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Global and Local Multiscale Analysis of Magnetic Susceptibility Data

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Abstract—Geophysical well-logs often show a complex behavior which seems to suggest a multifractal nature. Multifractals are highly intermittent signals, with distinct active bursts and passive regions which cannot be satisfactorily characterized in terms of just second-order statistics. They need a higher-order statistical analysis. In contrast with monofractals which have a homogeneous scaling, multifractals may include singularities of many types. Here we describe how a multiscale analysis can be used to describe the magnetic susceptibility data scaling properties for a deep well (KTB, Germany), down to about 9000 m. A multiscale analysis describes the local and global singular behavior of measures or distributions in a statistical fashion. The global analysis allows the estimation of the global repartition of the various Holder exponents. As such, it leads to the definition of a spectrum, $D(\alpha)$, called the singularity spectrum. The local analysis is related to the possibility of estimating the Lipschitz regularity locally, i.e., at each point of the support of a multifractal signal. The application of both approaches to the KTB magnetic susceptibility data shows a meaningful correlation between the sequence of Holder exponents vs. depth and the lithological units. The Holder exponents reach the highest values for gneiss units, intermediate ones for amphibolite units and the lowest values for variegated units. Faults are found to correspond to changes for H also when they are of intra-lithological type.

Key words: Geophysical log, magnetic susceptibility, wavelet, multiscale analysis.

Introduction

This paper deals with the multiscale analysis of potential fields and of their related physical quantity distributions, i.e., density and susceptibility, derived from well log measurements. As such it will be centered on two main themes: the first one is that of self-similar and/or self-affine fractals and the second is the multiscale wavelet analysis. The two theories are strictly interlaced since both help to unravel the scale-related complexity of geophysical signals, such as gravimetric, seismic or magnetometric ones.

Self-similarity, either exact or approximate, has entered *ex abrupto* to the common scientific language since the celebrated work of MANDELBROT (1982). The

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wavelet transform analysis was initially introduced by GOUPILLAUD *et al.* (1985) for the analysis of seismic signals and one of the more utilized wavelets is called "Morlet wavelet" after one of the authors of that paper. Morlet actually gave the first broad definition of a wavelet (GROSSMANN and MORLET, 1984). Some applications to potential fields are those by FEDI and QUARTA (1998), HORNBY *et al.* (1999), FEDI *et al.* (2000), SAILHAC *et al.* (2000), GUYODO *et al.* (2000).

The scaling properties of potential fields were extensively studied in the 1990s. These studies have opened new frontiers to the description of the potential field data structure and to the statistical description of the susceptibility and density distributions within the earth. In fact, the more common assumptions used in interpretation were those of homogeneous sources, in contrast to the evidence from well logs which show a complex behavior. The statistical distribution of the susceptibility and density was related to the observed field in a series of basic papers, such as NAIDU (1968) and SPECTOR and GRANT (1970). Naidu's model deals with randomly distributed sources and it is appropriate for describing a highly variable source distribution. Relatively large and homogeneous bodies may be instead better represented by the Spector and Grant statistical model, assuming ensembles of blocks of various sizes, depths and magnetisation/density.

GREGOTSKI *et al.* (1991), PILKINGTON and TODOESCHUCK (1993) and MAUS and DIMRI (1995) used fractal geometry to point out how the complexity of susceptibility and density logs could be interpreted in terms of scaling sources. The related magnetic and gravity fields also should be considered as scaling quantities, whose scaling exponent is related in a very simple fashion to the fractal dimension of the source parameters.

Such kind of analyzing power spectra is however ambiguous. In fact, as shown later by FEDI *et al.* (1997), the Spector and Grant model can also lead to a scaling power spectrum with a fixed scaling exponent of about 3. This is in agreement with the exponents estimated for fields relative to several regions on the Earth. BANSAL and DIMRI (1999) interpreted gravity data assessing the depth to the source by this approach.

Although the relative validity of any of these approaches can only be demonstrated by direct exploration results, the history of exploration geophysics shows that simple homogeneous sources may, to a first approximation, be enough to understand the physical properties of the sources. In this sense, QUARTA *et al.* (2000) suggested that an approach using both of these points of view is probably the more appropriate. For instance, let us consider the case of a sedimentary basin. The very simple modelling approach of homogeneous sources is strongly suggested by the fact that the anomaly that it generates is well correlated with its overall geometrical shape; nevertheless, well logs show unequivocally that the distribution of density is very different from a homogeneous one. Subsequently, both aspects should be considered in interpreting the fields.

Scaling Source Properties

In order to relate source properties to field properties, an approach explicitly based on the concept of scale is necessary. Consider again the Naidu and the Spector and Grant models. As regards the first, some authors (PILKINGTON and TODOESCHUCK, 1993; PILKINGTON *et al.*, 1994) considered a semi-infinite medium of depth to the top h and a magnetization power spectrum $E_{s(\rho)} = \rho^{-\beta_s}$, where ρ indicates radial frequencies, with the scaling exponent β_s in the fractal range, $3 < \beta_s < 5$. In this case, the zero-level ($h = 0$) field radial power spectrum is given by:

$$E_0(\rho) \approx \rho^{-\beta}, \tag{1}$$

where $\beta = \beta_S - 1$ (Fig. 1a).

