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A Constrained Nonlinear Inversion Approach to Quantitative Interpretation of Self-potential Anomalies Caused by Cylinders, Spheres and Sheet-like Structures

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Abstract — An interpretative method based on a nonlinearly mathematical optimization concept has been developed in this paper, in order to interpret self-potential anomalies (*SP*) due to horizontal cylinder, vertical cylinder, sphere and sheet-like structures. This interpretative method comprises three main steps. The first step is to formulate mathematically a nonlinearly constrained minimization problem (*NCMP*) to describe the geophysical problem related to the studied structure. The second one is to suggest an interior penalty function in order to convert the nonlinearly constrained minimization problem (*NCMP*) into a nonlinearly unconstrained minimization one (*NUMP*). The third step is to solve the converted nonlinearly unconstrained minimization problem (*NUMP*) by the well-known Hooke and Jeeves direct search algorithm in order to estimate the geophysical parameters of the studied structure, i.e., depth, polarization angle, electric dipole moment (magnitude of polarization) and geometric shape factor. The Hooke and Jeeves direct search algorithm is purposely chosen for being robust and its application to *SP* data allows a rapid convergence towards the optimal estimate of parameters. This interpretative method was first tested on theoretical synthetic models with different random noise, where a very close agreement was obtained between assumed and evaluated parameters.

The validity of the proposed interpretative method is also tested on practical field examples taken from Turkey, India and Germany, where available *SP* data existed and was previously analyzed by different interpretative methods. The agreement between the results obtained by the developed method and those obtained by other published methods is good.

Key words: Self-potential anomalies, mathematical optimization, penalty function, polarized structures, *SP* interpretation.

Introduction

The self-potential *SP* method, which is one of the oldest geophysical methods, plays an important role in many fields of applied geophysics, particularly in the exploration of metallic sulfides. Two quantitative categories are usually adapted for the interpretation of *SP* anomalies in order to determine principally the depth and the shape of a buried structure. Both of these two parameters are considered as the most important problem in exploration geophysics.

The first category includes 2-D and 3-D continuous modelling (GUPTASARMA, 1983; FURNESS, 1992 and 1993; SHI and MORGAN, 1996; PATELLA, 1997). Using the

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methods of this category, current density, resistivity, and rough estimation of the structure's depth are prerequisites for running the modelling process. The more accurate the parameters, the higher the close agreement between the theoretical curve and the *SP* observed data. Accordingly, the derived geological interpretation will be more realistic.

The second category includes fixed simple geometrical methods, oriented to determine the depth and the shape of the buried structure, depending on the residual and/or *SP* observed data. However, the obtained theoretical models by these methods may differ slightly from the real geological setting, nonetheless they are qualified as sufficient to determine the mentioned parameters. Thus, geological interpretation is reasonable, and close agreement between observed and computed anomalies is obtained.

The advantages of the fixed geometrical methods over the 2-D and 3-D continuous modelling are:

a. They do not require current density, resistivity and rough estimation on depth.
b. They require only *SP* data.
c. Interpretation of isolated *SP* anomalies is fast and accurate.

Different methods have been proposed to interpret *SP* data using a fixed simple geometrical structure (MEISER, 1962; PAUL, 1965; RAO *et al.*, 1970; BHATTACHARYA and ROY, 1981; ATCHUTA RAO and RAM BABU, 1983; MURTY and HARICHARAN, 1984, 1985; RAO and MOHAN, 1984; ABDELRAHMAN and SHARAFELDIN 1997; SHALIVAHAN *et al.*, 1998; ASFAHANI and TLAS, 2002; ASFAHANI *et al.*, 2004.

In the present paper, a new interpretative technique related to this fixed simple geometry and based on a nonlinearly constrained optimization concept is developed in order to interpret *SP* anomalies due to horizontal cylinder, vertical cylinder, sphere, and sheet-like structures. The method consists of three main steps:

1. The *SP* geophysical problem related to the studied structure is firstly described by formulating a nonlinearly constrained minimization problem (*NCMP*). This nonlinearly constrained minimization problem is to minimize a mathematical objective function $f(v)$ on an unbounded subset X contained in the real space of parameters. Where v is the vector of geophysical parameters and X is a subset defined by mathematical inequalities constraints of the form $g_i(v) \geq 0$ $(i = 1, ..., m)$ which the geophysical parameters are generally assumed to satisfy. Ignoring these mathematical constraints probably yields to error estimations of parameters in general. This explains the reasons behind formulating the geophysical problem as a nonlinearly constrained optimization problem. The objective function $f(v)$ is taken, in this research, as the squared Euclidean distance between observed points and synthetic potentials.
2. The (*NCMP*) is converted into a nonlinearly unconstrained minimization problem (*NUMP*) by using a proposed interior penalty function. The goal of using the penalty function is to eliminate the constraints $g_i(v) \geq 0$ $(i = 1, \ldots, m)$ of (*NCMP*)

and reactivate them anew in the target function of (*NUMP*). The target function of (*NUMP*) considers both the objective function of (*NCMP*) and the proposed interior penalty function. The penalty function concept was previously adopted for the interpretation of magnetic anomalies (ASFAHANI and TLAS, 2004).

3. The (*NUMP*) is solved by using Hooke and Jeeves direct search algorithm, very well known for optimizing numerical functions of several real variables.

The obtained solution of the (*NUMP*) includes the geophysical parameters of the studied structure such as: depth, polarization angle, amplitude coefficient and geometric shape factor. Two structural cases have been treated and interpreted by the proposed method.

The first case deals with the interpretation of *SP* anomalies due to a horizontal cylinder, vertical cylinder and a sphere. The validity of this method is tested on a theoretical synthetic example with different random noise of 2% and 4% and through a practical field example taken from Turkey.

The second case deals with the interpretation of *SP* anomalies due to a two-dimensional inclined sheet of finite depth extent. It is also tested on a theoretical synthetic example with different random noise of 2% and 4% and on practical field examples taken from India and Germany.

Hooke and Jeeves Direct Search Algorithm

The HOOKE and JEEVES direct search algorithm (1961) is one of the most widely known methods for optimizing a numerical function of several real variables on the real space $\mathbb{R}^n$. Here, n is the number of model parameters. We will now illustrate the algorithm for solving the following multi-variables unconstrained problem:

$$\begin{aligned}&\text{Minimize } \phi(v)\\&\text{Subject to } v \in \mathbb{R}^n,\end{aligned}$$

where the numerical function $\phi(v)$ is called the objective function of the problem and $v = (v_1, \ldots, v_n) \in \mathbb{R}^n$ is the vector of model parameters (decision variables).

The Hooke and Jeeves direct search algorithm consists of two distinct phases. The first one is an exploratory search phase which serves to establish a direction of improvement. The second one is a pattern move which extracts the current solution vector to another point in the solution space. The algorithm for minimizing a numerical function proceeds as follows:

Main Procedure

Initialization (Input): $\varepsilon > 0$ is the accuracy parameter; n linearly independent search directions $d^1, \ldots, d^n$. It is possible to take $d^j = h\, e^j$ $(j = 1, \ldots, n)$ where $h \in \mathbb{R}$ and e^j

denote the j-th unit vector, equal to the j-th column in the unit matrix I; $0 < \alpha < 1$ damping factor; $v \in \mathbb{R}^n$ is a given initial point.

Call the subroutine auxiliary procedure $\hat{v} =$ explore (v, h)

While$(|h| \geq \varepsilon)$**do**
{we always know two points v and $\hat{v}$}
If $\phi(\hat{v}) < \phi(v)$ **then**
$z = \hat{v} + (\hat{v} - v)$ (Pattern move or search)
$v = \hat{v}$
else
$h = \alpha h$
$z = v$
end
$\hat{v} =$ explore (z, h)
end

Auxiliary procedure (exploratory search) $\hat{v} =$ explore (v, h)

begin
$\hat{v} = v$
for $j = 1$ to n **do**
$\hat{\phi} = \min\{\phi(\hat{v} - h e^j), \phi(\hat{v}), \phi(\hat{v} + h e^j)\}$
$\hat{v} =$ Corresponding point
end
end

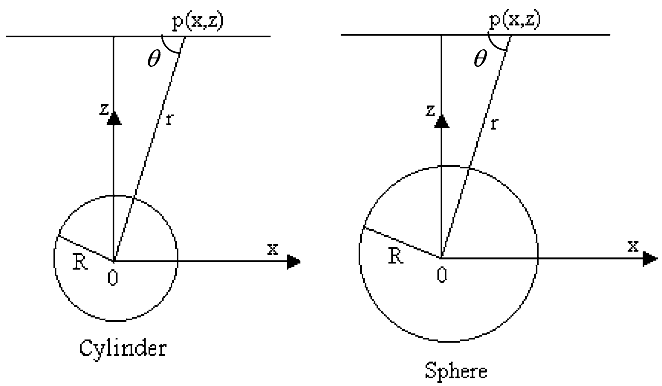

Figure 1
Cross sectional view of a polarized horizontal cylinder and sphere.

