Phys. perspect. 6 (2004) 372–389
1422-6944/04/040372–18
DOI 10.1007/s00016-003-0205-8

Physics in Perspective

The *Bild* Conception of Physical Theory: Helmholtz, Hertz, and Schrödinger

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Hermann von Helmholtz (1821–1894) criticized the objective conception of physical theory, denying that theoretical concepts are "images" of physical objects. Heinrich Hertz (1857–1894) and Erwin Schrödinger (1887–1961) used the term *Bild* to designate their conception of physical theory, meaning an intellectual construct whose relationship to phenomena was to be analyzed. The main features of their *Bild* conception were an outspoken anti-inductivism and an affirmation of a partial separation of physical theory and experimental observations. Once accepted, the *Bild* conception loosened the bonds that still justified the attempts at the end of the nineteenth century, such Helmholtz's and Hertz's, to unify physics through a generalized form of mechanics and opened the way to the innovations of Einstein's theory of relativity.

Key words: Hermann von Helmholtz; Heinrich Hertz; Erwin Schrödinger; *Bild* conception of physical theory.

Introduction

Hermann von Helmholtz (1821–1894) was one of the first scientists to criticize the objective conception of physical theory by denying that theoretical concepts describe real physical objects. He realized that Immanuel Kant's *a priori* forms of intuition should to be taken into account in analyzing problems that were emerging at the end of the nineteenth century in the new formulations of physics.

The objective conception of physical theory also was criticized by such physicists as Heinrich Hertz (1857–1894) and Ludwig Boltzmann (1844–1906), who adopted the Kantian term *Bild* to designate the new conception of physical theory, which they took to mean not a faithful image of nature but an intellectual construct whose relationship to empirical phenomena was to be analyzed. Later, Erwin Schrödinger (1887–1961) took the *Bild* conception of physical theory to be an established tradition in theoretical physics.

Among philosophers, Ernst Cassirer (1874–1945) devoted many pages to an analysis of the *Bild* conception of physical theory.[1] To him, a crisis in fundamentals charac-

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terized much of physics in the second half of the nineteenth century and still affected physical research. Since the time of Galileo and Newton, he argued, theories differed widely in their content, but their "*ontological* significance … was never seriously challenged…."[2] That challenge precipitated a crisis at the end of the nineteenth century, causing "reflective criticism in the natural sciences … [to become] urgent with ever mounting emphasis."[3]

I argue that the *Bild* conception of physical theory and its attendant modification of the way in which physicists viewed the relationship between theory and experiment were among the pressing issues that stimulated their reflective criticism. I restrict my discussion to the *Bild* conception of physical theory as prepared by Helmholtz and elaborated by Hertz and Schrödinger between the last quarter of the nineteenth and the middle of the twentieth century.

Helmholtz's Secularization of Kant's *Anschaulichkeit*

Hermann von Helmholtz (figure 1), in a lecture on "The Facts in Perception" that he delivered during the anniversary celebrations of the University of Berlin in 1878, posed the following philosophical questions:

> What is true in our intuition and thought? In what sense do our representations correspond to actuality? Philosophy and natural science encounter this problem from two opposite sides, it is a task common to both.[4]

With Helmholtz epistemology was given new life through his physiological investigations,[5] in particular through his optical researches and theory of vision as stimulated by the theory of the specific energies of sensory nerves of his teacher Johannes Müller (1801–1858):

> Excitation of the optic nerve produces only light sensations, no matter whether objective light – i.e. aether vibrations – impinges upon it, or [whether it is stimulated by] an electric current which we pass through the eye, or [by] pressure on the eyeball, or [by] straining of the nerve stem during rapid changes of the direction of vision.[6]

Starting from these premises, Helmholtz denied that the eye is a passive receptor of a presumably faithful image of the world. The problem was to discriminate between those stimulations that produce light sensations that are caused by "objective light" or "aether vibrations" and those that are caused by other means. However, because of Müller's discovery of the specific energies of sensory nerves, even those stimulations that are caused by "objective light" represent only a "report of what is peculiar to the external influence" and are not an image of this influence but only a symbolic representation of it.

