Relationships Between M_w and Other Earthquake Size Parameters in the Spanish IGN Seismic Catalog

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Abstract—An important part of the preparation and homogenization of the seismic catalog used in the recent seismic hazard assessment study of Spain is the development of relationships among the different scales used to measure earthquake size. The objective is to convert all earthquake size data in the original catalog to a single magnitude scale, the moment magnitude M_w, in order to have a set of events with an uniform comparable size measurement. These new relationships are based on regression analysis between M_w and the other units used in the catalog in different epochs. The reduced major axis regression scheme is used, because it is the most suitable method for symmetric treatment of the variables involved in the fits. The new relationships obtained for Spain, M_w as a function of m_{bLg}; M_w as a function of m_b; and M_w as a function of I_{max}, are presented and their applicability limits and accuracy are discussed. The results obtained could have other practical uses in regional seismicity analysis.

Key words: Seismicity analysis, homogeneous catalogs, magnitude conversion, moment magnitude, regression procedures.

1. Introduction

Knowledge of the seismicity of a region is of fundamental importance for such seismological applications as seismic hazard assessment or implementation of earthquake early warning systems. Seismic catalogs contain part of this knowledge. In general, any earthquake catalog covering a long period of time, subject to changes in seismic instrumentation or containing data from different sources, includes earthquake sizes measured in different units, i.e., values in different scales of magnitude for instrumental seismicity and different macroseismic intensity values for historical earthquakes. Furthermore, a national catalog must eventually be extended and merged with other catalogs to achieve better coverage of seismic sources beyond the borders of a country. The result is a quite inhomogeneous catalog, especially with regard to the magnitude scales involved. A seismic catalog containing a homogeneous estimation of earthquake sizes, in which all the values are directly comparable, is a practical necessity for many applications, for example hazard studies and other seismicity analysis (Braunmiller *et al.* 2005).

In particular, implementation of an earthquake early warning system requires knowledge of the possible earthquake sources that could affect the target area. In the initial stages, a homogeneous earthquake database is needed to furnish specific empirical relationships between earthquake early warning characteristics and final magnitude. Such characteristics as average predominant period and the peak ground displacement are determined from the early portion of the P wave signal and then used for rapid assessment of the magnitude and damage potential of the impending earthquake (Wu and Kanamori 2005a, b; Zollo *et al.* 2006; Satriano *et al.* 2011; Carranza *et al.* 2013).

However, in the process of catalog homogenization is not always possible to obtain all the different earthquake size measurements directly from seismic records. In the historical period only macroseismic intensity is known. For old events from the instrumental period seismograms may be missing,

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instrument responses may be unknown, or not all amplitude phase readings are available. In addition, classic magnitude scales for local and teleseismic earthquakes (M_L, m_b, M_S) and their succeeding modified versions, are measured in different period and bandwidth ranges (BORMANN et al. 2013). Therefore, magnitude homogenization by use of empirical relationships is almost unavoidable when converting these traditional magnitudes into equivalent values, even though this approach cannot be regarded as physically justified (BORMANN and DI GIACOMO 2011).

Magnitude M_w is the earthquake size parameter into which all other magnitudes from a seismic catalog are most commonly converted, because it is a non-saturating magnitude scale and is physically well defined. M_w is scaled with earthquake size through the scalar seismic moment M_0 (KANAMORI 1977), which represents the size of an earthquake as a dislocation phenomenon along a fault (proportional to the product of the rupture area and the average slip). However, it is difficult to routinely determine reliable M_w values for small to moderate earthquakes. Thus, it is widespread practice to develop empirical relationships between M_w and other earthquake size scales and obtain the equivalent M_w magnitudes. Examples of this approach can be found in the works of JOHNSTON (1996a, b), PAPAZACHOS et al. (1997), RUEDA and MEZCUA (2002), (BRAUNMILLER et al. 2005), SONLEY and ATKINSON (2005), CASTELLARO et al. (2006), RUEDA (2006), SCORDILIS (2006), GASPAR-ESCRIBANO et al. (2008) and (CHEN and TSAI 2008), among others.

Some authors criticize use of M_w for seismic hazard applications. They recognize that M_w is most appropriate for measuring earthquake size (as "average tectonic effect of an earthquake"; BORMANN and DI GIACOMO 2011) but maintain it does not contain any direct information about frequency content, which is "essential for a realistic assessment of the hazard of structural damage due to ground shaking" (CASTELLARO and BORMANN 2007). It has been suggested that the combined use of well-defined, complementary magnitudes, for example energy magnitude M_e and moment magnitude M_w, should be considered (BORMANN and DI GIACOMO 2011).

In addition, the trend toward increasingly use of M_w data in ground motion prediction equations (a crucial component of seismic hazard analysis) is another important argument supporting conversion of cataloged parameters of earthquake size into M_w.

In accordance with these ideas, the main motivation of this work was development of relationships between moment magnitude M_w and other magnitude scales and also macroseismic intensity to enable preparation of a homogeneous seismic catalog that could be used primarily in hazard studies but also in other practical applications related to regional seismicity. The work was part of the recently completed project "Updating Seismic Hazard Maps of Spain" (IGN-UPM WORKING GROUP 2013), and the seismic catalog of the National Geographical Institute of Spain (IGN) constitutes the main data source for this work.

Further, other authors involved in earthquake early warning development have used some of these relationships in their seismicity analysis to obtain specific empirical relationships between earthquake early warning characteristics and final magnitude for the South of the Iberian Peninsula (CARRANZA et al. 2013).

1.1. Data

The IGN seismic catalog is the result of sequential compilations and revisions conducted by, among others, MUNUERA (1963), MEZCUA (1982), MEZCUA and MARTÍNEZ-SOLARES (1983), and MARTÍNEZ-SOLARES and MEZCUA (2002).

The data used in this work were obtained from a rectangular geographic area spanning from 34°N to 45°N in latitude and from 13°W to 6°E in longitude. Events with hypocentral depths greater than 65 km are excluded from the analysis. The period covered extends from 1048 to June 2011.

