Pure appl. geophys. 165 (2008) 981–994
0033–4553/08/050981–14
DOI 10.1007/s00024-008-0333-9

Pure and Applied Geophysics

An Automatic Method of Direct Interpretation of Residual Gravity Anomaly Profiles due to Spheres and Cylinders

J. Asfahani,[1] and M. Tlas[1]

Abstract—We have developed a least-squares minimization approach to determine the depth and the amplitude coefficient of a buried structure from residual gravity anomaly profile. This approach is basically based on application of Werner deconvolution method to gravity formulas due to spheres and cylinders, and solving a set of algebraic linear equations to estimate the two-model parameters. The validity of this new method is demonstrated through studying and analyzing two synthetic gravity anomalies, using simulated data generated from a known model with different random error components and a known statistical distribution. After being theoretically proven, this approach was applied on two real field gravity anomalies from Cuba and Sweden. The agreement between the results obtained by the proposed method and those obtained by other interpretation methods is good and comparable. Moreover, the depth obtained by the proposed approach is found to be in very good agreement with that obtained from drilling information.

Key words: Gravity anomalies, inversion of field gravity anomalies, cylinder-like bodies, sphere-like bodies, systems of algebraic linear equations.

1. Introduction

The goal of gravity inversion is to point estimate the parameters (depth, amplitude coefficient, and shape factor) of gravity anomalies produced by simple geometric shaped structures (sphere, cylinder) from a set of given gravity observations. The gravity anomaly expression produced by a simple geometrically shaped model (sphere and cylinder) can be represented by analytical formula. This mathematical formula functions as both variables depth and shape factor with an amplitude coefficient related to the radius and the density contrast of the buried structure. Several numerical techniques have been presented and reported for interpreting gravity anomalies and estimating depths and amplitude coefficients of geological structures, assuming fixed simple source geometry as a sphere, a horizontal cylinder, or a vertical cylinder. These techniques include, for example, graphical methods (Nettleton, 1962, 1976), ratio methods (Bowin *et al.*, 1986; Abdelrahman *et al.*, 1989), Fourier transform (Odegard and Berg, 1965; Sharma and Geldart, 1968), Euler deconvolution (Thompson, 1982), neural network (Elawadi *et al.*, 2001), Mellin transform

[1] Atomic Energy Commission, P. O. Box 6091, Damascus, Syria. E-mail: jasfahani@aec.org.sy, scientific@aec.org.sy

(MOHAN *et al.*, 1986), least-squares minimization approaches (GUPTA, 1983; LINES and TREITEL, 1984; ABDELRAHMAN, 1990; ABDELRAHMAN *et al.*, 1991; ABDELRAHMAN and EL-ARABY, 1993; ABDELRAHMAN and SHARAFELDIN, 1995a), Werner deconvolution (HARTMAN *et al.*, 1971; JAIN, 1976), KILTY (1983) extended the Werner deconvolution technique to the analysis of gravity data, using both the residual anomaly and its first and second horizontal derivatives. KU and SHARP (1983) further refined the method by using iteration for reducing and eliminating the interference field and then applied Marquardt's nonlinear least-squares method to further refine automatically the first approximation provided by deconvolution. However, only a few techniques have been treated for the determination of shape of the buried structure. These techniques include, for example, Walsh transform (SHAW and AGARWAL, 1990), least-squares methods (ABDELRAHMAN and SHARAFELDIN, 1995b; ABDELRAHMAN *et al.*, 2001a,b), constrained and penalized nonlinear optimization technique (TLAS *et al.*, 2005). Generally, the determination of the depth, shape factor, and amplitude coefficient of the buried structure is performed by these methods from residual gravity anomaly, where, the accuracy of the results obtained by them depends on the accuracy in which the residual anomaly can be separated and isolated from the observed gravity anomaly.

In this paper, a simple and efficient interpretation method is proposed for the best estimate of gravity parameters, e.g., depth and amplitude coefficient of simple shaped bodies such as sphere, horizontal cylinder and vertical cylinder from residual gravity anomaly. The method is basically based on the application of Werner deconvolution method to a modified gravity formulas due to sphere and cylinders, and solving a system of algebraic linear equations. It is mentionable that the proposed technique also could be applicable with only some modifications to any anomaly described by a bell-shaped function.

The accuracy of such a method is demonstrated through theoretical gravity anomalies, where simulated data are generated by using a known model with random errors and a known statistical distribution. Being theoretically proven, the method is thereafter applied to real field gravity anomalies taken from Cuba and Sweden. The agreement between the results obtained by the proposed method and those obtained by other interpretation methods is good and comparable. In addition, the depth obtained by applying this method is found to be in very good agreement with that obtained from drilling information.

