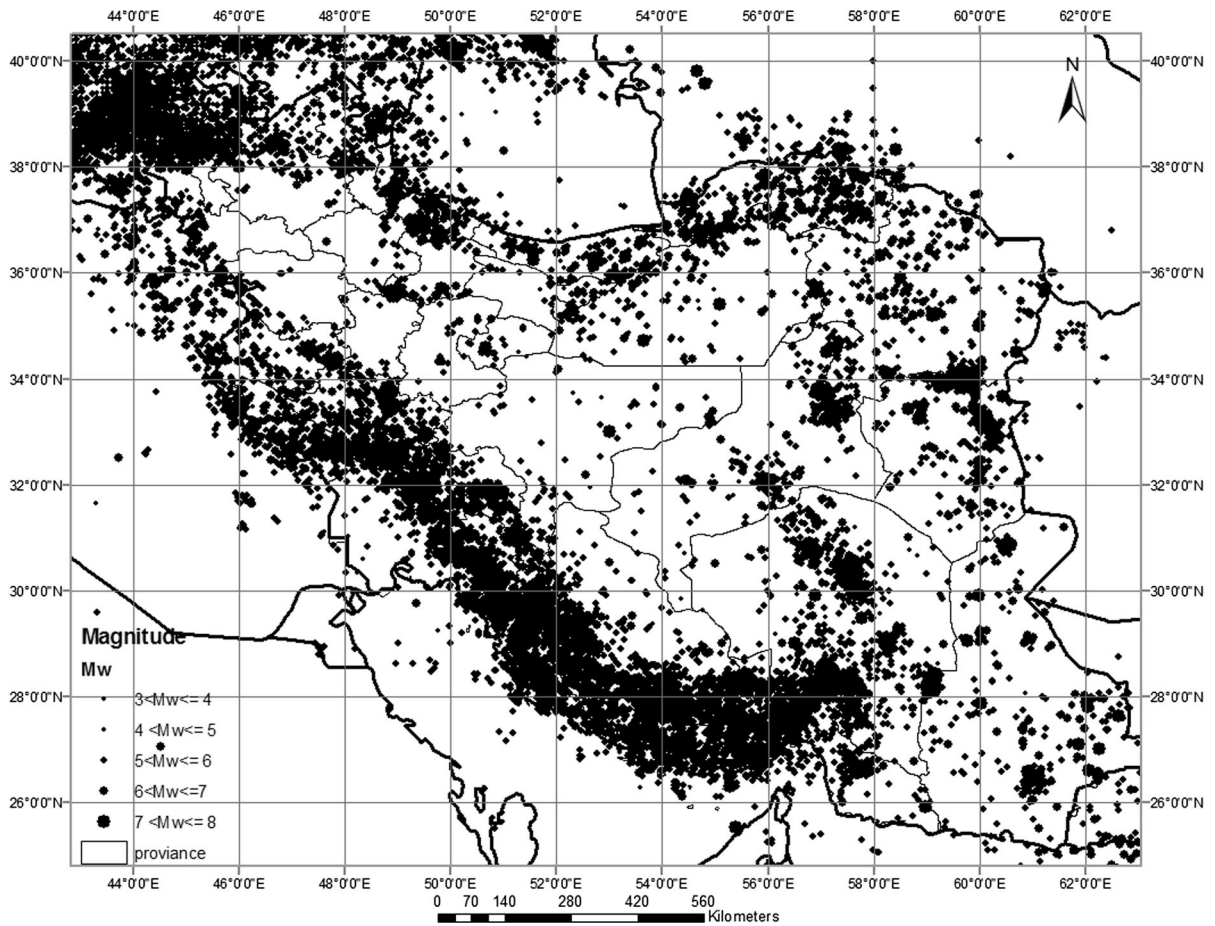


Figure 2

Distribution of earthquake occurrences in six seismotectonic zones of Iran before declustering the catalog

Table 1

Number of earthquakes with $M_w > 7$ in different seismotectonic zones of Iran between 1900 and 2015

Zone	Date	Time	Long.	Lat.	Depth (Km)	Moment magnitude M_w	Earthquake
Alborz	1962-09-01	19:20:50	49.81	35.71	10	7.2	Buin Zahra
	1990-06-21	00:30:14	49.41	36.96	15	7.3	Manjil Rudbar
Azerbaijan	1930-05-06	22:34:00	44.60	38.20	35	7.1	Salmas
	1976-11-24	12:22:20	44.04	39.12	22	7.3	Caldiran-Muradiye
Koppeh Dagh	1929-05-01	15:37:30	57.80	37.80	10	7.2	Koppeh Dagh
	1948-10-05	20:12:10	58.40	37.79	60	7.1	Ashgabat
Makran	1945-11-27	21:56:01	63.47	25.02	15	8.0	Balochistan
	2013-04-16	10:44:18	62.03	27.88	70	7.7	Saravan
Central Iran	1997-05-10	12:57:29	59.81	33.83	10	7.3	Qayen
	1968-08-31	10:47:39	58.97	34.11	25	7.2	Dashte-Bayaz
	1978-09-16	15:35:59	57.43	33.39	10	7.4	Tabas
	1979-11-27	17:10:30	59.73	33.96	33	7.1	Ardekul
	1909-01-23	17:10:30	53.00	33.00	33	7.4	Silakhor
	1981-07-28	17:22:20	57.79	30.01	14	7.1	Sirch

Table 2

Number of removed and remained events after using the Grünthal declustering method

Zone	Total number of events	Number of clusters	Number of events in final catalog	Number of events out of catalog
Zagros	8256	997	1509	6747 (81.72%)
Azerbaijan	7153	792	1555	5598 (78.26%)
Central Iran	2634	321	701	1933 (73.39%)
Alborz	840	149	270	570 (67.85%)
Makran	610	114	221	389 (63.77%)
Kopeh Dagh	586	119	232	354 (60.41%)

Table 3

The highest level of confidence for $m_0 = 4.8, 5.0, 5.5$ in original catalog of six seismotectonic zones of Iran

