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Nonlinearly Constrained Optimization Theory to Interpret Magnetic Anomalies Due to Vertical Faults and Thin Dikes

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Abstract—A new and simple method based on a nonlinearly mathematical optimization concept has been proposed in this research to interpret magnetic anomalies due to vertical faults and thin dikes. This proposed interpretative method consists of three main steps. The first step is to formulate nonlinearly constrained optimization problems to describe the geophysical problems related to the studied structures. The second step is to suggest an interior penalty function in order to convert these nonlinearly constrained optimization problems into nonlinearly unconstrained optimization ones. The third step is to solve the converted nonlinearly unconstrained optimization problems by using the famous Hooke and Jeeves's algorithm in order to estimate the geophysical parameters of the studied structures such as: depth, amplitude coefficient, and index parameter. The Hooke and Jeeves's algorithm is purposely chosen for being robust and also its application to magnetic data converges rapidly towards the optimal estimation of parameters. This method was first tested on theoretical models with different random noise, where a very close agreement was obtained between the assumed and evaluated parameters.

The validity of this new method was also tested on practical field examples taken from Australia, India, United States, and Brazil, where available magnetic data existed and was previously analyzed by different interpretative methods. The agreement between the results obtained by our developed method and those obtained by the other geophysical methods is good. The advantages of this newly proposed method, compared with the other published interpretative methods, also have been discussed and demonstrated.

Key words: Magnetic anomalies, mathematical optimization, penalty function, geomagnetic interpretation.

Introduction

Several methods have been developed for interpreting magnetic anomalies (total, vertical, or horizontal) caused by vertical faults and thin dikes' structures, in an attempt to estimate the depth, index parameter, and the amplitude coefficient of igneous rocks in the form of dikes and faults. An excellent review is given by NETTLETON (1976) and BLAKELY (1994). The methods include, for example, matching standardized curves (PARKAR, 1963), characteristic points and distance approaches (GRANT and WEST, 1965; ABDELRAHMAN, 1994), monograms (PRAKASA *et al.*, 1986), Hilbert transforms (MOHAN *et al.*, 1982), Fourier transform techniques (BHATTACHARYA, 1965), correlation factors between successive least-squares residual

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anomalies (ABDELRAHMAN and SHARAFELDIN, 1996), and least-squares minimization methods (SILVA, 1989).

In the present paper, a new technique based on a nonlinearly mathematical concept is developed in order to interpret magnetic anomalies due to vertical faults and thin dikes-like structures. This new method consists of three main steps:

1. The geophysical problem related to the studied structures is first described by formulating a nonlinearly constrained minimization problem (*NCMP*).
2. This (*NCMP*) is secondly converted into nonlinearly unconstrained minimization problem (*UNCMP*), by introducing an interior penalty function, which is originally proposed in this research work.
3. The (*UNCMP*) is thirdly solved by the very famous Hook and Jeeves's algorithm, known for minimizing the numerical function of several real variables.

The obtained solution of the (*UNCMP*) includes the geophysical parameters of the studied structures such as: depth, amplitude coefficient, and index parameter.

Two structural cases have been treated and interpreted by this newly proposed method:

The first case is the interpretation of magnetic anomalies due to vertical faults. The validity of this method is tested on synthetic examples with different random noise of 2% and 4% and through practical field examples taken from Australia and India.

The second case is the interpretation of magnetic anomalies due to thin dikes. Our new method, in this case, is also tested on synthetic examples with different random error of 2% and 4% and on practical field examples taken from the United States and Brazil.

Hooke and Jeeves's Algorithm

HOOKE and JEEVES's algorithm (1961) is one of the most widely known methods for minimizing a numerical function of several real variables on the real space R^n. We will now illustrate in brief the algorithm for solving the following multi-variables unconstrained problem:

$$\begin{aligned}&\text{Minimize } f(x)\\&\text{Subject to } x \in R^n,\end{aligned}$$

where the numerical function $f(x)$ is called the objective function of the problem and the real variables $x = (x_1, \ldots, x_n) \in R^n$ are also called decision variables.

The method of Hooke and Jeeves is based on the idea of determining search directions on the basis of information gained at successive points during the iteration. This method uses a cycle with two components, an exploratory phase and a pattern move. In the exploratory phase the algorithm starts at a point $x^{(i)}$ and it explores the possibility of a better point for the objective function by moving a fixed step h along directions parallel to the coordinate axis. When a better point $x^{(i+1)}$ is found in this phase, a pattern move of the same fixed distance h is made along the direction $x^{(i+1)} - x^{(i)}$ to the new point $x^{(i+2)}$ and the cycle is repeated. The method uses for each move a fixed step rather than a line search.

This algorithm, which can be easily coded, has a good robustness, and it also does not require differentiation of the objective function with respect to the decision variables. For more details about this algorithm, the reader is invited to see one of the following excellent references: HOOKE and JEEVES (1961), NASH (1990), and PHILLIPS *et al.* (1976). The appendix explains in more detail the steps of this algorithm and provides the readers with sufficient information to allow implementation and coding of this algorithm.