2. *Gravity Problem Formulation due to a Sphere Model*

The general expression of a gravity effect (V) for a sphere-like structure at any point on the free surface along the principal profile in a Cartesian coordinate system (Fig. 1) is given by GUPTA (1983) as:

$$V(x_i) = k\frac{z}{(x_i^2 + z^2)^{3/2}} \quad (i = 1, 2, \ldots, N), \tag{1}$$

where, z is the depth from the surface to the center of the sphere body, x_i $(i = 1, \ldots, N)$ is the horizontal position coordinate, and k is the amplitude coefficient given by

$k = 4/3\ \pi G\rho R^3$, where ρ is the density contrast, G is the universal gravitational constant, and R is the radius.

For simplification, V_i is used in the rest of this paper instead of $V(x_i)$ ($i = 1, \ldots, N$).

Multiplying the two sides of equation (1) by the mathematical term $(x_i^2 + z^2)^{3/2}$, it can be found:

$$V_i\left(x_i^2 + z^2\right)^{3/2} = k\,z \quad (i = 1, 2, \ldots, N). \tag{2}$$

Squaring both sides of equation (2), the following equation is obtained

$$V_i^2\left(x_i^2 + z^2\right)^{3} = k^2\,z^2 \quad (i = 1, 2, \ldots, N). \tag{3}$$

Arranging equation (3), it can result:

$$\left(V_i^2 x_i^2 + V_i^2 z^2\right)\left(x_i^4 + z^4 + 2x_i^2 z^2\right) = k^2\,z^2 \quad (i = 1, 2, \ldots, N),$$

or

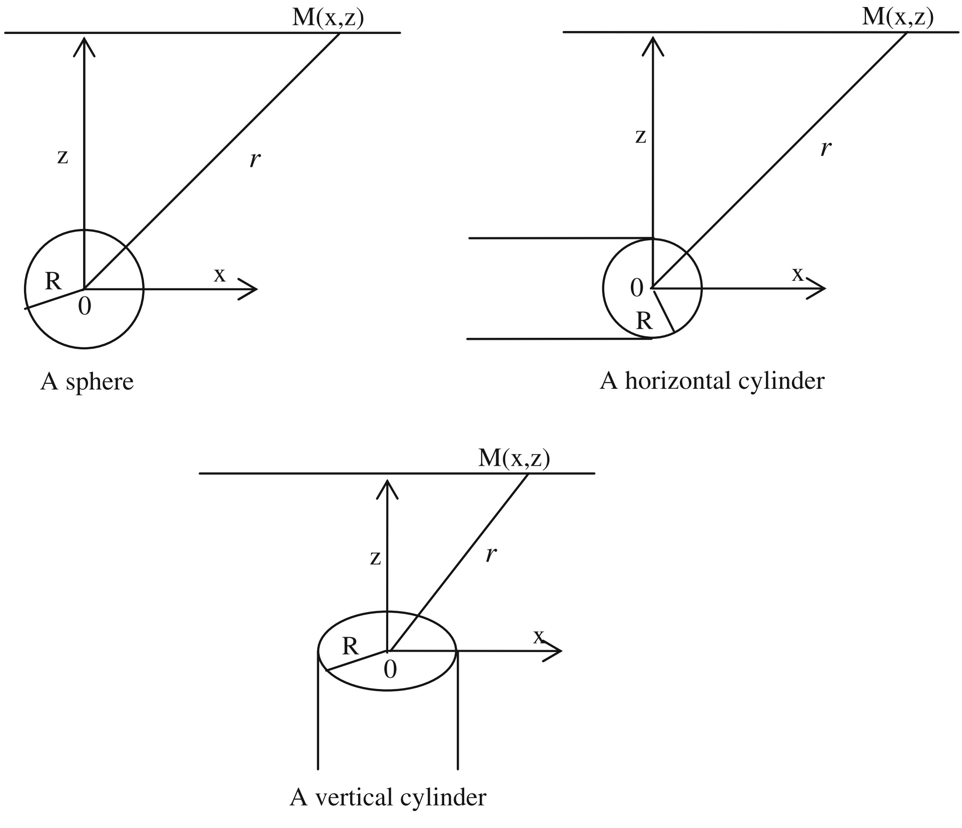

Figure 1
The diagrams for simple geometrical structures (sphere, horizontal cylinder, and vertical cylinder).