Zone	m_0	Number	\bar{m}	b -value	m_{max}	Level of confidence ($1 - \alpha_c$)
Alborz	4.8	243	5.10	1.46	7.3	0.05
	5.0	106	5.37	1.17	7.3	0.20
	5.5	28	5.99	0.88	7.3	0.52
Azerbaijan	4.8	853	5.08	1.56	7.3	0.20
	5.0	394	5.31	1.39	7.3	0.22
	5.5	78	5.90	1.08	7.3	0.59
Kopeh Dagh	4.8	169	5.20	1.07	7.2	0.37
	5.0	96	5.45	0.97	7.2	0.51
	5.5	31	5.99	0.88	7.2	0.63
Makran	4.8	220	5.14	1.27	8.0	0.02
	5.0	106	5.40	1.03	8.0	0.08
	5.5	33	5.99	0.89	8.0	0.18
Central Iran	4.8	664	5.14	1.28	7.4	0.27
	5.0	303	5.44	0.99	7.4	0.72
	5.5	98	6.04	0.79	7.4	0.96
Zagros	4.8	2309	5.05	1.77	6.9	0.36
	5.0	955	5.27	1.61	6.9	0.57
	5.5	170	5.84	1.28	6.9	0.94

(Marzocchi and Sandri 2003), the parameter β is estimated according to Aki (1965)

$$\beta = \frac{1}{\bar{m} - m_0}, \tag{2}$$

with

$$\bar{m} = \frac{1}{N} \sum_{i=1}^n m_i. \tag{3}$$

The lower magnitude bound m_0 is assumed to be known and is identified with the completeness magnitude of the earthquake catalog. The upper magnitude m_{max} is the parameter to be estimated.

The cumulative distribution function of the doubly truncated Gutenberg–Richter relation is defined as

$$F_{\beta m_{max}}(m) = \int_{m_0}^m f(x) dx = \frac{\exp(-\beta m_0) - \exp(-\beta x)}{\exp(-\beta m_0) - \exp(-\beta m_{max})}; \tag{4}$$

$$m_0 \leq m \leq m_{max}.$$

The likelihood function for n observed magnitudes m_i ($i \in 1, \dots, n$) is

$$L(\{m_1, m_2, \dots, m_n\} | \beta, m_{max}) = \frac{\beta^n \exp(-\beta n \bar{m})}{[\exp(-\beta m_0) - \exp(-\beta m_{max})]^n}; \tag{5}$$

$$m_0 \leq m \leq m_{max}.$$

The probability density function of the random variable $\mu = \max\{m_i\}$ for n independent earthquake magnitudes is

$$Pr(\mu \leq z) = [F_{\beta m_{max}}(z)]^n. \tag{6}$$

Table 4

The highest confidence level for $m_0 = 4.8, 5.0, 5.5$ in declustered catalog of six seismotectonic zones of Iran

Zone	m_0	Number	\bar{m}	b -value	m_{\max}	Level of confidence ($1 - \alpha_c$)
Alborz	4.8	112	5.163	1.19	7.3	0.11
	5.0	62	5.43	1.00	7.3	0.27
	5.5	20	6.04	0.80	7.3	0.52
Azerbaijan	4.8	394	5.13	1.32	7.3	0.18
	5.0	229	5.33	1.29	7.3	0.22
	5.5	56	5.92	1.03	7.3	0.55
Kopeh Dagh	4.8	117	5.18	1.15	7.2	0.18
	5.0	62	5.49	0.89	7.2	0.5
	5.5	24	6.02	0.83	7.2	0.61
Makran	4.8	130	5.16	1.20	8.0	0.02
	5.0	71	5.4	1.00	8.0	0.07
	5.5	22	6.03	0.82	8.0	0.18
Central Iran	4.8	282	5.24	0.98	7.4	0.55
	5.0	170	5.51	0.84	7.4	0.81
	5.5	69	6.06	0.77	7.4	0.91
Zagros	4.8	771	5.13	1.30	6.9	0.76
	5.0	462	5.33	1.30	6.9	0.79
	5.5	133	5.82	1.36	6.9	0.81

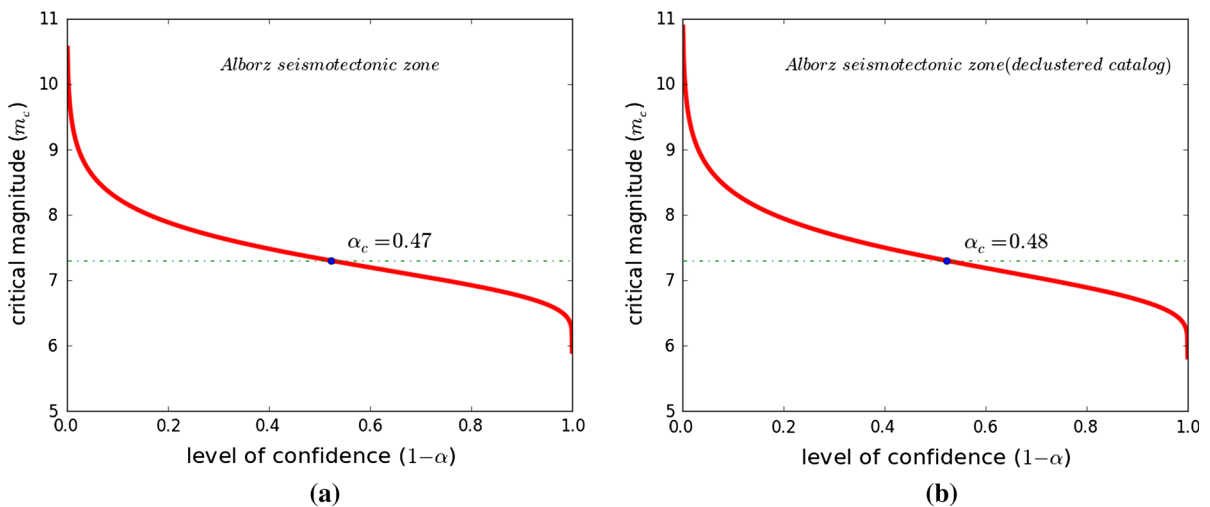


Figure 3

Illustration of Eq. (7) with $m_0 = 5.5, b = 0.88, n = 28$ in (a) original and $m_0 = 5.5, b = 0.8, n = 20$ in (b) declustered catalogs of the Alborz seismotectonic zone. The point indicates the highest confidence level ($1 - \alpha_c$) and the lowest probability of error (α_c) in which the confidence interval is finite

Because of a possible violation of the assumption of independent events, we also decluster the catalog using the method of Grünthal (1985) and apply our analysis to both, the original catalog and the declustered catalog. The parameters of the catalogs are

shown in Table 2. The result indicates that Zagros and Kopeh Dagh seismotectonic zones have the most and the least clusters, respectively.