> Inasmuch as the quality of our sensation gives us a report of what is peculiar to the external influence by which it is excited, it may count as a symbol of it, but not as an *image*. For from an image one requires some kind of alikeness with the object of which it is an image….[7]

Fig. 1. Hermann von Helmholtz (1821-1894). *Source*: Herman von Helmholtz, *Wissenschaftliche Abhandlungen*, Dritter Band (Leipzig: Johann Ambrosius Barth, 1895), frontispiece.

In the case of vision, the only way that the eye can connect the external influence to the subjective *lux* we experience is through the regularity by which a concept like aether vibrations obeys permanent and determinable laws that correspond to those of the sensation of *lux* in the eye. This "parallelism of laws" holds in general:

> Every law of nature asserts that upon preconditions alike in a certain respect, there always follow consequences which are alike in a certain other respect. Since like things are indicated in our world of sensations by like signs, an equally regular sequence will also correspond in the domain of our sensations to the sequence of like effects by [the] law of nature [that like effects follow from] … like causes.[8]

Helmholtz's "parallelism of laws" led him to embrace Kant's philosophy, but not as mediated by the Romantic philosophy of Friedrich Schelling (1775–1854) and Arthur Schopenhauer (1788-1860), much less by the *Naturphilosophie* of Georg Hegel (1770–1831), which Helmholtz vehemently opposed.[9] Rather, his fascination with general epistemological problems of perception and Kant's doctrine of *a priori* forms of intuition was stimulated by Müller's investigations of the physiology of the senses and his own researches on the physiology of vision. As he declared:

> investigations into the physiology of the senses, which were in particular completed and critically sifted by Johannes Müller and then summarized by him in the law of *specific energies of the sensory nerves*, have now brought the fullest confirmation [of Kant's doctrine], one can almost say to an unexpected degree.[10]

This confirmation, however, carried with it a modification of Kant's doctrine, introducing a distinction that was foreign to Kant. To Helmholtz, the Kantian *a priori* forms of intuition had to be divided into two kinds, a general one and a narrower one. The truly *a priori* feature of every sensory perception is a *general* form of intuition that is devoid of any empirical content, as exemplified by the spatial perception of place or by the visual perception of an aggregate of colored surfaces. The *narrower* form of intuition then concerns particular types of geometric space or the particular colors that appear at a given time.

> Everything our eye sees, it sees as an aggregate of coloured surfaces in the visual field – that is its [general] form of intuition. The particular colours which appear on this or that occasion, their arrangement and sequence – this is the result of external influences and is not determined by any law of our makeup. Similarly from the fact that space is a [general] form of intuiting, nothing whatever follows about the facts expressed by the axioms. If such propositions are taken to be not empirical ones, but to belong instead to the necessary form of intuition, then this is a further particular [narrower] specification of the general form of space; and *those grounds which allowed the conclusion that the form of intuition of space is transcendental, do not necessarily for that reason already suffice to prove, at the same time, that the axioms too are of transcendental origin.*[11]

Applying this distinction between the Kantian *a priori* forms of intuition to sensations in general, Helmholtz argued that there also are differences in form between those associated with the different senses, like sight and sound, which he called differences in *modality*,[*] and those that characterize sensations of the same type, which he called differences in *quality*. Thus, for instance, optical modalities are distinguished from acoustical ones by their characteristic features and thus belong to the general *a priori* form of intuitions: A truly general *a priori* form of intuition "must be devoid of content and free to an extent sufficient for absorbing any content whatsoever that can enter the relevant form of perception."[12] By contrast to the narrower form of intuition, the truly general Kantian *a priori* form of intuition is thus compatible with various systems of axioms. By contrast, the axioms that characterize Euclidean as distinguished

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[*] Helmholtz compared the different perceptual characteristics of the eye and ear and concluded that the eye's incapability to detect beat frequencies represented its *a priori* modality. As he wrote: "Could the optic nerve at all follow in sensation the enormously rapid beats of light oscillations, then every mixed colour would act as a dissonance." See Helmholtz, "Facts in Perception" (ref. 4), p. 121. One can argue that in Helmholtz's conception the eye's inability to distinguish between pure and mixed colors should not be considered as an imperfection or inability but as the modality that distinguishes vision from our other senses, for instance, hearing. See Chevalley,"Complementarieté" (ref. 64).

from non-Euclidean geometry are not truly *a priori* because both geometries concern differences in quality between the same spatial perceptions and not their fundamental differences in modality.