It is readily apparent that macroseismic data cover by far the largest time period (approx. 1000 years) whereas instrumental records cover only the last 90 years. On the basis of the temporal division established by MARTÍNEZ-SOLARES and MEZCUA (2002), the catalog can be divided in three main periods: historical, pre-instrumental, and instrumental.

The historical period considered here is that until 1923 (although scarce instrumental recordings have been obtained by use of mechanical seismographs

operating in Spain since 1898). Earthquake information comes from old texts describing the effects of earthquakes. Therefore, the unit characterizing earthquake size for this epoch is macroseismic intensity. After the revision by MARTÍNEZ-SOLARES and MEZCUA (2002) all intensities from the IGN catalog were given on the European Macroseismic Scale (EMS-98).

During the pre-instrumental period, from 1924 to 1962 (the year of installation of the World Wide Standard Seismograph Network), information about earthquakes was is the form of macroseismic data and instrumental measurements.

The first magnitude estimates based on local seismograms were calculated by Spanish seismological observatories by use of their own magnitude formulas. The interesting evolution of the formulas used by these observatories and by the different seismological services of Spain is described by LÓPEZ and MUÑOZ (2003).

Magnitudes for this epoch were eventually estimated by MEZCUA and MARTÍNEZ SOLARES (1983) in their catalog revision. Because of the scarce availability of information about instrument response, a magnitude scale based on signal durations was used.

The instrumental period from 1963 until now contains instrumental and complementary macroseismic data. Four magnitude scales have been used in this period, because of changes in instrumentation and magnitude formulas. Two are Lg wave magnitudes: m_{bLg}(MMS) from MEZCUA and MARTÍNEZ SOLARES (1983) and m_{bLg}(L) from the LÓPEZ (2008) formula, which is the formula currently in use. The third is a body wave magnitude m_b(VC) based on the Veith and CLAWSON (1972) formula, and the fourth is the moment magnitude M_w proposed by KANAMORI (1977), and based on the seismic moment released.

Three sub-periods, corresponding to the times these magnitudes were used, can be distinguished. Between 1962 and 1998 only m_{bLg}(MMS) was used. It is based on the magnitude definition of NUTTLI (1973) and is composed of two equations corresponding to two epicentral distance ranges. Between 1998 and 2002, and coexisting with m_{bLg}(MMS), the m_b(VC) magnitude scale was introduced for earthquakes located beyond 200 km from the Spanish coastline or for hypocentral depths greater than 30 km. From 2002, m_b(VC) continued in use, with scale m_{bLg}(MMS) replaced by scale m_{bLg}(L), and M_w scale was introduced. The m_{bLg}(L) magnitude scale is calibrated on the basis of the local Richter magnitude, such that for a period of 1 s and at reference distance of 100 km, both give the same value (LÓPEZ 2008). M_w is routinely calculated for moderate earthquakes (RUEDA and MEZCUA 2005) and is based on the moment tensor inversion methodology proposed by DREGER and HELMBERGER (1993).

Table 1 summarizes the different earthquake size estimates contained in the IGN catalog, including their notation and period of use. More detailed information can be obtained from Ref. (IGN 2014).

Only 126 earthquakes with M_w directly calculated by use of the IGN (denoted "M_w(IGN)") were available in the seismic catalog until June 2011. To improve this basic information to obtain a more comprehensive data set, the M_w data estimated by STICH et al. (2003, 2010), were added to the initial catalog. These magnitudes correspond to the period 1984–2008 and to earthquakes with epicenters in the Iberian Peninsula and nearby zones. The number of data added (denoted "M_w(St)") is 217.

To ensure the consistency of both data sets, a simple check was performed—linear regression analysis was conducted using a subset of events for which both M_w(IGN) and M_w(St) data were available (91 in total). The results are presented in Fig. 1a, which shows an almost one-to-one relationship between the estimates (with slope very close to 1 and intercept very close to 0), after exclusion of two outlier points by use of Chauvenet's criterion.

Table 1

Notation and period of use of earthquake size parameters contained in the IGN catalog

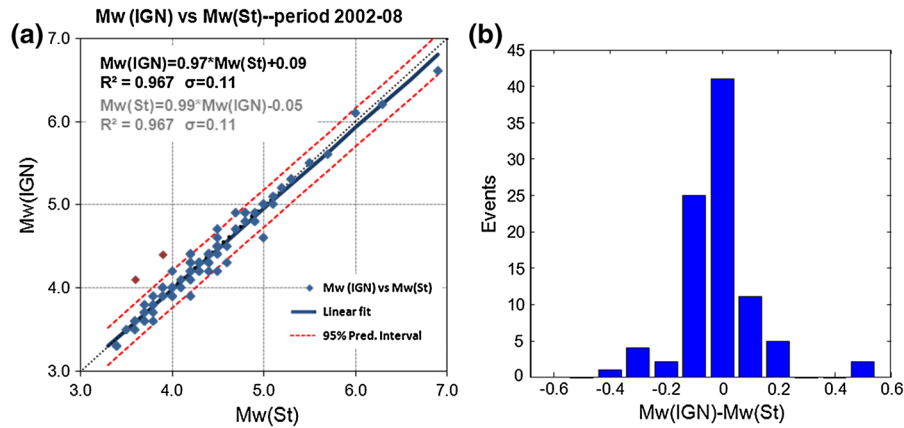| Size parameter	Definition	Period
I_{max}	Epicentral or maximum intensity	All
m_D	Duration magnitude	Before 1962
m_{bLg}(MMS)	Lg magnitude	1962–2002
m_b(VC)	Body wave magnitude	1998 to date
m_{bLg}(L)	Lg magnitude	2002 to date
M_w(IGN)	Moment magnitude (calculated by IGN)	2002 to date

Figure 1

a Comparison of M_w from IGN and M_w from STICH *et al*. (2003, 2010) for common earthquakes in the period 2002–2008. **b** Histogram of differences $M_w(IGN) - M_w(St)$

Table 2

Number of data pairs used for the regression, range, and period covered for each data subset

Relationship	N data	Range	Period
$M_w - I_{max}$	99	III–(IX–X)	1984–2011
$M_w - m_{bLg}(MMS)$	56	3.1–7.3	1984–2001 + 5 events in 1964–1978
$M_w - m_b(VC)$	98	3.7–6.3	1998–2011
$M_w - m_{bLg}(L)$	89	3.0–5.4	2002–2011

The average of the differences $M_w(IGN) - M_w(St)$ is -0.03 ± 0.11 (Fig. 1b). These small differences between the magnitude estimates indicate they can be regarded as an uniform dataset. Thus, the final M_w catalog resulting from combining $M_w(IGN)$ and $M_w(St)$ data sets contains 252 M_w values.