We now determine the uncertainty of the maximum magnitude in terms of confidence intervals for

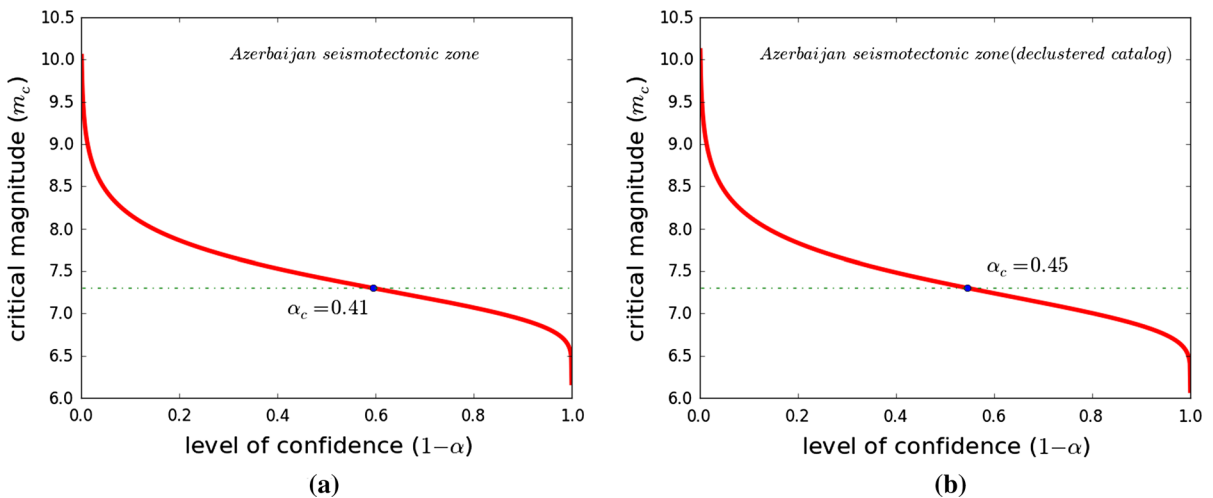


Figure 4

Illustration of Eq. (7) with $m_0 = 5.5, b = 1.08, n = 78$ in (a) original and $m_0 = 5.5, b = 1.03, n = 56$ in (b) declustered catalogs of the Azerbaijan seismotectonic zone. The point indicates the highest confidence level ($1 - \alpha_c$) and the lowest probability of error (α_c) in which the confidence interval is finite

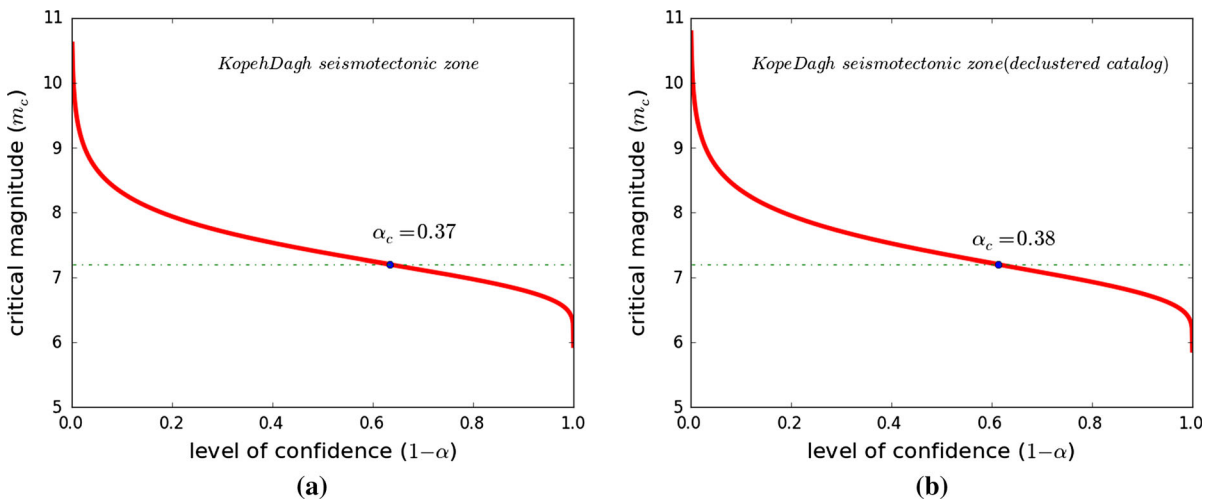


Figure 5

Illustration of Eq. (7) with $m_0 = 5.5, b = 0.88, n = 31$ in (a) original and $m_0 = 5.5, b = 0.83, n = 24$ in (b) declustered catalogs of the Kopeh Dagh seismotectonic zone. The point indicates the highest confidence level ($1 - \alpha_c$) and the lowest probability of error (α_c) in which the confidence interval is finite

predefined levels of confidence $1 - \alpha$. Previous works by Holschneider et al. (2011) and Pisarenko (1991) show that the frequentist confidence interval for high levels of confidence $1 - \alpha$ corresponding to low error probabilities α is often unlimited. This leads to the trivial result that m_{\max} lies between μ and ∞ . In

general, the confidence interval can be expressed as $[\mu, \psi]$ with $\mu = \max\{m_i\}$ and $\psi = \psi[\mu(m_1, \dots, m_n)]$.

