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Estimation of Gravity Parameters Related to Simple Geometrical Structures by Developing an Approach Based on Deconvolution and Linear Optimization Techniques

J. Asfahani[1] and M. Tlas[1]

Abstract—An easy and practical method for interpreting residual gravity anomalies due to simple geometrically shaped models such as cylinders and spheres has been proposed in this paper. This proposed method is based on both the deconvolution technique and the simplex algorithm for linear optimization to most effectively estimate the model parameters, e.g., the depth from the surface to the center of a buried structure (sphere or horizontal cylinder) or the depth from the surface to the top of a buried object (vertical cylinder), and the amplitude coefficient from the residual gravity anomaly profile. The method was tested on synthetic data sets corrupted by different white Gaussian random noise levels to demonstrate the capability and reliability of the method. The results acquired show that the estimated parameter values derived by this proposed method are close to the assumed true parameter values. The validity of this method is also demonstrated using real field residual gravity anomalies from Cuba and Sweden. Comparable and acceptable agreement is shown between the results derived by this method and those derived from real field data.

Key words: Gravity anomaly, sphere-like structure, cylinder-like structure, deconvolution technique, simplex algorithm, linear optimization.

1. Introduction

Geological structures in mineral and petroleum exploration can be approximated by simple geological structures such as faults, spheres, cylinders, sheets, or dikes. According to this approximation, many methods have been introduced for interpreting gravity field anomalies due to simple geometric models in an attempt to best-estimate the gravity parameter values, e.g., the depth to a buried object and the amplitude coefficient. These interpretation methods include graphical methods (Nettleton 1962, 1976), ratio methods (Bowin et al. 1986; Abdelrahman et al. 1989), the Fourier transform (Odegard and Berg 1965; Sharma and Geldart 1968), Euler deconvolution (Thompson 1982), a neural network (Elawadi et al. 2001), the Mellin transform (Mohan et al. 1986), least-squares minimization approaches (Gupta 1983; Lines and Treitel 1984; Abdelrahman 1990; Abdelrahman et al. 1991; Abdelrahman and El-Araby 1993; Abdelrahman and Sharafeldin 1995a), and Werner deconvolution (Hartmann et al. 1971; Jain 1976). Kilty (1983) extended the Werner deconvolution technique to the analysis of gravity data using both the residual anomaly and its first and second horizontal derivatives; Ku and Sharp (1983) further refined the method by using iteration for reducing and eliminating the interference field and then applied Marquardt's non-linear least squares method to further refine automatically the first approximation provided by deconvolution. Salem and Ravat (2003) presented a new automatic method for interpretation of magnetic data, called AN-EUL. Their method is based on a combination of the analytic signal and the Euler deconvolution method (AN-EUL). With the AN-EUL method, both the location and the approximate geometry of a magnetic source can be deduced. Fedi (2007) described the theory for gravity and magnetic fields and their derivatives for any order and proposed a method called depth from extreme points (DEXP) to interpret any potential field. The DEXP method allows estimating of source depths, density, and structural index from the extreme points of a three-dimensional (3D) field scaled according to specific power laws of the altitude. Salem and Smith (2005) presented an alternative method to estimate both the depth and model type using the first order local wave number approach without the need for third order derivatives of the field.

[1] Atomic Energy Commission, P. O. Box 6091, Damascus, Syria. E-mail: cscientific@aec.org.sy

In their method, normalization of the first order local wavenumber anomalies is achieved and a generalized equation to estimate the depth of some two-dimensional (2D) magnetic sources regardless of the source structure is obtained. SILVA and BARBOSA (2003) derived the analytical estimators for the horizontal and vertical source position in 3D Euler deconvolution as a function of the x, y, and z derivatives of the magnetic anomaly within a data window. BARBOSA et al. (1999) proposed a new criterion for determining the structural index based on the correlation between the total magnetic field anomaly and the estimates of an unknown base level. SALEM et al. (2008) developed a new method for the interpretation of gridded magnetic data based on derivatives of the tilt angle, where a simple linear equation, similar to the 3D Euler equation, can be obtained. Their method estimates both the horizontal location and the depth of magnetic bodies, but without specifying prior information about the nature of the sources. FEDI et al. (2009) proposed a new method based on a 3D multiridge analysis of the potential field. The new method assumes a 3D subset in the harmonic region and studies the behavior of the potential field ridges, which are built by joining extreme points of the analyzed field computed at different altitudes.