The input data subsets used for the regressions are summarized in Table 2.

A geographical representation of these data is shown on the maps in Fig. 2. Magnitude $m_{bLg}(MMS)$ constitutes the most extended distribution. The geographical distribution of magnitude $m_{bLg}(L)$ corresponds to earthquakes located inland or close to the coastline. Magnitude $m_{bLg}(L)$ data corresponding to events occurring more than 100 km from the coast are excluded from the regressions to minimize biasing effects, because of Lg phase blockage, leading to unrealistically low $m_{bLg}(L)$ estimates.

Magnitude $m_b(VC)$ characterizes distant earthquakes, propagating principally along oceanic paths, and deep earthquakes. It corresponds to earthquakes located in the Azores-Gibraltar zone and in the North of Africa.

Intensity I_{max} (or I_0) represents the maximum intensity in the epicentral zone, which may or may not coincide with the actual intensity of the epicenter. A total of 99 $I_{max} - M_w$ pairs, in the North of Africa and in the Iberian Peninsula, are contained in the subset for performing the regression.

The IGN dataset is completed with M_w magnitude estimates of earthquakes that occurred before 1984, obtained from specific studies. These include UDÍAS and LÓPEZ ARROYO (1970), LÓPEZ ARROYO and UDÍAS (1972), FUKAO (1973), PONDRELLI *et al*. (1999) and from the International Seismological Centre catalog (ISC 2010). In particular, for the 1969 February 28th earthquake the magnitude $m_{bLg} = 7.3$ given in the IGN catalog coincides with the determination given by the US Coast and Geodetic Survey agency (USCGS) and the value $M_w = 7.8$ was obtained from the seismic moment estimated by FUKAO (1973).

1.2. Fitting Method

It is assumed that the relationship between the different units of earthquake size is linear. This seems justified, as long as none of the magnitudes shows
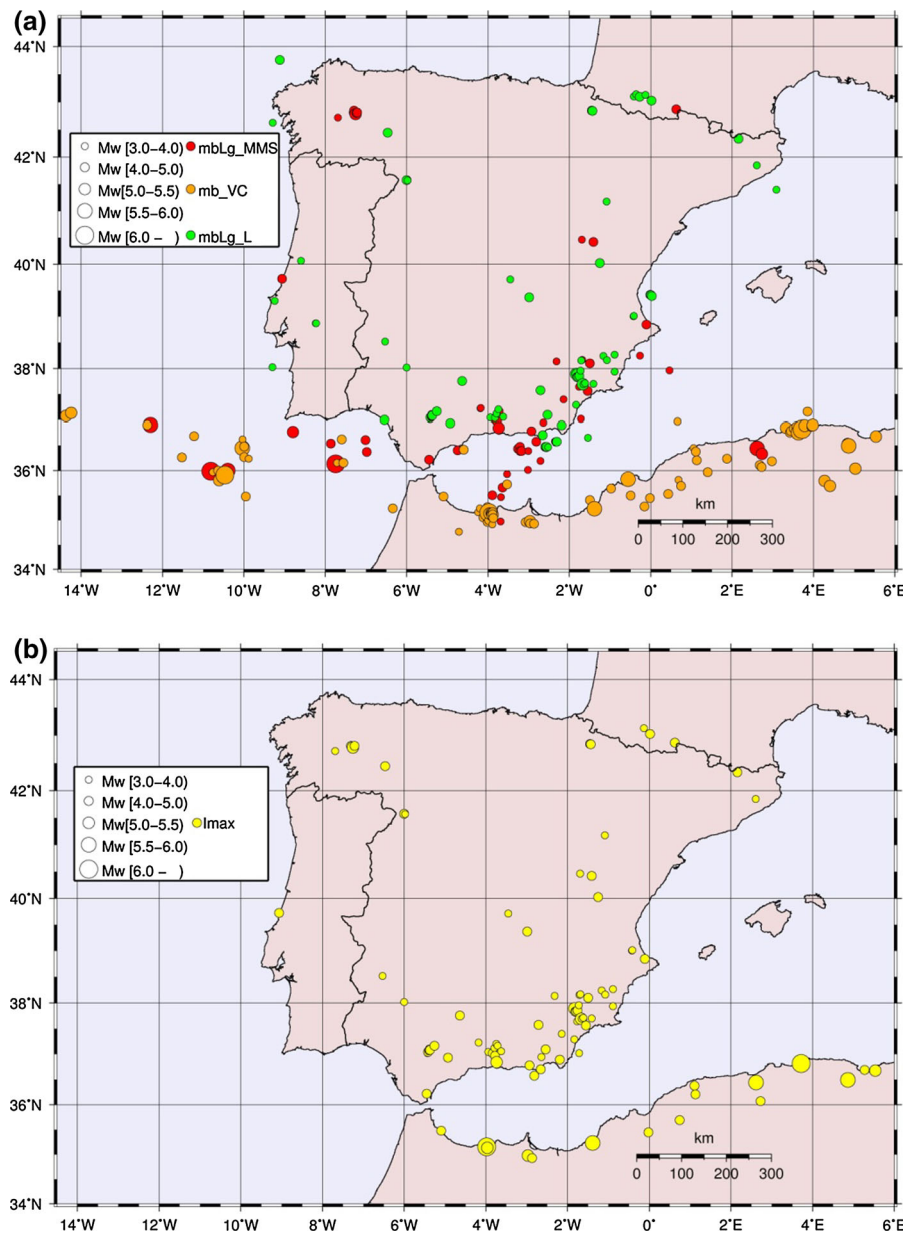
Figure 2

M_w magnitudes and geographic distribution of data subsets used for each relationship. **a** m_{bLg}(MMS), m_b(VC) and m_{bLg} (L). **b** I_{max}

evidence of strong saturation (CASTELLARO and BOR-MANN 2007).