Based on Holschneider et al. (2011) and Pisarenko (1991), there is a critical magnitude m_c

$$m_c = m_0 - \beta^{-1} \log(1 - \alpha^{\frac{1}{n}}), \tag{7}$$

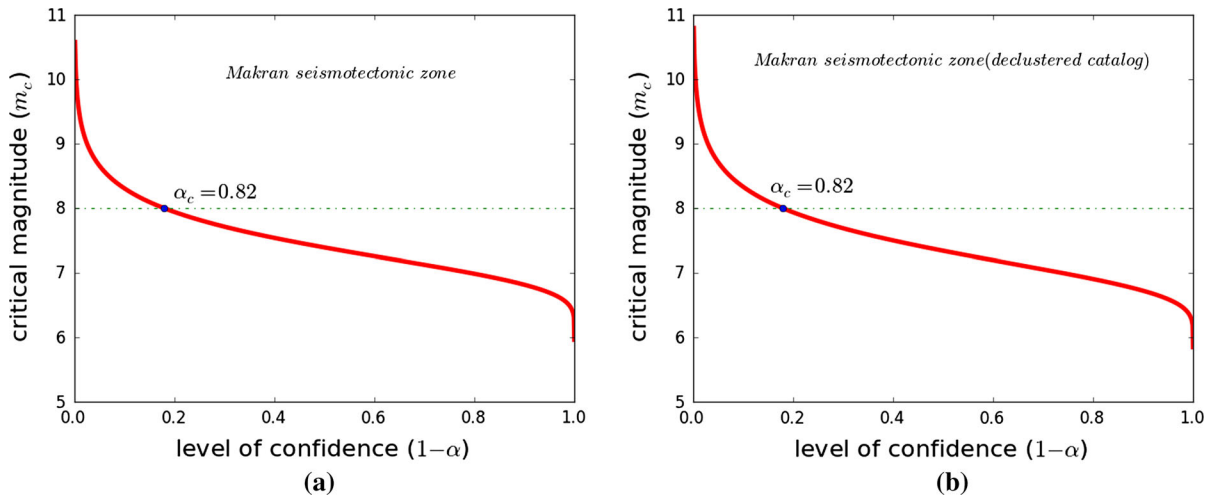


Figure 6

Illustration of Eq. (7) with $m_0 = 5.5, b = 0.89, n = 33$ in (a) original and $m_0 = 5.5, b = 0.82, n = 22$ in (b) declustered catalogs of the Makran seismotectonic zone. The *point* indicates the highest confidence level ($1 - \alpha_c$) and the lowest probability of error (α_c) in which the confidence interval is finite

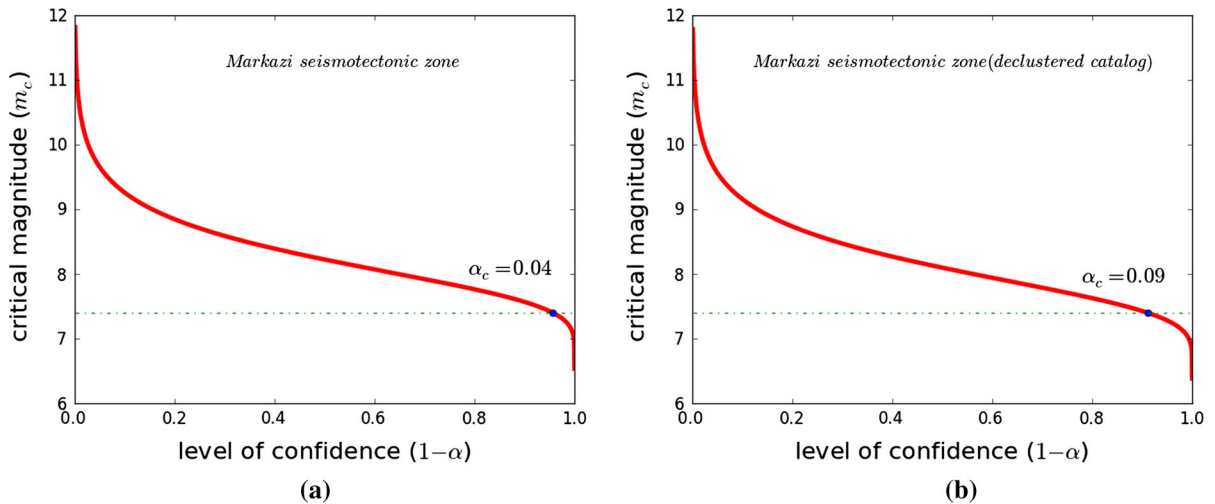


Figure 7

Illustration of Eq. (7) with $m_0 = 5.5, b = 0.79, n = 98$ in (a) original and $m_0 = 5.5, b = 0.77, n = 69$ in (b) declustered catalogs of the Central Iran seismotectonic zone. The *point* indicates the highest confidence level ($1 - \alpha_c$) and the lowest probability of error (α_c) in which the confidence interval is finite

such that for $\mu > m_c$ the upper bound of the confidence interval diverges. For $\mu < m_c$, this upper bound is

$$\psi(\mu) = m_0 - \beta^{-1} \log \left[\frac{\exp[-\beta(\mu - m_0)] - 1}{\alpha_c^{\frac{1}{n}}} + 1 \right]. \quad (8)$$

The right side of Eq. (8) shows the smallest confidence interval in agreement with the findings of Pisarenko (1991). Pisarenko (1991) uses Eq. (8) to construct the so-called fiducial $\psi_\mu(m_{\max})$. Using Eq. (7), the highest confidence level, $1 - \alpha_c$, for which the confidence interval is finite, is calculated