This algorithm has a good robustness and is easily inserted in a code. It does not require differentiality of the objective function with respect to the decision variables. HOOKE and JEEVES (1961), BAZARAA and SHETTY (1979), NASH (1990), and PHILLIPS *et al.* (1976) provide more details about this algorithm.

Geophysical Problem Formulation due to Cylinder and Sphere

Following BHATTACHARYA and ROY (1981), the general expression describing the self-potential anomaly *SP* at a point on the *x* axis of an arbitrary polarized structure is given by the following equation (Fig.1):

$$V(x_i, z, \theta, k, q) = k\frac{x_i \cos\theta + z\sin\theta}{(x_i^2 + z^2)^q} \quad (i = 1, \ldots, N), \tag{1}$$

where z denotes the depth center of the body, θ denotes the polarization angle, k denotes the electric dipole moment (magnitude of polarization), x_i denotes the position coordinate, and q denotes the geometric shape factor whose value is as follows:

$$q = \begin{cases} 0.5 & \text{for vertical cylinder} \\ 1 & \text{for horizontal cylinder} \\ 1.5 & \text{for sphere} \end{cases}$$

The evaluation of the parameters (z, θ, k, q) could be obtained by solving the following nonlinearly constrained minimization problem:

$$\begin{aligned} &\text{Minimize } f(z, \theta, k, q) = \sum_{i=1}^{N} [L(x_i) - V(x_i, z, \theta, k, q)]^2 \\ &\text{Subject to} \qquad z \geq 0 \\ &\qquad\qquad\qquad -90^\circ \leq \theta \leq +90^\circ \\ &\qquad\qquad\qquad 0.5 \leq q \leq 1.5 \\ &\qquad\qquad\qquad -\infty < k < +\infty \end{aligned} \tag{NCMP}$$

where $L(x_i) \quad (i = 1, \ldots, N)$ are the observed values of the self-potential anomaly at the points $x_i \quad (i = 1, \ldots, N)$. This problem is very difficult to solve in the domain of convex nonlinearly constrained programming because the feasible region of (*NCMP*)

$$X = \left\{(z, \theta, k, q) \in \mathbb{R}^4 / z \geq 0, -90^\circ \leq \theta \leq +90^\circ, 0.5 \leq q \leq 1.5, \text{ and } -\infty < k < +\infty\right\}$$

is not bounded in the real space $\mathbb{R}^4$. To avoid this difficulty, the mathematical problem (*NCMP*) is converted into a nonlinearly unconstrained minimization one by introducing a new objective function $\phi(z, \theta, k, q)$. This function $\phi(z, \theta, k, q)$ considers both the objective function $f(z, \theta, k, q)$ of (*NCMP*) and a suggested interior penalty function (logarithmic form), which is defined by the bounded constraints of the studied problem (*NCMP*). This new objective function is defined as follows:

$$\phi(z,\theta,k,q) = f(z,\theta,k,q) - r \times \sum_{i=1}^{m} \mathrm{Ln}(g_i),$$

where $g_1 = z$, $g_2 = 90 - \theta$, $g_3 = 90 + \theta$, $g_4 = q - 0.5$, $g_5 = 1.5 - q$, m is the number of constraints g_i $(i = 1, \ldots .m)$ and r (penalty factor) is an arbitrary positive real number chosen to be close to zero, and is taken as equal to $1/N(N = 262$ discrete points) in the cylinder and sphere interpretation. The choice of penalty factor as equal to the inverse of the number of sampled data is not a restriction in the work, because we can choose any number from the interval $(0, 1)$ and the convergence of the method is not affected by this choice.