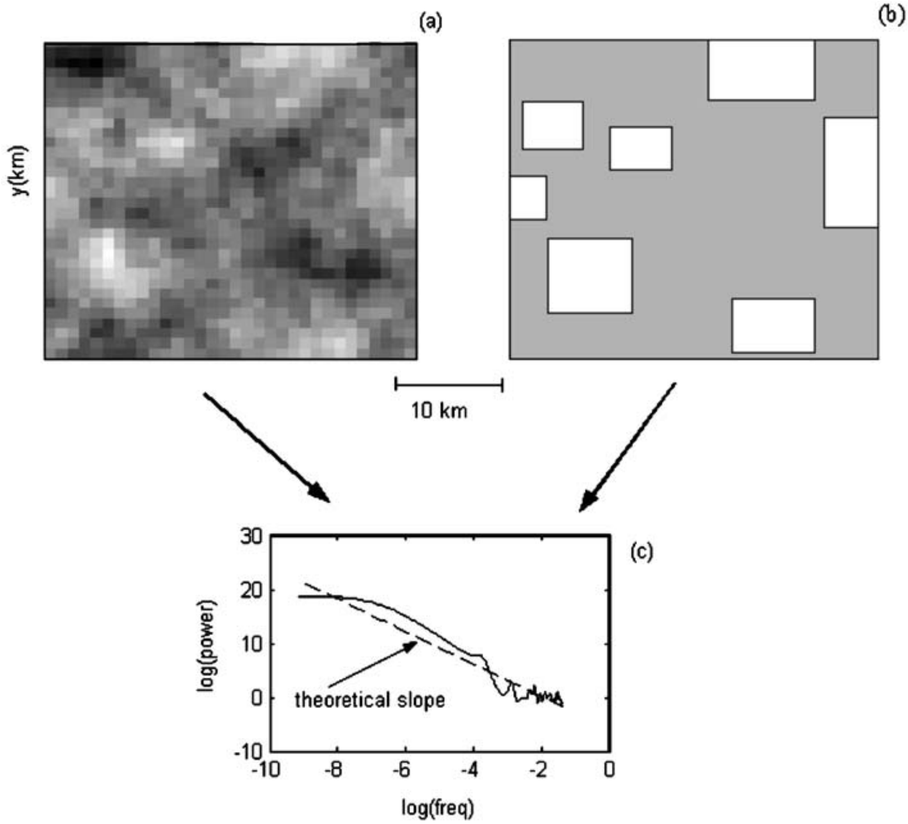

Figure 1
Red radial power spectra of a synthetic magnetic field (c, $\beta = 3$) may originate from different source distributions. The first case (a, horizontal section) is that of Gaussian random noise, with a specific scaling exponent, $\beta_s = 4$; a second case (b, horizontal section) occurs when the distribution of the susceptibility is piecewise correlated, corresponding to the presence of several homogeneous blocks.

FEDI *et al.* (1997) showed that the Spector and Grant model also leads to a scaling relationship, with a scaling exponent in the fractal range, $\beta = 2.95$. At zero-level we have:

$$E_0(\rho) = \left(\frac{\mu_0}{2}\right)^2 \bar{k}^2 C(\rho, \bar{l}, \bar{m}) T(\rho, \bar{t}) \approx \rho^{-\beta}, \tag{2}$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ S I), $C(\rho, \bar{l}, \bar{m})$ and $T(\rho, \bar{t})$ are spectral factors related to the average sizes $\bar{l}, \bar{m}$ and $\bar{t}$ of the block ensemble and $\bar{k}$ is the ensemble average susceptibility. All the source parameters are assumed to be uniformly random distributed. At first sight one might not understand how an uncorrelated block susceptibility distribution produces a red field power spectrum. In fact, some papers (e.g., PILKINGTON and TODOESCHUCK, 1993; MAUS and DIMRI, 1994) refer to the Spector and Grant source spectrum as a white one. The simple explanation is that one has to consider not the block ensemble susceptibility distribution, which is white, but the effective and continuous susceptibility distribution within the whole source-space. The latter is no longer uncorrelated, but piecewise correlated because of the homogeneity within each block of the ensemble (Fig. 1b)

But the difference between the two models is just a question of scale: if the smallest scale represented in the measurement set is less than three-four times the size of a dense set of homogeneous prisms (QUARTA *et al.*, 2000) the Spector and Grant spectrum reduces to a layered Naidu scheme, with $\beta_s = 0$. In fact, since the block ensemble susceptibilities are uniformly random distributed, the distribution of the magnetization tends to become random overall instead of partially homogeneous.

We conclude that any analysis of a given phenomenon and its complexity depends strongly on the scale at which it is actually observed.

General Fractal Noises as Multifractals

Fractal models arise often in many scientific disciplines, such as physics, chemistry, astronomy and biology. They are geometrical objects exhibiting an irregular structure at any scale. Self-similar fractals show a structure that is similar at any scale. The structure of deterministic fractals usually may be constructed through a few simple steps. Real world phenomena may, however, be rarely described using such simple models. Nevertheless similarity can hold on all scales in a statistical sense, leading to the notion of random fractals. Fractional Brownian motions (fBm) have played a central role in many fields. fBm is the unique Gaussian process with stationary increments and has the following scaling property for all scales $a > 0$

$$B(at) \equiv a^H B(t), \tag{3}$$

where $\equiv$ denotes an equality of finite-dimensional distributions.

It follows from (3) that $B(0) = 0$ and that B and its increments are zero-mean. The parameter H, $0 < H < 1$ is known as the Hurst parameter. A covariance analysis shows that the fBm are nonstationary processes. Despite this, their increments are stationary so that it is possible to define a generalized power spectrum having a power-law decay, with an exponent $\beta = 2H + 1$ and $1 < \beta < 3$. Furthermore (TURCOTTE, 1992) the self-similarity implies that any individual realization of the process is a fractal curve with a fractal dimension D given by: $D = 2 - H$. Realizations of fractional Brownian motions are everywhere singular, i.e., $\forall\ t \in R$ and for $0 \leq \alpha < 1$, the following relationship is verified:

$$|B_H(t) - B_H(t - \Delta t)| \leq K|\Delta t|^{\alpha}, \tag{4}$$

where K is some positive constant.

The Lipschitz regularity of a given signal at some point t_0 is the superior bound of all α verifying the above equation. If the Lipschitz regularity is $\alpha < 1$ at some point t_0, the signal is not differentiable at t_0 and α will characterize the singularity type. Note that the Lipschitz regularity, i.e., the Holder exponent, may be extended to the more general case of $\alpha > 1$ and that, when considering tempered distributions, negative exponents may be also taken into consideration (MALLAT and HWANG, 1992), as in the case of a Dirac $\delta(x)$ distribution ($\alpha = -1$).