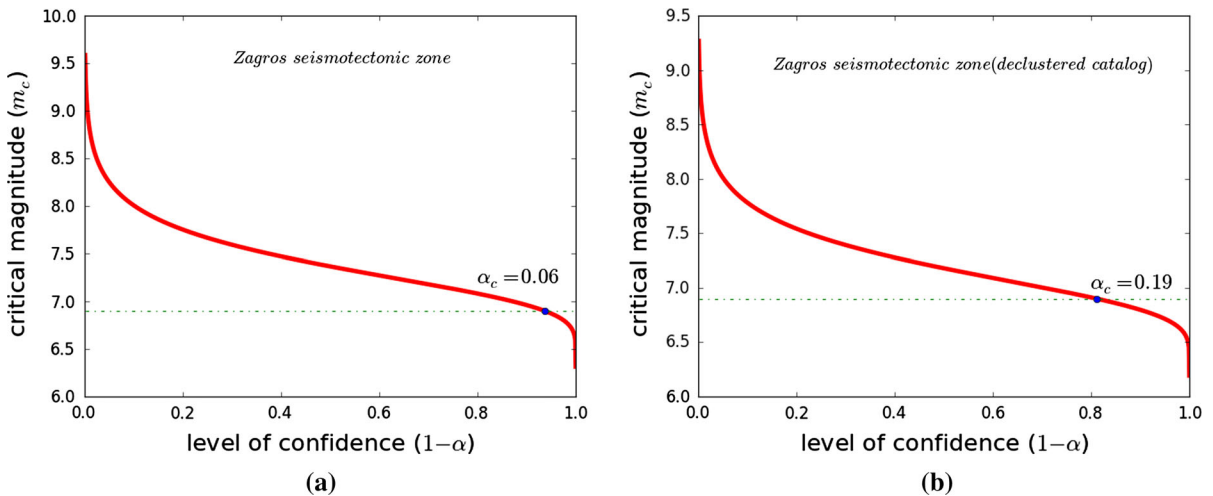


Figure 8

Illustration of Eq. (7) with $m_0 = 5.5, b = 1.28, n = 170$ in (a) original and $m_0 = 5.5, b = 1.36, n = 133$ in (b) declustered catalogs of the Zagros seismotectonic zone. The point indicates the highest confidence level ($1 - \alpha_c$) and the lowest probability of error (α_c) in which the confidence interval is finite

Table 5

Different levels of confidence and confidence intervals in original catalogs of six seismotectonic zones of Iran

Zone	Magnitude of completeness (m_0)	Level of confidence ($1 - \alpha$)	Maximum observed magnitude	Upper bound of the confidence interval $\psi(\mu)$
Alborz	5.5	0.5	7.3	8.66
		0.45	7.3	8.11
		0.4	7.3	7.87
Azerbaijan	5.5	0.58	7.3	8.59
		0.55	7.3	8.16
		0.5	7.3	7.88
Koppeh Dagh	5.5	0.6	7.2	8.38
		0.55	7.2	7.97
		0.5	7.2	7.77
Makran	5.5	–	8.0	–
		–	8.0	–
		–	8.0	–
Central Iran	5.5	0.9	7.4	8.11
		0.85	7.4	7.9
		0.80	7.4	7.79
Zagros	5.5	0.9	6.9	7.5
		0.85	6.9	7.29
		0.8	6.9	7.19

for the original and the declustered catalog in the six seismotectonic zones of Iran. For some values less than $1 - \alpha_c$, the finite confidence interval is calculated based on the information gained from the earthquake catalog. These calculations have been performed for the magnitude of completeness equal to 4.8, 5.0, 5.5, and the result is shown in Tables 3

and 4. Figures 3, 4, 5, 6, 7 and 8 show the results for the highest level of confidence in both catalogs with the magnitude of completeness equal to 5.5. The upper bound of the confidence interval is calculated for different levels of confidence less than the critical one, $1 - \alpha_c$; results are provided in Tables 5 and 6.

Table 6

Different levels of confidence and confidence intervals in declustered catalog of six seismotectonic zones of Iran

Zone	Magnitude of completeness (m_0)	Level of confidence ($1 - \alpha$)	Maximum observed magnitude	Upper bound of the confidence interval $\psi(\mu)$
Alborz	5.5	0.5	7.3	8.79
		0.45	7.3	8.18
		0.4	7.3	7.93
Azerbaijan	5.5	0.5	7.3	8.18
		0.45	7.3	7.89
		0.4	7.3	7.74
Kopeh Dagh	5.5	0.6	7.2	8.93
		0.55	7.2	8.15
		0.5	7.2	7.87
Makran	5.5	–	8.0	–
		–	8.0	–
		–	8.0	–
Central Iran	5.5	0.85	7.4	8.25
		0.80	7.4	8.0
		0.75	7.4	7.87
Zagros	5.5	0.8	6.9	7.96
		0.75	6.9	7.46
		0.7	6.9	7.31

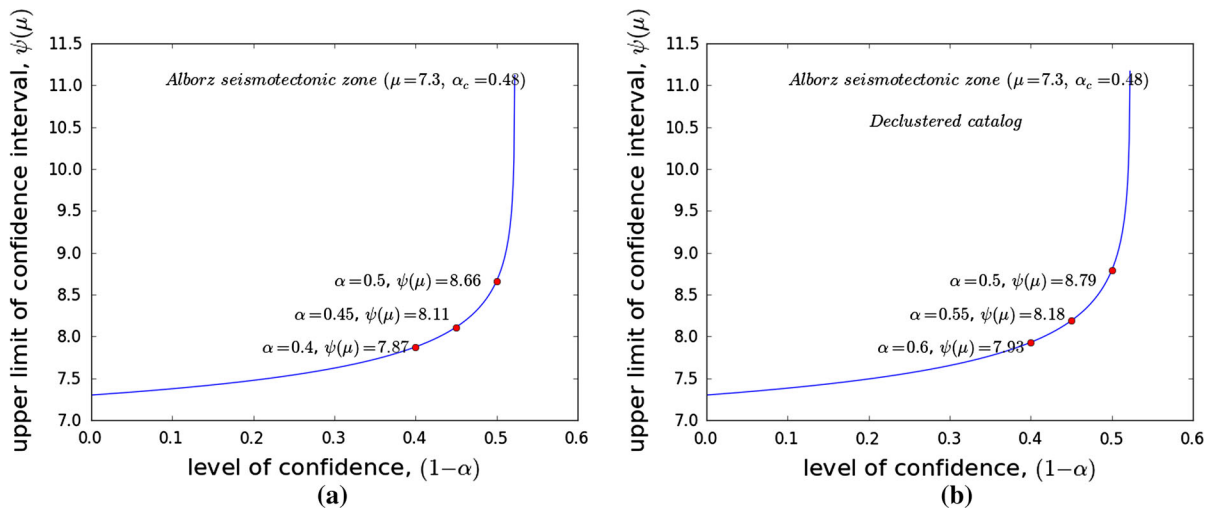


Figure 9

Illustration of Eq. (8) with $m_0 = 5.5, b = 0.88, n = 28$ in (a) original and $m_0 = 5.5, b = 0.8, n = 20$ in (b) declustered catalogs of the Alborz seismotectonic zone

4. Discussion

In the current study, the upper bound of the confidence interval of the maximum possible earthquake magnitude m_{\max} in different seismotectonic zones of Iran is calculated. This calculation is performed for different predefined levels of confidence based on a

complete earthquake catalog. To study the confidence interval of the maximum magnitude in different seismotectonic zones of Iran, the uniform earthquake catalog between 1900 and 2015 in Iran and the surrounding area is gathered. It is divided into six sub-catalogs in six seismotectonic zones of Alborz, Azerbaijan, Central Iran, Zagros, Makran, Kopeh

