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Chaos in the Solar System

J. Laskar

Abstract. The implications of the chaotic evolution of the Solar System are briefly reviewed, both for the orbital and rotational motion of the planets. In particular, Why Venus spins backward ? can be now understood while considering the transition through a highly chaotic state during its history; chaotic state that the Earth itself would have experienced in absence of the Moon, while the large variations of Mars' obliquity were probably at the origin of considerable climate variations that may have left some geological traces on its surface.

The limits of predictability for a precise solution of the planetary orbits is an obstruction to the use of the astronomical insolation computations as an absolute geological time scale through paleoclimates reconstructions beyond a few tens of millions of years. On the opposite, as the paleoclimate geological records increase in duration and quality, they may provide an ultimate constraint for the dynamical model of the Solar system.

1 The chaotic motion of the Solar System

The orbital motion of the planets in the Solar System is chaotic. After Pluto (Sussman and Wisdom, 1988), the evidence that the motion of the whole Solar system is chaotic was established with the averaged equations of motion (Laskar, 1989, 1990), and confirmed later on by direct numerical integration (Sussman and Wisdom, 1992). For a review, see (Laskar, 1996), and for some historical considerations (Laskar, 1992a). The most immediate expression of this chaotic behavior is the exponential divergence of trajectories with close initial conditions. Indeed, the distance of two planetary solutions, starting in the phase space with a distance $d(0) = d_0$, evolves approximately as $d(T) \approx d_0 e^{T/5}$ or in a way which is even closer to the true value,

$$d(T) \approx d_0 10^{T/10} \tag{1}$$

where T is expressed in million of years. Thus, an initial error of 10^{-10} leads to an indeterminacy of $\approx 10^{-9}$ after 10 millions of years, but reaches the order of 1 after 100 millions of years. When specific planets are considered, these results may vary, as the coupling between some of the planets is small, and the outer planets (Jupiter, Saturn, Uranus, Neptune) are more regular than the inner ones (Mercury, Venus, Earth, Mars). For the Earth the above formula is a very good approximation of the divergence of the orbits (Fig.1) (Laskar, 1999a).
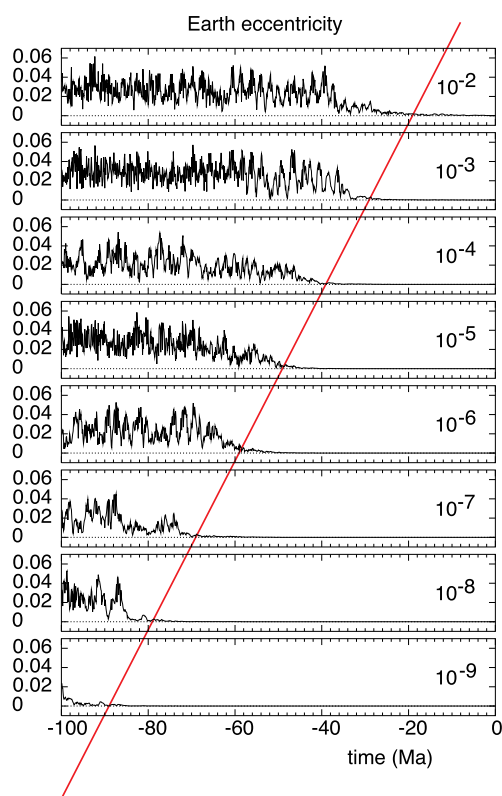
Figure 1: Error in the eccentricity of the Earth resulting from an initial change of 10^{-n} radian in the perihelion of the Earth at the origin. After about $n \times 10$ millions of years, the exponential divergence of the orbits dominates, and the solutions are no longer valid. Error in eccentricity is plotted versus time (in Ma). (Laskar, 1999)