However, only a few techniques have addressed the task of determining the shape of a buried structure. These techniques include, for example, the Walsh transform (SHAW and AGARWAL 1990), least-squares methods (ABDELRAHMAN and SHARAFELDIN 1995b; ABDELRAHMAN et al. 2001a, b), and a constrained and penalized nonlinear optimization technique (TLAS et al. 2005). Generally, determining the depth, shape factor, and amplitude coefficient of a buried structure is performed by these methods based on the residual gravity anomaly, where the accuracy of the results depends on the accuracy in which the residual anomaly can be separated and isolated from the observed gravity anomaly.

Recently, ASFAHANI and TLAS (2012) proposed an efficient approach to interpret residual gravity anomalies in order to estimate the gravity parameters, e.g., depth, amplitude coefficient. and geometric shape factor of simple buried bodies, such as a sphere, horizontal cylinder, or vertical cylinder. The method is based on non-convex and nonlinear Fair

function minimization, adaptive simulated annealing, and a stochastic optimization algorithm. The main advantage of this approach is that the buried body shape is considered an unknown factor and can be estimated as an independent parameter. However, this approach suffers from a discrepancy and has some disadvantages because it sometimes necessitates the use of multi-starting or initial parameter guesses in order to assure global convergence or to reach the global minima of the objective function.

A recent publication by TLAS and ASFAHANI (2014) focused on interpretation of magnetic anomalies based on the deconvolution technique to transform the non-convex and the nonlinear minimization problem into a linear one, to avoid being trapped in a local minima of the objective function and, hence, obtain a solution using a linear optimization algorithm for definitively reaching the global minima of the minimization problem.

In the present paper, a new practical interpretation methodology is proposed for interpreting residual gravity field anomalies and accurately estimating model parameter values, e.g., the depth to the top or to the center of a body and the amplitude coefficient related to a buried sphere or a cylinder-like structure. The method also uses the deconvolution technique to avoid the local minima, where the nonlinear optimization problem describing the suitable simple geometric-shaped model of a structure is transformed into a linear optimization one. The linear problem is thereafter solved by the very well-known algorithm in linear optimization called the simplex algorithm of Dantzig (PHILLIPS et al. 1976) in order to definitely reach the global minima.

The reliability and capability of the proposed interpretative method is demonstrated using synthetic data sets corrupted by different white Gaussian random noise levels of 7 and 10 %. The results acquired show that the estimated parameter values derived by this method are very close to the assumed true parameter values.