$$V_i^2 x_i^6 + 3V_i^2 x_i^2 z^4 + 3V_i^2 x_i^4 z^2 + V_i^2 z^6 = k^2 z^2 \quad (i = 1, 2, \ldots, N). \tag{4}$$

Equation (4) is not linear in the function of parameters z, and k. In order to avoid this non linearity, new variables q_1, q_2, q_3, q_4 are introduced and defined as follows:

$$q_1 = z^2, \tag{5}$$

$$q_2 = z^4, \tag{6}$$

$$q_3 = z^6, \tag{7}$$

$$q_4 = k^2 z^2. \tag{8}$$

Introducing these new variables into equation (4), it can be found

$$V_i^2 x_i^6 + 3V_i^2 x_i^4 q_1 + 3V_i^2 x_i^2 q_2 + V_i^2 q_3 - q_4 = 0 \quad (i = 1, 2, \ldots, N). \tag{9}$$

Equation (9) is now linear in function of variables q_1, q_2, q_3, q_4.

The global optimal solution of the linear system of equations (9) is found by minimizing the following mathematical objective function onto the real space $\mathbb{R}^4$. In mathematical form, it can be written

$$q = \arg\min \varphi(q) = \sum_{i=1}^{N} \left(V_i^2 x_i^6 + 3V_i^2 x_i^4 q_1 + 3V_i^2 x_i^2 q_2 + V_i^2 q_3 - q_4\right)^2.$$

$$\textit{subject to} \quad q \in \mathbb{R}^4$$

This mathematical nonlinear program is simply solved by finding the unique solution of the following system of linear equations: $\partial\varphi(q)/\partial q_i = 0 \quad (i = 1, \ldots, 4)$. This system of linear equations could be written in matrix form as:

$$Aq = b, \tag{10}$$

where A is a squared matrix of 4 $\times$ 4 dimensions given as follows

$$A = \begin{bmatrix} 3 \times \sum_{i=1}^{N} x_i^8 V_i^4 & 3 \times \sum_{i=1}^{N} x_i^6 V_i^4 & \sum_{i=1}^{N} x_i^4 V_i^4 & -\sum_{i=1}^{N} x_i^4 V_i^2 \\ 3 \times \sum_{i=1}^{N} x_i^6 V_i^4 & 3 \times \sum_{i=1}^{N} x_i^4 V_i^4 & \sum_{i=1}^{N} x_i^2 V_i^4 & -\sum_{i=1}^{N} x_i^2 V_i^2 \\ 3 \times \sum_{i=1}^{N} x_i^4 V_i^4 & 3 \times \sum_{i=1}^{N} x_i^2 V_i^4 & \sum_{i=1}^{N} V_i^4 & -\sum_{i=1}^{N} V_i^2 \\ 3 \times \sum_{i=1}^{N} x_i^4 V_i^2 & 3 \times \sum_{i=1}^{N} x_i^2 V_i^2 & \sum_{i=1}^{N} V_i^2 & -N \end{bmatrix}$$

q and b are vectors of four dimensions given as:

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -\sum_{i=1}^{N} x_i^{10} V_i^4 \\ -\sum_{i=1}^{N} x_i^{8} V_i^4 \\ -\sum_{i=1}^{N} x_i^{6} V_i^4 \\ -\sum_{i=1}^{N} x_i^{6} V_i^2 \end{bmatrix}.$$

The linear system of algebraic equations (10) could be easily solved by one of the direct methods (Gauss, Cholesky, Householder) or by one of the iterative methods (Jacobi, Gauss-Seidel, Jacobi and Gauss-Seidel).

The system (10) has a unique solution q, and the causative body of the anomaly $V(x)$ can probably be represented by a spherical model.

The parameters related to the causative sphere body are computed as follows.

From equations (5), (6) and (7), it can be easily found that the depth (z) from the surface to the sphere center is given by:

$$z = \frac{|q_1|^{\frac{1}{2}} + |q_2|^{\frac{1}{4}} + |q_3|^{\frac{1}{6}}}{3}. \tag{11}$$

Using equations (8) and (11), the amplitude coefficient (k) can be given by:

$$k = \pm \frac{\sqrt{|q_4|}}{z}. \tag{12}$$

The sign of k can be assigned by using the accordance between the field data anomaly and the computed one.