Geophysical Problem Formulation Due to Vertical Faults Model

The general expression of the total, vertical and horizontal field components of the magnetic anomaly due to a vertical fault is formulated using the following notations: The edge of the vertical fault (Fig. 1) is at a depth of z units directly below the origin (0) and extends to a depth of z_b units (in the z direction). The fault extends to infinity in the strike direction (along $-y$ to $+y$ axis) and along the positive x. The strike ζ is the clock-wise angle from the magnetic north to the positive y axis. δ is the dip of the sides of the faulted block measured from the horizontal. k is the magnetic susceptibility contrast between the faulted block and its surroundings. r_1 and r_2 are the distances between the point of observation $P(x)$ and the upper and lower edges of the faulted block, respectively. ψ_1 and ψ_2 are the angles made by the vertical with the lines joining the point of observation and the upper and lower edges of the faulted block, respectively.

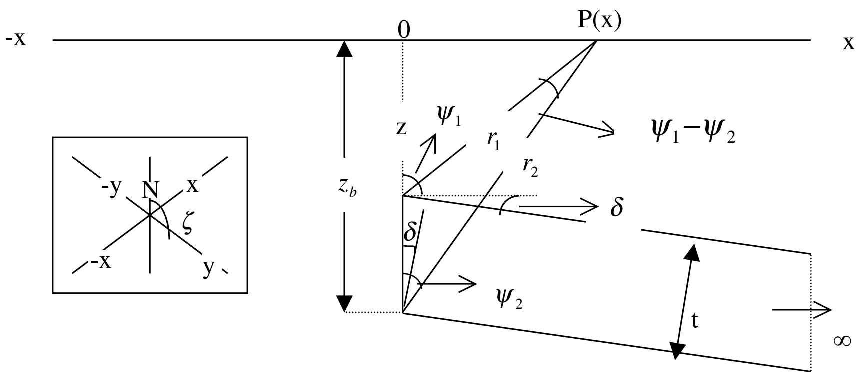

Figure 1
Cross-sectional view of a two-dimensional vertical fault model with the notation used.

The magnetic anomalies of the vertical (ΔZ) and horizontal ($\Delta H'$) intensity over a vertical fault are given by ATCHUTA RAO and RAM BABU (1983) as follows:

$$\Delta Z = 2kT_0' \cos\delta \left[\cos(I_0' - \delta)\mathrm{Ln}\frac{\sin\psi_1}{\sin\psi_2} + \sin(I_0' - \delta)(\psi_1 - \psi_2)\right] \quad (1)$$

and

$$\Delta H' = 2kT_0' \cos\delta \left[\sin(I_0' - \delta)\mathrm{Ln}\frac{\sin\psi_1}{\sin\psi_2} - \cos(I_0' - \delta)(\psi_1 - \psi_2)\right]. \quad (2)$$

T_0' and I_0' are the values of the effective total intensity and effective inclination, respectively, in the vertical plane perpendicular to the strike of the vertical fault. T_0' and I_0' are related to T_0 (true total intensity) and I_0 (true inclination) as follows:

$$I_0' = \tan^{-1}\frac{\tan I_0}{\sin\zeta} \quad (3)$$

and

$$T_0' = T_0 \frac{\sin I_0}{\sin I_0'}, \quad (4)$$

ΔH, the commonly measured component of horizontal intensity in the direction of the magnetic north is obtained using the relation:

$$\Delta H = \Delta H' \sin\zeta. \quad (5)$$

The total intensity anomaly (ΔT) in the direction of the undisturbed field is calculated from ΔZ and ΔH using the relation:

$$\Delta T = \Delta Z \sin I_0 + \Delta H \cos I_0. \quad (6)$$

Equation (6) may also be written as (GAY, 1965)

$$\Delta T = \frac{\sin I_0}{\sin I_0'}[\Delta Z \sin I_0' + \Delta H' \cos I_0']. \quad (7)$$

From equations (1), (2) and (7) the equation for the total field anomaly (ΔT) may be written as:

$$\Delta T = 2kT_0' \cos\delta \frac{\sin I_0}{\sin I_0'} \left[\sin(2I_0' - \delta)\mathrm{Ln}\frac{\sin\psi_1}{\sin\psi_2} - \cos(2I_0' - \delta)(\psi_1 - \psi_2)\right]. \quad (8)$$

Equations (1), (2) and (8) are similar in form and the expressions for ΔZ, ΔH and ΔT may be rewritten in the following fashion:

$$\Delta Z = 2kT_0' \cos\delta \left[\cos(I_0' - \delta)\mathrm{Ln}\frac{\sin\psi_1}{\sin\psi_2} + \sin(I_0' - \delta)(\psi_1 - \psi_2)\right], \quad (9)$$

$$\Delta H = 2kT_0' \cos\delta \sin\zeta \left[\cos(I_0' - \delta - 90) \mathrm{Ln} \frac{\sin\psi_1}{\sin\psi_2} + \sin(I_0' - \delta - 90)(\psi_1 - \psi_2) \right]. \quad (10)$$

and

$$\Delta T = 2kT_0' \cos\delta \frac{\sin I_0}{\sin I_0'} \left[\cos(2I_0' - \delta - 90) \mathrm{Ln} \frac{\sin\psi_1}{\sin\psi_2} + \sin(2I_0' - \delta - 90)(\psi_1 - \psi_2) \right]. \quad (11)$$