The validity of this method is also demonstrated using real field gravity anomalies from Cuba and Sweden. Comparable and acceptable agreement is shown between the results derived by this proposed method and those obtained by other interpretation methods.
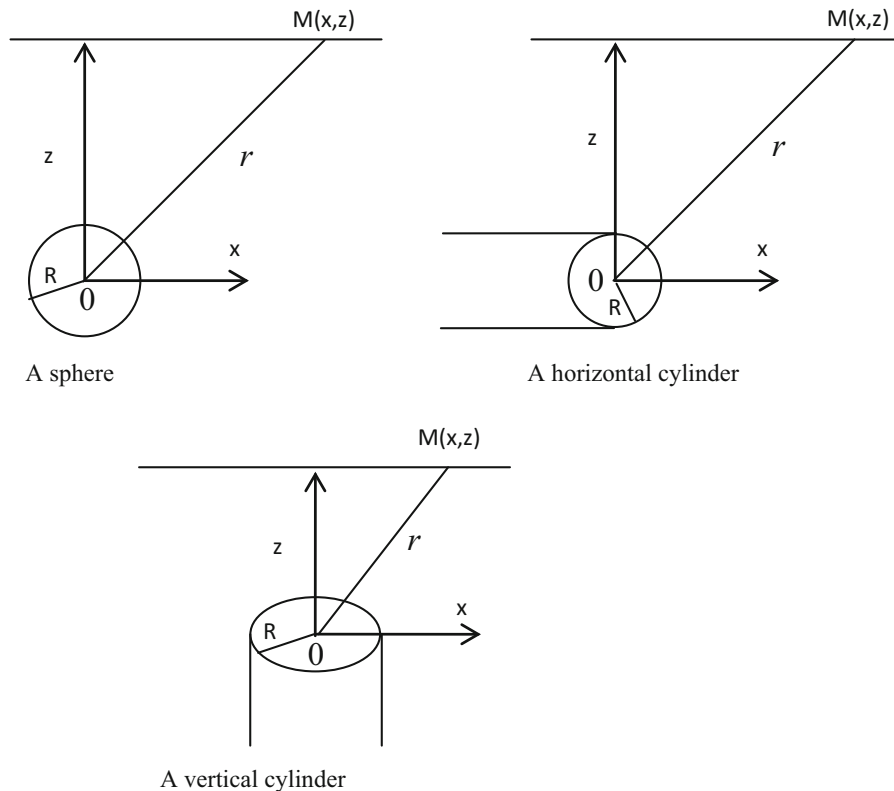
A sphere

A horizontal cylinder

A vertical cylinder

Figure 1
Diagrams of simple geometrical structures (*sphere*, *horizontal cylinder*, and *vertical cylinder*)

Moreover, the depth obtained by such a method is found to be in high accordance with that obtained from real field data.

2. Theory

Theoretical residual gravity anomalies due to various geological models are treated in this paper.

2.1. Interpretation of a Residual Gravity Anomaly due to a Sphere Model

The general expression of a residual gravity anomaly (V) at any point $M(x)$ along the x-axis of a sphere–like structure in a Cartesian coordinate system (Fig. 1) can be given according to GUPTA (1983) as:

$$V(x_i) = k \frac{1}{(x_i^2 + z^2)^{\frac{3}{2}}} \quad (i = 1, \ldots, N) \qquad (1)$$

where z is the depth to the center of the buried sphere body and k is the amplitude coefficient given by $k = \frac{4}{3}\pi G \rho R^3 z$, where ρ is the density contrast, G is the universal gravitational constant, R is the radius, and $x_i(i = 1, \ldots, N)$ is the horizontal position coordinate.

The set of Eq. (1) consists of N nonlinear equations as a function of the parameters k and z. To avoid this nonlinearity, the following proposed deconvolution technique will be used. First and for simplification, V_i will be used instead of $V(x_i)$ $(i = 1, \ldots, N)$ in the rest of this paper.

By multiplying the two sides of Eq. (1) by the term $(x_i^2 + z^2)^{\frac{3}{2}}$ and re-arranging it, it can be found

$$V_i^2 x_i^6 + 3V_i^2 x_i^4 q_1 + 3V_i^2 x_i^2 q_2 + V_i^2 q_3 - q_4 = 0 \quad (i = 1, \ldots, N) \qquad (2)$$

where

$$q_1 = z^2 \qquad (3)$$

$$q_2 = z^4 \tag{4}$$

$$q_3 = z^6 \tag{5}$$

$$q_4 = k^2 \tag{6}$$

The optimal solution (q_1, q_2, q_3, q_4) of the set of linear Eq. (2) can be found by solving the following nonlinear optimization problem onto the real space R^4:

$$\min_{q \in R^4} f(q) = \sum_{i=1}^{N} \left(V_i^2 x_i^6 + 3V_i^2 x_i^4 q_1 + 3V_i^2 x_i^2 q_2 + V_i^2 q_3 - q_4 \right)^2 \tag{7}$$

Subject to $q_1, q_2, q_3, q_4 \geq 0$

will surely be the global minima of the convex non-linear optimization problem (7).