Finally, the fact that fractional Brownian motions satisfy (MALLAT, 1998)

$$|B_H(t) - B_H(t - \Delta t)| \propto |\Delta t|^{H}, \tag{5}$$

implies that the Holder exponent of fBm is equal to the Hurst parameter.

Since fBm are everywhere singular with the same Holder exponent, they may also be called "homogeneous fractals" or "monofractals".

Monofractals are characterized by burstiness and long-range dependence (LRD). The latter implies that the data show a positive correlation, differently from a pure Gaussian noise $\beta = 0$. H controls the LRD of fBm and its local spikiness.

The statistical self-similarity of fBm has proved useful for signal modeling, since it efficiently captures signal features such as burstiness and LRD, nevertheless models based on fBm can be too restrictive to adequately characterize many types of signals. In fact, highly intermittent signals, with distinct active bursts and passive regions cannot be satisfactorily represented only in terms of second-order statistics, but need higher-order statistics. In other words, power spectra or variogram analysis may be useful for the characterization of the signal up to the second-order statistics, but may fail in describing more complex structures. One has now to ask what does change in the signal if H is allowed to vary across scales and/or across times. The answer is that the signals will show inhomogeneous spikiness, or in other words, will exhibit a varying degree of intermittence with sudden bursts of high frequency activity and large outliers. Compare for instance a self-similar signal (Fig. 2a) generated according to the network traffic multifractal model of CROUSE

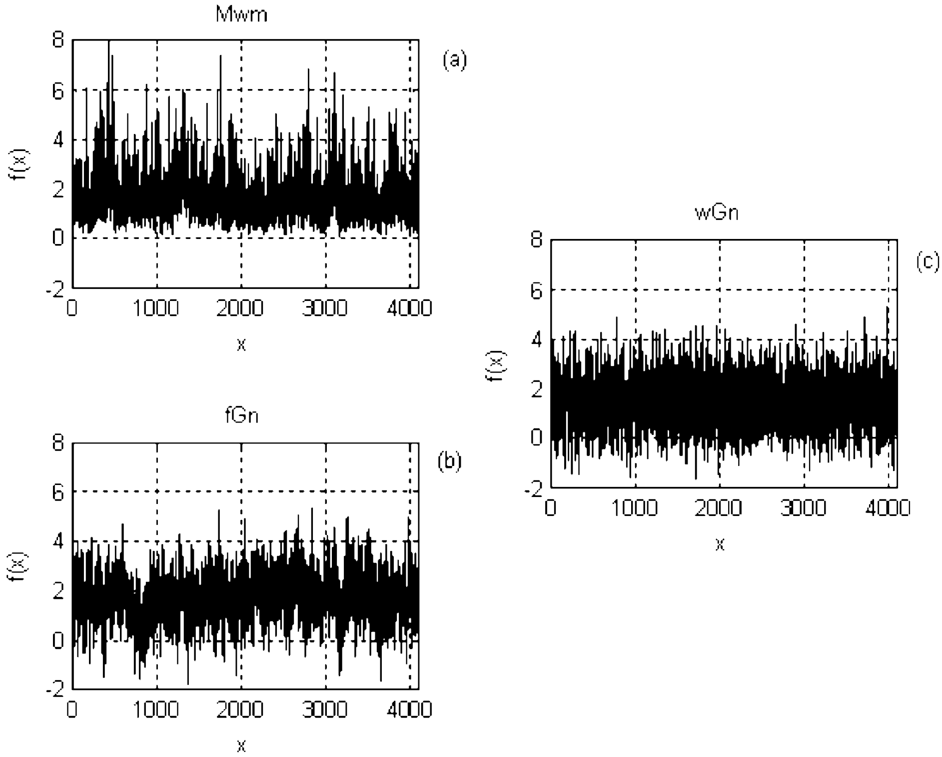

Figure 2
A comparison between several noisy signals. A self-similar multifractal (a) appears more complex than a fractal Gaussian noise (fGn) (b) or a white noise (c). Note that the multifractal and the fGn have the same global scaling exponent, mean and variance.

et al. (1999) and the realizations of a monofractal process characterized by $H = 0.8$. In order to better approximate the multifractal, we do not consider such nonstationary fBm, but its increment process, called fractal Gaussian noise (fGn) (Fig. 2b). This is indeed a stationary process, with a scaling exponent $\beta < 1$ (Davis *et al.*, 1994). The global scaling exponent, the mean and the variance are the same for all the processes, however the structure of the signals is very different and fGn clearly fails to model positive data, showing in fact a considerable number of negative values. A similar reasoning may be made after comparison with a white Gaussian noise (Fig. 2c).

Many signals exist which have positive increments and hence, differently from fBm, are not Gaussians. For those with stationary increments it is still possible to define a generalized power spectrum with the same power law decay as the fBm (Mallat, 1998). They are called general fractal noises (gFn). We stress that from the spectral point of view gFn cannot be distinguished from fBm, although a strong difference holds. In fact, realizations of gFn may include singularities of many types. In this sense, they belong to the class of "nonhomogeneous" fractals or "multifractals."

Multiscale Wavelet Analysis

Multifractal signal models are therefore positive measures or distributions possessing self-similarity but nonhomogeneous scaling. They were first introduced by MANDELBROT (1974) to explain turbulence phenomena and, since then, they have been utilized in very different contexts (MANDELBROT, 1989). As stated by STANLEY and MEAKIN (1988) multifractal scaling provides a quantitative description of a broad range of heterogeneous phenomena that can distinguish between different regions which have different fractal properties. Multifractal phenomena seem to be associated with systems where the underlying physics is governed by a random multiplicative process. The typical construction of a multifractal starts at a coarse scale and develops the details of the process on finer scales iteratively, in a multiplicative fashion. The simplest of this process is the binomial multifractal (FEDER, 1988). We have two ways to study multifractals, which are respectively of local and global nature. The first one allows a local estimation of the Lipschitz regularity, i.e., at each point of the support of a multifractal signal. This is possible especially if the singularities appear isolated. Otherwise, the estimation is more difficult, due to interference effects which may occur especially at large scales and also due to the finite numerical resolution. The second one consists in defining a procedure to estimate the global repartition of the various Holder exponents. As such, it leads to the definition of a spectrum, $D(\alpha)$, called singularity spectrum.