Using this new function $\phi(z,\theta,k,q)$, the problem (*NCMP*) becomes as follows:

$$\begin{aligned} &\text{Minimize } \phi(z,\theta,k,q) \\ &\text{Subject to } (z,\theta,k,q) \in \mathbb{R}^4, \end{aligned} \qquad (NUMP)$$

where the objective function of (*NUMP*) is:

$$\phi(z,\theta,k,q) = \sum_{i=1}^{N} \left[L(x_i) - k \frac{x_i \cos\theta + z \sin\theta}{(x_i^2 + z^2)^q} \right]^2 - r \times [\mathrm{Ln}\, z + \mathrm{Ln}\,(90-\theta) + \mathrm{Ln}(90+\theta) + \mathrm{Ln}(q-0.5) + \mathrm{Ln}(1.5-q)].$$

The mathematical problem (*NUMP*) is then solved by using Hooke and Jeeves direct search algorithm to estimate the geophysical parameters (z,θ,k,q). This interpretative method is initially tested on a theoretical synthetic example with different random errors in order to demonstrate its efficiency and stability, and secondly validated on *SP* field data taken from Turkey.

Theoretical Synthetic Example

A theoretical synthetic example has been studied using the following assumed parameters: $z = 4$ units, $\theta = 30°$, $k = -1000$ and $q = 0.5$. These assumed parameters have been used in the equation (1) of $V(x_i, z, \theta, k, q)$ in order to generate the corresponding theoretical curve. New random data are regenerated by applying a continuous uniform distribution with maximum random error of 2% and 4% respectively, on the theoretical curve. The real goal of using the continuous uniform generator is to regenerate very close data similar to the field measurements. This explains the advantages of studying synthetic examples with different random errors generated by random generators in order to validate the method.

Both regenerated random data are thereafter subjected to interpretation by the proposed interpretative method, where the evaluated parameters are shown in Table 1.

Table 1

Theoretical synthetic example with 2% and 4% random noise

Geophysical parameters	Assumed parameters	Evaluated parameters with 2% random noise	Evaluated parameters with 4% random noise
z (unit)	4	4.010	3.976
$\theta°$	30	29.966	30.138
k	-1000	-1024.771	-1028.884
q	0.5	0.501	0.497

Results show a close agreement between assumed and evaluated parameters, which attests to the efficiency, the stability and the validity of the proposed method for the *SP* data interpretation.

Field Example

Figure 2 shows the Suleynankoy self-potential anomaly, Ergani copper district, 65 km SE of Elazig in eastern Turkey. This field example was interpreted by YUNGUL (1950) and BHATTACHARYA and ROY (1981) by taking into consideration the spherical target model ($q = 1.5$) and the horizontal cylinder target model ($q = 1$). ABDELRAHMAN and SHARAFELDIN (1997), ABDELRAHMAN *et al.* (1997) and SHALIVAHAN *et al.* (1998) treated this *SP* data by assuming a spherical target model only. They used in their interpretation the traditional least-squares technique over only one

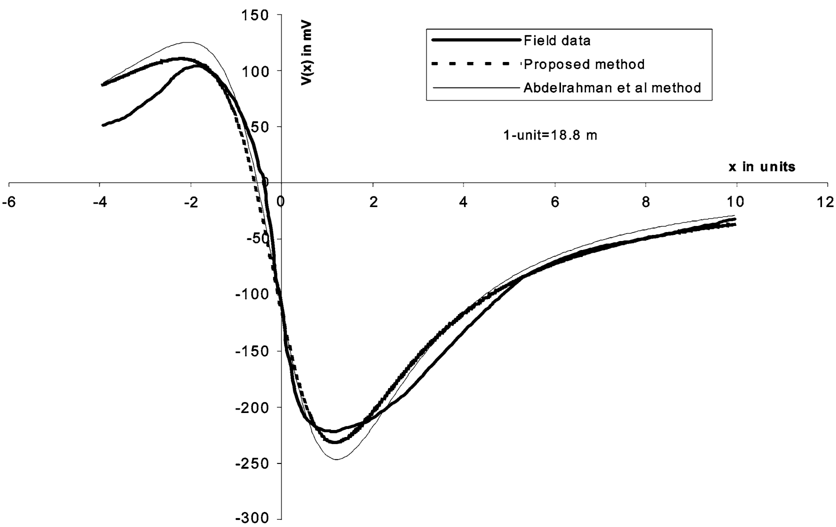

Figure 2
SP anomaly over a polarized copper ore body formation in the Ergani district, Turkey. The theoretical curves for our method ($q = 1.194$), and for the ABDELRAHMAN *et al.* (1997) method ($q = 1.356$) are shown.