The KKT optimality conditions (8) for the nonlinear optimization problem (7) are satisfied through solving the following linear optimization problem:

$$\left. \begin{array}{ll} \min \ \sum_{i=1}^{4} u_i & \\ \text{subject to } \frac{\partial f(q)}{\partial q_i} - u_i = 0 & \forall i = 1, \ldots, 4 \\ q_i, u_i \geq 0 & \forall i = 1, \ldots, 4 \end{array} \right\} \tag{9}$$

Or

$$\left. \begin{array}{l} \min \quad u_1 + u_2 + u_3 + u_4 \quad \text{subject to} \\ 3\left(\sum_{i=1}^{N} V_i^4 x_i^8\right) q_1 + 3\left(\sum_{i=1}^{N} V_i^4 x_i^6\right) q_2 + \left(\sum_{i=1}^{N} V_i^4 x_i^4\right) q_3 - \left(\sum_{i=1}^{N} V_i^2 x_i^4\right) q_4 - u_1 = -\left(\sum_{i=1}^{N} V_i^4 x_i^{10}\right) \\ 3\left(\sum_{i=1}^{N} V_i^4 x_i^6\right) q_1 + 3\left(\sum_{i=1}^{N} V_i^4 x_i^4\right) q_2 + \left(\sum_{i=1}^{N} V_i^4 x_i^2\right) q_3 - \left(\sum_{i=1}^{N} V_i^2 x_i^2\right) q_4 - u_2 = -\left(\sum_{i=1}^{N} V_i^4 x_i^8\right) \\ 3\left(\sum_{i=1}^{N} V_i^4 x_i^4\right) q_1 + 3\left(\sum_{i=1}^{N} V_i^4 x_i^2\right) q_2 + \left(\sum_{i=1}^{N} V_i^4\right) q_3 - \left(\sum_{i=1}^{N} V_i^2\right) q_4 - u_3 = -\left(\sum_{i=1}^{N} V_i^4 x_i^6\right) \\ 3\left(\sum_{i=1}^{N} V_i^2 x_i^4\right) q_1 + 3\left(\sum_{i=1}^{N} V_i^2 x_i^2\right) q_2 + \left(\sum_{i=1}^{N} V_i^2\right) q_3 - (N) q_4 - u_4 = -\left(\sum_{i=1}^{N} V_i^2 x_i^6\right) \\ q_1, q_2, q_3, q_4, u_1, u_2, u_3, u_4 \geq 0 \end{array} \right\} \tag{10}$$

The quadratic objective function $f(q)$ of the nonlinear optimization problem (7) is convex onto the non-negative orthant of the real space R^4. This mathematically means that for any solution $(q_1, q_2, q_3, q_4) \in R^4$ that satisfies the following optimality conditions of Karush-Kuhn-Tuker (KKT) (PHILLIPS *et al.* 1976):

$$\left. \begin{array}{ll} q_i \geq 0 & \forall i = 1, \ldots, 4 \\ q_i \frac{\partial f(q)}{\partial q_i} = 0 & \forall i = 1, \ldots, 4 \\ \frac{\partial f(q)}{\partial q_i} \geq 0 & \forall i = 1, \ldots, 4 \end{array} \right\} \tag{8}$$

where (q_1, q_2, q_3, q_4) are structural variables, while (u_1, u_2, u_3, u_4) are artificial variables (surplus variables). The artificial variables (u_1, u_2, u_3, u_4) are added only in order to measure the deviations between the two sides of Eq. (10) (requirements of the simplex algorithm), and also to define the objective function of the linear optimization program (10). These variables (u_1, u_2, u_3, u_4) will be dropped when the optimal values of structural variables (q_1, q_2, q_3, q_4) are reached.