Now we see that the above three equations are similar and can be presented as a general equation of the form

$$\Delta F = C_F \left[\cos\theta_F \left(\frac{1}{R} \mathrm{Ln} \frac{\sin\psi_1}{\sin\psi_2} \right) + \sin\theta_F \left(\frac{\psi_1 - \psi_2}{R} \right) \right], \quad (12)$$

where ΔF is the anomaly in the corresponding component of the magnetic field, C_F is the amplitude coefficient which is a function of all variables except dip, θ_F is the index parameter which is related to the effective inclination of polarization I_0' and the angle of the dip δ, and R is the thickness parameter defined as

$$R = \frac{z_b - z}{z}. \quad (13)$$

Since the range of the angle δ is from 0^0 to $+90^0$ and the angle I_0' is from $-90°$ to $+90°$, then the range of the angle θ_F is from $-270°$ to $90°$ (ATCHUTA RAO and RAM BABU, 1983).

The equivalents of C_F and θ_F are given in Table 1 for the three components of ΔF.

From equation (12) it may be observed that the shape of the magnetic anomaly for a vertical fault depends only on θ_F, the index parameter, which is a function of the strike and dip of the faulted block and the inclination of the inducing vector. The amplitude of the anomaly depends on the coefficient C_F, which is a function of all the variables except dip.

Our proposed method depends mainly on equation (12), knowing that $\mathrm{tg}\psi_1 = \frac{x}{z}$ and $\mathrm{tg}\psi_2 = \frac{x}{z_b}$ (Fig. 1), then the general equation describing the vertical faults model becomes as follows:

Table 1

Characteristics amplitude coefficient of C_F and index parameter in total ΔT, vertical ΔZ, and horizontal ΔH magnetic anomalies due to faulted structure (ATCHUTA RAO and RAM BABU, 1983)

Anomaly in ΔF	Amplitude coefficient C_F	Index parameter θ_F
ΔT Total field	$2kT_0' \frac{t}{z} \frac{\sin I_0}{\sin I_0'}$	$2I_0' - \delta - 90$
ΔZ Vertical field	$2kT_0' \frac{t}{z}$	$I_0' - \delta$
ΔH Horizontal field	$2kT_0' \frac{t}{z} \sin\zeta$	$I_0' - \delta - 90$

$$\Delta F(x,z,z_b,C_F,\theta_F) = C_F\frac{z}{z_b - z}\left[\cos\theta_F\left(\mathrm{Ln}\left|\sin\mathrm{arctg}\frac{x}{z}\right| - \mathrm{Ln}\left|\sin\mathrm{arctg}\frac{x}{z_b}\right|\right)\right.$$
$$\left. + \sin\theta_F\left(\mathrm{arctg}\frac{x}{z} - \mathrm{arctg}\frac{x}{z_b}\right)\right] \quad (14)$$

The evaluation of the parameters (z, z_b, C_F, θ_F) could be obtained by solving the following nonlinearly constrained minimization problem:

$$\text{Minimize} \sum_{i=1}^{i=N}[L(x_i) - \Delta F(x_i, z, z_b, C_F, \theta_F)]^2$$
$$\text{Subject to } z \le z_b$$
$$-270^\circ \le \theta_F \le 90^\circ$$
$$z, z_b \ge 0$$
$$-\infty < C_F < +\infty, \quad (NCMP)$$

where $L(x_i)(i = 1, \ldots, N)$ are the observed values of the magnetic anomaly at the points $x_i(i = 1, \ldots, N)$. This problem is very difficult to solve in the domain of convex nonlinearly constrained programming because the feasible region of $(NCMP)$

$$X = \left\{(z, z_b, \theta_F, C_F) \in R^4/z \le z_b, -270^\circ \le \theta_F \le 90^\circ, (z, z_b) \ge 0,\right.$$
$$\left.\text{and } -\infty < C_F < +\infty\right\}$$

is not bounded in the real space R^4. To avoid this mentioned difficulty the $(NCMP)$ is converted into an unconstrained nonlinearly minimization problem by introducing the following interior penalty function, considering both the objective function and constraints of the studied problem.

$$\phi(x_i,z,z_b,\theta_F,C_F) = \sum_{i=1}^{i=N}[L(x_i) - \Delta F(x_i,z,z_b,\theta_F,C_F)]^2 - r\times[\mathrm{Ln}\,z + \mathrm{Ln}\,z_b + \mathrm{Ln}(z_b - z)$$
$$+ \mathrm{Ln}(270 + \theta_F) + \mathrm{Ln}(90 - \theta_F)],$$

where r is a positive real number chosen to be close to zero and not equal to zero, and in this work r is taken as equal to 10^{-4} (in a practice case, r is normally taken as $r = 1/N$).

Using this penalty function, the problem $(NCMP)$ becomes as follows:

$$\text{Minimize } \phi(z, z_b, \theta_F, C_F)$$
$$(z, z_b, \theta_F, C_F) \in R^4 \quad (UNCMP)$$

This $(UNCMP)$ then will be solved by using Hooke and Jeeves's algorithm, which directly allows obtainment of the values of geophysical parameters (z, z_b, θ_F, C_F).

Field Examples

The main objective of the new technique proposed in this research is to estimate the geophysical parameters of the vertical magnetic fault (z, z_b, θ_F, C_F) identified in equation (14). This scheme is applied to both theoretical synthetic data and real field data.