3. *Gravity Problem Formulation due to a Vertical Cylinder Model*

The gravity effect (V) of a vertical cylinder-like structure at any point on the free surface along the principal profile in a Cartesian coordinate system (Fig. 1) is given also by GUPTA (1983) as:

$$V(x_i) = k \, \frac{1}{(x_i^2 + z^2)^{\frac{1}{2}}} \quad (i = 1, 2, \ldots, N), \tag{13}$$

where z is the depth from the surface to the top of the body, and k is the amplitude coefficient given by $k = \pi G \rho R^2$.

Multiplying the two sides of equation (13) by the term $\left(x_i^2 + z^2\right)^{\frac{1}{2}}$ and squaring the two sides, it can be found:

$$V_i^2\left(x_i^2+z^2\right)=k^2 \quad (i=1,2,\ldots,N). \tag{14}$$

Arranging equation (14), it can be concluded:

$$V_i^2x_i^2+V_i^2z^2-k^2=0 \quad (i=1,2,\ldots,N). \tag{15}$$

The nonlinearity of equation (15) in function of parameters z and k is avoided by introducing new variables q_1, q_2, defined as follows

$$q_1=z^2, \tag{16}$$

$$q_2=k^2. \tag{17}$$

Introducing these new variables into equation (15), it can result:

$$V_i^2x_i^2+V_i^2q_1-q_2=0 \quad (i=1,2,\ldots,N). \tag{18}$$

The global optimal solution of the system of linear equations (18) is reached by minimizing the following objective function onto the real space $\mathbb{R}^2$. Mathematically, it can be written

$$q=\arg\min\phi(q)=\sum_{i=1}^{N}\left(V_i^2x_i^2+V_i^2q_1-q_2\right)^2$$
$$\textit{subject to} \quad q\in\mathbb{R}^2.$$

This nonlinear program is solved in order to find the unique solution of the following system of linear equations: $\partial\phi(q)/\partial q_i=0 \quad (i=1,2)$. This system of linear equations could be written in matrix form as:

$$Aq=b, \tag{19}$$

where A is a squared matrix of 2×2 dimensions given as follows:

$$A=\begin{bmatrix}-\sum_{i=1}^{N}V_i^4 & \sum_{i=1}^{N}V_i^2\\ -\sum_{i=1}^{N}V_i^2 & N\end{bmatrix}$$

q and b are vectors of two dimensions given as:

$$q=\begin{bmatrix}q_1\\ q_2\end{bmatrix},\quad b=\begin{bmatrix}\sum_{i=1}^{N}x_i^2\,V_i^4\\ \sum_{i=1}^{N}x_i^2\,V_i^2\end{bmatrix}.$$

The solved linear system of algebraic equations (19) has a unique solution q, and the causative body of the anomaly $V(x)$ can probably be represented by a vertical cylinder model.

The unique solution q could be analytically given as follows

$$q_1 = \frac{1}{Q} \times \left\{ N\left(\sum_{i=1}^{N} x_i^2 V_i^4\right) - \left(\sum_{i=1}^{N} V_i^2\right)\left(\sum_{i=1}^{N} x_i^2 V_i^2\right) \right\}$$

$$q_2 = \frac{1}{Q} \times \left\{ \left(\sum_{i=1}^{N} V_i^2\right)\left(\sum_{i=1}^{N} x_i^2 V_i^4\right) - \left(\sum_{i=1}^{N} V_i^4\right)\left(\sum_{i=1}^{N} x_i^2 V_i^2\right) \right\}$$

where,

$$Q = -N\sum_{i=1}^{N} V_i^4 + \left(\sum_{i=1}^{N} V_i^2\right)^2.$$

The parameters related to the causative vertical cylinder will be computed as follows: From equation (16), the depth to the top of the body is found and given by:

$$z = \sqrt{|q_1|}. \tag{20}$$

From equations (16) and (17) the amplitude coefficient can be given by

$$k = \pm\sqrt{|q_2|}. \tag{21}$$

The sign of k also can be assigned by using the accordance between the field data anomaly and the evaluated one.

It is noted that equation (13) is an approximation valid for Z > R, and the complete and exact formulation is given by NABIGHIAN (1962), and NAGY (1965).

4. *Gravity Problem Formulation due to a Horizontal Cylinder Model*

The gravity effect (V) of a horizontal cylinder-like structure at any point on the free surface along the principal profile in a Cartesian coordinate system (Fig. 1) is given also by GUPTA (1983) as:

$$V(x_i) = k\,\frac{z}{(x_i^2 + z^2)} \quad (i = 1, 2, \ldots, N), \tag{22}$$

where, z is the depth from the surface to the center of the body, and k is the amplitude coefficient given by $k = 2\pi G\rho R^2$.