1.1   Secular resonances

In (Laskar, 1990, 1992b), it was demonstrated that this chaotic behavior arise from multiple secular resonances in the inner solar system. In particular, the critical argument associated to

$$(s_4 - s_3) - 2(g_4 - g_3) \tag{2}$$

where g_3, g_4 are related to the precession of the perihelion of the Earth and Mars, $s_3 s_4$ are related to the precession of the node of the same planets, is presently in in a librational state, but can evolve in a rotational state, and even move to libration in a new resonance, namely

$$(s_4 - s_3) - (g_4 - g_3) = 0 \ , \tag{3}$$

showing that a region of relatively strong instability extend over these two resonances. This result was questioned by Sussman and Wisdom (1992) which did not recover such a large variation for the secular frequencies, but reached only $2(s_4 - s_3) - 3(g_4 - g_3) = 0$ instead of (3). Recently, in our group, we performed precise new numerical integrations of the complete equations of motion of the Solar System, including all 9 main planets, the Moon as a separate object, Earth and solar oblateness, tidal dissipation in the Earth Moon system, and the effect of general relativity. This new solution was also adjusted and compared to the JPL ephemeris DE406 (Standish, 1998). Different numerical integrations were conducted, with inclusion or not of the oblateness of the Sun ($J_2 = 10^{-7}$) (num2002 and num2001 in Fig.2). The secular equations were also slightly readjusted and integrated over the same time (sec2001 in Fig.2). In all cases, the resonant argument corresponding to $(s_4 - s_3) - 2(g_4 - g_3)$ was in libration over 30 Myr. The transition to circulation occurred at 30 Myr and about 40 Myr for sec2001 and num2002, while no transition was observed for num2001 (over 100 Myr). In both cases when the transition occurs, the orbit reaches the resonance $(s_4 - s_3) - (g_4 - g_3) = 0$ before 100 Myr, comforting the conclusions of (Laskar, 1990, 1992b). The other important resonant argument $((g_1 - g_5) - (s_1 - s_2))$ that is at the origin of the chaotic behavior of the inner planets (Laskar, 1990, 1992) presents a similar behavior.

The chaotic motion of the solar system is mainly due to the interactions of the secular resonances in the precessing motion of the inner planets, while the secular motion of the outer planets is very regular. In order to evaluate the possible chaotic diffusion of the planetary orbits over the age of the Solar system, numerous numerical integrations have been conducted over several billions of years (Gyr), extending even the age of the Solar system. From these integrations, it appears that Venus and the Earth present some moderate chaotic diffusion, while the lightest planets, Mars and Mercury, can experience very large changes in their orbit eccentricity, allowing even for collision between Mercury and Venus in less than 5 Gyr (Laskar, 1994). From these integrations, it appears that the chaotic diffusion of the orbits in the earlier stages of the solar system formation could provide some clues for the planetary distribution in semi major axis (Laskar, 1997, 2000).

1.2   The outer planets

The secular motion of the outer planets is very regular. Nevertheless, there exists also some instabilities among these planets, due to resonances of high order in their orbital motion around the Sun (mean motion), although these instabilities do not lead to significant changes in the orbits (Murray and Holman, 1999).

A global view of the short period dynamics of the outer planets is given in figure 3 using frequency analysis (Laskar, 1999b, Laskar and Robutel, 2001, Robutel, 2003). In each plot, three of the planets initial conditions are fixed, and the initial conditions of the remaining one (respectively Saturn, Uranus, or Neptune) are changed in a large area of the phase space, spanning semi-major axis and
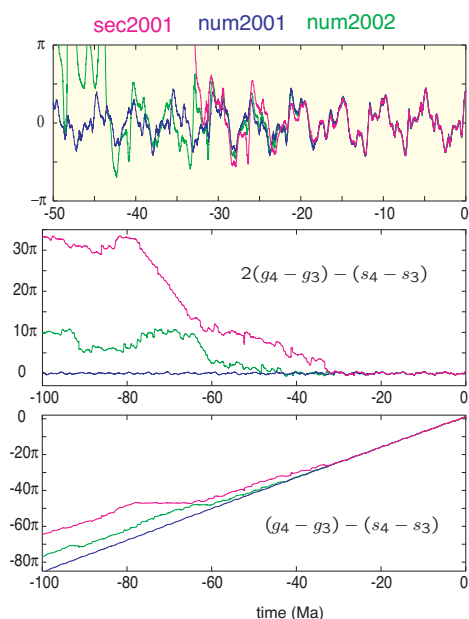
Figure 2: Evolution of he critical angle related to $2(g_4 - g_3) - (s_4 - s_3)$ (top and middle) and $(g_4 - g_3) - (s_4 - s_3)$ (bottom) for three different solutions for the Solar system over 100 Myr. sec2001 is obtained with an averaged system, while num2001 and num2002 are two direct numerical integrations with very close dynamical models.