The linear optimization problem (10) is thereafter solved by the Simplex algorithm that starts automatically with initial parameter guesses; zeros for

(q_1, q_2, q_3, q_4) and right hand side values of Eq. (10) for (u_1, u_2, u_3, u_4). The Simplex algorithm will find the optimal values of $(q_1, q_2, q_3, q_4, u_1, u_2, u_3, u_4) \in R^8$ which satisfy the KKT optimality conditions (8) for the nonlinear optimization problem (7) in a bounded number of iterations.

For more details about the simplex algorithm of Dantzig for linear optimization problems, readers are referred to: PHILLIPS *et al.* (1976), HILLIER and LIEBERMAN (1986) and BRADLEY *et al.* (1977).

After knowing the optimal values of q_1, q_2, and q_3, the best estimate of the depth to the center of the buried sphere body (z) can be easily found by using simultaneously Eqs. (3, 4, and 5) as:

$$z = \frac{(q_1)^{\frac{1}{2}} + (q_2)^{\frac{1}{4}} + (q_3)^{\frac{1}{6}}}{3} \qquad (11)$$

The best estimate of the amplitude coefficient (k) can easily be obtained by using Eq. (6) as:

$$k = \mp\sqrt{q_4} \qquad (12)$$

The sign of k can be easily assigned by using the statistical criterion of preference called the root mean square error (RMSE; COLLINS 2003), based on the minimal value, between the field data anomaly and the computed one, using the estimated values of z and k. The mathematical formula of this criterion is given as:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}\left(V_i(\text{Observed}) - V_i(\text{Computed})\right)^2}{N}}, \qquad (13)$$

where $V_i(\text{Observed})$ and $V_i(\text{Computed})$ $(i = 1, \ldots, N)$ are the observed and the computed values at the point $x_i\,(i = 1, \ldots, N)$, respectively.

2.2. Interpretation of a Residual Gravity Anomaly due to a Vertical Cylinder Model

The residual gravity effect (V) of a vertical cylinder-like structure at any point on the free surface along the principal profile in a Cartesian coordinate system (Fig. 1) is also given according to GUPTA (1983) as:

$$V(x_i) = k\frac{1}{(x_i^2 + z^2)^{\frac{1}{2}}} \quad (i = 1, \ldots, N) \qquad (14)$$

where z is the depth from the surface to the top of the body and k is the amplitude coefficient given by $k = \pi G\rho R^2$. Following the same manner as in the sphere model, the parameters related to the gravity anomaly produced by the vertical cylinder can be estimated by the following equations:

$$z = \sqrt{q_1} \qquad (15)$$

$$k = \mp\sqrt{q_2} \qquad (16)$$

where q_1 and q_2 are obtained by solving the following linear optimization problem using the simplex algorithm:

$$\left. \begin{array}{l} \min\ u_1 + u_2 \quad \text{subject to} \\ \left(\sum_{i=1}^{N} V_i^4\right)q_1 - \left(\sum_{i=1}^{N} V_i^2\right)q_2 - u_1 = -\left(\sum_{i=1}^{N} V_i^4 x_i^2\right) \\ \left(\sum_{i=1}^{N} V_i^2\right)q_1 - (N)q_2 - u_2 = -\left(\sum_{i=1}^{N} V_i^2 x_i^2\right) \\ q_1, q_2, u_1, u_2 \geq 0 \end{array} \right\} \qquad (17)$$