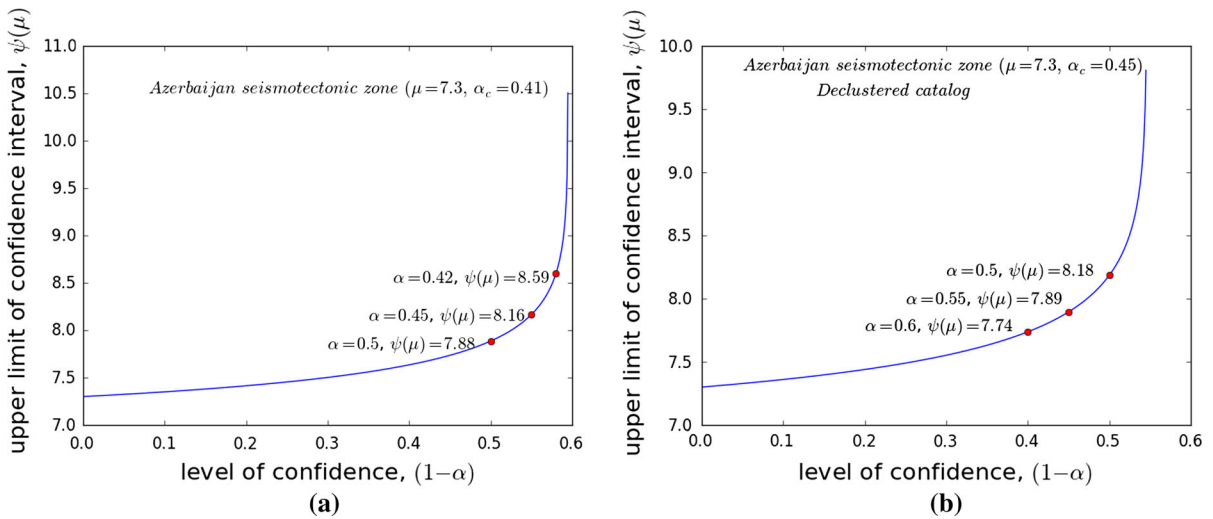


Figure 10

Illustration of Eq. (8) with $m_0 = 5.5, b = 1.08, n = 78$ in (a) original and $m_0 = 5.5, b = 1.03, n = 56$ in (b) declustered catalogs of the Azerbaijan seismotectonic zone

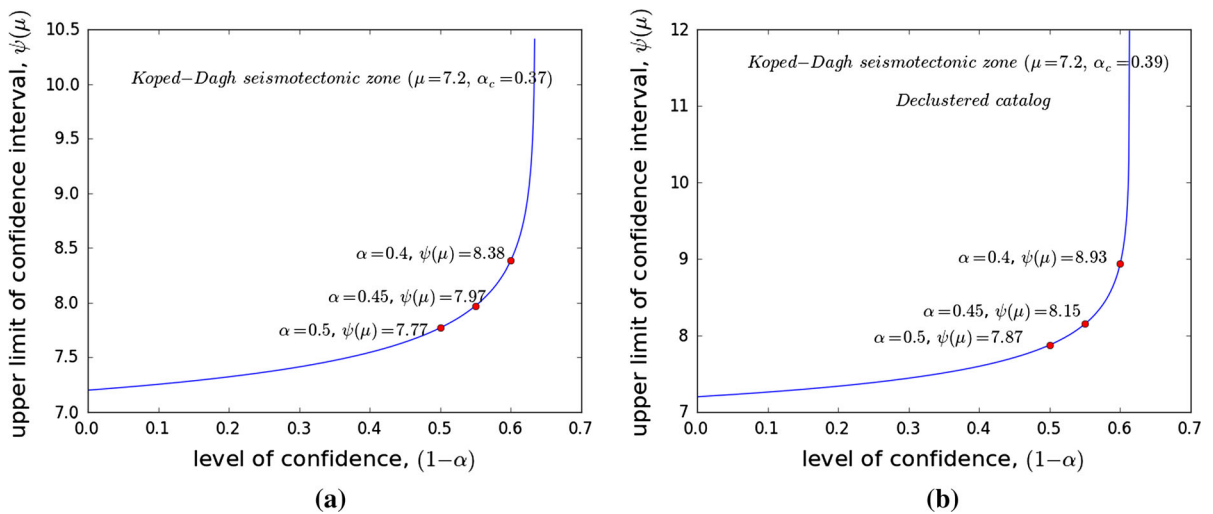


Figure 11

Illustration of Eq. (8) with $m_0 = 5.5, b = 0.88, n = 31$ in (a) original and $m_0 = 5.5, b = 0.83, n = 24$ in (b) declustered catalogs of the Kopeh Dagh seismotectonic zone

Dagh and both original and declustered catalogs are considered. To have independent events to a certain degree, the method of Grünthal (1985) is applied to decluster the original catalog. This allows to evaluate the influence of a possible violation of the assumption of the independence of events. It is assumed that seismic events in the studied region follow the

Gutenberg–Richter relation with known magnitude of completeness m_0 and the Richter- b value calculated from Aki’s formula.

Because the confidence interval is known to be unbounded in many cases, we calculate the highest confidence level with finite confidence interval $1 - \alpha_c$.