excentricity in a regular mesh around their current position. In each case, an integration is conducted over 1 Myr, and a stability index, obtained as the variation of the mean motion frequency with time (Laskar, 1990, Laskar and Robutel, 2001) is reported, from very stable (blue) to very unstable (red), while close encounter or ejection is denoted by white dots. The resonances are identified and reported in black. It is thus very clear that although the motion of the outer planets are essentially stable, they are numerous mean motion resonances and small chaotic zones that in the vicinity of the present solar system solution (see also Michtchenko and Ferraz-Mello, 2001).
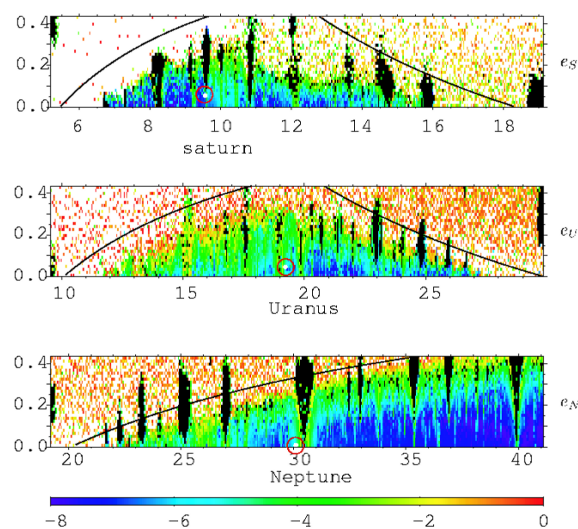
Figure 3: Global dynamics of the giant planets. The abscissa and ordinate correspond to the initial semi major axis and eccentricity of respectively Saturn (top), Uranus (middle) and Neptune (bottom), while in each case the initial conditions of the other planets are taken at their actual values. The current position of each planet is given by a circle. The colors correspond to a stability index obtained by frequency analysis on two consecutive time interval of 1 Myr, and the black regions the mean motion resonances locations (Robutel, 2003).

2 Obliquity of the planets

The planetary perturbations induce also some effect on the tilt (obliquity) of their equatorial plane over their orbital plane . For the Earth, as the precession frequency of the axis is far from the main orbital secular frequencies of the precession motion of the ecliptic, the obliquity presents only small variations of about 1.3 degrees around the mean value of 23.3 degrees (Laskar *et al.*, 1993a). In the absence of the Moon, the situation would be very different, as multiple resonances then occur between the precession of the axis and the precession of the orbital plane, and a very large chaotic zone would exist, ranging from 0 to about 85 degrees (Laskar *et al.*, 1993b). This is also what will happen as the Moon recedes from the Earth due to tidal dissipation (Néron de Surgy and Laskar, 1997). This situation is very similar for all inner planets (Laskar and Robutel, 1993, Laskar, 1995).
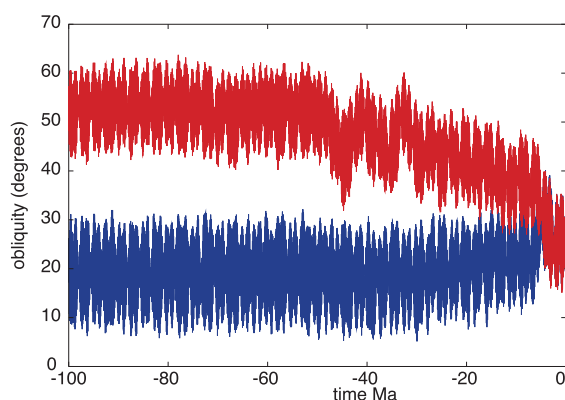
Figure 4: Two different solutions for Mars obliquity over 100 Myrs, obtained with direct integration of the planetary orbits, and with initial conditions of the planet spin within the uncertainty of the most recent determinations (Folkner *et al.*, 1997, Laskar *et al.*, 2002)