Table 2

Geophysical parameters with different values of the geometric shape factor

Evaluated parameters	$q = 0.5$ (Vertical cylinder)	$q = 1$ (Horizontal cylinder)	$q = 1.5$ (Sphere)	q is a variable, estimated by our method to be 1.194
$z(m)$	7.200	27.410	47.508	35.686
$\theta°$	57.181	21.199	14.439	17.655
k	−184.748	−491.782	−2805.85	−928.211
Standard error over 262 discrete points	49.178	17.082	17.177	15.617

variable, mainly the depth of structure, without taking into consideration any mathematical restrictions on the geophysical parameters.

The *SP* anomaly has been digitized over a length of 262 m at an interval of 18.8 m (1 unit = 18.8 m) and subjected to interpretation by applying the newly proposed technique. The interpretation is carried out in four cases of q (shape factor), where $q = 0.5$ (vertical cylinder), $q = 1$ (horizontal cylinder), $q = 1.5$ (sphere) and q as a continuous real variable in the closed interval [0.5, 1.5]. The obtained results are summarized in Table 2.

Standard error (*SE*) is used in this research as a statistical criteria in order to compare between the obtained results shown in Table 2. This *SE* is given by the following mathematical relationship:

$$SE = \sqrt{\frac{\sum_{i=1}^{N} \left[L(x_i) - V(x_i, z, \theta, k, q)\right]^2}{N - w}},$$

where w is the number of independent parameters, in our studied model $w = 4$ (i.e., z, θ, k, q) and N is the number of discrete points x_i.

The highest value of standard error $SE = 49.178$ is obtained for $q = 0.5$ as shown in Table 2, which indicates clearly that the shape of the structure corresponding to the *SP* anomaly could not be modeled as a vertical cylinder. The lowest value of standard error $SE = 15.617$ is obtained for $q = 1.194$, suggesting that the source must be either a horizontal cylinder or a 3-*D* source with a hemi-cylinder roof and a root buried at a depth of 35.686 m. The random errors committed in the estimation of geophysical parameters z, θ, k and q for $q = 1.194$ are evaluated to be equal to 1.439 m, 0.584°, 104.990 and 0.033, respectively.

The standard error $SE = 17.082$ obtained for $q = 1$ (horizontal cylinder) is slightly less than the standard error $SE = 17.177$ obtained for $q = 1.5$ (sphere), which justifies a structure of near horizontal cylinder model responsible for the *SP* anomaly.

A comparison study has been carried out for different values of the shape factor (q) according to different interpretative methods as shown in Table 3.

Table 3

Interpretation of SP anomaly profile over a polarized copper ore body in Ergani district, Turkey

Evaluated parameters	ABDELRAHMAN and SHARAFELDIN (1997)	ABDELRAHMAN *et al.* (1997)	ASFAHANI and TLAS (2002)	Present method
z (m)	42	38.780	39.386	35.686 ± 1.439
$\theta°$	13	14.670	16.844	17.655 ± 0.584
k	−2458	−1549.360	−1208.785	-928.211 ± 104.990
q	1.5	1.356	1.269	1.194 ± 0.033
Standard error over 262 discrete points	21.016	18.264	15.924	15.617

Table 3 also shows the standard error (*SE*), which is computed for each interpretative method, ABDELRAHMAN and SHARAFELDIN (1997) for $q = 1.5$, ABDELRAHMAN *et al.* (1997) for $q = 1.356$, ASFAHANI and TLAS (2002) for $q = 1.269$, and the present method for $q = 1.194$. The lowest value of *SE* is obtained for the new proposed method (15.617), where a slight improvement is reached in comparison with our previous method (ASFAHANI and TLAS, 2002). This difference in *SE* between our two methods is due to the difference in formulation of mathematical models, which describe the same geophysical problem, and to the different algorithms applied to solve these mathematical models.

According to the *SE* shown in Table 3, the proposed method is therefore qualified as the best one for the interpretation of *SP* data.

Geophysical Problem Formulation due to a Two-dimensional Inclined Sheet of Finite Depth Extent

The geometry of the inclined sheet of finite depth extent is shown in Figure 3. The upper and lower edges of the sheet are situated at depth h and H units, respectively below the ground surface. The expression of the *SP* anomaly due to the sheet along a profile perpendicular to its strike is given by ROY and CHOWDHURY (1959) as follows:

$$U(r_1, r_2) = \frac{\rho I}{2\pi} \mathrm{Ln} \frac{r_1^2}{r_2^2},$$

where I: is the current per units length, ρ: is the resistivity of the surrounding medium, r_1, r_2: are the distances of the edges of the sheet from the point of observation.