A synthetic example has been treated in order to show the efficiency and the stability of the proposed method. The assumed model parameters are: $z = 7$ units, $z_b = 14$ units, $\theta_F = 40°$, and $C_F = 200$ gammas. A theoretical curve is generated using the assumed parameters in equation (14). A new random data is regenerated depending on the theoretical curve where each point of this random data has random error "generated by the uniform distribution" which does not exceed 2% of its value. These random data have been treated by the suggested method where the computed parameters are: $z = 7.001$ units, $z_b = 13.998$ units, $\theta_F = 40.05°$, and $C_F = 203.976$ gammas for a random noise of 2% and are: $z = 7.004$ units, $z_b = 13.993$ units, $\theta_F = 40.05°$, and $C_F = 207.862$ gammas for a random noise of 4% (Table 2).

The results of this synthetic example show a valid and close agreement between assumed and computed parameters, which consequently highly prove the efficiency of the proposed method in this research.

The field magnetic anomalies from Australia and India have been analyzed and interpreted by using this proposed method:

1. The first example presented in (Fig. 2) is the reinterpretation of the total field magnetic anomaly on the western margin of Perth basin published by QURESHI and NALAYE (1978).

The evaluated parameters of this anomaly obtained by our approach are:

$$z = 7.52 \pm 0.66 \text{ kms}, \quad z_b = 13.97 \pm 1.11 \text{ kms}, \quad \theta_F = 39.78 \pm 0.22°, \text{and}$$
$$C_F = 200.29 \pm 19.22 \text{ gammas},$$

where the values of 0.66, 1.11, 0.22^0 and 19.22 are the maximum standard errors committed in the estimation of the geophysical parameters z, z_b, θ_F and C_F, respectively. These values are estimated by using the inverse Hessian matrix of the function $\phi(z, z_b, \theta_F, C_F)$.

Table 2

Synthetic example with 2% and 4% random noise

Parameters	Assumed parameter	Computed parameters with 2% random error subjected to ΔF	Computed parameters with 4% random error subjected to ΔF
z in units	7	7.001	7.004
z_b in units	14	13.998	13.993
θ_F in degrees	40	40.05	40.05
C_F in gammas	200	203.976	207.862

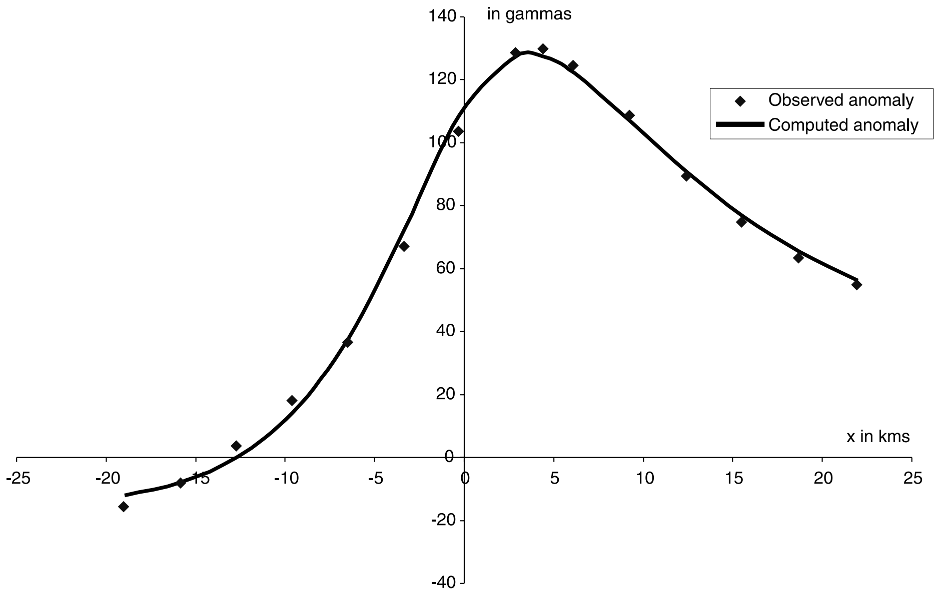

Figure 2
Interpretation of the total magnetic field anomaly (ΔT) on the western margin of the Perth basin, Australia.

The theoretical magnetic anomaly has been computed according to these evaluated parameters as shown in Figure 2. The comparison between observed and theoretical computed anomalies indicates clearly the close agreement between them. Table 3 shows a comparison between the results of our interpretation method and those obtained by QURESHI and NALAYE (1978), and by ATCHUTA RAO and RAM BABU (1983).