For Helmholtz, therefore, the specification that Kant introduced into our intuition of space limits the general *a priori* form of intuition, because "the axioms of [Euclidean] geometry limit the form of intuition of space in such a way that it can no longer absorb every thinkable concept, if geometry is at all supposed to be applicable to the actual world...."[13] Helmholtz thus took the axioms of Euclidean geometry to represent the narrower form of intuition, a specification in quality that is not *a priori* because it limits our perception of space to three dimensions, which is too narrow (too full of empirical content) to represent all possible experience.

Helmholtz thought that with his distinction he had gone beyond Kant: "Here Kant was not critical enough in his critique...."[14] He believed that his advance followed from his physiological investigations, because the "processes [related to the modality features of perceptions] had to remain still unformulable in words, and unknown and inaccessible to philosophy, as long as ... [Kant had] investigated only cognitions finding their expression in language."[15] One thus can argue that through his physiological investigations of cognition, Helmholtz introduced a new theme into Kantian philosophy: to consider that processes of cognition are accessible only through observations, and not through linguistic analysis, he opened up a new avenue of inquiry that still is pursued fruitfully in psychophysical researches.

Helmholtz was convinced that by his physiological investigations he had uncovered a correlation between the presumed external physical agents and their observable effects on our sensations or perceptions. To him, an ideal description of those effects thus should embody a perfect correlation between physical concepts and their corresponding perceptions. He emphasized that his view was supported by no less a figure than Johann Wolfgang von Goethe (1749–1842).* He also took his view to confirm the statement of Gustav Kirchhoff (1824–1887) that "the task of the most abstract amongst the natural sciences, namely mechanics, [is] to *describe completely and in the simplest manner* the motions occurring in nature."

According to Moritz Schlick, Helmholtz supported a physiological and psychological interpretation of those forms of intuition that Kant had assigned to a transcendental aspect of knowledge.[17] One thus can argue that by limiting the transcendental character of the Kantian *a priori* forms of intuition, Helmholtz diminished their cognitive import and thus contributed, so to speak, to their "secularization." Helmholtz's use of the term "description" referred only to physical phenomena and not to the presumed underlying agents of our perceptions, so that he is considered to be an important early representative of the German phenomenological tradition in physics.

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* "I take it as a favourable sign that we find Goethe, here and further on, together with us on the same path." See Helmholtz, "Facts in Perception" (ref. 4), p. 143. Helmholtz believed that he was faithful to Goethe's views, because his conception of a necessarily symmetrical correspondence between concepts and perceptions agreed with Goethe's desire for a unification of the two. He also agreed with Goethe that one should demand from science only that it give an artistic arrangement of the facts and form no abstract concepts going beyond this.

Robert S. Cohen and Yehuda Elkana note that the principal intellectual heir of Helmholtz's Kantianism was Heinrich Hertz.[18] One certainly can find agreement and continuity between Helmholtz's and Hertz's theories and conceptions. But in their epistemologies there are also remarkable notes of discordance between the two. In fact, in Helmholtz's preface to Hertz's *Principles of Mechanics* of 1894, in a passage often ignored in the historical literature,[19] he inserted a clear statement of the difference between his and Hertz's epistemologies. He remarked that his favorite student had adopted mechanical representations instead of the "simple representation of physical facts and laws by "systems of differential equations," the latter being the phenomenological approach that Helmholtz had embraced. For this reason, he bracketed Hertz with Lord Kelvin (1824–1907) and James Clerk Maxwell (1831–1879) as representatives of the same mechanistic school of thought.