From Figure 3, it is shown that:
$r_1^2 = x^2 + h^2$ and $r_2^2 = (x - a)^2 + H^2$, where $a = \frac{H-h}{\tan \beta}$, β is the inclination angle and x is the distance of the point of observation from the origin.

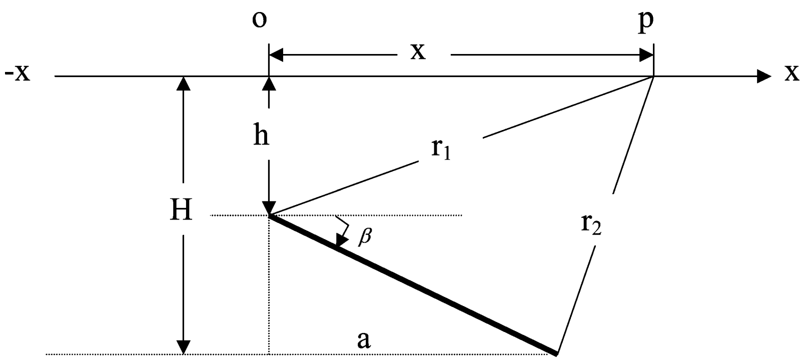

Figure 3
Cross section of an inclined sheet of finite depth extent.

Replacing the expressions of r_1, r_2 and $p = \dfrac{\rho I}{2\pi}$ in the equation of $U(r_1, r_2)$, the following equation could be obtained:

$$U(x, h, H, \beta, p) = p \text{ Ln } \frac{x^2 + h^2}{(x - a)^2 + H^2}. \tag{2}$$

The evaluation of the parameters (h, H, β, p) could be obtained by solving the following nonlinearly constrained minimization problem:

$$\begin{aligned} &\text{Minimize } g(h, H, \beta, p) = \sum_{i=1}^{N} [L(x_i) - U(x_i, h, H, \beta, p)]^2 \\ &\text{Subject to} \quad h \leq H \\ &\qquad 0 \leq \beta \leq 180^\circ \\ &\qquad h \geq 0 \\ &\qquad H \geq 0 \\ &\qquad -\infty < p < +\infty. \end{aligned} \tag{$(NCMP)_1$}$$

The mathematical problem $(NCMP)_1$ is converted into a nonlinearly unconstrained minimization problem by introducing a new objective function $\varphi(h, H, \beta, p)$. The function $\varphi(h, H, \beta, p)$ takes into consideration both the objective function $g(h, H, \beta, p)$ of $(NCMP)_1$ and a suggested interior penalty function, which is defined by the bounded constraints of $(NCMP)_1$. This new objective function is defined as follows:

$$\begin{aligned} \varphi(h,H,\beta,p) &= g(h,H,\beta,p) - r \times [\text{Ln}\,h + \text{Ln}\,H + \text{Ln}(H-h) + \text{Ln}\,\beta + \text{Ln}(180-\beta)] \\ &= \sum_{i=1}^{N} \left[L(x_i) - p\,\text{Ln}\,\frac{x^2+h^2}{(x-a)^2+H^2} \right]^2 - r \times [\text{Ln}\,h + \text{Ln}\,H + \text{Ln}(H-h) + \text{Ln}\,\beta \\ &\quad + \text{Ln}(180-\beta)], \end{aligned}$$

where r (penalty factor) is taken as equal to $1/N(N = 198$ discrete points) in the sheet interpretation.

Using this new objective function, the problem $(NCMP)_1$ becomes as follows:

$$\begin{aligned} &\text{Minimize } \varphi(h, H, \beta, p) \\ &\text{Subject to } (h, H, \beta, p) \in \mathbb{R}^4. \end{aligned} \qquad (NUMP)_1$$

The mathematical problem $(NUMP)_1$ is then solved by using the Hooke and Jeeves direct search algorithm, which allows the values of geophysical parameters (h, H, β, p) to be obtained. The proposed method is initially tested on a theoretical synthetic example with different random errors and secondly practiced on two field examples taken from India and Germany.

