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η-Ricci solitons on para-Sasakian manifolds

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Abstract. The object of this paper is to study a special type of metrics called η-Ricci solitons on para-Sasakian manifolds. We give the existence of para-Sasakian η-Ricci solitons in our settings. In addition, the non-existence of certain geometric characteristics of para-Sasakian η-Ricci solitons are studied. Finally, we discuss 3-dimensional and conformally flat para-Sasakian η-Ricci solitons.

Mathematics Subject Classification. 53C21, 53C44, 53C25.

Keywords. Para-Sasakian manifold, η-Ricci solitons, Ricci-semisymmetric manifold, conformally flat manifolds.

1. Introduction

A *Ricci soliton* is a natural generalization of Einstein metric. A Ricci soliton (g, V, λ) is defined on a pseudo-Riemannian manifold (M, g) by

$$(\pounds_V g)(X, Y) + 2S(X, Y) + 2\lambda g(X, Y) = 0, \tag{1.1}$$

where $\pounds_V g$ denotes the Lie derivative of Riemannian metric g along a vector field V, λ is a constant, and X, Y are arbitrary vector fields on M. A Ricci soliton is said to be *shrinking*, *steady*, and *expanding* according as λ is *negative*, *zero*, and *positive*, respectively. Theoretical physicists have also been taking interest in the equation of Ricci soliton in relation with string theory, and the fact that equation (1.1) is a special case of Einstein field equations. The Ricci soliton in Riemannian Geometry was introduced [13] as self-similar solution of the Ricci flow. Recent progress on Riemannian Ricci solitons may be found in [9]. Also, Ricci solitons have been studied extensively in the context of pseudo-Riemannian Geometry; we may refer to [1,6–8,15] and references therein.

As a generalization of Ricci solitons, the notion of η-*Ricci solitons* was introduced by Cho and Kimura [12]. This notion has also been studied in [5] for Hopf hypersurfaces in complex space forms. An η-Ricci soliton is a tuple (g, V, λ, μ), where V is a vector field on M, λ and μ are constants, and g is a Riemannian (or pseudo-Riemannian) metric satisfying the equation

$$\pounds_V g + 2S + 2\lambda g + 2\mu\eta \otimes \eta = 0, \tag{1.2}$$

where S is the Ricci tensor associated to g. In this connection we mention the works of Blaga (see, [2–4]) with η-Ricci solitons. In particular, if $\mu = 0$ then the notion η-Ricci soliton (g, V, λ, μ) reduces to the notion of Ricci soliton (g, V, λ).

In 1976, Sato [17] introduced the notion of almost paracontact structure (ϕ, ξ, η) on a differentiable manifold. This structure is an analogue of the almost contact structure. An almost contact manifold is always odd-dimensional but an almost paracontact manifold could be of even dimension as well. Takahashi [20] defined almost contact manifolds (in particular, Sasakian manifolds) equipped with an associated pseudo-Riemannian metric.

In 1985, Kaneyuki and Williams [14] defined the notion of almost paracontact structure on pseudo-Riemannian manifold of dimension $(2n + 1)$. Later, Zamkovoy [22] showed that any almost paracontact structure admits a pseudo-Riemannian metric with signature $(n + 1, n)$.

In the present paper, we study para-Sasakian η-Ricci solitons. The paper is organised as follows: Sect. 2 is devoted to preliminaries on para-Sasakian manifolds. In Sect. 3, we study para-Sasakian η-Ricci soliton and its existence in our settings. Here, it is proved that, on a para-Sasakian manifold $M(\phi, \xi, \eta, g)$ the symmetric parallel $(0, 2)$-tensor field $\alpha = \pounds_\xi g + 2S + 2\mu\eta \otimes \eta$ is parallel with respect to Levi–Civita connection, as a result, the pair $(M, (g, \xi, \lambda, \mu))$ is a para-Sasakian η-Ricci soliton. Also, we show that a para-Sasakian Ricci soliton $(M, (g, \xi, \lambda))$ is expanding. In Sect. 4, the non-existence of certain geometric characteristics of para-Sasakian η-Ricci solitons are studied. Sections 5 and 6 contain the study of 3-dimensional and conformally flat para-Sasakian η-Ricci solitons, respectively.