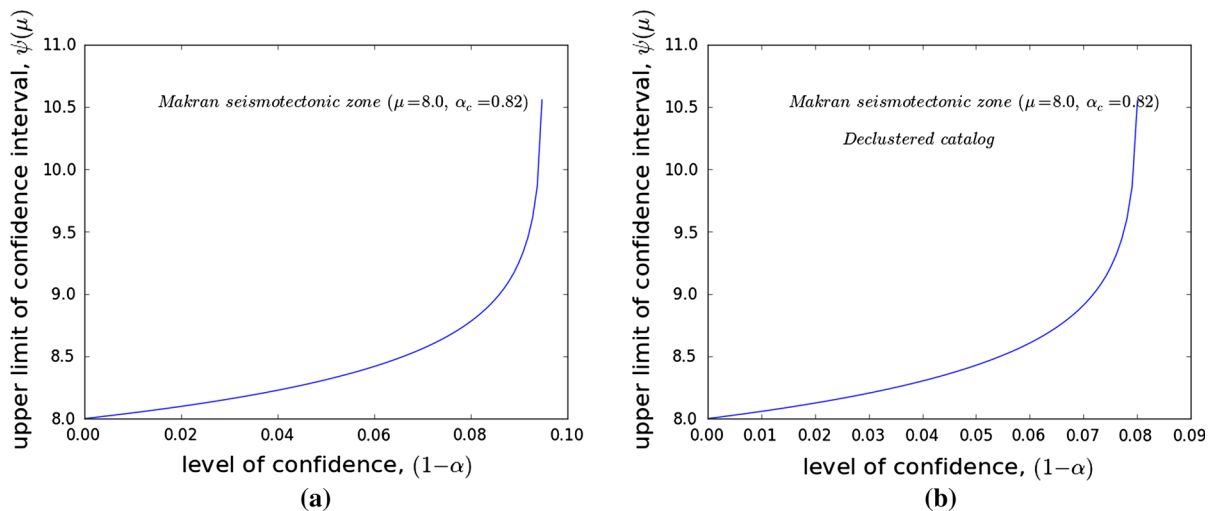


Figure 12

Illustration of Eq. (8) with $m_0 = 5.5, b = 0.98, n = 33$ in (a) original and $m_0 = 5.5, b = 0.82, n = 22$ in (b) declustered catalogs of the Makran seismotectonic zone

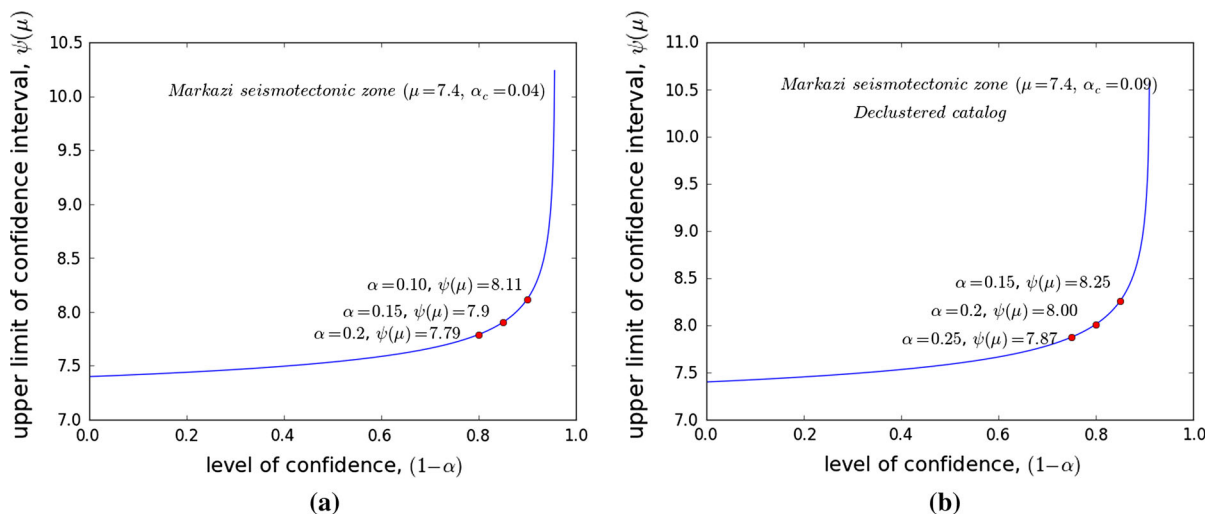


Figure 13

Illustration of Eq. (8) with $m_0 = 5.5, b = 0.79, n = 98$ in (a) original and $m_0 = 5.5, b = 0.77, n = 69$ in (b) declustered catalogs of the Central Iran seismotectonic zone

The critical value of the confidence level $1 - \alpha_c$ is calculated for the original and a declustered version of the catalog in different magnitudes of completeness equal to 4.8, 5.0 and 5.5. Based on the calculation shown in Tables 3 and 4, it is deduced that increasing the magnitude of completeness resulted in the higher value of the confidence level but the removal of a higher percentage of data must

be considered in this case. For $m_0 = 5.5$, $1 - \alpha_c$ in the original catalog is higher than the declustered catalog in all seismic zones. For $m_0 = 4.8$ and $m_0 = 5.0$, more significant results for $1 - \alpha_c$ are shown in declustered catalog of Zagros seismotectonic zone and it seems logical based on the existence of numerous clusters with magnitude less than 5 in the mentioned zone.

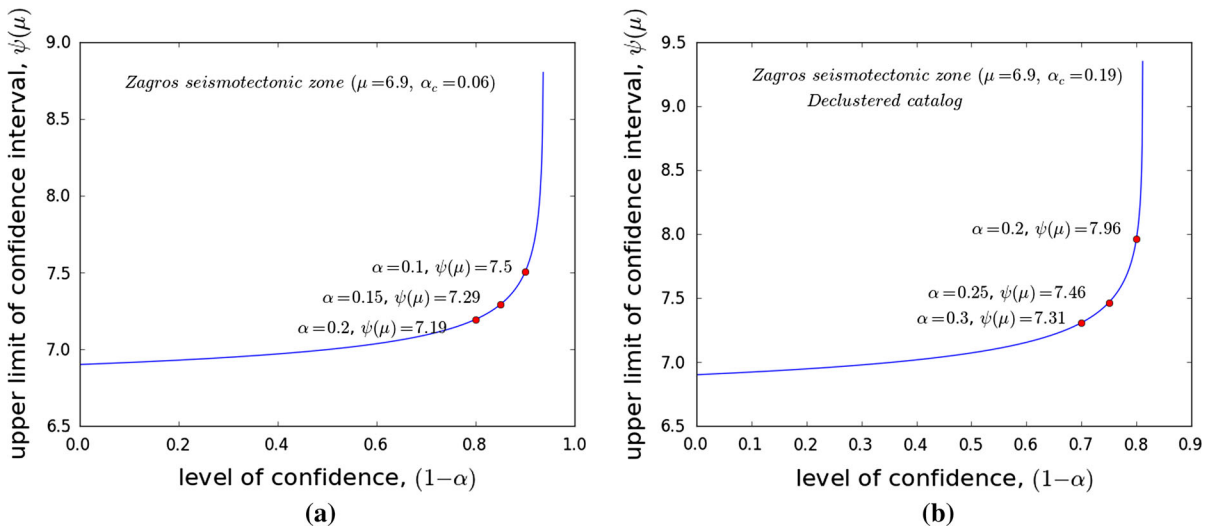


Figure 14

Illustration of Eq. (8) with $m_0 = 5.5, b = 1.28, n = 170$ in (a) original and $m_0 = 5.5, b = 1.36, n = 133$ in (b) declustered catalogs of the Zagros seismotectonic zone