Ordinary least-squares regression of Y on X OLS($Y|X$), or standard regression has been a widely used method to find relationships between two magnitude scales. This method enables the best regression line to be obtained when three basic assumptions are valid (DRAPER and SMITH 1998; GORGAS *et al.* 2009):

1. the true relationship between the variables X and Y is linear;
2. the observed values of the dependent variable Y are independent from point to point (i.e. independent of the X value) and follow a normal distribution for each X, with common constant variance (homoscedasticity); and
3. the values of the independent variable X are measured without error or are fixed by design, i.e.

the X-values are not random and do not have a distribution associated.

Therefore, for each X we have a random sample of Y-values.

Condition (2) is equivalent, in error terms, to assuming that the values of dependent variable Y are subjected to errors which have zero mean and finite common variance and that these errors do not depend on the independent variable X. Thus, all variability (error) is assumed to be on Y. This implies asymmetric treatment of the variables. OLS($Y|X$) minimizes the sum of the squares of the vertical deviations, furnishing a line that, given X, enables prediction of the Y value and that should never be inverted.

Although standard regression is widely used, the X values are rarely error free. Actually, errors in both variables X and Y may be significant and may vary from point to point. This would be true for magnitude conversions, because different magnitudes can be affected by uncertainties of comparable size and, then, estimates from standard regression could be biased. Thus it seems more appropriate to use an alternative regression model that takes into account errors in the independent variable, i.e. based on symmetrical treatment of the variables.

An interesting comparison of regression methods is presented in Isobe *et al.* (1990) and in Babu and Fiegelson (1992). They used, with the OLS regression (of Y on X and of X on Y), three methods that treat the variables symmetrically: orthogonal regression (OR), ordinary least-squares bisector regression (OLS bisector), and reduced major axis regression (RMA). In the OR method (also known as major axis regression) the solution corresponds to minimizing the sum of the squares of the perpendicular distances of the observed points from the fitting line. The solution for the OLS bisector method is the line that intersects the two ordinary least square lines OLS($Y|X$) and OLS($X|Y$). The slope is given by the tangent of the arithmetic mean of the angles of inclination of such lines. The RMA method, also called as geometric mean regression, minimizes the sum of the products of horizontal and vertical distances to the fitted line, i.e. the sum of the areas

of right triangles formed by the data points and the fitted line. In this case, the slope of the line is the geometric mean of the slopes of the two least-squares regressions.

Isobe *et al.* (1990) and Babu and Fiegelson (1992) include, for each method, the formulas for slope and intercept and the asymptotic formulas for their variances. In their analysis, by use of a Monte Carlo data set simulation, the bisector method and the reduced major axis method give the highest accuracy. Finally, they recommend the use of the bisector method and reject the reduced major axis regression, alluding to the observation that the slope estimator does not depend on the correlation coefficient. Tofallis (2000, 2003) argues that this is a rather strange objection because the correlation is a measure of the strength of the linear relationship and should be independent of the slope.

In the development of magnitude conversion relationships, Johnston (1996a) used a different approach to this problem, a weighted ordinary least-squares procedure. The weights of the observations include the uncertainty from the independent variable, which is approximated analytically and added as an indirect contribution to the total observational uncertainty.

A general, unified procedure for estimating the best fit line when both observables (Xi, Yi) are subject to errors was proposed by York (1969) and York *et al.* (2004). This interesting approach comprises an iterative process from which it is easy to derive simplified solutions or closed forms for such special cases as OLS, OR, and RMA regressions.

Castellaro *et al.* (2006) and Castellaro and Bormann (2007) used a generalized orthogonal regression (GOR) procedure to deal with magnitude conversion problem. This procedure is designed to account for errors in X values. Assuming that both x and y are measured with errors, that these errors are independent for each other, and that they have a normal distribution, a maximum likelihood estimator of the slope and intercept can be calculated (Eqs. 1 and 2). Both can be expressed as a function of the x and y sample variances, S_{xx} and S_{yy} respectively, of the sample covariance of x and y, S_{xy}, and of the ratio

of measurement error variances for both variables $\eta = \sigma_{\delta y}^2/\sigma_{\delta x}^2$, where δ_x and δ_y are the measurement errors of x and y respectively. They also give formulas for estimation errors on the on the regression coefficients.

$$\hat{b} = \frac{S_{yy} - \eta S_{xx} + \sqrt{\left(S_{yy} - \eta S_{xx}\right)^2 + 4\eta S_{xy}^2}}{2S_{xy}} \quad (1)$$

$$\hat{a} = \bar{y} - \hat{b}\bar{x} \quad (2)$$

Interestingly this algebraic solution is related to distinct regression procedures depending on the unknown η value. Thus, the OLS($Y|X$) and OLS($X|Y$) estimates can be obtained for $\eta = \infty$ and $\eta = 0$, respectively. The orthogonal (or major axis) regression estimate is derived from $\eta = 1$, and if $\eta = \sigma_y^2/\sigma_x^2$ (i.e., the ratio of measurements error variances is proportional to the overall variation in Y and X), the reduced major axis regression (RMA) estimate is obtained.

The value of η is rarely known. Usually, the variances of magnitude values given in seismic catalogs are not quantified. CASTELLARO and BORMANN (2007) obtained η values between approximately 0.11 and 9 by considering that in most cases the regression slope spans the range 0.7–1.3 and the magnitude errors span the range 0.1–0.3 magnitude units.

MCARDLE (2003) suggests that RMA seems to be less sensitive to misspecification of η and recommends using this method if the error in x is larger than one third that in y, this is for $\eta < 9$. Otherwise it would be safe (giving non-misleading results) to use OLS($Y|X$).