Multiplying the two sides of equation (22) by the term $(x_i^2 + z^2)$ and by arranging them, it can be found:

$$V_i x_i^2 + V_i z^2 - k\,z = 0 \quad (i = 1, 2, \ldots, N). \tag{23}$$

The nonlinearity of equation (23) is avoided by introducing new variables q_1, q_2 defined as follows:

$$q_1 = z^2, \tag{24}$$

$$q_2 = kz. \tag{25}$$

Introducing these new variables into equation (23), it can be concluded:

$$V_i x_i^2 + V_i q_1 - q_2 = 0 \quad (i = 1, 2, \ldots, N.) \tag{26}$$

Equation (26) is now linear in function of variables q_1, q_2.

The global optimal solution of the linear system of equations (26) is obtained by minimizing the following objective function on the real space $\mathbb{R}^2$. Mathematically, it can be written:

$$q = \arg\min \psi(q) = \sum_{i=1}^{N} \left(V_i x_i^2 + V_i q_1 - q_2 \right)^2$$
$$\textit{subject to} \quad q \in \mathbb{R}^2.$$

Solving this nonlinear program is equivalent to solving the following system of linear equations: $\partial\psi(q)/\partial q_i = 0 \quad (i = 1, 2)$. This system of linear equations can be described in matrix form as:

$$Aq = b, \tag{27}$$

where A is a squared matrix of 2×2 dimensions given as follows:

$$A = \begin{bmatrix} -\sum\limits_{i=1}^{N} V_i^2 & \sum\limits_{i=1}^{N} V_i \\ -\sum\limits_{i=1}^{N} V_i & N \end{bmatrix}$$

q and b are vectors of two dimensions given as:

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \quad b = \begin{bmatrix} \sum\limits_{i=1}^{N} x_i^2 V_i^2 \\ \sum\limits_{i=1}^{N} x_i^2 V_i \end{bmatrix}.$$

The linear system of algebraic equations (27) has a unique solution q, and the causative body of the anomaly $V(x)$ can be represented by a horizontal cylinder model.

The unique solution q could be analytically given as follows:

$$q_1 = \frac{1}{Q} \times \left\{ N\left(\sum_{i=1}^{N} x_i^2 V_i^2 \right) - \left(\sum_{i=1}^{N} V_i \right)\left(\sum_{i=1}^{N} x_i^2 V_i \right) \right\}$$

$$q_2 = \frac{1}{Q} \times \left\{ \left(\sum_{i=1}^{N} V_i \right)\left(\sum_{i=1}^{N} x_i^2 V_i^2 \right) - \left(\sum_{i=1}^{N} V_i^2 \right)\left(\sum_{i=1}^{N} x_i^2 V_i \right) \right\}$$

where,

$$Q = -N\sum_{i=1}^{N} V_i^2 + \left(\sum_{i=1}^{N} V_i\right)^2.$$

The parameters related to the causative body, in this case, are computed as follows: The depth (z) to the center of the body is obtained by using equation (24) as follows:

$$z = \sqrt{|q_1|}. \tag{28}$$

Equations (24) and (25) are used to compute the amplitude coefficient as follows:

$$k = \frac{q_2}{z}. \tag{29}$$

5. *Interpretation of a Synthetic Gravity Anomaly due to a Sphere Model with Adding Different Random Errors*

A synthetic gravity anomaly $V(x_i)$ $(i = 1, \ldots, N)$ due to a sphere-like structure is generated from equation (1), using the following assumed gravity parameters: the depth from the surface to the center of the body $z = 25$ unit length, and the amplitude coefficient $k = 100$ mGal.

Two new synthetic gravity anomalies are randomly regenerated from the synthetic gravity anomaly $V(x_i)$ $(i = 1, \ldots, N)$, by using the continuous uniform distribution with maximum random errors of 5% and 10%, respectively. The continuously uniform distribution is purposely used in order to regenerate randomly two gravity anomalies, which resemble well the real observed field measurements.

Both regenerated synthetic gravity anomalies are thereafter interpreted by the proposed method, where the evaluated gravity parameters are presented in Table 1.

The results of Table 1 show good agreement between assumed and evaluated gravity parameters, which obviously indicates the high efficiency of the newly proposed method.

Being theoretically tested and proven, however, this method has limitations, which could be summarized as follows:

Table 1

Interpretation of a synthetic gravity anomaly due to a sphere model with an additional 5% and 10% random errors

Gravity parameters	Assumed gravity parameters	Evaluated gravity parameters with an additional 5% random error	Evaluated gravity parameters with an additional 10% random error
z (unit length)	25	25.7	26.5
k (mGal)	100	104.4	111.9

1. The geometric shape factor of the body should be known a priori (a horizontal cylinder, a vertical cylinder, or a sphere).
2. The method requires field data no more contaminated by gross errors because it is based on a least-squares minimization technique which is very sensible to gross errors in a general case.