2. Preliminaries

An *almost paracontact structure* on a manifold M of dimension n is a triplet (ϕ, ξ, η) consisting of a (1,1)-tensor field ϕ, a vector field ξ, a 1-form η satisfying:

$$\phi^2 = I - \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0, \tag{2.1}$$

$$\eta \cdot \phi = 0, \quad rank(\phi) = n - 1, \tag{2.2}$$

where I denotes the identity transformation. A pseudo-Riemannian metric g on M is compatible with the almost paracontact structure (ϕ, ξ, η) if

$$g(\phi X, \phi Y) = -g(X, Y) + \eta(X)\eta(Y). \tag{2.3}$$

In such case, (ϕ, ξ, η, g) is called an *almost paracontact metric structure*. By (2.1)–(2.3), it is clear that $g(X, \xi) = \eta(X)$ for any compatible metric. Any almost paracontact structure admits compatible metrics. The fundamental 2-form Φ of an almost paracontact structure (ϕ, ξ, η, g) is defined by $\Phi = g(X, \phi Y)$, for all tangent vector fields X, Y. If $\Phi = d\eta$, then the manifold

(M, ϕ, ξ, η, g) is called a *paracontact metric manifold* associated to the metric g.

In this case, the paracontact metric structure is normal and the structure is called para-Sasakian. Equivalently, a paracontact metric structure (ϕ, ξ, η, g) is para-Sasakian if

$$(\nabla_X \phi)Y = -g(X, Y)\xi + \eta(Y)X \tag{2.4}$$

for any $X, Y \in \Gamma(TM)$, where ∇ is Levi–Civita connection of g.

From (2.4), it follows that

$$\nabla_X \xi = -\phi X. \tag{2.5}$$

Also in an n-dimensional para-Sasakian manifold, the following relations hold:

$$\eta(R(X, Y)Z) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X), \tag{2.6}$$

$$R(\xi, X)Y = -g(X, Y)\xi + \eta(Y)X, \tag{2.7}$$

$$S(X, \xi) = -(n-1)\eta(X), \tag{2.8}$$

for any $X, Y, Z \in \Gamma(TM)$. Here R is the Riemannian curvature tensor and S is Ricci tensor defined by $S(X, Y) = g(QX, Y)$, where Q is Ricci operator.

3. η-Ricci solitons on para-Sasakian manifolds

Let $M(\phi, \xi, \eta, g)$ be an n-dimensional para-Sasakian manifold and let $(M, (g, \xi, \lambda, \mu))$ be a para-Sasakian η-Ricci soliton. Then the relation (1.2) implies

$$(\pounds_\xi g)(X, Y) + 2S(X, Y) + 2\lambda g(X, Y) + 2\mu\eta(X)\eta(Y) = 0$$

or

$$2S(X, Y) = -(\pounds_\xi g)(X, Y) - 2\lambda g(X, Y) - 2\mu\eta(X)\eta(Y) \tag{3.1}$$

for any $X, Y \in \Gamma(TM)$.

On a para-Sasakian manifold M, from (2.5) and the skew-symmetric property of ϕ, we obtain

$$(\pounds_\xi g)(X, Y) = g(\nabla_X \xi, Y) + g(X, \nabla_Y \xi) = 0. \tag{3.2}$$

By plugging (3.2) in (3.1), we have

$$S(X, Y) = -\lambda g(X, Y) - \mu\eta(X)\eta(Y). \tag{3.3}$$

Thus, we conclude that $(M, (g, \xi, \lambda, \mu))$ is an η-Einstein manifold. Thus we state:

Theorem 3.1. *A para-Sasakian η-Ricci soliton $(M, (g, \xi, \lambda, \mu))$ is an η-Einstein manifold.*

In particular, if $\mu = 0$ in (3.3), then it reduces to

$$S(X, Y) = -\lambda g(X, Y). \tag{3.4}$$

Thus the pair $(M,(g,\xi,\lambda))$ is an Einstein one. So we have the following corollary:

Corollary 3.1. *A para-Sasakian Ricci soliton* $(M,(g,\xi,\lambda))$ *is an Einstein manifold.*

Ricci solitons exhibit rich geometric properties. An important geometrical object in studying Ricci soliton is to be a symmetric (0, 2)-tensor field which is parallel with respect to the Levi-Civita connection. In this connection, we state the existence of η-Ricci solitons in our settings.