2.1 Mars obliquity

Mars presents a very large chaotic zone for its obliquity ranging from 0 to more than 60 degrees (Laskar and Robutel, 1993, see also Touma and Wisdom, 1993). On Figure 4 are plotted two possible solutions for the past evolution of the obliquity of Mars, obtained using our latest numerical integration for the orbital motion of all the planets, and initial conditions and parameters within the uncertainty of the best known values for Mars rotational parameters (Folkner *et al.*, 1997). It is clear that the climatic history of the planet would be very different in the two situations. It is presently particularly important to understand what are the possible past evolutions of the obliquity (and thus climate) of Mars in the past, as the recent Martian spacecrafts provide very detailed observations of the Martian surface (Malin and Edgett, 2001), and give some accurate account of the weather on the planet. In particular, the polar caps of Mars present crevasses that reveal the layered features of the ice (Fig. 5). This layers are thought to be induced by climate variations on Mars surface, and the succession of layers can indeed be related to the insolation variations on its surface (Laskar *et al.*, 2002), although the possible relation from insolation to surface feature on Mars remains very uncertain.

2.2 Venus final states

In 1962, using radar measurements, the slow retrograde rotation of Venus was discovered. Since, the understanding of this particular state becomes a challenge as many uncertainties remain in the dissipative models of Venus' rotation. Various
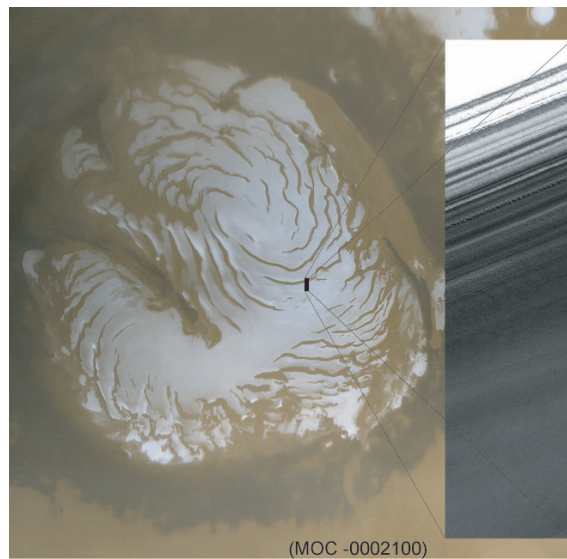
(MOC -0002100)

Figure 5: Mars North polar ice cap, and detailed view of a crevasse, showing the layered nature of the ice. The picture was taken by the Mars Orbital Camera (MOC) on april 13, 1999 (Malin and Edgett, 2001)

hypothesis were proposed for its evolution, aiming to search wether Venus was born with a direct or retrograde rotation. The most favored scenario assumes that its axis was actually tilted down during its past evolution as a result of core mantle friction and atmospheric tides (for a detailed review, see Correia *et al.*, 2003 and references therein). Nevertheless, this requires high values of the initial obliquity, and it was proposed that Venus was strongly hit by massive bodies which would have tilted it significantly or started its rotation backward (Dones and Tremaine, 1993).

The discovery of a very large chaotic zone in the obliquity evolution of Venus allowed new possible scenario for driving Venus to 180 degrees obliquity (Laskar and Robutel, 1993, Yoder,1997), but some difficulties remained. Recently, we have shown that due to the dissipative effects, there are only 4 possible final states for Venus' rotation, and only 3 of them are really reachable. When the planetary perturbations are added, most of the initial conditions lead to the two states corresponding to the present configuration of Venus, one with period -243.02 days and nearly 0 degree obliquity, and the other with opposite period and nearly 180 degree obliquity. We thus demonstrate that a large impact is not necessary to have a satisfying scenario for the reverse rotation of Venus (Fig. 5) (Correia and Laskar, 2001, 2003, Correia *et al.*, 2003), and that most initial conditions can lead to the present reverse rotation. On the opposite, we demonstrate that the