6. *Interpretation of Gravity Field Anomalies*

The proposed method has been adapted for interpreting residual gravity anomalies related to three different types of structures, e.g., a sphere, a vertical cylinder, and a horizontal cylinder.

The standard error (σ) is used in this paper as a statistical preference criteria in order to compare the observed and evaluated values. This σ is given by the following mathematical relationship:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} \left(V_i(\text{observed}) - \mathrm{V_i}(\text{evaluated})\right)^2}{N}}, \tag{30}$$

where V_i(observed) and V_i(evaluated) $(i = 1, \ldots, N)$ are the observed and the evaluated values at the points x_i $(i = 1, \ldots, N)$, respectively.

Two residual gravity field anomalies taken from Cuba and Sweden have been reinterpreted in order to examine the applicability and stability of the proposed method. The geometric shaped model is assumed to be known *a priori* such as sphere, horizontal cylinder or vertical cylinder.

6.1. *The Chromites Gravity Anomaly*

A normalized residual gravity anomaly measured over a chromites deposit in Camaguey province, Cuba (Robinson and Coruh, 1988) is shown in Figure 2. Several researchers have used a spherical model to interpret this anomaly (Robinson and Coruh, 1988; Salem *et al.*, 2003). The results showed that a sphere model located at a depth of 21 m probably approximates the source of this anomaly. The application of the proposed method to these field data by using a sphere model, a vertical cylinder, and a horizontal

Table 2

Interpretation of gravity field anomaly over a chromites deposit in Camaguey province, Cuba

Gravity parameters	Horizontal cylinder model	Vertical cylinder model	Sphere model
z (m)	17.71	7.14	18.91
k (mGal)	18.24	8.71	157.34
σ (mGal)	0.02	0.6	0.3

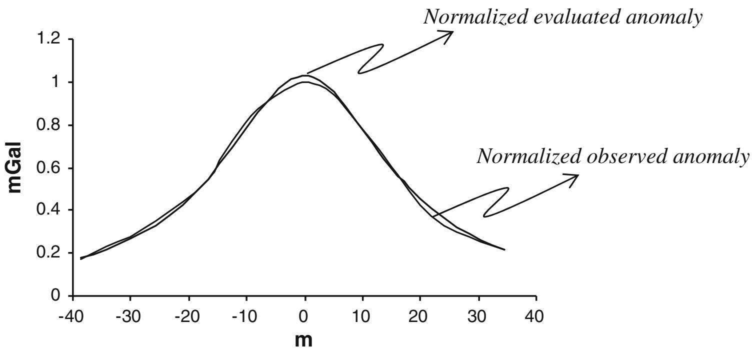

Figure 2
Normalized residual gravity field anomaly over a chromites deposit, Camaguey province, Cuba. The evaluated curve by the proposed method is shown for a horizontal cylinder model.

model respectively, yields to the estimation of the amplitude coefficient and the depth from the surface to the body as shown in Table 2.

Table 2 shows that the highest value of standard error $\sigma = 0.6$ mGal is obtained for the vertical cylinder, meaning that, the residual gravity anomaly cannot be modeled as a vertical cylinder. The second highest value of standard error $\sigma = 0.3$ mGal is obtained for the sphere, which means the residual gravity anomaly is also not preferably to be modeled as a sphere. The smallest value of standard error $\sigma = 0.02$ mGal is obtained for the horizontal cylinder, meaning that the gravity anomaly is preferably to be modeled as a horizontal cylinder.

The depth obtained in this case ($z = 17.71\ m$) is found to be in very good agreement with the information obtained from drill-hole information ($z = 21$ m).

6.2. *The Karrbo Gravity Anomaly*

A residual gravity anomaly measured over the two-dimensional pyrrhotite ore, Karrbo, Vastmanland, Sweden (SHAW and AGARWAL, 1990) is shown in Figure 3. This anomaly is interpreted by applying the proposed method and assuming *a priori* a spherical model, a vertical cylinder model, and a horizontal cylinder model, respectively. The interpretation yields to the estimation of the amplitude coefficient and the depth from the surface to the body as shown in Table 3.