In this work the RMA method with the formulation of ISOBE et al. (1990) was used to estimate the variances of the slope, b, and intercept, a (Annex 1). The expression for the slope is merely the standard deviation of the y-values divided by the standard deviation of the x-values. The sign of the slope is given by the sign of the correlation coefficient. Likewise this expression is related to the two ordinary least-squares regression lines—it is the geometric mean of the slopes of these two lines (Eq. 3). The resulting line is symmetric in respect of the two variables and it is scale invariant.

$$b_{RMA} = \text{sign}\left(s_{xy}\right)\sqrt{\frac{S_{yy}}{S_{xx}}} = \text{sign}\left(s_{xy}\right)\sqrt{b_1 b_2} \quad (3)$$

$$a_{RMA} = \bar{y} - b_{RMA}\bar{x} \quad (4)$$

where S_{xx} and S_{yy} are sample variances of x and y; S_{xy} is the sample covariance of x and y; $r = S_{xy}/(S_{xx} S_{yy})$ is the sample correlation coefficient.

Further, the uncertainties of the original earthquake size data are incorporated in the study by use of a Monte Carlo process. Given that data uncertainties are unknown for the most of the catalog, an average uncertainty value is assigned to each original earthquake size on the basis of expert judgment. These values, which are listed on Table 3, depend principally on a quality index defined in the catalog for macroseismic data and on the epoch in which the magnitudes were used. In this case the most appropriate probability distribution for representing data uncertainty for each unit of size is a triangular distribution (ISO/IEC 2008), which, when it is symmetrical, is defined by a mean value and a standard deviation (the average uncertainty assigned). Considering this probability distribution, a total of 5000 Monte Carlo simulations were conducted for each dataset. The fitting variables, a and b, and their variances were analytically calculated by use of the RMA method for each of these samples and their mean values were taken as the final results. As a consequence, the values of coefficients a and b, and their uncertainties, were slightly re-adjusted in respect of the values calculated for the original sample, and the covariance of a and b was empirically estimated.

Table 3

Uncertainty assigned to the earthquake size measurements used in the study

Earthquake size measurement	Average assigned uncertainty
I_{max} (quality A)	0.5
I_{max} (quality B)	1.0
I_{max} (quality C)	1.5
m_{blg} (MMS) period 1962–1985	0.3
m_{bLg} (MMS) period 1985–2002	0.2
m_b(VC)	0.2
m_{bLg}(L)	0.2
M_w (specific studies <1985)	0.3
M_w (IGN, IAG)	0.1

Figure 3
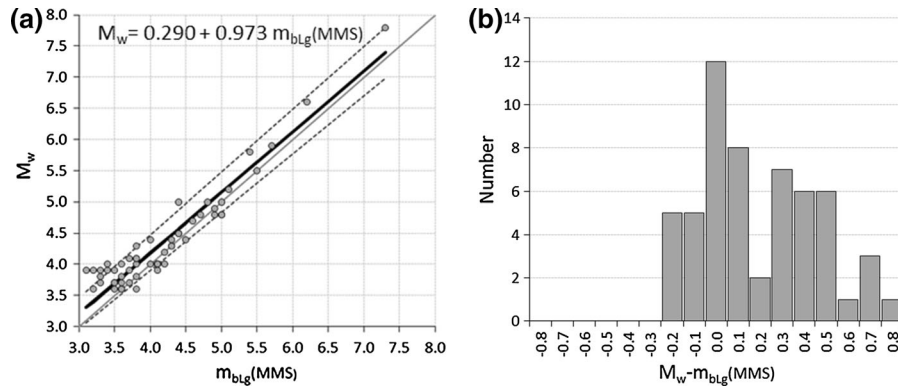**a** M_{w} as a function of m_{bLg}(MMS) data, regression line, and confidence limit $\pm 1\sigma$. **b** Differences histogram $M_{\mathrm{w}} - m_{\mathrm{bLg}}$(MMS)

2. Results

The RMA regression method was used to fit M_{w} as a function of all other earthquake size estimates. The results are shown in Figs. 3, 4, 5, 6 and discussed in the paragraphs below.

The relationship obtained for $M_{\mathrm{w}} - m_{\mathrm{bLg}}$(MMS) is shown in Fig. 3a. It is close to the one-to-one line. However, the dispersion in the magnitude range 3.0–4.0 forces the fitted line to be placed above this diagonal line. This is probably related to a large uncertainty in the determination of low magnitudes of M_{w}. A histogram of differences $M_{\mathrm{w}} - m_{\mathrm{bLg}}$(MMS) is presented in Fig. 3b. It shows a predominance of positive values, with an average value of 0.19 ± 0.10 magnitude units, which is mainly because of the contribution of the low magnitude range aforementioned. As a consequence, the relationship is indicative of slight underestimation of M_{w} values from the m_{bLg}(MMS) values in the IGN catalog. If, however, the fit is made without regard to the 1969 earthquake data, whose m_{bLg}(MMS) value could be regarded as being in the magnitude saturation range, the slope is slightly reduced and the resulting M_{w} values tend to match m_{bLg}(MMS) values from magnitude 6.5. However, it was decided to keep the original adjustment, preserving these earthquake magnitudes.

The result for the m_{b}(VC) magnitude is shown in Fig. 4. It has the largest deviation from the one-to-one relationship among all regressions presented. The histogram of differences $M_{\mathrm{w}} - m_{\mathrm{b}}$(VC), presented in Fig. 4b, shows a clear displacement to negative

values, with a mean value -0.43 ± 0.19 magnitude units. This is indicative of systematic overestimation of M_{w} values from m_{b} values.

The relationship obtained for the m_{bLg}(L) magnitude, which is shown in Fig. 5a, is also fairly close to the one-to-one relationship line, but now the fitted line crosses the former at magnitudes of approximately 4.0–4.4. The histogram of differences $M_{\mathrm{w}} - m_{\mathrm{bLg}}$(L), shown in Fig. 5b, has a good symmetrical appearance centered at 0 units. The average value is 0.06 ± 0.03.

The relationship fitted for maximum intensity is presented in Fig. 6a. There are few large intensity values. The maximum values of this subset correspond to earthquakes in northern Africa (Boumerdas, Algeria 2003/05/21, M_{w} 6.8; and Tammassint, Morocco 2004/02/24, M_{w} 6.3). The number of events of this subset as a function of time is represented in Fig. 6b.