2. The second example (Fig. 3) is an aero-magnetic anomaly recorded at 2500 ft over a suspected deep seated fault southwest of Dehri, Bihar, India. The area is covered by Uindhyan and sediments in contact with Bijawar rocks. The magnetic

Table 3

Interpretation of magnetic field anomaly on the western margin of the Perth basin, Australia

Parameters	QURESHI and NALAYE (1978)	ATCHUTA RAO and RAM BABU (1983)	Present method
Depth (z) to top in kms	5.80–6.85	6.26	7.52 ± 0.66
Depth (z_b) to bottom in kms	15.55–17.00	15.45	13.97 ± 1.11
Index parameter (θ_F) in degrees	30	40	39.78 ± 0.22
C_F in gammas	–	–	200.29 ± 19.22

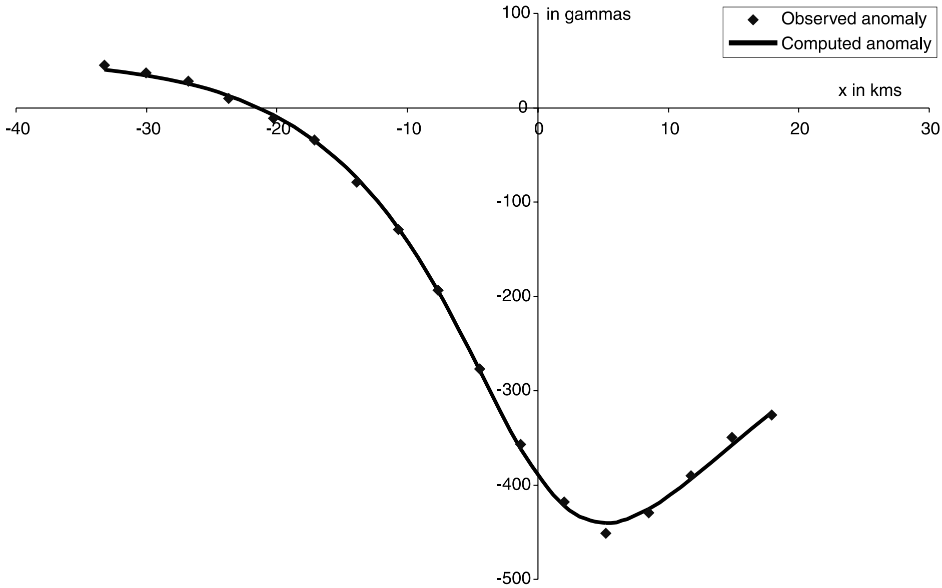

Figure 3
Interpretation of the aeromagnetic anomaly (ΔT), southwest of Dehri, Bihar, India.

anomaly is assumed to be due to magnetization contrast, in the deeper layers of the crust. The anomaly is analyzed and interpreted using our newly proposed method for vertical fault model, and the evaluated parameters of this anomaly are:

$$z = 10.16 \pm 0.25 \text{ kms}, \quad z_b = 25.43 \pm 0.63 \text{ kms}, \quad \theta_F = -141.73 \pm 0.13^\circ,$$
$$\text{and } C_F = 813.86 \pm 24.21 \text{ gammas}.$$

The theoretical magnetic anomaly has been computed according to these evaluated parameters as shown in Figure 3. A close agreement is noticed between the observed and computed anomaly, which proves the efficiency of the proposed method. The anomaly is also analyzed using both the method of QURESHI and NALAYE (1978), and the method of ATCHUTA RAO and RAM BABU (1983). The results of these methods are comparable, Table 4.

Geophysical Problem Formulation Due to Thin Dikes Model

The general expression ΔF, for the magnetic anomaly in total, vertical, or horizontal field at a point $p(x)$ along the x axis (Fig. 4) of an arbitrary magnetized thin dike (2-D) is given as ATCHUTA RAO *et al.*, (1980), and ABDELRAHMAN and SHARAFELDIN (1996):

Table 4

Interpretation of magnetic field anomaly on the south west of Dehri, Bihar, India

Parameters	Qureshi and Nalaye (1978)	Atchuta Rao and Ram Babu (1983)	Present method
Depth (z) to top in kms	7.50	8.00	10.16 ± 0.25
Depth (z_b) to bottom in kms	30.00	32.00	25.43 ± 0.63
Index parameter (θ_F) in degrees	−133	−130	−141.73 ± 0.13
C_F in gammas	–	–	813.86 ± 24.21

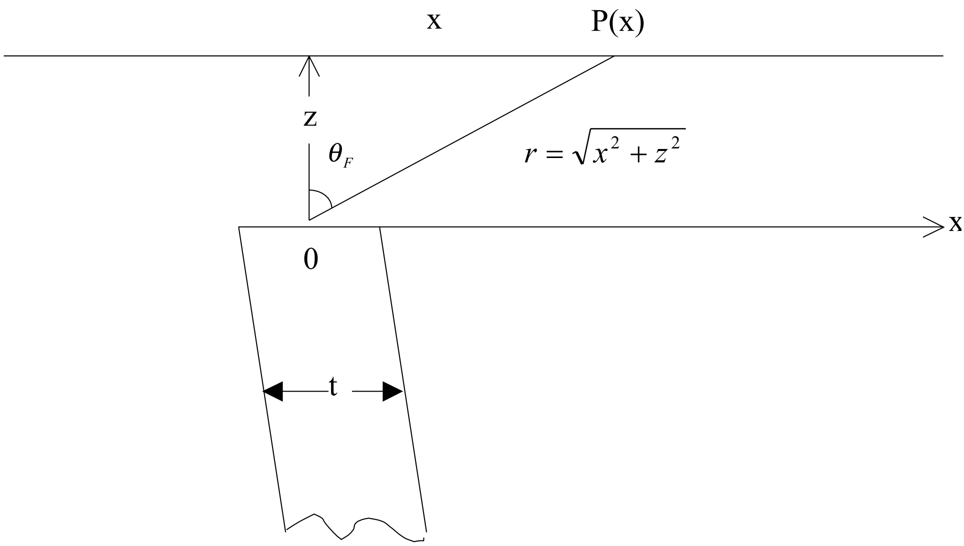

Figure 4
The thin dike model (2-D).

$$\Delta F(x, z, C_F, \theta_F) = C_F z \frac{x \sin\theta_F + z \cos\theta_F}{x^2 + z^2},$$

where z: is the depth to the top of the body, C_F: is the amplitude coefficient, and θ_F: is the index parameters.