2.3. Interpretation of Residual Gravity Anomaly due to a Horizontal Cylinder Model

The residual gravity effect (V) of a horizontal cylinder-like structure at any point on the free surface along the principal profile in a Cartesian coordinate system (Fig. 1) is also given according to Gupta (1983) as:

$$V(x_i) = k\frac{1}{(x_i^2 + z^2)} \quad (i = 1, \ldots, N) \qquad (18)$$

where z is the depth from the surface to the center of the body and k is the amplitude coefficient given by $k = 2\pi G\rho R^2 z$. Following the same manner as in the sphere model, the parameters related to the gravity anomaly produced by horizontal cylinder can be estimated by the following equations:

$$z = \sqrt{q_1} \qquad (19)$$

$$k = q_2 - q_3 \qquad (20)$$

where q_1, q_2 and q_3 are obtained by solving the following linear optimization problem using the simplex algorithm:

$$\left.\begin{array}{l}\min \quad u_1 + u_2 \quad \text{subject to} \\ \left(\sum_{i=1}^{N} V_i^2\right)q_1 - \left(\sum_{i=1}^{N} V_i\right)q_2 + \left(\sum_{i=1}^{N} V_i\right)q_3 - u_1 = -\left(\sum_{i=1}^{N} V_i^2 x_i^2\right) \\ \left(\sum_{i=1}^{N} V_i\right)q_1 - (N)q_2 + (N)q_3 - u_2 = -\left(\sum_{i=1}^{N} V_i x_i^2\right) \\ q_1, q_2, q_3, u_1, u_2 \geq 0 \end{array}\right\}$$
$$(21)$$

2.4. Interpretation of a Synthetic Gravity Anomaly due to a Horizontal Cylinder Model with Different Levels of Gaussian Random Noise

A synthetic gravity anomaly $V(x_i)$ ($i = 1, \ldots, N$) due to a horizontal cylinder-like structure is generated from Eq. (18) using the following model parameter values: depth to the center of the structure, $z = 15$ m; and the amplitude coefficient, $k = 1{,}500$ mGal. m^2 (Fig. 2).

Based on this generated synthetic anomaly, two additional gravity anomalies are regenerated; by perturbing it with different Gaussian random noise maximum levels of 7 and 10 %, respectively.

Figure 2 shows the data set of the synthetic gravity anomaly of a profile 23 m in length digitized at a 1-m sampling step generated with a maximum 10 % level of Gaussian random noise.
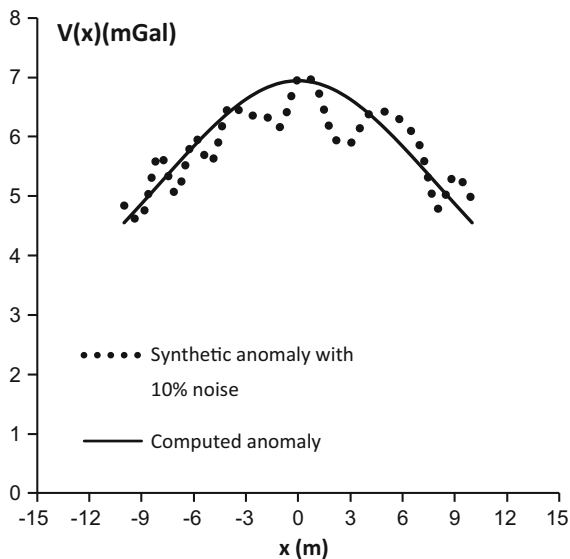

Figure 2
Diagrams of the computed anomaly and synthetic data set due to an horizontal cylinder, adding a maximum of 10 % random noise (sampling step is 1 m)

Both regenerated gravity anomalies are consequently interpreted by the proposed method by considering a priori that the sources causing these anomalies are a horizontal cylinder, a sphere, and a vertical cylinder. The results acquired for the three models are shown in Table 1.

Table 1 shows clearly that the obtained minimal RMSE for the two anomalies are related to the horizontal cylinder model. This means that the residual gravity anomaly must be preferably modeled as a horizontal cylinder. The results presented in Table 1 show also that the estimated parameter values derived by this proposed interpretation method are very close to the true parameter values. This clearly proves the efficiency and capability of this proposed interpretation method.