present observed retrograde spin state of Venus can be attained by two different processes that cannot be discriminated by the observation of the present rotational state of Venus. In the first scenario, the axis is tilted towards 180 degrees while its rotation rate slows down, while in the second one, the axis is driven towards 0 degree obliquity and the rotation rate decreases, stops, and increases again in the reverse direction, being accelerated by the atmospheric tides, until a final equilibrium value.
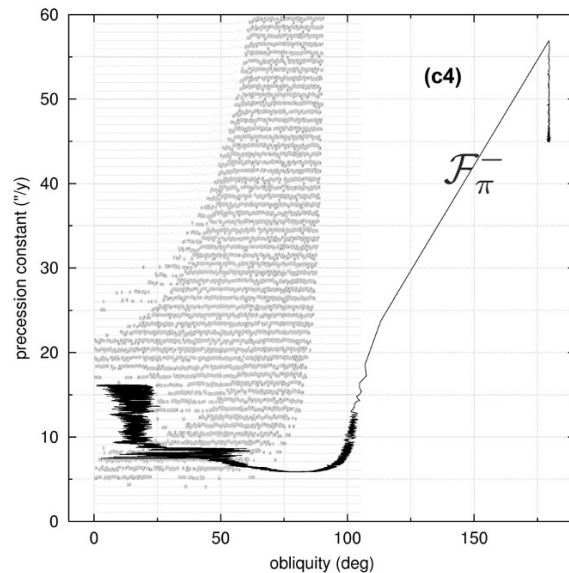

Figure 6: examples of the possible evolution of Venus spin axis during its history. The precession frequency (in arcsec/years) is plotted versus obliquity (in degrees). The initial obliquity is 1 degree, and initial period 3 days. The initial precession frequency is about 16 arcsec/yr, but due to tidal dissipation and core-mantle friction, the planet slows down and the precession frequency decreases. The obliquity then enters a zone of larger chaos (in grey) where the variations of the obliquity increases, until the dissipations drives the obliquity out of the chaotic zone, for a high obliquity. The various dissipative effects can then drive the axis towards 180 degrees (Correia and Laskar, 2003).

3   Earth paleoclimates and chaos

For Mars, the chaotic variations of the obliquity are large, and certainly induce some dramatic effects on its climate. For example, it is probable that at large obliquity (> 45 degrees), the polar caps entirely sublimate, the ice been eventually

redisposed in the equatorial regions (Levrard, 2003). For the Earth, the situation is very different. The variation of the obliquity is also chaotic, but mainly because it is driven by the chaotic orbital motion of the planet that acts as a forcing term on the obliquity. The variations of the obliquity and orbital parameters are small, but they induce significative changes of the insolation on Earth at a given latitude, that are thought to be at the origin of large climatic variations in the past (Hays *et al.*, 1976). Even more, the astronomical solution is used to provide an absolute time scale for the geological paleoclimate records, over a few tens of millions of years, when the effect of the chaotic behavior of the solution is not yet sensitive (e.g. Schackleton *et al.*, 1990, 1999). The quality of the geological data is now becoming sufficient to allow on the other hand to constrain some geophysical parameters of the Earth that are not well known, by comparison of the computed astronomical evolution of the insolation with the geological data (Lourens *et al.*, 2001, Pälike and Shackleton, 2001). As the age of good quality geological records, obtained over long period of time, is now exceeding 30 to 40 millions of years, it becomes interesting to search whether it would be possible to use these geological data to trace back the chaotic diffusion of the solar system in the past.