By the same manner as presented in the case of vertical fault model, the evaluation of the parameters (z, θ_F, C_F) could be obtained by solving the following nonlinearly constrained minimization problem. Knowing that the range of the angle θ_F is from $-90°$ to $+90°$ (Abdelrahman and Sharafeldin, 1996):

$$\text{Minimize } \sum_{i=1}^{i=N} [L(x_i) - \Delta F(x_i, z, C_F, \theta_F)]^2$$

$$\text{Subject to } 0 \leq z$$
$$-90^\circ \leq \theta_F \leq 90^\circ$$
$$-\infty < C_F < +\infty. \qquad (NCMP)$$

The (*NCMP*) is converted to an unconstrained nonlinearly minimization problem by introducing the following interior penalty function:

$$\phi(x_i, z, \theta_F, C_F) = \sum_{i=1}^{i=N} [L(x_i) - \Delta F(x_i, z, \theta_F, C_F)]^2 - r \times [\text{Ln } z + \text{Ln}(90 + \theta_F) + \text{Ln}(90 - \theta_F)].$$

Using the penalty function, the problem (*NCMP*) could be written as follows:

$$\text{Minimize } \phi(x_i, z, \theta_F, C_F)$$
$$(z, \theta_F, C_F) \in R^3. \qquad (UNCMP)$$

Hooke and Jeeves's algorithm is then used to solve this (*UNCMP*), in order to obtain directly the geophysical parameters of the dike model (z, θ_F, C_F).

Field Examples

The efficiency of the proposed method for interpreting the magnetic anomalies related to thin dikes (2-D) has been tested on both theoretical, synthetic and real field data.

A synthetic example has been treated with 2% and 4% random noise respectively, where the assumed model parameters are: $z = 70$ units, $\theta_F = -60^\circ$, and $C_F = 600$ nT.

A theoretical curve is generated using the assumed parameters in the general expression ΔF of thin dikes. A new random data is regenerated depending on the theoretical curve where each point of this random data has a random error "generated by the uniform distribution" which does not exceed 2% of its value. These random data have been treated by the suggested method where the computed parameters are: $z = 70.001$ units, $\theta_F = -59.997^\circ$, and $C_F = 612.001$ nT for a random noise of 2% and are: $z = 70.005$ units, $\theta_F = -59.995^\circ$, and $C_F = 624.003$ nT for a random noise of 4% (Table 5).

Table 5 shows the assumed and computed parameters of this theoretically studied model, where very close agreement between them is noticed, which attests and clearly proves the validity of such a method.

Two field magnetic anomalies from the USA and Brazil have been interpreted by our proposed method.

Table 5

Synthetic example with 2% and 4% random noise

Parameters	Assumed parameter	Computed parameters with 2% random error subjected to ΔF	Computed parameters with 4% random error subjected to ΔF
z in units	70	70.001	70.005
θ_F in degrees	−60	−59.997	−59.995
C_F in nT	600	612.001	624.003

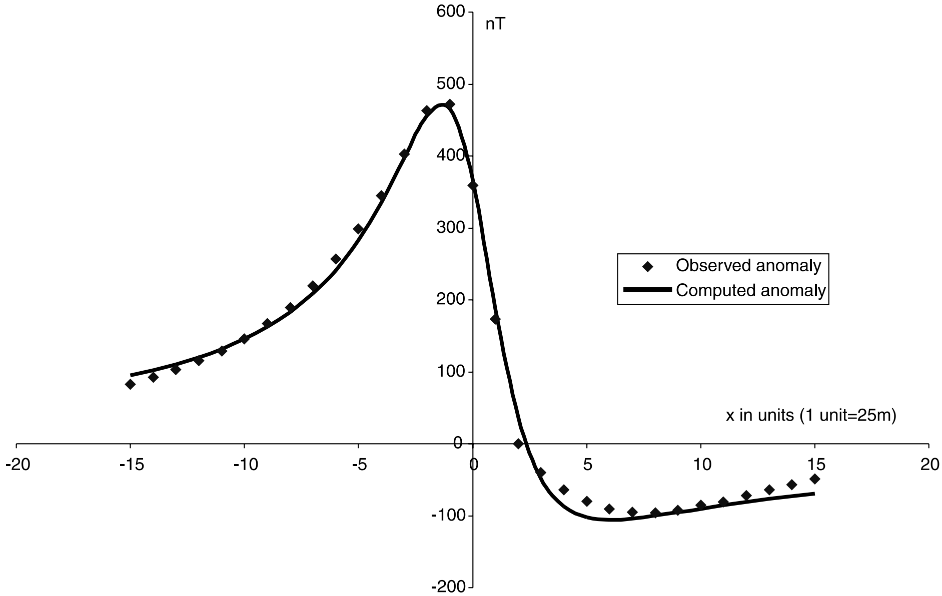

Figure 5
Vertical magnetic anomaly (ΔZ) over Pima Copper mine in Arizona, USA.