Moreover, it is noticed from Table 1 that the amplitude coefficient parameter (k) is found to be more sensitive to random noise than the depth parameter (z). With k being a multiplier factor, this explains the sensitivity and the differences between the estimated and the true model values.

3. Applications

Residual gravity field anomalies over various geological structures were interpreted by the proposed method. The field gravity anomalies were interpreted according to three different geological structures, e.g., a sphere, a horizontal cylinder, and a vertical cylinder. The resulting model with the lowest RMSE was selected as the best model for estimating the parameters of the interpreted residual gravity anomaly.

3.1. Interpretation of the Chromites Residual Gravity Field Anomaly

Figure 3 shows a normalized residual field gravity anomaly measured over a chromites deposit in Camaguey Province, Cuba (ROBINSON and CORUH 1988). This gravity anomaly was reinterpreted by the proposed method. The results acquired are shown in Table 2, which includes the results of assuming a priori that the source of this anomaly wa a sphere, a vertical cylinder or a horizontal cylinder model.

Table 1

Interpretation of a synthetic gravity anomaly with different maximum levels of Gaussian random noise

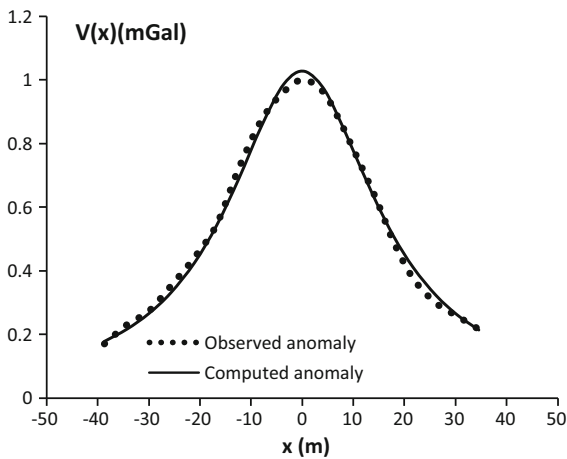| Source shape	Model parameters	True model parameter values	Estimated model parameter values with a maximum of 7 % random noise	Estimated model parameter values with a maximum of 10 % random noise
Horizontal cylinder	z (m)	15	14.7	13.8
	k (mGal. m^2)	1,500	1,469.8	1,322.2
	RMSE (mGal)	–	0.27	0.41
Sphere	z (m)	15	7.5	7.3
	k (mGal. m^3)	1,500	13,439	12,912
	RMSE (mGal)	–	14.8	15.6
Vertical cylinder	z (m)	15	9.5	8.8
	k (mGal. m)	1,500	64.4	61.2
	RMSE (mGal)	–	9.2	12.9

Figure 3

Normalized residual gravity field anomaly over a chromites deposit, Camaguey Province, Cuba. The evaluated curve by the proposed method is done for the *horizontal cylinder model*

Table 2 shows precisely that the minimal RMSE was obtained for the horizontal cylinder, meaning the residual gravity anomaly is preferably modeled as a horizontal cylinder. The depth obtained in this case ($z = 17.71$ m) is found to be in a good agreement with that obtained from borehole information ($z = 21$ m) (ROBINSON and CORUH 1988). The computed gravity anomaly was drawn according to these estimated horizontal cylinder model parameters, as shown in Fig. 3. The comparison between field and computed anomalies clearly indicates close agreement, attesting to the capability and validity of the method.