> English physicists – *e.g.* Lord Kelvin in his theory of vortex-atoms, and Maxwell, in his hypothesis of systems of cells with rotating contents, on which he bases his attempt at a mechanical explanation of electromagnetic processes – have evidently derived a fuller satisfaction from such explanations than from *the simple representation of physical facts and laws in the most general form, as given in systems of differential equations*. For my own part, I must admit that I have adhered to the latter mode of representation [by systems of differential equations] and have felt safer in so doing; yet I have no essential objections to raise against a method which has been adopted by three physicists of such eminence.[20]

The mechanical explanations that Helmholtz attributed to Hertz as the characteristic feature of Hertz's epistemology are related to Hertz's conception of physical theory as a *Bild*, a theoretical model whose concepts do not necessarily correspond to observables, since it includes hidden theoretical quantities.

Hertz's *Bild* Conception of Physical Theory

In the introduction to his *Electric Waves*, Hertz (figure 2) examined the various "modes of representation" that Maxwell gave of his electromagnetic theory, as well as its representation as a limiting case of Helmholtz's theory and its representation in his own work. He concluded that these modes of representation,

> however different in form – have substantially the same inner significance. This common significance of the different modes of representation (and others can certainly be found) appears to me to be the undying part of Maxwell's work. This, and not Maxwell's peculiar conceptions or methods, would I designate as "Maxwell's theory." To the question, "What is Maxwell's theory?" I know of no shorter or more definite answer than the following: – Maxwell's theory is Maxwell's system of equations.[21]

In this oft-quoted statement, Hertz identifies Maxwell's theory with Maxwell's system of equations, but he also identifies that system of equations with the common and inner significance of a plurality of modes of representation of the same theory. This latter identification has often been neglected by commentators.

Fig. 2. Heinrich Hertz (1857–1894). *Credit*: Photo Deutsches Museum München; courtesy of American Institute of Physics Emilio Segrè Visual Archives, Physics Today Collection.

Hertz's statement that "Maxwell's theory is Maxwell's system of equations" does not preclude that its different "modes of representation" are to be taken into account in a more general meaning of what constitutes a physical theory. Thus, Hertz continues:

> Every theory which leads to the same system of equations, and therefore comprises the same possible phenomena, I would consider as being a form or special case of Maxwell's theory.... Hence in this sense and in this sense only, may the two theoretical dissertations in the present volume be regarded as representations of Maxwell's theory. In no sense can they claim to be a precise rendering of Maxwell's ideas. On the contrary, it is doubtful whether Maxwell, were he alive, would acknowledge them as representing his own views in all respects.[22]

Hertz therefore contends that a physical theory is comprised of both its fundamental equations and its modes of representation – and only its fundamental equations are its "undying part"; its modes of representation can change over time. Its fundamental equations, that is, its mathematical structure, is the common denominator of its modes of representation, that is, of its physical interpretations.

The mathematical structure of a theory, just as Kant's *a priori* categories, is certain and eternal but, in the absence of its modes of representation it is a purely formal theory, devoid of physical content. That is the price that mathematics has to pay to live its eternal life in the realm of pure forms. Hertz thus elevates mathematics to a highly significant role in physical theory,* but by separating the mathematical structure of a theory from its modes of representation he has profoundly challenged the conception of a physical theory as an indivisible unity of the two – a conception accepted by Maxwell and other nineteenth-century mathematical physicists.[23]

In fact, to Hertz the various modes of representation of a physical theory play the role of interpreting its mathematical symbols and thus constitute its empirical content; they correlate its mathematical symbols to its observables. This correlation, however, is not one-to-one; there is a certain amount of freedom in it, since different modes of representation are possible. Hertz's position thus is in conflict with Helmholtz's parallelism of laws between concepts and perceptions, because such parallelism does not determine unambiguously a physical theory, which may have different modes of representation.

The images [*Bilder*] that we may form of things are not determined without ambiguity by the requirement that the consequents of the images must be the images of the consequents. Various images of the same objects are possible, and these images may differ in various aspects.

A physical theory thus is not determined completely on the empirical level; different modes of representation can correspond to the same phenomenon; in other words, different images or *Bilder* can correspond to the same object. This has been called the underdetermination problem in physical theory.[25]

Hertz believed that Helmholtz's parallelism of laws not only was indeterminate but in general even impossible if theory were limited to describing observable quantities.