In both cases, techniques based on the continuous wavelet transform (CWT) may be used (MALLAT, 1989; FLANDRIN, 1999) and, in particular, on the wavelet transform local maxima, as described in a landmark paper of MALLAT and HWANG (1992).

As it is known, for any $f \in L^2(\Re)$ its continuous wavelet transform Wf is defined as the integral transform (DAUBECHIES, 1992):

$$Wf(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(x)\bar{\Psi}\left(\frac{x-b}{a}\right) dx, \tag{6}$$

where $\bar{\Psi}$ is the complex conjugate of a fixed function $\Psi \in L^2(\Re)$, called the mother wavelet or analyzing wavelet, whose main properties are: a) a compact support (to obtain localization in space) ; b) a zero-mean for the wavelet and for higher-order moments (to oscillate like a wave).

Wf is built with translations and dilations of Ψ, the two parameters a and b controlling the dilations and translations. a is related to the scale and b to the position. At a given scale, the parameter b assures the space-localization of the features of the signal specific to that scale.

The continuous wavelet transform allows for a localized decomposition of measured physical quantities (in our case, well logs and field data) into their multiscale constituents. This naturally leads to unravel considerably more complexity than which we can deal with other techniques, such as the Fourier analysis. In particular, the scaling behavior of a process carries over the local scaling properties of its wavelet coefficients.

Care, however, must be taken in choosing an appropriate analyzing wavelet. We must initially be sure that the analyzing wavelet possesses a number of vanishing moments adequate to study a given signal. The analyzing wavelet must also be more regular (or smoother) than the process under study. Otherwise the analysis will be biased by its own properties instead of reflecting those of the signal.

Referring to MALLAT (1998) for a full explanation of the theory, the spectrum $D(\alpha)$ is obtained by the inverse Legendre transform of the "mass exponent" function $\tau(q)$, computed with a linear regression of $\log(\Sigma_n\ |Wf(x_n,\ a)|^q\ )$ versus $\log a$, where q are suitable exponents. The typical multifractal spectrum is strictly concave and the maximum of $D(\alpha)$ will reach the dimension of the support of the measure. As an example of D, first consider the integral measure $M(x)$, as obtained by integration of a binomial multifractal measure μ (Fig. 3a). Its singularity spectrum (Fig. 3b) defines a notably large interval of values, with Holder exponents from 0.25 to 2.75 approximately. Consider instead a monofractal process, with $H = 0.5$ (Fig. 4a) and its Legendre spectrum (Fig. 4b). We note that in this monofractal case, the Holder exponents are all close in their values to the assumed exponent 0.5. We may therefore conclude that the two processes are clearly distinguished by their spectra.

The procedure for the *local approach* is based instead on computing the local Holder exponent with a linear regression of $\log|Wf(x,a)|$ as a function of $\log a$ at the finest scales, avoiding those large scales which could be affected by consecutive singularities.

As an example, consider a signal consisting of an isolated singularity at some point t_0 (Fig. 5a) with a local Lipschitz-Holder regularity $\alpha = -1$. Following the above steps, the local multiscale analysis, based on the CWT maxima of the signal (Fig. 5b), generates the correct Holder exponent (Fig. 5c).

Global Scaling Properties of Sources from Well Logs and Potential Field Measurements

The problem faced in this section is to study, within the more general theory of multifractals, the complexity shown by well-log measurements. Well-log measurements have given evidence of a predominant scaling behavior of the spatial distribution of physical parameters such as magnetic susceptibility, density and

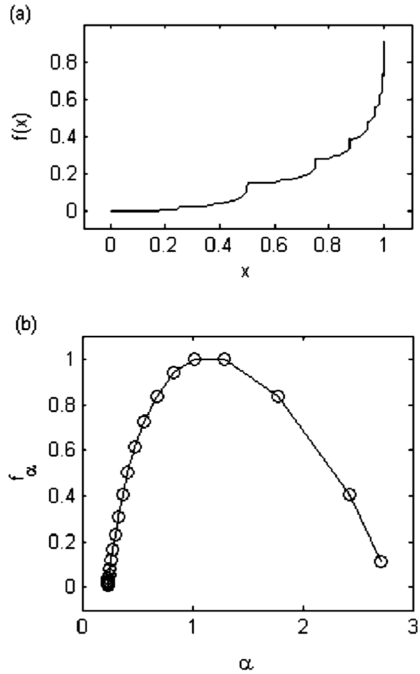

Figure 3
Singularity spectrum of a multifractal binomial process for $P = 0.15$, where P indicates the distribution fraction. The measure $M(x)$ is shown (a), obtained by integration of the multifractal binomial measure $\mu(x)$ on the region [0, x]. The corresponding Legendre spectrum (b) has extreme values 0.25 and 2.75.

others (WALDEN and HOSKEN, 1985; TODOESCHUCK and JENSEN, 1989; HERRMANN (1992); PILKINGTON and TODOESCHUCK (1993).

Magnetic susceptibility logs from shallow boreholes (< 300 m) were analysed by PILKINGTON and TODOESCHUCK (1993). The power spectra of the analyzed logs were scaling with an exponent β ranging from 1.32 to 1.96 for sedimentary rocks and from 2.08 to 2.72 for igneous rocks. PILKINGTON *et al.* (1994) explored further the horizontal distribution of susceptibility for two large datasets in Sierra Nevada (USA) and Saskatchewan (Canada). They found scaling exponents of about 3 for these surface 2-D data sets of susceptibility. MAUS and DIMRI (1994) analyzed a vertical profile of rock susceptibilities from drill cores of the KTB pilot drill hole (Germany) and estimated $\beta = 0.4$ from the power spectrum. LEONARDI and KUMPEL (1996) explored susceptibility logs again from the German KTB borehole (down to 4000 m) and found scaling exponents from 1.4 to 0.3. Extending the analysis of KTB data down to 9000 m, ZHOU and THYBO (1998, their Fig. 5) obtained an average value of about 1.5 and observed variability with respect to depth, with estimates of β varying from 0.5 to 2.