Theoretical Synthetic Example

A theoretical synthetic example has been studied, using the following assumed parameters $h = 4$ units, $H = 10$ unit, $\theta = 30°$ and $p = 200$ mV. These parameters have been used in equation (2) of $U(x, h, H, \beta, p)$ in order to generate the corresponding theoretical curve. New random data are regenerated by applying the continuous uniform distribution with maximum random error of 2% and 4%, respectively on the theoretical curve. Both regenerated random data are thereafter subjected to interpretation by the proposed interpretative method, where the evaluated parameters are shown in Table 4.

Results show the goodness between assumed and evaluated parameters, which indicates obviously the suitability of this method for interpreting *SP* field data related to inclined sheet-like structure.

Field Examples

Two field examples from India and Germany have been interpreted by the suggested interpretative method.

1. The first anomaly shown in Figure 4 is taken across a mineralized belt in the Kalava fault zone, 52 km south of Karnool in Cuddapah basin, Andhra, Pradesh, India (SANKER NARAYAN *et al.*, 1982). The earlier drilling over some anomaly

Table 4

Theoretical synthetic example with 2% and 4% random noise

Geophysical parameters	Assumed parameters	Evaluated parameters with 2% random noise	Evaluated parameters with 4% random noise
h (unit)	4	3.999	4.01
H (unit)	10	9.999	10.03
$\beta°$	30	29.851	30.023
p (mV)	200	203.949	208.049

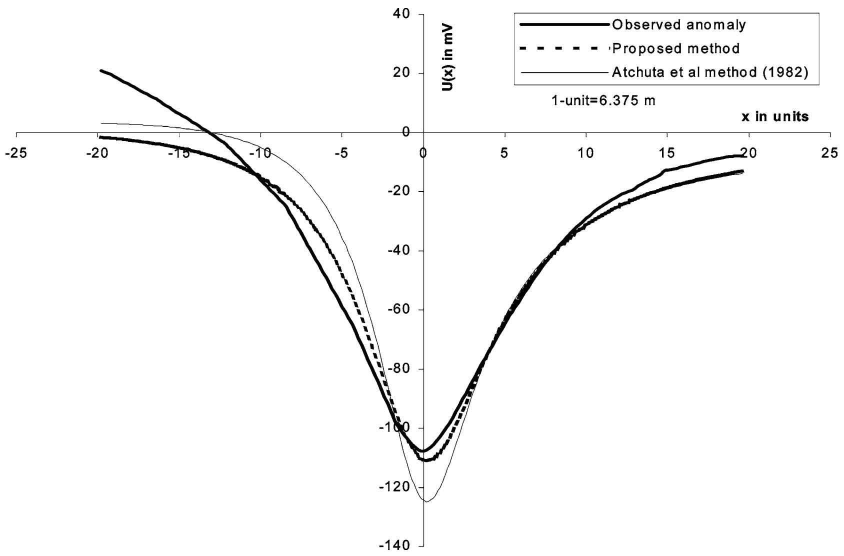

Figure 4
SP anomaly over a sulfide body in the Kalava fault zone (Cuddapah basin, India). The theoretical curves for our method and the ATCHUTA *et al.* (1982) method are shown.

locations in this area, which was carried out by the geophysical survey of India, encountered carbonaceous shales with sulfide mineralization. These might be the sources causing the *SP* anomaly. The *SP* profile with a length of 255 m has been digitized at an interval of 6.375 m (1 unit = 6.375) and subjected to interpretation by the proposed technique.

The obtained results are shown in Table 5, which also includes the interpretation results obtained by ATCHUTA *et al.* (1982), for the same *SP* anomaly.

It is to noteworthty that the *SE* of our suggested method (8.090) is less than that of ATCHUTA *et al.* (1982), which attests to the accuracy and the preciseness of the proposed method.

Table 5
Interpretation of SP anomaly profile over a sulfide body in the Kalava fault zone, Cuddapah basin, India

Evaluated parameters	ATCHUTA *et al* method (1982)	Present method
h (m)	15.9	21.699 ± 2.579
H (m)	41.2	47.789 ± 4.511
β^0	110	102.585 ± 1.723
p (mV)	63.68	69.498 ± 18.153
Standard error over 198 discrete points	10.974	8.090