The expressions of these relationships, the uncertainties of their coefficients, and the range of usage of each are summarized in Table 4. In Fig. 7 we present, together, the three relationships developed for magnitude scales. This figure gives a clear idea of the relative location and range of each magnitude relationship.

Finally, we should emphasize that in conversion into M_{w} by use of these relationships for the IGN catalog, only one earthquake size parameter is considered for each earthquake. When different earthquake size data are available for an event, magnitude is usually preferred to intensity. When
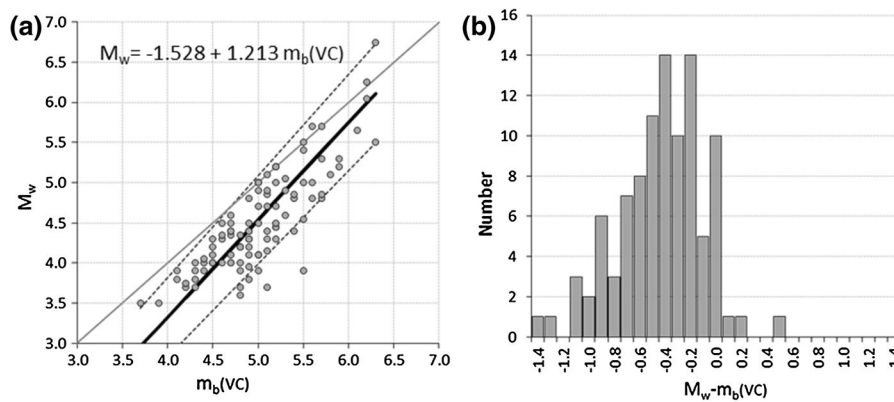
Figure 4
a M_w as a function of m_b(VC) data, regression line, and confidence limit $\pm 1\sigma$. b $M_w - m_b$(VC) histogram

Figure 5
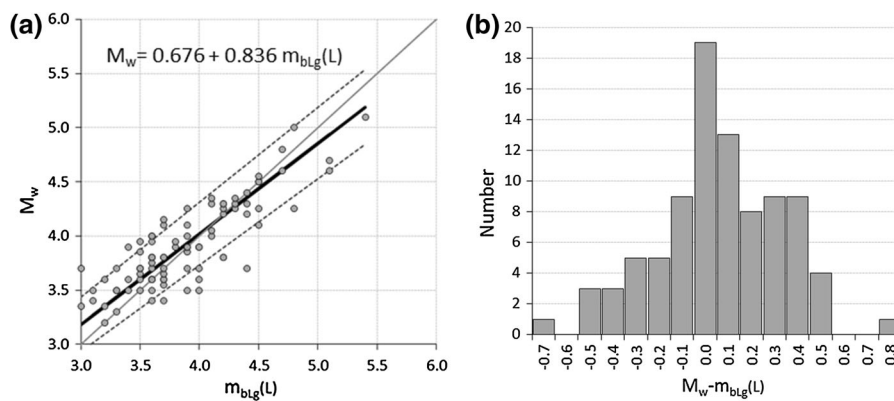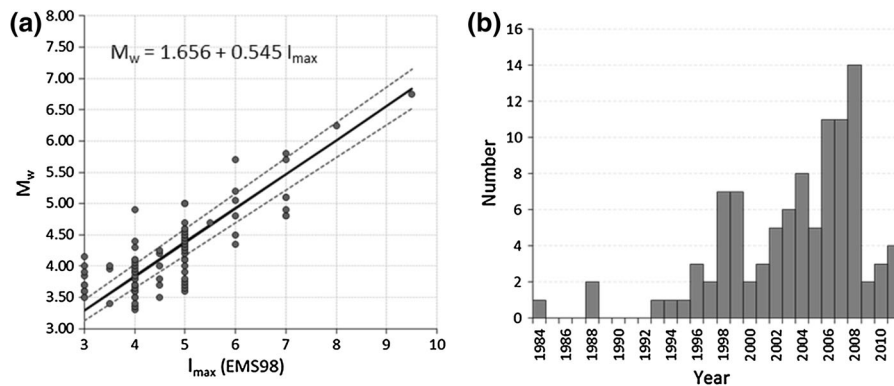a M_w as a function of m_{bLg} (L) data, regression line, and confidence limit $\pm 1\sigma$. b $M_w - m_{bLg}$(L) histogram

Figure 6
a M_w as a function of I_{max} data, regression line, and confidence limit $\pm 1\sigma$. b Number of events versus time for I_{max} data subset

different magnitudes (m_b and m_{bLg}) are available, the magnitude scale designated, in the catalog, as preferred (or most appropriate) is the one selected for this conversion.

3. Comparisons With Other Relationships

The relationships obtained in this study were compared with others published elsewhere, including

Table 4

RMA fits, uncertainty data, and usage range of the relationships developed in this study

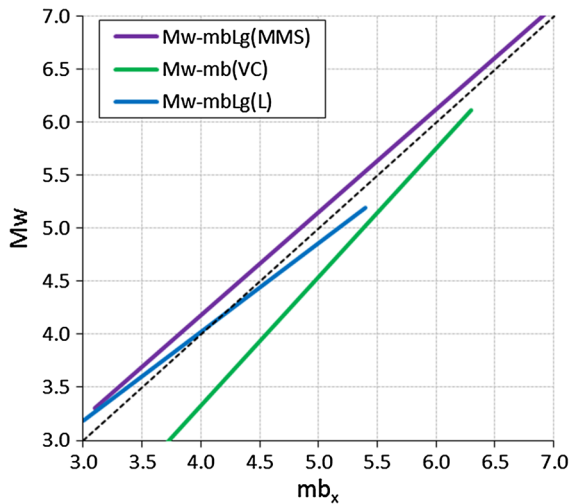| $Y = a + bx$	σ_a	σ_b	σ_{ab}	Range
$M_w = 1.656 + 0.545\, I_{max}$	0.144	0.030	−0.0001	III–(IX–X)
$M_w = 0.290 + 0.973\, m_{bLg(MMS)}$	0.208	0.049	−0.0005	3.1–7.3
$M_w = -1.528 + 1.213\, m_{b(VC)}$	0.385	0.077	−0.0001	3.7–6.3
$M_w = 0.676 + 0.836\, m_{bLg(L)}$	0.202	0.052	−0.0001	3.0–5.4

Figure 7

Combined representation of the developed relationships, M_w as a function of m_{bLg}(MMS); M_w as a function of m_b(VC), and M_w as a function of m_{bLg}(L)

relationships used in previous seismic hazard studies in Spain. It is acknowledged that some of these models may not be directly comparable, given the types of magnitudes concerned, although it is conceivable that both m_{bLg} magnitudes reported by IGN are close to a local magnitude. Irrespective of this, the main objective of these comparisons was to set up a reference for the developed relationships.