Table 2

Interpretation of the chromites residual gravity anomaly, Camaguey Province, Cuba

Model parameters	Horizontal cylinder	Vertical cylinder	Sphere
z (m)	17.71	7.2	36.4
k	321.85 (mGal m^2)	8.7 (mGal m)	2.7×10^4 (mGal m^3)
RMSE (mGal)	0.0189	0.08	0.21

3.2. Interpretation of the Karrbo Residual Gravity Field Anomaly

Figure 4 shows a residual field gravity anomaly of a profile 25.6 m in length measured over the elongated pyrrhotite ore, Karrbo, Vastmanland, Sweden (SHAW and AGARWAL 1990). This anomaly has been also reinterpreted by the proposed method. The results acquired are shown in Table 3, which includes the results of assuming a priori that the source of this anomaly is a sphere, a vertical cylinder, or a horizontal cylinder model. Table 3 shows precisely that the minimal RMSE was obtained for the horizontal cylinder, meaning that the residual gravity anomaly is preferably modeled as a horizontal cylinder. The depth obtained in this case ($z = 4.7$ m) is found to be in good agreement with the depth reported by TLAS *et al.* (2005) ($z = 4.82$ m), SHAW and AGARWAL (1990) ($z = 5.8$ m), and EL-ARABY (2000) ($z = 5.23$ m). The computed gravity anomaly was drawn according to these estimated horizontal cylinder model parameters, as shown in Fig. 4. Comparison between
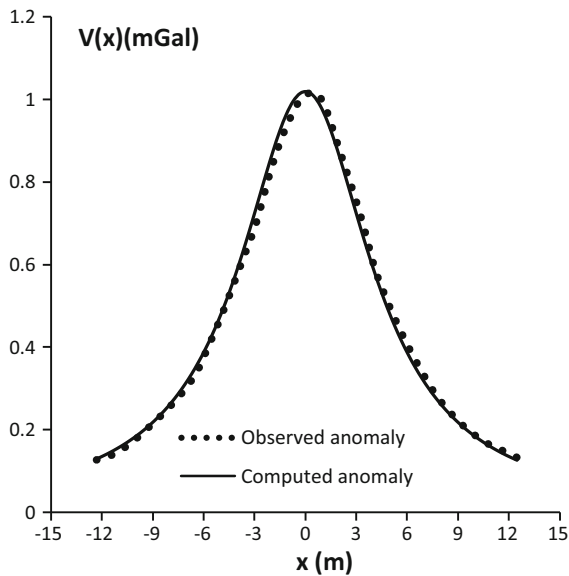
Figure 4
Residual gravity field anomaly over the 2D pyrrhotite ore, Karrbo, Vastmanland, Sweden. The evaluated *curve* by the proposed method is done for horizontal model

Table 3

Interpretation of the pyrrhotite ore residual gravity anomaly, Karrbo, Sweden

Model parameters	Horizontal cylinder	Vertical cylinder	Sphere
z (m)	4.7	1.6	6.0
k	22.5 (mGal. m^2)	2.1 (mGal. m)	297.9 (mGal. m^3)
RMSE (mGal)	0.016	0.10	0.20

field and computed anomalies clearly indicates close agreement, attesting to the capability and validity of the suggested method.

4. Conclusion

Herewith a new approach is proposed for interpretation of residual gravity anomalies due to simple geometrically shaped models such as a sphere, a vertical cylinder and/or a horizontal cylinder. The proposed method is based on both the deconvolution technique to avoid the local minima and on the simplex algorithm for linear programming to best-estimate the model parameters values. The new method was successfully tested first on synthetic data sets corrupted by different white Gaussian random noise levels of 7 and 10 % to demonstrate its reliability and capability. The synthetically obtained results show that the estimated parameter values derived by this method are very close to the assumed true parameter values. The validity of this method was then tested on field data sets, showing acceptable agreement between the results derived by this method and those obtained by other interpretation methods.

In addition, the depth obtained by such a proposed method was found to be in a good agreement with that obtained from the real field data.

Furthermore, the new proposed method is simple and easily applied, and is, therefore, strongly recommended for routine analysis of field gravity anomalies, in an attempt to determine the best parameter estimates for the studied structures.

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