Table 3

Interpretation of gravity field anomaly over the two-dimensional pyrrhotite ore, Karrbo, Vastmanland, Sweden

Gravity parameters	Horizontal cylinder model	Vertical cylinder model	Sphere model
z (m)	4.7	1.57	5.8
k (mGal)	4.79	2.14	13.20
σ (mGal)	0.03	0.44	0.34

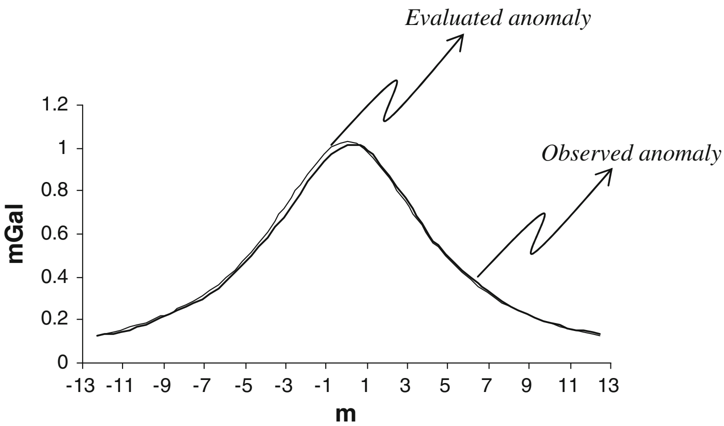

Figure 3
Residual gravity field anomaly over the two-dimensional pyrrhotite ore, Karrbo, Vastmanland, Sweden. The evaluated curve by the proposed method is shown for a horizontal model.

It is noteworthy that the highest value of standard error $\sigma = 0.44$ mGal is obtained for the vertical cylinder, meaning that, the residual gravity anomaly cannot be modeled as a vertical cylinder. The second highest value of standard error $\sigma = 0.34$ mGal is obtained for the sphere, meaning that the gravity anomaly also cannot be modeled as a sphere. The smallest value of standard error $\sigma = 0.03$ mGal is obtained for the horizontal cylinder, which means that the gravity anomaly is preferably to be modeled as a horizontal cylinder.

The depth obtained ($z = 4.7$ m) in this case is found to be in very good agreement with that obtained by Tlas *et al.* (2005) ($z = 4.82$ m).

7. Conclusion

A new original and accurate method is proposed to interpret real field gravity anomalies and to estimate gravity parameters related to simple shaped bodies such as sphere, horizontal cylinder and vertical cylinder. The proposed approach is basically based on the application of Werner deconvolution technique to a modified gravity formula due to spheres and cylinders, and on solving a set of algebraic linear equations in order to estimate the two gravity model parameters (depth, and amplitude coefficient). The efficiency of such a proposed method is demonstrated through synthetic anomalies, by using simulated data generated from a known model with different random errors of 5% and 10%, and a known statistical distribution. After being theoretically proven, the method was thereafter applied to two real field gravity anomalies taken from Cuba and Sweden. The agreement between the results obtained by the proposed method and those obtained by other interpretation methods is good and comparable. Moreover, the depth of the studied structures obtained by the developed method is found to be in a very good agreement with that obtained from drilling information. The algorithm developed in this paper is designed to treat methodologically with an integral manner the three mentioned structures (sphere, vertical and horizontal cylinders). A decision based on the standard error (σ) is taken at the end of the mathematical process in the algorithm, in order to select the preferable structure that can represent well the studied gravity field anomaly.

This interpretation method can be easily put in a code, robust, and furthermore, the convergence towards the optimal estimation of the parameters is assured and rapidly reached.

This method is therefore recommended for routine analysis of gravity anomalies in an attempt to determine the parameters related to the studied structures, and could also extended to be applicable to interpret any gravity anomaly if it is described by a bell-shaped function.

Acknowledgments

Dr. I. Othman, Director General of the Atomic Energy Commission of Syria is thanked for his interest and continuous encouragement to achieve this work. Special thanks are due to the anonymous reviewers and to Professor Pierre Keating from the Geological Survey of Canada for their constructive suggestions and remarks that considerably improved the paper quality.

References

Abdelrahman, E.M., El-Araby, T.M., El-Araby, H.M., and Abo-Ezz, E.R. (2001a), *Three least-squares minimization approaches to depth, shape, and amplitude coefficient determination from gravity data*, Geophys. *66*, 1105–1109.

Abdelrahman, E.M., El-Araby, T.M., El-Araby, H.M., and Abo-Ezz, E.R., (2001b), *A new method for shape and depth determinations from gravity data*, Geophys. *66*, 1774–1780.