Consider that the symmetric (0, 2)-tensor field $\alpha = \pounds_\xi g + 2S + 2\mu\eta\otimes\eta$ is parallel with respect to the Levi–Civita connection associated to g. Then

$$\alpha(\xi,\xi) = (\pounds_\xi g)(\xi,\xi) + 2S(\xi,\xi) + 2\mu\eta(\xi)\eta(\xi), \tag{3.5}$$

which yields by virtue of (3.2) and (3.3) that

$$\lambda = -\frac{1}{2}\alpha(\xi,\xi). \tag{3.6}$$

Recently, Tarafdar and De proved that a second order symmetric parallel tensor on a para-Sasakian manifold is a constant multiple of the associated tensor, and that on a para-Sasakian manifold there is no non-zero parallel 2-form [19]. That is,

$$\alpha(X,Y) = \alpha(\xi,\xi)g(X,Y). \tag{3.7}$$

Equating (3.6) and (3.7), one can conclude that

$$\alpha(X,Y) = -2\lambda g(X,Y), \tag{3.8}$$

for any $X, Y \in \Gamma(TM)$. Therefore, $\pounds_\xi g + 2S + 2\mu\eta\otimes\eta = -2\lambda g$. This relation defines an η-Ricci soliton on M. Hence, in conclusion we have

Theorem 3.2. *Let* $M(\phi,\xi,\eta,g)$ *be a para-Sasakian manifold. If the symmetric (0,2)-tensor field* $\alpha = \pounds_\xi g + 2S + 2\mu\eta\otimes\eta$ *is parallel with respect to the Levi–Civita connection of* g*, then the relation* (3.1) *defines an* η*-Ricci soliton on* M*.*

For $\mu = 0$ it follows from (3.1) that $\alpha = \pounds_\xi + 2S$. If α is parallel with respect to the Levi–Civita connection associated to g, then it follows that $\pounds_\xi g + 2S = -2\lambda g$. This defines a Ricci soliton on M. Immediately, we have this corollary:

Corollary 3.2. *Let* $M(\phi,\xi,\eta,g)$ *be a para-Sasakian manifold. If the symmetric (0,2)-tensor field* $\alpha = \pounds_\xi + 2S$ *is parallel with respect to the Levi–Civita connection of* g*, then the relation* (3.1) *defines a Ricci soliton on* M *for* $\mu = 0$.

By using (3.2) and (3.3) in (3.5), we have

$$\alpha(\xi,\xi) = -2(n-1), \tag{3.9}$$

where $\mu = 0$ is used.

If we equate (3.8) and (3.9), we obtain $\lambda = n-1$. That is, λ is positive as $n > 1$. Therefore, $(M,(g,\xi,\lambda))$ is expanding. Hence we are able to state the following result:

Corollary 3.3. *An n-dimensional* $(n > 1)$ *para-Sasakian Ricci soliton* $(M, (g, \xi, \lambda))$ *is expanding.*

4. Non-existence of certain kinds of para-Sasakian η-Ricci solitons

In this section we will establish the non-existence of certain geometric characteristics of para-Sasakian η-Ricci solitons. Let $M(\phi, \xi, \eta, g)$ $(n > 1)$ be a para-Sasakian manifold. Now we have

$$(R(X,Y)\cdot S)(U,V) = -S(R(X,Y)U,V) - S(U,R(X,Y)V),$$

which implies that

$$(R(X,\xi)\cdot S)(U,V) = -S(R(X,\xi)U,V) - S(U,R(X,\xi)V). \quad (4.1)$$

In a para-Sasakian η-Ricci soliton $(M,(g,\xi,\lambda,\mu))$, the relation (3.3) holds. By making use of (3.3) in (4.1) we obtain

$$\begin{aligned}(R(X,\xi)\cdot S)(U,V) = &-\lambda[g(R(X,\xi)U,V) - g(U,R(X,\xi)V)]\\ &-\mu[\eta(R(X,\xi)U)\eta(V) - \eta(U)\eta(R(X,\xi)V)]. \end{aligned}\quad (4.2)$$