In order to tackle such a challenging problem it is important, as the quality of geological records decreases with age, to search in the solution some features that would have an important implication on the observed data. Such factor could actually be the resonant argument

$$(s_4 - s_3) - 2(g_4 - g_3) \,, \tag{4}$$

and its evolution with time. It would be indeed fascinating to be able to retrieve in the geological data the transitions from libration to circulation of this argument, as well as its eventual transition to the resonance

$$(s_4 - s_3) - (g_4 - g_3) = 0 \,. \tag{5}$$

The direct observation of the individual arguments related to g_3, g_4, s_3, s_4 is certainly out of reach. But it may be possible to follow the evolution in time of the differences $g_3 - g_4$ and $s_3 - s_4$. Indeed, $s_3 - s_4$ appears as a beat of about 1.2 million of years in the solution of obliquity, as the result of the beat between the $p + s_3$ and $p + s_4$ components of the obliquity, where p is the precession frequency of the axis (see Laskar, 1999a) (Fig. 7). In a similar way, $g_3 - g_4$ appears as a beat with period of about 2.4 Myr in the climatic precession (Fig. 7). One understands that because of the occurrence of these beats, the detection of the resonant state in the geological data can be possible, as one has to search now for phenomena of large amplitude in the geological signal. Indeed, in the newly collected data from Ocean Drilling Program Site 926, the modulation of 1.2 Myr of the obliquity appears clearly in the spectral analysis of the paleoclimate record (Zachos *et al.*, 2001).

The search for the determination of these resonant angles in the geological data is just starting, but we may expect that a careful analysis, and new data
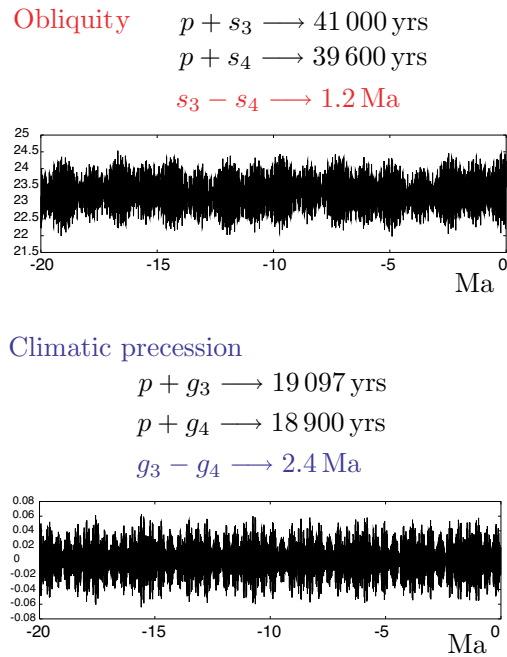
Obliquity $p + s_3 \longrightarrow 41\,000\,\text{yrs}$
$p + s_4 \longrightarrow 39\,600\,\text{yrs}$

$s_3 - s_4 \longrightarrow 1.2\,\text{Ma}$

Climatic precession

$p + g_3 \longrightarrow 19\,097\,\text{yrs}$
$p + g_4 \longrightarrow 18\,900\,\text{yrs}$

$g_3 - g_4 \longrightarrow 2.4\,\text{Ma}$

Figure 7: The main components of the obliquity of the Earth are given by two terms of period close to 40 000 years ($p + s_3$ and $p + s_4$) than induce a beat of about 1.2 Myr period. For the climatic precession ($e \sin \omega$, where e is the excentricity and ω the longitude of perihelion from the moving equinox), the main terms are of period close to 20 000 years, and the beat $g_3 - g_4$ has a period of about 2.4 Myrs.

spanning the last 60 to 100 Myr, will allow to determine the possible succession of resonant states in the past, allowing, for example, to discriminate between the solutions displayed in figure 2. Of particular importance would be the detection of the first transition from libration to circulation of the resonant argument $(s_4 - s_3) - 2(g_4 - g_3)$. This program, if completed, will provide some extreme constraint for the gravitational model of the Solar System. Indeed, the observation of a characteristic feature of the solution at 40 to 100 Myr in the past, because of the exponential divergence of the solutions, will provide a constraint of 10^{-4} to 10^{-10} on the dynamical model of the Solar System.

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J. Laskar
Astronomie et Systèmes Dynamiques
CNRS-UMR8028, IMCCE-Observatoire de Paris
77, Av Denfert-Rochereau
F-75014 Paris
France