Among available published magnitude relationships, some are regional relationships (denoted local models) also based on the IGN seismic catalog, for example those of Rueda and Mezcua (2002), Gaspar-Escribano *et al.* (2008), and Rueda (2006). Others are either regional relationships (Braunmiller *et al.* 2005; Castellaro *et al.* 2006) or global models based on international catalogs (Johnston 1996a; Scordilis 2006).

Figure 8 shows the relationship obtained for m_{bLg}(MMS) in this study with those obtained by

Rueda and Mezcua (2002), Gaspar-Escribano *et al.* (2008), Johnston (1996a) and Castellaro *et al.* (2006). The first was developed for a small data subset of earthquakes mainly located in Spain. The second was developed for earthquakes in the Mediterranean–Iberian Peninsula region, taking as independent variable a generic IGN magnitude, referred to, in this paper, as the m(IGN) magnitude (i.e. without distinction between m_{bLg} and m_b). Both are quadratic equations based on ordinary least-squares regression (OLS). The relationship of Johnston (1996a) for m_{bLg} is for global earthquakes in stable continental regions; the relationship for local magnitude Castellaro *et al.* (2006) was developed using the generalized orthogonal regression method with Italian data from the period 1981–1996. It is apparent there is some approximation of the local models to the relationship developed in this work. This relationship also seems in good agreement with the model of Castellaro *et al.* (2006) for M_L magnitude.

There is, however, a marked difference from the Johnston (1996a) relationship, which, for m_{bLg} magnitudes in the range 3.5–5.5, gives lower M_w values than all the other relationships. In this sense, Vilanova and Fonseca (2007) report that the m_{bLg}(MMS) magnitudes from IGN are more consistent with the m_b magnitudes from the ISC catalog than with the m_{bLg} magnitudes used in Johnston (1996a). In fact, it can be seen that the relationship obtained by Johnston (1996a) for m_b, which is represented in Fig. 9 (together with m_b comparisons), is much closer to the relationship for m_{bLg}(MMS) obtained in this study. Mezcua and Martínez-Solares (1983) explain that during the period 1964–1980, and even some years afterwards, some magnitude values included in the catalog were not calculated but obtained from international agencies

such as ISC and NEIS (National Earthquake Information Service), especially for large and distant earthquakes (Martínez-Solares personal communication).

The M_w –m_b(VC) relationship developed in this work is compared with several relationships for m_b in Fig. 9, including the RUEDA (2006) relationship, which was derived from an earthquake data set from the IGN catalog for the period 2002–2005. This author also considered as independent variable a generic IGN magnitude (without further distinction between m_b and m_{bLg}). The other relationships shown in Fig. 9, corresponds to JOHNSTON (1996a), BRAUNMILLER et al. (2005), CASTELLARO et al. (2006) and SCORDILIS et al. (2006).

The local relationship RUEDA (2006) is a specific approximation to the M_w – m_b relationship developed in this study for magnitudes larger than 5.0; both are far from the other relationships, which seem to be more grouped. However, there is agreement between the slope of the relationship presented in this work and that of the regional relationships of CASTELLARO et al. (2006) and BRAUNMILLER et al. (2005). This may be further evidence of the already recognized need to perform re-calibration of the magnitude m_b(VC) reported by IGN. In this comparison, the relationship of JOHNSTON (1996a) is the closest to the one-to-one line, and the global relationship of

SCORDILIS (2006) is located over this line throughout the entire range, which implies that these m_b values underestimate the M_w values.

Figure 10 shows comparisons of different relationships with the M_w – m_{bLg}(L) relationship. We considered previous local relationships of RUEDA (2006) and GASPAR-ESCRIBANO et al. (2008) and the relationships proposed by BRAUNMILLER et al. (2005) and CASTELLARO et al. (2006) for local magnitude.

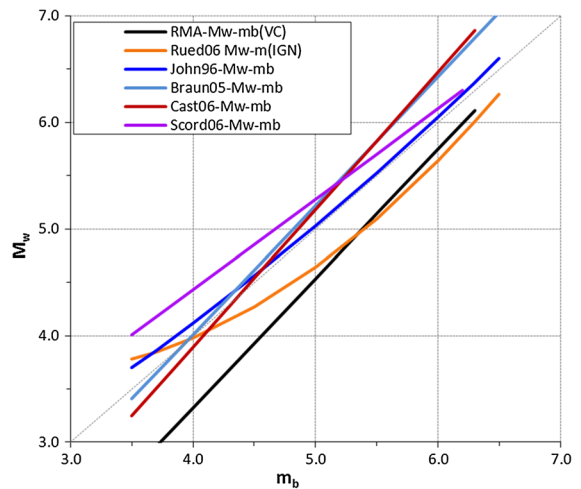The best coincidence occurs for the relationship RMA developed in this study and that developed in