In view of (2.7), (4.2) reduces to

$$(R(X,\xi)\cdot S)(U,V) = \mu[-g(X,U)\eta(V) - g(X,V)\eta(U) + 2\eta(X)\eta(U)\eta(V)]. \quad (4.3)$$

Setting $V = \xi$ in (4.3) we get

$$(R(X,\xi)\cdot S)(U,\xi) = \mu[-g(X,U) + \eta(X)\eta(U)]. \quad (4.4)$$

Suppose that a para-Sasakian η-Ricci soliton $(M,(g,\xi,\lambda,\mu))$ $(n > 1)$ is Ricci-semisymmetric. Then we have $R(X,Y)\cdot S = 0$. That is,

$$(R(X,\xi)\cdot S)(U,\xi) = 0. \quad (4.5)$$

Equating (4.4) and (4.5), we get $\mu[-g(X,U)+\eta(X)\eta(U)] = 0$ or, equivalently, $\mu[g(\phi X,\phi U)] = 0$. It gives that $\mu = 0$. This is a contradiction. Therefore, $(M,(g,\xi,\lambda,\mu))$ cannot be Ricci-semisymmetric.

On the other hand, from (3.3) we have

$$QY = -\lambda Y - \mu\eta(Y)\xi. \quad (4.6)$$

Taking covariant derivative of (4.6) with respect to X, we obtain

$$(\nabla_X Q)(Y) = -\mu[(\nabla_X\eta)(Y)\xi - \eta(Y)\nabla_X\xi]. \quad (4.7)$$

Applying ϕ^2 on both sides of the above equation and then using (2.1), one can get

$$\phi^2(\nabla_X Q)(Y) = \mu\eta(Y)\phi^2(\nabla_X\xi). \quad (4.8)$$

If we use (2.1) and (2.5) in the right hand side of (4.8), then it gives

$$\phi^2(\nabla_X Q)(Y) = -\mu\eta(Y)\phi X. \quad (4.9)$$

Assume that a para-Sasakian η-Ricci soliton $(M,(g,\xi,\lambda,\mu))$ is ϕ-Ricci symmetric [18]. Then, the Ricci operator Q satisfies

$$\phi^2(\nabla_X Q)(Y) = 0, \tag{4.10}$$

for all vector fields X and Y on M and $S(X,Y) = g(QX,Y)$.

Then by virtue of (4.9) and (4.10), we get $\mu\eta(Y)\phi X = 0$. This follows that $\mu = 0$. Again, a contradiction. Hence $(M,(g,\xi,\lambda,\mu))$ cannot be ϕ-Ricci symmetric.

Next, consider a para-Sasakian η-Ricci soliton $(M,(g,\xi,\lambda,\mu))$ $(n > 1)$ which is projective Ricci-semisymmetric. Then the condition $\mathcal{P}(X,Y)\cdot S = 0$ implies that

$$S(\mathcal{P}(\xi,X)Y,Z) + S(Y,\mathcal{P}(\xi,X)Z) = 0, \tag{4.11}$$

for any vector fields X, Y, Z on M and $\mathcal{P}$ denotes the Projective curvature tensor defined by

$$\mathcal{P}(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}\{S(Y,Z)X - S(X,Z)Y\}. \tag{4.12}$$

By using (3.3), (4.11) gives

$$\begin{aligned} &-\lambda[g(\mathcal{P}(\xi,X)Y,Z) + g(Y,\mathcal{P}(\xi,X)Z)] \\ &-\mu[\eta(\mathcal{P}(\xi,X)Y)\eta(Z) + \eta(Y)\eta(\mathcal{P}(\xi,X)Z] = 0. \end{aligned} \tag{4.13}$$