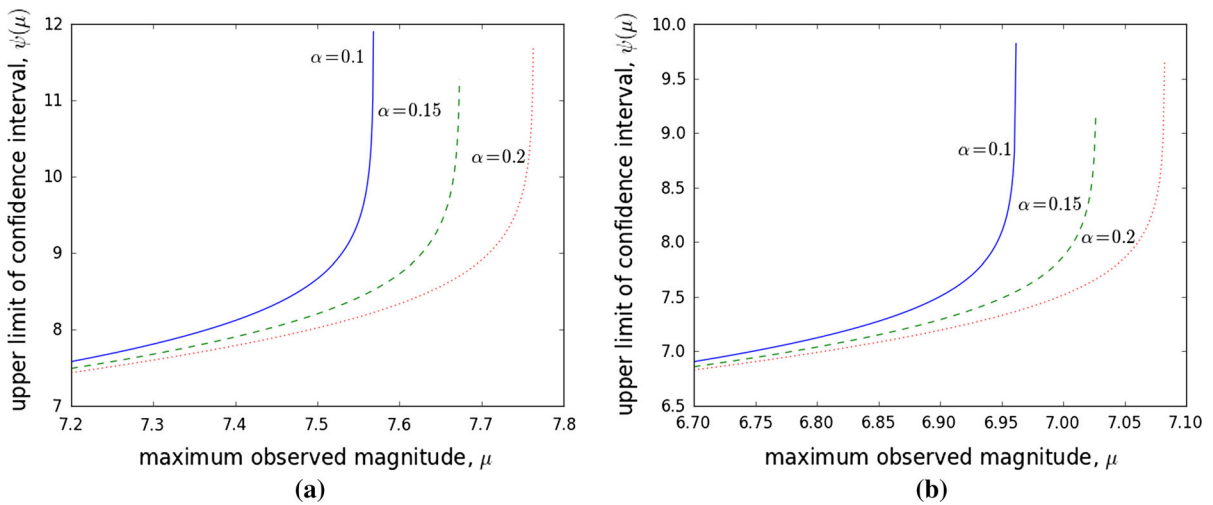


Figure 15

Illustration of Eq. (8) in original catalogs of (a) the Central Iran and (b) the Zagros seismotectonic zones in $m_0 = 5.5$

Figures 3, 4, 5, 6, 7 and 8 show the variation of the critical threshold m_c with the level of confidence $1 - \alpha$. The lowest value of the error probability α_c in which the confidence interval is finite is shown in the related figures. The same result has been shown in Figs. 9, 10, 11, 12, 13 and 14 which shows the variation of the upper bound of the confidence interval based on the level of confidence $1 - \alpha$. As it

is shown, for the values less than the critical value of confidence level $1 - \alpha_c$, the confidence interval is finite and for the value higher than it the trivial result of infinity will achieve (see Fig. 15).

After calculation of the critical value of the confidence level $1 - \alpha_c$ in different seismotectonic zones of Iran for $m_0 = 5.5$, we use three different values of the confidence level lower than the critical one to

calculate the finite confidence intervals. Tables 5 and 6 show the results of confidence intervals for different values of $1 - \alpha$ in both catalogs for six seismotectonic zones. For different numbers of events, maximum observed magnitudes and the Richter- b values, the highest confidence level $1 - \alpha_c$ in the original and the declustered catalog of each zone is calculated and the result shows the highest value in the Central Iran and the Zagros seismotectonic zones. The confidence levels in the Central Iran seismotectonic zone, for the completeness magnitude of $m_0 = 5.5$, $m_0 = 5.0$ and $m_0 = 4.8$ in the original and the declustered catalog are equal to 0.96, 0.72, 0.27 and 0.91, 0.81, 0.55, respectively. These values are equal to 0.94, 0.57, 0.36 and 0.81, 0.79, 0.76 for the original and the declustered catalog in Zagros seismotectonic zone for $m_0 = 5.5$, $m_0 = 5.0$ and $m_0 = 4.8$, respectively. It should be noted, even though, increasing the magnitude of completeness would probably result in more independency of events, but reduction of the number of events may cause high uncertainty in the estimation of m_{\max} . Calculations of the confidence interval in the original catalog of the Central Iran indicate that with $m_0 = 5.5$, the confidence interval ranges are [7.4, 8.11], [7.4, 7.9], [7.4, 7.79] for 90, 85 and 80% of all cases. The same results are equal to [6.9, 7.5], [6.9, 7.29], [6.9, 7.19] for 90, 85 and 80% of all cases in the Zagros seismotectonic zone. The upper bound of the confidence interval for three different values of the confidence level in Central Iran and Zagros seismotectonic zones are illustrated in Fig. 15. Calculation of the confidence interval in Kopeh Dagh, Alborz and Azerbaijan seismotectonic zones are not that much promising and still too low due to the lack of sufficient data in the Makran seismotectonic zone, the seismic catalog is not qualified enough to have a good estimation in this zone and the current estimation is not meaningful. Results of the calculated confidence interval for $m_0 = 5.5$ in original catalogs of all seismotectonic zones of Iran are more acceptable in comparison with different values of the magnitude of completeness. Since for $m_0 = 5.5$, declustering the catalog does not have any effect on the seismicity parameters in all seismotectonic zones; to calculate the confidence interval, it is recommended to use the original catalog without any

declustering. Getting the reasonable confidence interval with the low probability of error needs qualified statistics of more events, and the current catalog with few events, high uncertainty and small number of large events specially in Makran, Kopeh Dagh, Alborz and Azerbaijan does not result in a meaningful level of confidence with finite confidence interval. Reduction in the probability of error may result in a reasonable level of confidence, but it causes the divergence of the confidence interval between the maximum observed magnitude and infinity which means unbounded confidence intervals.

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