> If we try to understand the motions of bodies around us, and to refer them to simple and clear rules, paying attention only to what can be *directly observed*, our attempt will in general fail.... We soon become aware that *the totality of things visible and tangible* do not form an universe conformable to law, in which the same results always follow from the same conditions. We become convinced that the manifold of the actual universe must be greater than the manifold of the universe which is directly revealed to us by our senses.[26]

Hertz concludes that only by introducing *hidden quantities* can Helmholtz's parallelism of laws attain the status of a general principle in physical theory.** But such hidden quantities, that is, concepts that correspond to no perceptions, introduce an even stronger element of underdetermination into physical theory, inasmuch as they introduce a new element of freedom in the choice of theoretical concepts. However, to limit underdetermination, Hertz also introduced strong internal and formal (nonempirical)

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* Hertz's statement about the mathematical structure of Maxwell's theory has not yet received adequate analysis. I call attention here to Hertz's oft-quoted statement that mathematics is at times cleverer than its author to emphasize the power of the logical or mathematical aspects of theories, in the sense of a neo-Kantian emphasis on logical consistency as an *a priori* requirement of theories.

** Hertz's construction of a general theoretical system encompassing mechanics and electrodynamics implied the concept of a hidden substance, that is, the ether, as its unifying element. This was Hertz's unfinished program in his *Principles of Mechanics* (ref. 20).

requirements for the validation of a physical theory. One of the most important was that of simplicity,[27] which one can require inasmuch as theories are *Bilder* of our own creation.

> It is true we cannot *a priori* demand from nature simplicity, nor can we judge what in her opinion is simple. But with regard to images [*Bilder*] of our own creation we can lay down requirements. We are justified in deciding that if our images are well adapted to the things, the actual relations of the things must be represented by simple relations between the images.[28]

Simplicity requires a minimum number of axioms for deducing physical laws,* and by shaping his theory of mechanics through this and other requirements, Hertz was inspired by Kant:

> The subject-matter of the first book [of my *Principles of Mechanics*] is completely independent of experience. *All the assertions made are* a priori *judgments in Kant's sense*. They are based upon the laws of internal intuition of, and upon the logical forms followed by, the person who makes the assertions; with his external experience they have no other connection than these intuitions and forms may have.[29]

He continued in orthodox Kantian fashion:

> The time of the first book is the time of our internal intuition.... [In] itself it is always an independent variable.
>
> The space of the first book is the space as we conceive it. It is therefore the space of Euclid's geometry, with all the properties which this geometry ascribes to it.[30]

Note that Hertz limits his conception of space to three-dimensional Euclidean geometry, and in so doing he again is in conflict with Helmholtz's view that the Euclidean system of axioms pertains not to the modality but to the quality of our perception of space, which to Helmholtz is not truly *a priori*. Hertz thus has reinstated a Kantian orthodoxy at the cost of contradicting Helmholtz's criticism of Kant's *a priori* forms of intuition.

Hertz's Kantian orthodoxy, however, was a necessary logical component of the researches that led him to his discovery of electric waves. Thus, note that the propagation of electric force could be explained not only by the contiguous action of Hertz's electric waves but also by a different "mode of representation," Helmholtz's action-at-a-distance theory. Experiment alone was unable to decide between the two. A striking confirmation of this is that Ludwig Boltzmann (1844–1906), more than ten years after Hertz's celebrated experiments, still considered that contiguous action, "however *a priori* likely it may seem to some, still goes completely beyond the facts and to date remains well beyond what can be elaborated in detail."[31]

Boltzmann was right – no crude experimental fact could prove contiguous action directly. Hertz, however, would have contended that by introducing an *a priori* assump-

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* Hertz's criterion of simplicity is consistent with his requirement that the images be "logically permissible" and "correct."

tion his concept of contiguous action could find experimental support. That assumption was primarily his principle of the uniqueness and independent existence of electric forces.[32] That principle and his axiomatic approach to theory guided his experiments on electromagnetic waves.[33]

Hertz himself noted this connection between his philosophical commitment to this principle and his experimental results:

> by the experiments above sketched the propagation in time of a supposed action-at-a-distance is for the first time proved. This fact forms the philosophic result of the experiments; and, indeed, in a certain sense the most important result.[34]