Abdelrahman, E.M. and Sharafeldin, S.M. (1995a), *A least-squares minimization approach to depth determination from numerical horizontal gravity gradients*, Geophys. *60*, 1259–1260.

Abdelrahman, E.M. and Sharafeldin, S.M. (1995b), *A least-squares minimization approach to shape determination from gravity data*, Geophys. *60*, 589–590.

Abdelrahman, E.M. and El-Araby, T.M. (1993), *A least-squares minimization approach to depth determination from moving average residual gravity anomalies*, Geophys. *59*, 1779–1784.

Abdelrahman, E.M., Bayoumi, A., I., and El-Araby, H.M. (1991), *A least-squares minimization approach to invert gravity data*, Geophys. *56*, 115–118.

Abdelrahman, E.M. (1990), *Discussion on "A least-squares approach to depth determination from gravity data" by Gupta, O.P.*, Geophys. *55*, 376–378.

Abdelrahman, E.M., Bayoumi, A.I., Abdelhady, Y.E., Gobash, M.M., and El-Araby, H.M. (1989), *Gravity interpretation using correlation factors between successive least-squares residual anomalies*, Geophys. *54*, 1614–1621.

Bowin, C., Scheer, E., and Smith, W. (1986), *Depth estimates from ratios of gravity, geoid and gravity gradient anomalies*, Geophys. *51*, 123–136.

ELAWADI, E., SALEM, A., and Ushijima, K. (2001), Detection of cavities from gravity data using a neural network, Exploration Geophys. *32*, 75–79.

GUPTA, O.P. (1983), *A least-squares approach to depth determination from gravity data*, Geophys. *48*, 357–360.

HARTMANN, R.R., TESKEY, D., and FRIEDBERG, I. (1971), *A system for rapid digital aeromagnetic interpretation*, Geophys. *36*, 891–918.

JAIN, S. (1976), *An automatic method of direct interpretation of magnetic profiles*, Geophys. *41*, 531–541.

KILTY, T. K. (1983), *Werner deconvolution of profile potential field data*, Geophys. *48*, 234–237.

KU, C.C. and SHARP, J.A. (1983), *Werner deconvolution for automatic magnetic interpretation and its refinement using Marquardt's inverse modeling* Geophys. *48*, 754–774.

LINES, L. R. and TREITEL, S. (1984), *A review of least-squares inversion and its application to geophysical problems*, Geophys. Prospect. *32*,159–186.

MOHAN, N.L., ANANDABABU, L. and ROA, S. (1986) *Gravity interpretation using the Melin transform*, Geoptys. *51*, 114–122.

NABIGHIAN, M. N. (1962), *The gravitational attraction of a right rectangular cylinder at points external to it*, Geofis. Pura e Appl. *53*, 45–51.

NAGY, D. (1965), *The evaluation of Heumans's Lamda function and its application to calculate the gravitational effects of a right circular cylinder*, Pure Appl. Geophys. *62*, 3–12.

NETTLETON, L.L. (1962), *Gravity and magnetics for geologists and seismologists*, AAPG *46*, 1815–1838.

NETTLETON, L.L. *Gravity and Magnetic in Oil Prospecting* (Mc-Graw Hill Book Co. 1976).

ODEGARD, M.E. and BERG, J. W. (1965), *Gravity interpretation using the Fourier integral*, Geophys. *30*, 424–438.

ROBINSON, E.S., and CORUH, C. *Basic Exploration Geophysics* (Wiley, New York, NY, 1988) 562 pp.

SALEM, A., ELAWADI, E. and USHIJIMA, K. (2003), *Depth determination from residual gravity anomaly data using a simple formula*, Comp. Geosci. *29*, 801–804.

SHARMA, B. and GELDART, L.P. (1968), *Analysis of gravity anomalies of two-dimensional faults using Fourier transforms*, Geophys. Prospect. *16*, 77–93.

SHAW, R.K. and AGARWAL, S.N.P. (1990) *The application of Walsh transform to interpret gravity anomalies due to some simple geometrically shaped causative sources: A feasibility study*, Geophys. *55*, 843–850.

THOMPSON, D.T. (1982), *EULDPH-a new technique for making computer-assisted depth estimates from magnetic data*, Geophys. *47*, 31–37.

TLAS, M., ASFAHANI, J. and KARMEH, H. (2005), *A versatile nonlinear inversion to interpret gravity anomaly caused by a simple geometrical structure*, Pure Appl. Geophys. *162*, 2557–2571.

(Received July 26, 2007, accepted January 24, 2008)
Published Online First: May 10, 2008