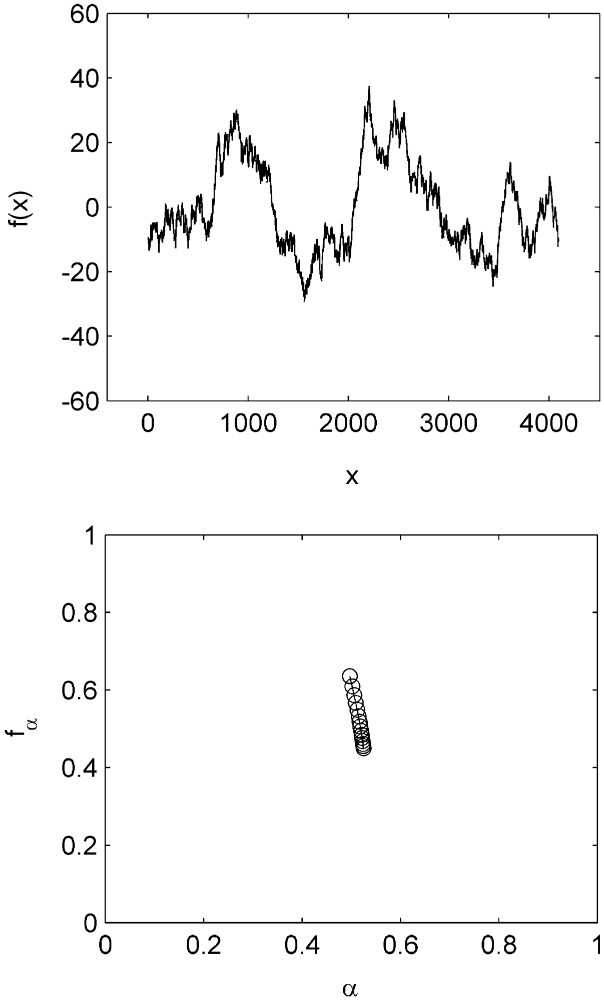

Figure 4
A self-affine signal (a) of $H = 0.5$ ($D = 1.5$) and its singularity spectrum (b). The singularity spectrum clearly shows the homogeneous scaling property of such signal.

For isotropic scaling sources, MAUS and DIMRI (1994, 1995) illustrated important relationships between the scaling exponents of sources and those of the related potential fields. A good approximation for the scaling exponent of any 3-k dimensional subset of the distribution is $\beta_s - k$, where β_s is the scaling exponent of the susceptibility distribution. Furthermore, assuming a half-space of sources, a good approximation for the scaling exponent (β) of the 2-D magnetic field, reduced to the pole and at source-level, is: $\beta = \beta_s - 1$.

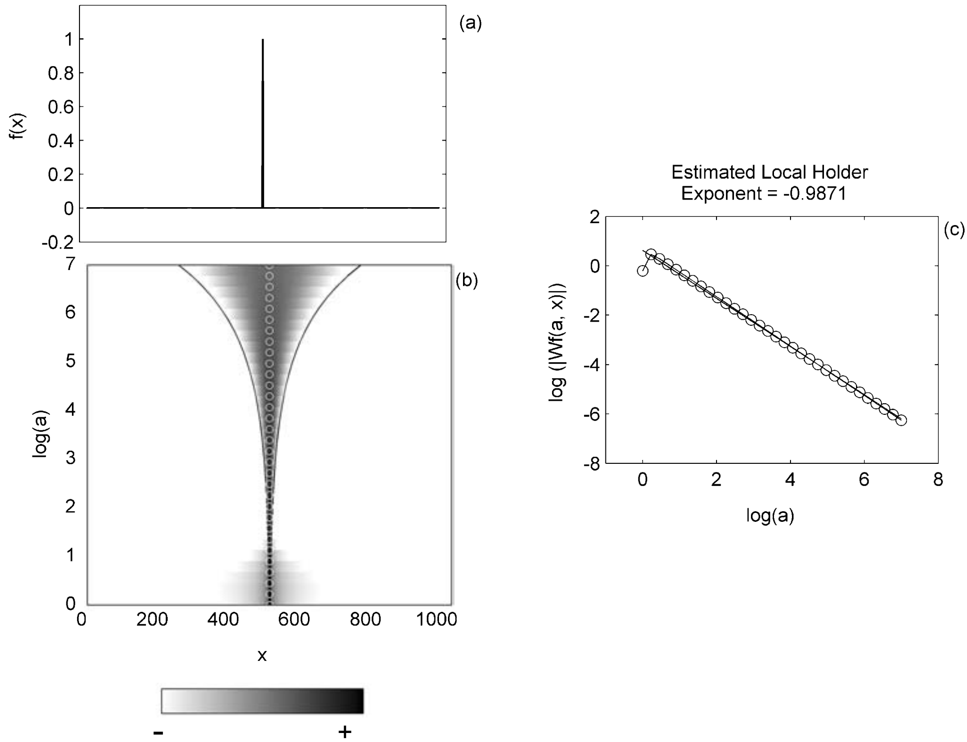

Figure 5
An isolated and spiky singularity (a) which has a local Lipschitz-Holder regularity $\alpha = -1$ and its CWT (b). The CWT shows a characteristic cone at the point where the singularity lies (Fig. 6b). A local multiscale analysis gives the correct Holder exponent (Fig. 6c). Circles indicate CWT maxima.