Using (2.7) and (3.3) in (4.12) we obtain

$$\begin{aligned} \mathcal{P}(\xi,X)Y = &\left(\frac{\lambda}{n-1} - 1\right) g(X,Y)\xi + \left(1 - \frac{(\lambda+\mu)}{n-1}\right)\eta(Y)X \\ &+ \left(\frac{\mu}{n-1}\right)\eta(X)\eta(Y)\xi \end{aligned} \tag{4.14}$$

and

$$\eta(\mathcal{P}(\xi,X)Y) = \left(\frac{\lambda}{n-1} - 1\right)\{g(X,Y) - \eta(X)\eta(Y)\}. \tag{4.15}$$

Taking account of (4.14) and (4.15) in (4.13) we get

$$\mu[g(X,Y)\eta(Z) + g(X,Z)\eta(Y) - 2\eta(X)\eta(Y)\eta(Z)] = 0. \tag{4.16}$$

Setting $Z = \xi$ in (4.16), we have $\mu[g(X,Y) - \eta(X)\eta(Y)]$. It implies that $\mu = 0$. Therefore, $(M,(g,\xi,\lambda,\mu))$ $(n > 1)$ cannot be projective Ricci-semisymmetric. Thus, the above results can be stated as follows:

Theorem 4.3. *A para-Sasakian η-Ricci soliton $(M,(g,\xi,\lambda,\mu))$ $(n > 1)$ cannot be Ricci-semisymmetric, ϕ-Ricci symmetric nor projective Ricci-semisymmetric.*

5. 3-Dimensional para-Sasakian η-Ricci solitons

In a 3-dimensional para-Sasakian manifold M^3 (ϕ, ξ, η, g), the curvature tensor $\acute{R}$ of type (0, 4) has the following form [16]

$$\begin{aligned}\acute{R}(X,Y,Z,W) = &\{S(Y,Z)g(X,W) - S(X,Z)g(Y,W)\\ &+S(X,W)g(Y,Z) - S(Y,W)g(X,Z)\}\\ &+\frac{r}{2}\{g(X,Z)g(Y,W) - g(Y,Z)g(X,W)\},\end{aligned} \tag{5.1}$$

where $\acute{R}(X,Y,Z,W) = g(R(X,Y)Z,W)$.

By virtue of (3.3), the Eq. (5.1) can be written as

$$\begin{aligned}\acute{R}(X,Y,Z,W) = &-\left(\frac{r}{2}+2\lambda\right)\{g(Y,Z)g(X,W) - g(X,Z)g(Y,W)\}\\ &-\mu\{\eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(Z)g(Y,W)\\ &+\eta(X)\eta(W)g(Y,Z) - \eta(Y)\eta(W)g(X,Z)\}.\end{aligned} \tag{5.2}$$

Contracting (3.3), for n = 3 we get

$$r = -(3\lambda + \mu). \tag{5.3}$$

Using (5.3) in (5.2) we obtain

$$\begin{aligned}\acute{R}(X,Y,Z,W) = &\,p^2\{g(Y,Z)g(X,W) - g(X,Z)g(Y,W)\}\\ &+p\acute{p}\{\eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(Z)g(Y,W)\\ &+\eta(X)\eta(W)g(Y,Z) - \eta(Y)\eta(W)g(X,Z)\},\end{aligned} \tag{5.4}$$

where

$$p = \sqrt{\frac{(\mu-\lambda)}{2}}$$

and

$$\acute{p} = -\mu\sqrt{\frac{2}{(\mu-\lambda)}}.$$

The Eq. (5.4) leads to

$$\acute{R}(X,Y,Z,W) = B(X,W)B(Y,Z) - B(Y,W)B(X,Z), \tag{5.5}$$

where

$$B(X,Y) = pg(X,Y) + \acute{p}\eta(X)\eta(Y). \tag{5.6}$$

It is known that a type of n-dimensional Riemannian or semi-Riemannian manifold whose curvature tensor $\acute{R}$ of type (0, 4) satisfies the condition (5.5), where B is a symmetric tensor field of type (0, 2) is called *special manifold* with the associated symmetric tensor B and is denoted by the symbol $(\psi B)_n$.