1. The first anomaly shown in Figure 5 is a vertical magnetic anomaly from the Pima Copper mine, Arizona, USA (PARKAR GAY, 1963). A magnetic profile 750 meters long was digitized at an interval of 25 meters. This magnetic anomaly was subjected to the interpretation by the method developed in this research. The evaluated parameters obtained for this anomaly are: $z = 71.50 \pm 1.78$ m, $\theta_F = -50.46 \pm 0.68°$, and $C_F = 577.61 \pm 9.23$ nT. Figure 5 also shows the theoretically computed field anomaly obtained according to the mentioned evaluated parameters, which obviously indicates the close agreement between observed and computed anomalies. ABDELRAHMAN and SHARAFELDIN (1996) interpreted the same anomaly and estimated the depth of the magnetic dike responsible for it to be 66 meters, while GAY (1963) has estimated this depth to be 70 m (Table 6).

Table 6

Interpretation of magnetic field anomaly on Pima Copper mine, Arizona, USA

Parameters	ABDELRAHMAN (1997)	ABDELRAHMAN and SHARAFELDIN (1996)	GAY (1963)	Present method
z in meters	62.1	66	70	71.50 ± 1.78
θ_F in degrees	–	−53	−50	−50.46 ± 0.68
C_F in nT	–	596.5	–	577.61 ± 9.23

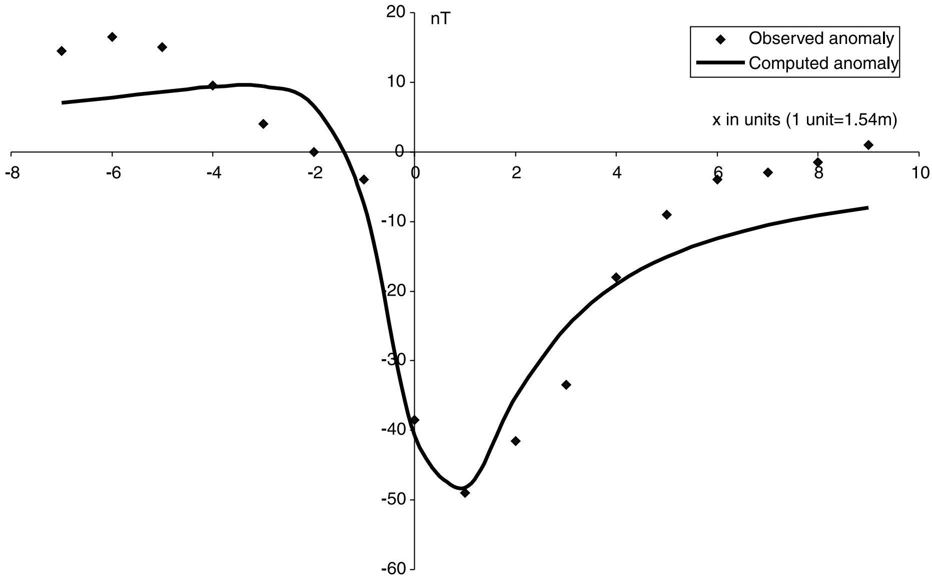

Figure 6
Total magnetic anomaly (ΔT) over an outcropping dike in the Parnaiba basin, Brazil.

2. The second anomaly shown in Figure 6 is a total magnetic anomaly above a Mesozoic diabase dike intruded into Paleozoic sediments in the Parnaiba basin, Brazil (SILVA, 1989). This anomaly profile 24.64 meters in length was digitized at an interval of 1.54 meters. The interpretation of this anomaly gives the following evaluated parameters: $z = 2.26 \pm 0.09$ m, $\theta_F = 47.11 \pm 1.13^\circ$, and $C_F = -59.81 \pm 1.54$ nT. Figure 6 also shows an acceptable agreement between observed and computed anomalies. According to ABDELRAHMAN and HASSANEIN's (2000) results, the depth of the magnetic dike causing this anomaly is 2.1 meters, while this depth is estimated to be 3.5 meters, according to ABDELRAHMAN and SHARAFELDIN (1996) and SILVA (1989) where this result is considered as an overestimate of the real depth, (Table 7).

Table 7

Interpretation of magnetic field anomaly on Parnaiba basin, Brazil

Parameters	ABDELRAHMAN and SHARAFELDIN (1996)	SILVA (1989)	ABDELRAHMAN and HASSANEIN (2000)	Present method
z in meters	3.5	3.5	2.1	2.26 ± 0.09
θ_F in degrees	33.3	–	–	47.11 ± 1.13
C_F in nT	−58.6	–	–	−59.81 ± 1.54