This scheme allows a simple way to study the source statistical properties from its measured field. Scaling exponents of about 3 were actually estimated from a series of aeromagnetic surveys (GREGOTSKI *et al.*, 1991; PILKINGTON and TODOESCHUCK, 1993), suggesting a scaling exponent of about 2 for susceptibility (1-D) well logs and of about 3 for 2-D susceptibility datasets. The above-mentioned estimates for well logs often correspond to the above rules, however the agreement is substantially worse at large depths, as in the case of the KTB borehole.

This clearly indicates that the isotropic scaling half-space of sources is too simplified a model to reflect the real geology. The measured fields are a superposition of effects related to different rocks. Also within the same mineralogy, any change in temperature, pressure and composition can produce variations in density or magnetization, which in turn are reflected in the fields. Consequently, a more complicated behavior of the scaling properties is expected, which leads us to consider for these signals the more general theory of multifractals.

Multiscale Analysis of Well-logs Measurements and Discussion

As described in the previous section, the global analysis provides all the information required to assess the fractal nature of a signal through the singularity spectrum, but does not help to find and classify the singularities in a local sense. This can be achieved by a local multiscale analysis, since the local Lipschitz regularity of $f(x)$ at any singular point may be estimated from the decay of the wavelet transform along the maxima line that converges to that point. This task may be problematic when singularities are not isolated, but MALLAT and HWANG (1992, Theorem 5) give additional criteria, consisting in testing the sign at which the modulus maxima occur within the cone of influence of a given point x_0. We will perform both kinds of analysis on a deep susceptibility well log which has performed down to a depth of about 9 km in West Germany (KTB main hole, drill section HB1; data are from the Web-site http://icdp.gfz-potsdam.de/html/ktb). The analysis of the data of such a deep well gives us a good chance to detect significant scaling variations with depth. The susceptibilities were measured from rock cuttings at a 2 m sampling interval.

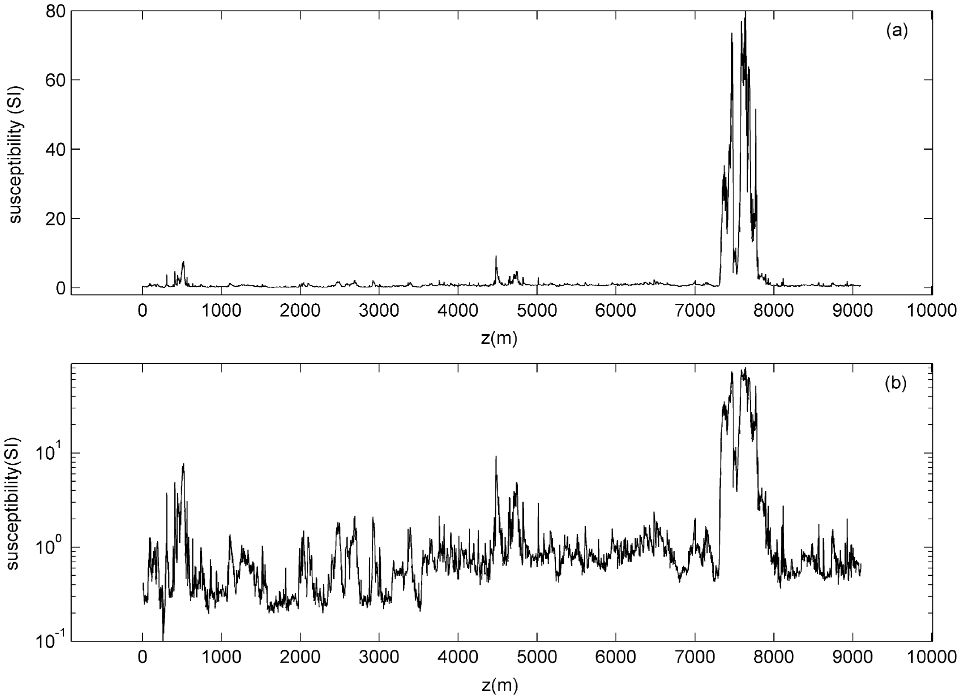

Figure 6

Susceptibility versus depth for the KTB well log (West Germany). For the sake of clarity the data are shown on a linear (a) and a semilogarithmic scale (b).

The data, shown in Figure 6, immediately suggest a probable multifractal nature. Firstly, they are in the typical nonnegative format of measures (Davis *et al.*, 1994). Furthermore, large outliers appear, together with a high degree of intermittency. This may be defined as the ratio between the relative active (sudden bursts) and passive regions (quiet zones) in a signal. Unfortunately, we have shown that modeling with monofractals is rather insensitive to these properties (see Fig. 2). The more general approach of multifractals may instead allow significant improvements in the understanding of the source properties.

The global scaling exponent for the KTB susceptibility data, shown in Figure 7, is $H = 0.32$, corresponding to $\beta = 1.64$. The estimation seems consistent either using a Fourier power spectrum (a) or a wavelet power spectrum (b). It does not correspond to the value found by Maus and Dimri (1995) for KTB susceptibility data ($\beta = 0.4$), but matches the average value estimated by Zhou and Thybo (1998). As mentioned, the latter authors computed the scaling exponent from power spectra relative to windows