By virtue of (5.5), (5.6), the following theorem is stated:

Theorem 5.4. *A 3-dimensional para-Sasakian η-Ricci soliton $(M^3, (g, \xi, \lambda, \mu))$ is a $(\psi B)_3$ with associated symmetric tensor B given by (5.6).*

6. Conformally flat para-Sasakian η-Ricci solitons

It is known that in case of a conformally flat para-Sasakian manifold M (ϕ, ξ, η, g) $(n > 3)$, the curvature tensor $\acute{R}$ of type $(0, 4)$ has the following form [21]:

$$\begin{aligned}\acute{R}(X,Y,Z,W) = \frac{1}{(n-2)}&[S(Y,Z)g(X,W) - S(X,Z)g(Y,W)\\&+ S(X,W)g(Y,Z) - S(Y,W)g(X,Z)]\\&+ \frac{r}{(n-1)(n-2)}[g(X,Z)g(Y,W) - g(Y,Z)g(X,W)].\end{aligned} \tag{6.1}$$

Contracting (3.3) over X and Y we get $r = -(n\lambda + \mu)$, where r denotes the scalar curvature of the manifold. By making use of the value of r and (3.3), Eq. (6.1) can be rewritten as

$$\begin{aligned}\acute{R}(X,Y,Z,W) = \frac{\mu - \lambda(n-2)}{(n-1)(n-2)}&[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)]\\&- \frac{\mu}{n-2}[\eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(Z)g(Y,W)\\&+ \eta(X)\eta(W)g(Y,Z) - \eta(Y)\eta(W)g(X,Z)].\end{aligned} \tag{6.2}$$

The substitution

$$B(X,Y) = qg(X,Y) + \acute{q}\eta(X)\eta(Y) \tag{6.3}$$

with

$$q = \sqrt{\frac{\mu - \lambda(n-2)}{(n-1)(n-2)}}$$

and

$$\acute{q} = -\frac{\mu}{n-2}\sqrt{\frac{(n-1)(n-2)}{\mu - \lambda(n-2)}}$$

leads to

$$\acute{R}(X,Y,Z,W) = B(X,W)B(Y,Z) - B(Y,W)B(X,Z). \tag{6.4}$$

The above result is stated in the following theorem:

Theorem 6.5. *A conformally flat para-Sasakian η-Ricci soliton $(M, (g, \xi, \lambda, \mu))$ is $(\psi B)_n$ with associated symmetric tensor B given by (6.4).*

In 1972, Chen and Yano [11] introduced the notion of a manifold of quasi-constant curvature as follows:

A non-flat para-Sasakian manifold $M(\phi, \xi, \eta, g)$ $(n > 3)$ is said to be of quasi-constant curvature if its curvature tensor $\acute{R}$ of type $(0, 4)$ satisfies the following condition [10]:

$$\begin{aligned}\acute{R}(X,Y,Z,W) = a&[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)]\\&+ b[A(Y)A(Z)g(X,W) - A(X)A(Z)g(Y,W)\\&+ A(X)A(W)g(Y,Z) - A(Y)A(W)g(X,Z)],\end{aligned} \tag{6.5}$$

where a and b are scalars of which $b \neq 0$ and A is non-zero 1-form such that $g(X, U) = A(X)$ for all X, U being a unit vector field. Such an n-dimensional manifold was denoted by the symbol $(QC)_n$.

Putting

$$B(X,Y) = \sqrt{a}g(X,Y) + \frac{b}{\sqrt{a}}\eta(X)\eta(Y) \tag{6.6}$$

it follows from (6.5) that

$$\acute{R}(X,Y,Z,W) = B(X,W)B(Y,Z) - B(Y,W)B(X,Z). \tag{6.7}$$

From (6.7) it could be seen that a $(QC)_n$ is $(\psi B)_n$. Comparing (6.2) with (6.5) it can be concluded that a conformally flat para-Sasakian η-Ricci soliton $(M, (g, \xi, \lambda, \mu))$ $(n > 3)$ is a manifold of quasi-constant curvature. Thus, we state the following:

Theorem 6.6. *A conformally flat para-Sasakian η-Ricci soliton $(M, (g, \xi, \lambda, \mu))$ $(n > 3)$ is a manifold of quasi-constant curvature.*

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