Conclusion

In this paper, the effectiveness of the newly proposed nonlinearly constrained optimization technique to interpret magnetic anomalies due to vertical faults and thin dikes models has been demonstrated. The geophysical problems related to the studied structures are described by firstly formulating nonlinearly constrained minimization problems (*NCMP*). This (*NCMP*) is converted into nonlinearly unconstrained minimization ones (*UNCMP*) by suggesting an interior penalty function. The geophysical parameters of the studied structures such as: depth, amplitude coefficient, and index parameter have been directly obtained by solving (*UNCMP*), and applying Hooke and Jeeves's algorithm. The well-known Hooke and Jeeves's algorithm is, easily converted to code, and is also chosen for being robust, and the convergence towards the optimal estimation of parameters is rapidly reached. This interpretative method is very well validated with theoretical synthetic data with random noise, in which very close agreement has been found between assumed and computed parameters. The application of this method on four examples taken from Australia, India, USA, and Brazil resulted in good agreement between observed anomalies and optimal solutions. This easy and accurate method can therefore be used for routine analysis of magnetic anomalies to determine the geophysical parameters, and may be extended to gravity and self-potential anomalies related to sphere and cylinder-like structures. The advantages of this new interpretative method over previous geophysical techniques which use characteristics points and distances, standard curves, and monograms are: 1) the method is not subjected to human errors in computing the model parameters. 2) the model parameters in both cases treated in this paper (z, z_b, θ_F, C_F) for faults and (z, θ_F, C_F) for dikes are determined in the same time through the finding of an optimal estimation (multi-parameters determination).

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Appendix

The Hooke and Jeeves algorithm consists of two distinct phases. The first is an exploratory search phase, which serves to establish a direction of improvement, and the second is a pattern move, which extracts the current solution vector to another point in the solution space. Using function minimization for illustrative purposes, the algorithm proceeds as follows: First, an initial solution vector is chosen $x^{(0)} = (x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})$. The initial value of the objective function is given by $f(x^{(0)})$. Label this point Set 1. An initial exploratory search is now conducted about this point in order to find a direction of objective function improvement. Define a perturbation vector $P = (\Delta x_1, \Delta x_2, \ldots, \Delta x_n)$, which will be used to systematically change the current solution vector. Choosing each variable in turn, an objective evaluation is made at $x_k^{(0)} \pm \Delta x_k; k = 1, 2, \ldots, n$. In particular, suppose that $f(x)$ is evaluated at $x_1^{(0)} = (x_1^{(0)} + \Delta x_1, x_2^{(0)}, \ldots, x_n^{(0)})$. If an improvement is found in $f(x)$ at $f(x_1^{(0)})$ namely $f(x_1^{(0)}) < f(x^{(0)})$, then the current value of the objective function is updated to $f(x_1^{(0)})$. If this move fails to improve the objective function then the vector $x_1^{(0)} = (x_1^{(0)} - \Delta x_1, x_2^{(0)}, \ldots, x_n^{(0)})$ is tried. This procedure is followed for each decision variable in turn, until the last decision variable has been changed. The final solution vector is accepted as a point in space, which indicates a direction of objective function improvement. Call this point $x^{(1)}$ and label it as Base 1. The pattern move phase is now implemented and consists of moving from $x^{(0)}$ through $x^{(1)}$ to a new point $x^{(2)}$ defined by: $x^{(2)} = x^{(0)} + 2(x^{(1)} - x^{(0)})$. Call this point Base 2.

The point $x^{(2)}$ is not immediately accepted. Before a decision is made to change the current accepted solution to Base 2, another exploratory search is conducted pertaining to Base 2. Performing this search as was done previously, a new point $x^{(3)}$ will be established. At this time a comparison is made between $f(x^{(3)})$ and the Base 1 solution vector. If $f(x^{(3)}) < f(x^{(1)})$, then $x^{(3)}$ is accepted as the new solution and labelled Base 1. The point from which additional moves will now be made is updated to $x^{(1)}$. Hence, $x^{(1)}$ is now labelled Set 1. We are now ready to make another pattern move from point $x^{(1)}$ (Set 1) through point $x^{(3)}$ (Base 1) to a point $x^{(4)}$ (Base 2). Exploratory searches will now be conducted regarding Base 2 to determine if the pattern move was a success. This sequence of moves is repeated until an exploratory search about the point Base 2 fails to yield objective function improvement. If this occurs, the pattern search is said to be a failure. When this occurs, the solution vector at Base 1 is returned to the original status of Set 1, and the procedure begins anew around the point Set 1 as if it were the initial solution vector. If an exploratory search about Set 1 fails to yield an improved solution vector, then the change vector

$P = (\Delta x_1, \Delta x_2, \ldots, \Delta x_n)$ should be reduced to $P = (\Delta x_1/2, \Delta x_2/2, \ldots, \Delta x_n/2)$ and another exploratory search conducted. When every component of P becomes less than a predetermined increment, the process terminates and Set 1 is accepted as the optimal solution.

In general, after the initial exploratory search, a point $x^{(k)}$ is labelled Set 1. A point $x^{(k+2)}$ is labelled Base 1. A projection is made from $x^{(k)}$ through $x^{(k+2)}$ to a point $x^{(k+3)}$, labelled Base 2. If an exploratory search about point $x^{(k+3)}$ is successful, then point $x^{(k+1)}$ is accepted as Set 1, point $x^{(k+3)}$ is accepted as Base 1, and the process repeated. If an exploratory search concerning Base 2 results in failure, then Base 1 is treated as if it were the initial solution vector, relabelled Set 1, and the entire procedure started anew.

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