Figure 9
Comparison of RMA M_w as a function of m_b(VC) with other relationships
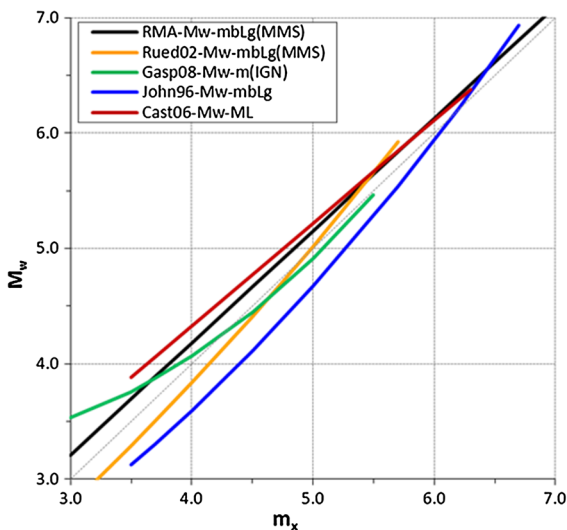

Figure 8
Comparison of RMA M_w as a function of m_{bLg}(MMS) with other relationships
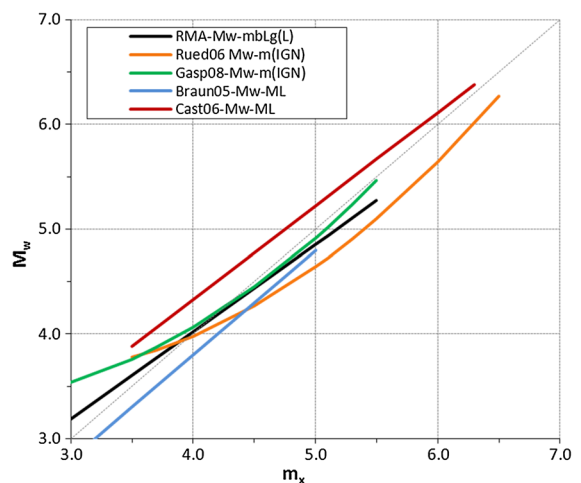

Figure 10
Comparison of RMA M_w as a function of m_{bLg}(L) with other relationships
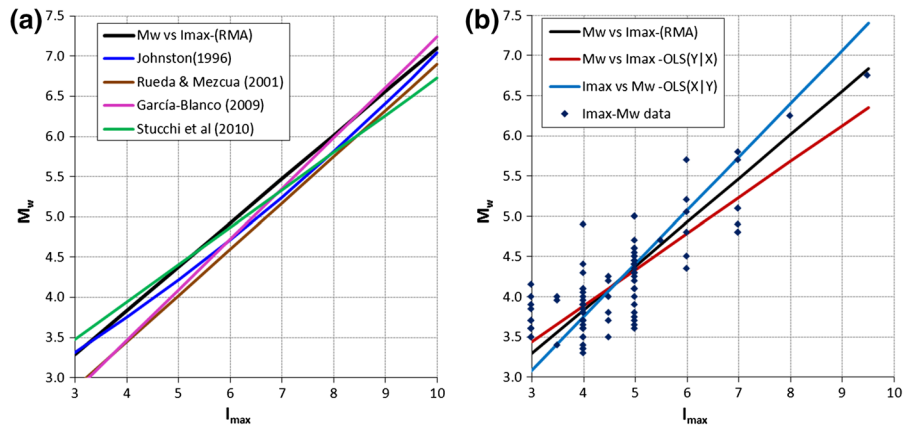
Figure 11
a Comparison of RMA M_w as a function of I_{max} with other relationships for intensity. **b** M_w as a function of I_{max} relationships for regression methods RMA, OLS($Y|X$), and OLS($X|Y$)

the study by GASPAR-ESCRIBANO *et al.* (2008), both of which are adjacent to the diagonal of the graph (one-to-one relationship). The regional data selection conducted by GASPAR-ESCRIBANO *et al.* (2008) implicitly involves selecting a predominant type of magnitude.

The different relationships for intensity are compared in Fig. 11a. Along with the RMA relationship derived in this study, the figure includes the local relationships of RUEDA and MEZCUA (2001) and GARCÍA-BLANCO (2009), the global relationship for stable continental regions obtained by JOHNSTON (1996b), and the European regional relationship of STUCCHI *et al.* (2010). The relationship obtained in this work lies slightly above the others. It must be taken into account that the data subset used does not contain many large intensity values compared with some of the other relationships. Figure 11b shows the three regression lines, RMA, OLS, and OLS inverse obtained for intensity, together with the data points of this subset. In this case, the relationship corresponding to the standard regression [OLS($Y|X$)] most closely resembles the relationship obtained by STUC-CHI *et al.* (2010) with European data.

Some of the previous relationships used above for comparative analysis were used for preparation of the European–Mediterranean Earthquake Catalog (EMEC) recently produced by GRÜNTHAL and WA-HLSTRÖM (2012). For the polygon of the Iberian Peninsula, in particular, the selected relationships were the RUEDA and MEZCUA (2002) relationship for

m_{bLg} magnitude (compared in Fig. 8) and the MEZCUA (2002) relationship for intensity. This is updated by the GARCÍA-BLANCO (2009) relationship, used in Fig. 10, which gives similar results.

4. Conclusions

The principal result of this study is a set of regression relationships that enable conversion of the magnitude types reported by the IGN Spanish catalog into moment magnitude M_w. These relationships, their uncertainties, and their usage range are given in Table 4.

For applications such as seismic hazard assessment, is preferable to have a unified measure of earthquake size to characterize seismic sources. Specifically, a final application of these results was the preparation of a homogeneous catalog to be used in the project "Updating Seismic Hazard Maps of Spain" (IGN-UPM WORKING GROUP 2013).

The moment magnitude was chosen as the unified size measurement and the RMA (Reduced Major Axis) regression as the method of obtaining these relationships, given that errors in both variables were considered at least comparable. An advantage of this method is that it considers a symmetrical treatment of both variables.

The relationship for the m_{bLg}(MMS) magnitude is closer to the relationship from JOHNSTON (1996a) for the m_b magnitude than to that for the m_{bLg} magnitude,

probably, as a consequence of a specific magnitude heterogeneity in the IGN catalog for the m_{bLg}(MMS) usage period.

The largest deviations of magnitude values relative to moment magnitude were found for the magnitude m_b(VC), which seems to substantially overestimate the values provided by M_w.

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