Schrödinger on the *Bild* Conception of Physical Theory

Six decades later, Erwin Schrödinger (1887–1961, figure 3) introduced entirely new elements into the *Bild* conception of physical theory. His views were conditioned by his creation of wave mechanics and his proof of the mathematical equivalence of his theory and Werner Heisenberg's matrix mechanics in 1926. Subsequently, he rebelled more and more against the Copenhagen interpretation of quantum mechanics as embodied in Heisenberg's uncertainty relations and Niels Bohr's principle of complementarity of 1927. To him, as he put it two years later, Heisenberg's uncertainty relations implied the impossibility of a space-time description of microphysical processes and "changed our conception [of our physical world image and] even what is to be understood by a physical world image."[35] In yet another two years, he cited the indistinguishability of particles in Bose-Einstein statistics as a manifestation of a general ontological crisis in microphysics, which created for him a major incentive to consider a radical innovation in his conception of microphysical systems.[36] Then, in 1935, he published his cat paradox,[37] which challenged the statistical interpretation of quantum mechanics, just as Albert Einstein challenged it that same year in the Einstein-Podolsky-Rosen paper.* Schrödinger, however, differentiated his position from Einstein's by refusing to accept the concept of a statistical ensemble as necessary and sufficient for a theory of measurement.[38]

All of these developments entered into the novel views that Schrödinger presented in the 1950s.[39] Some historians have interpreted his abandonment of field theory and return to fundamental problems in quantum theory in 1952 in a negative light as stemming from his inability to work out a full-fledged field theory.[40] I see it, however, in a positive light. In my view, he returned to quantum theory to explore the possibility of formulating a continuous theory of quantum mechanics, reexamining his analysis of Bose-Einstein statistics. He now argued that the fundamental difficulty in Bose-Ein-

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* It is noteworthy that the completeness problem was not original with Einstein but had concerned physicists since the time of Maxwell and Helmholtz. Maxwell, for example, was convinced that the completeness of a theory in its most general form was an indispensable requisite for testing it; see Daniel M. Siegel, "Completeness as a goal in Maxwell's electromagnetic theory, *Isis* **66** (1975), 361–368. Perhaps one should take account of this historical dimension when discussing the Einstein-Podolsky-Rosen paper.

Fig. 3. Erwin Schrödinger (1887–1961) in 1956 in Alpbach, Austria. *Credit*: American Institute of Physics Emilio Segrè Visual Archives.

stein statistics was its retention of the particle concept while at the same time maintaining that the particle has lost its distinguishability or individuality.* To Schrödinger, these difficulties were a manifestation of a general ontological crisis in microphysics and led him to conclude that only a wave theory could meet the requirement of continuity and thus satisfy the conditions for a complete theory. He accepted that space-time discontinuities and casual gaps may appear here and there on the observational level to account for Heisenberg's uncertainty relations, for example, but he was convinced that such causal gaps could be compatible with a purely continuous theory provided that it was given a suitable ontological foundation.

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* Schrödinger was convinced that another symptom of the difficulties of Born's probabilistic interpretation of the wave function was its vagueness as to whether the wave function gives information about an ensemble of particles or only one particle.

Schrödinger thus proposed to represent the wave equation in n-dimensional space and to employ the technique of second quantization, maintaining that this was the proper mathematical technique to determine the information carried by the waves.[41] He argued that the indistinguishability of particles in Bose-Einstein statistics and the results of second quantization "are intimately connected … [and] have not turned up suddenly. Their roots lie far back, but their bearing was only very gradually recognized."[42] In fact, indistinguishability leads to the absence of "what has been called a particle" and thus to renouncing the particle concept, and second quantization also eliminates the particle concept.*

In his 1950 article, "What is an elementary particle?" Schrödinger discussed the significance of Heisenberg's uncertainty relations,[43] criticizing the Copenhagen interpretation of quantum mechanics because, on the one hand, it presupposed a particle ontology but, on the other hand, it forbade complete knowledge of all of a particle's properties, such as its position and momentum simultaneously. Since therefore the concept of a particle, whose motion can be described in space-time, needs revision, Schrödinger proposed to take advantage of the unaltered validity of the wave concept, because "the concept of waves is unavoidable" in diffraction experiments both with electrons and with photons.[44] Moreover, such diffraction experiments are fatal for a particle description, because in the two-slit experiment, for example, the superposition principle does not yield the correct particle density distribution, but it does yield the correct wave intensity pattern.