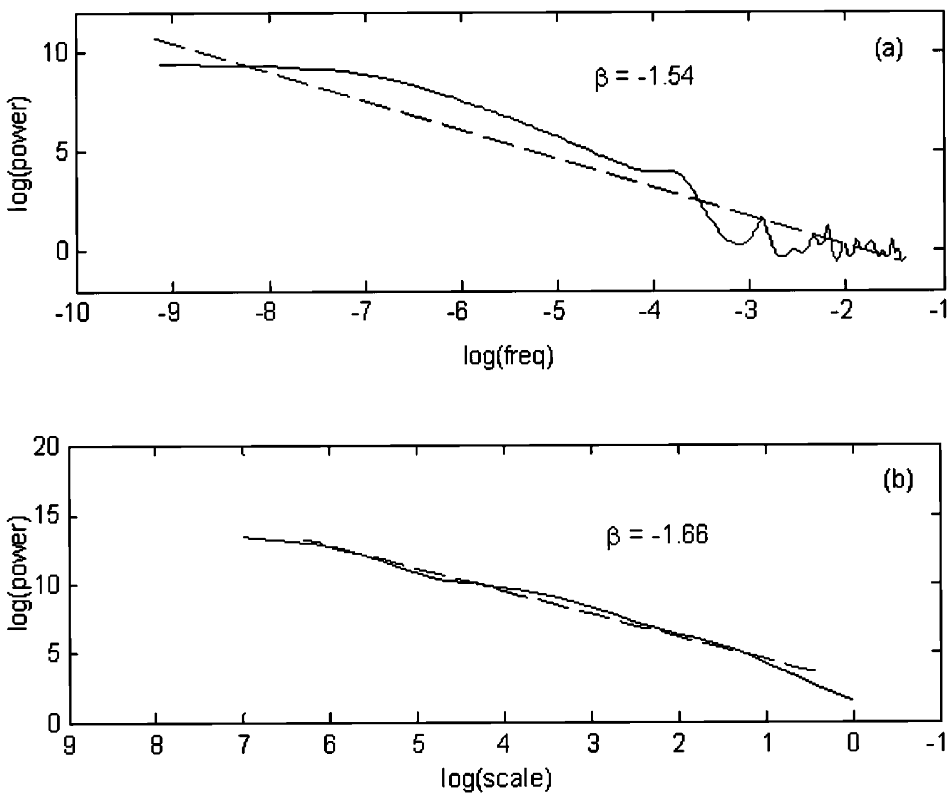

Figure 7
The global scaling exponent of the KTB susceptibility data (Fig. 7) is about 1.6 from either a Fourier (a) or a wavelet (b) power spectrum estimation.

of 1024 and 256 m, and estimates for both the window sizes indicate that the scaling exponent changes significantly with depth. The discrepancy with the result obtained by MAUS and DIMRI (1994) could be caused either by differences in the effectively analyzed datasets or by their different depth range (which was down to 4 km).

We have thus compared several estimations obtained within just second order statistics. Let us now see if a global multiscale analysis will help us to establish whether the data belong to a monofractal or a multifractal class. We recall that the typical multifractal spectrum is strictly concave and spans a large interval of Holder exponents, while in the case of monofractals, the spectrum shows Holder exponents very close to each other. The computation of the singularity spectrum for the KTB log susceptibility leads to a spectrum with a clear concave shape (Fig. 8). It is defined in a somewhat large interval of Holder exponents, between 0 and 0.7 approximately. Therefore, it appears to be multifractal. These values correspond to the scaling exponent range estimated by ZHOU and THYBO (1998). Note however that the Legendre spectrum evaluation does not often produce stable results. Also, in this case we found instability in performing the analysis, depending on the type of analyzing wavelet used and on the range of considered q.

Moreover, the singularity spectrum cannot provide the distribution of the Holder exponent with depth. This is similar to power spectra which completely lack any space localization. Therefore, we turn now to the local singularity analysis, in an attempt to assess the behavior of the average scaling properties with the depth.

We know (MALLAT, 1998) that this kind of analysis is not strictly defined for studying multifractal singularities, which vary from point to point, due to the limitations of any numerical analysis of discrete datasets. It is therefore important to assess the feasibility of a local analysis for non-isolated singularities. We test this kind of analysis on a simple case of geological value, i.e., that of piecewise self-similar processes (GONCALVES *et al.*, 1998), by which the profile is divided into several parts, each one having an homogeneous scaling. Such a case would correspond to the simplified succession assumed in geophysics, of layers having different

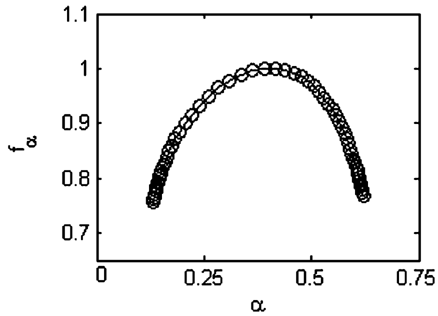

Figure 8
The singularity spectrum for the KTB susceptibility log. Note its concave shape and that it is defined over an interval of Holder exponents between approximately 0 and 0.7.

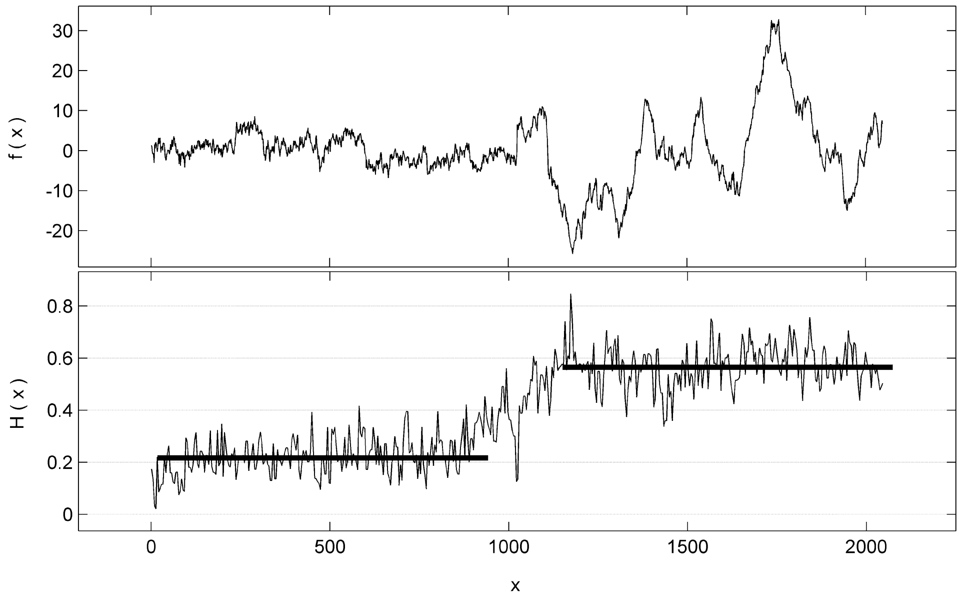

Figure 9
A multifractal signal which is piecewise homogeneous with respect to the Holder exponent (a). The right and left zone correspond respectively to $H = 0.2$ and $H = 0.6$. This is clearly evidenced by a multiscale local analysis (b).

lithologies, temperature, pressure and so on. As shown in Figure 9, the multiscale analysis produces good results, with a transition zone at the boundary between the two layers.