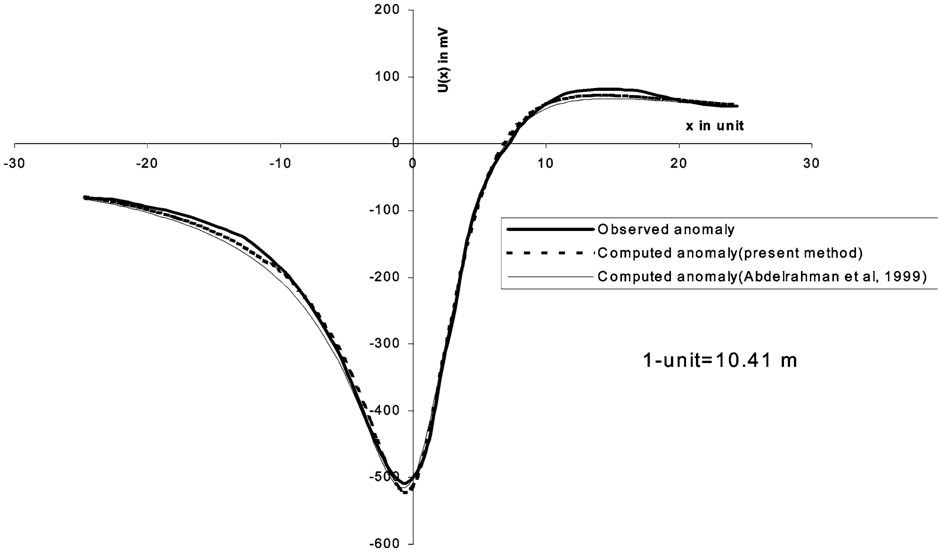

Figure 5
SP anomaly over a graphite ore body, southern Bavarian woods, Germany. The theoretical curves for our method and the ABDELRAHMAN *et al* (1999) method are shown.

2. The second anomaly shown in Figure 5 is an *SP* anomaly over a graphite deposit in the southern Bavarian woods of Germany. The *SP* measurements were performed and described by MEISER (1962), where the anomaly is represented as a result of a polarized sheet. This anomaly profile of 520.5 m length was digitized at an interval of 10.41 m (1 unit = 10.41 m) and subjected to interpretation by our technique. The sheet-evaluated parameters obtained by the interpretation are shown in Table 6.

A comparison study between our obtained results and those obtained by ABDELRAHMAN *et al.*'s method (1999) indicates clearly the superiority of the newly proposed technique, and its suitability for interpreting *SP* data related to inclined sheet-like structure.

Table 6

Interpretation of SP anomaly profile over a graphite ore body,southern Bavarian woods, Germany

Evaluated Parameters	ABDELRAHMAN *et al.* method (1999)	Present method
h (m)	35.8	29.514 ± 0.569
H (m)	66.3	67.936 ± 0.701
$\beta°$	49.5	47.825 ± 0.430
p (mV)	363.6	269.884 ± 8.508
Standard error over 99 discrete points	12.273	7.681

Conclusion

A new interpretative technique, based on a nonlinearly constrained optimization concept is proposed in this research to interpret self-potential *SP* anomalies due to cylinder, sphere and sheet-like structures. According to this technique, the geophysical problem related to the mentioned structure is mathematically formulated as a nonlinearly constrained minimization problem (*NCMP*). The (*NCMP*) is converted into a nonlinearly unconstrained minimization one (*NUMP*) by using an interior penalty function. Hooke and Jeeves direct search algorithm is thereafter used for solving the (*NUMP*), which finally allows the best estimate of the geophysical parameters related to the studied structure such as: depth, amplitude coefficient, polarization angle and geometric shape factor. The justification for using the well-known Hooke and Jeeves direct search algorithm is for being easy to be put in a code, robustness and that the convergence towards the optimal estimate of the vector of parameters is rapidly reached. This interpretative method is validated very well through theoretical synthetic data with different random noise, where very close agreement has been found between assumed and evaluated parameters. The validity of this method is tested and proved on three field examples taken from Turkey, India and Germany, where good agreements have been obtained between observed and computed anomalies. The standard error (*SE*) used as a statistical criterion and applied on the interpretative results of the three interpreted field examples indicates clearly the goodness and the high-level efficiency of the proposed interpretative technique. The advantages of the proposed method in comparison with other different interpretative methods of fixed geometry are demonstrated. In fact, this method can be generalized for interpreting *SP* field data of various geometries after the construction of the suitable mathematical model related to the field data. Therefore, this easy and accurate method can be used for routine analysis of *SP* anomalies in an attempt to determine the geophysical parameters related to the studied structures.

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