Application of local multiscale analysis to KTB susceptibility data (k) is rather satisfying (Fig. 10), since it yields Holder exponent estimations consistent with those retrieved by the global analysis and may therefore aid our understanding of how the values are distributed with depth. Comparing our results with those of ZHOU and THYBO (1998), we obtain substantial differences. In fact, they estimated a rather noninformative and oscillating behavior of the scaling exponent, which is here replaced by several rather homogeneous zones (Fig. 10). The first zone (from 0 to 3500 m approximately), is relative to a sequence consisting of variable units: gneiss, variegated and amphibolite units. The susceptibility and the Holder exponent vary too: gneiss units are characterized by low susceptibilities (about 0.5 SI) and by Holder exponents of about 0.4. The only noticeable exception corresponds to fault 3, where both susceptibility and Holder exponent change abruptly (k$\sim$1 SI; H$\sim$ 0). The variegated units have higher susceptibilities (from 1 to 9 SI) and lower Holder exponents (around 0). As we will see later, variegated units are well identified by the Holder exponent throughout the entire well. Finally, amphibolites correspond to susceptibilities of about 1 SI and to a 0.2 H, approximately.

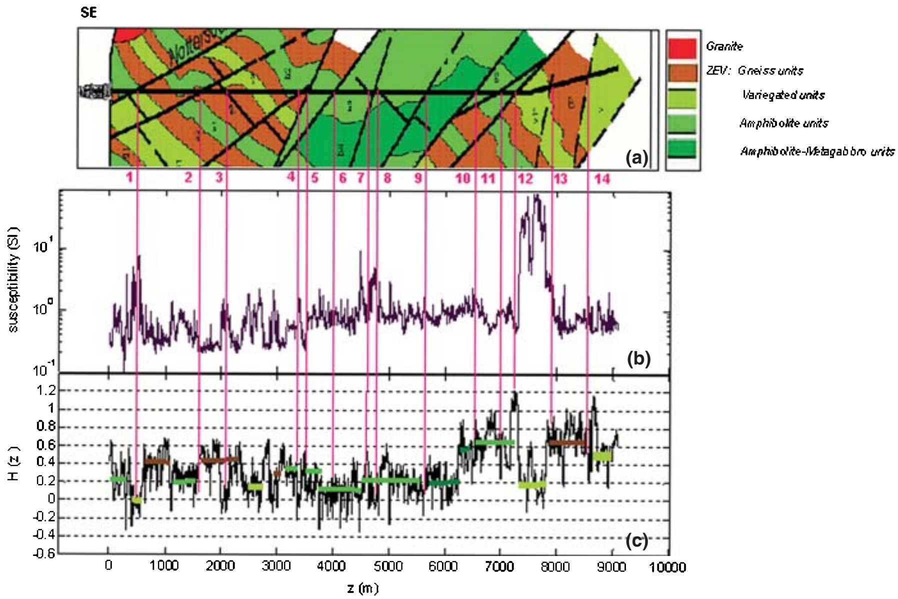

Figure 10
Local multiscale analysis of the KTB susceptibility data. The analysis of susceptibilities (b) puts in evidence several rather homogeneous zones (c), which seem well correlated to the lithologic units shown in (a). The lithologic sketch is modified from http://icdp.gfz-potsdam.de/html/ktb.

From 3000–7500 m the well intersects amphibolite and amphibolite-metagabbro units. From the susceptibility point of view, this relative lithologic homogeneity is confirmed by rather constant values of about 1 SI, except in the region of faults 7, 8, where it is higher (5–6 SI). This region is also characterized by relatively constant H values (about 0, 0.1). The final part (6500–7500 m), however, has considerably higher H values (from 0.6 to 1). Note that this change of regularity is not accompanied by an analogous variation of susceptibility, except a slow decay between faults 10 and 11, but that it seems well correlated to the presence of the faults 10, 11 ,12. Note also that fault 6 (completely within amphibolite units) does not correspond to any no change of either susceptibility or regularity.

The final part of the well, from 7500 down to 9000 m, crosses variegated and gneiss units. The first of them has very high susceptibilities (about 80 SI) and again a 0.2 H value. Recalling the similar behavior of it for this unit within the first 3500 m, the regularity seems therefore well characterize it, better than susceptibilities which vary largely, of about an order of magnitude. Gneiss units have compatible, but slightly higher, values for both H and k, if compared with the shallow layers of the well. The last unit of the well, again a variegated unit, seems instead to have susceptibilities and regularity values more compatible with those of gneiss units than with those of the other variegated units within the well.

Conclusions

We may conclude that the regularity analysis yields valuable information which may be used to complete and/or compare that obtained from a simple susceptibility analysis. As shown in Figure 10c, one may identify in a relatively clear way, the occurrence of several units from their regularity value; it being higher for gneiss units, intermediate for amphibolite units and lower for variegated units. Looking at Figures 10b and 10c this task appears easier than simply analysing the susceptibilities. This general correspondence with lithological units locally breaks down due to the occurrence of faults, which introduce nonnegligible changes for H also when they are of intra-lithological type, such as faults 3, 6, 7 and 11. The effect of faults is sometimes stronger on regularities (10, 11, 12) than on susceptibilities; nevertheless a combined analysis of both seems effective in identifying them.

A more exhaustive study of the well should surely be advantageous of taking into account the role of temperature, pressure and other physical and chemical parameters, especially for depths where the interpretation is less clear (from 8500 to 9000 m, for instance). In any case, the complexity of the geology seems well outlined by a multifractal study of the susceptibility well-log data.

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