The principle of continuity excludes instantaneous action-at-a-distance, although it does not prescribe a precise relativistic limit for the velocity of a causal action; it requires only that a causal connection has an unambiguous meaning at infinitesimal space-time distances (local causality). Thus, the possibility of defining such distances is a prerequisite for an unambiguous definition of causality, so that the principle of continuity is a necessary (but not sufficient) condition for a causal connection between two events. To Schrödinger, therefore, the principle of continuity is a precondition for any theory that claims to be precise, logical, and complete, because thinking or theorizing has exigencies of its own: "from an incomplete description – from a picture with gaps in space and time – one cannot draw clear and unambiguous conclusions; it leads to hazy, arbitrary, unclear thinking…."[45] Thus, the principle of continuity has a more fundamental status than causality because it undergirds our mental framework, by which we seek to establish a causal connection between phenomena. Such perfect continuity in thought is essentially different from the imperfect

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* In second quantization, the occupation numbers of states and not the coordinates of particles are independent variables. Thus, by substituting states for particles, one introduces a model that represents a physical situation without pretending that the model presupposes a particle ontology. Schrödinger emphasized on many occasions that the derivation of Bose-Einstein statistics, in which the particles are indistinguishable, indicated to him that the concept of the classical particle needs revision. This strengthened his conviction that second quantization was the most appropriate mathematical technique for his theory. One can conclude that he accepted this technique, which was developed by his Copenhagen opponents, because it confirmed to him that the referents of Bose-Einstein statistics are not the time-honored classical particles that just had been deprived of some of their particulate properties.

continuity in physical measurements, which can be made only to a certain limit of accuracy.[46]

We thus see that a remarkable feature of Schrödinger's novel conception of physical theory is that causality must remain valid on the theoretical level even though it may break down on the observational level. As an example, Schrödinger mentioned "Bohr's famous theory of spectral lines in 1913, [which] had to assume that the atom makes a *sudden* transition from one state into another state …, [but no] information about the atom during this transition can be offered."[47] The transition is not observable in principle, that is, there is a gap in the observation language, in this case in the language used to describe spectral lines. A gap in continuity at the observational level and an "incompleteness in description" of the observation language, however, does not forbid the construction of a theory in which the principle of continuity remains intact.

> [We] do give a complete description, continuous in space and time without leaving any gaps, conforming to the classical ideal – a description of *something*. But we do not claim that this "something" is the observed or observable facts….[48]

Since the concepts in a complete theory – one in which the principle of continuity is valid – do not refer to observations, a complete theory assumes the form of a conceptual model or *Bild*, and the language on the theoretical level now plays merely an interpretative role. Logical coherence and completeness of language on the theoretical level thus are in contrast to the incompleteness and causal gaps at the observational level. Thus, there can be no one-to-one correspondence between the two languages.

> The gaps, eliminated from the wave picture, have withdrawn to the connection between the wave picture and the observable facts. The latter are *not* in one-to-one correspondence with the former.[49]

Schrödinger thus highlights an opposition between the theoretical level and the observational or descriptive level. By stretching this opposition to its extreme, he arrives at the striking conclusion that the "observed facts … appear to be *repugnant* to the classical ideal of a continuous description in space and time."[50]

A complete theory can be formulated only by adopting an absolutely clear and precise model or *Bild*. That, however, carries a price: the renunciation of a description of what nature really is:

> we do not claim that this "something" [the model or *Bild*] is the observed or observable facts; and still less do we claim that we thus describe what nature (matter, radiation, etc.) really *is*. In fact we use this picture (the so-called wave picture) in full knowledge that it is *neither*.[51]

As another example, Schrödinger added:

> It is true that in thinking about the atom, in drafting theories to meet the observed facts, we do very often draw geometrical pictures on the black-board, or on a piece of paper, or more often just only in our mind, the details of the picture being given by a mathematical formula with much greater precision ... but the geometrical shapes ... are not anything that could be directly observed in the real atoms. The pictures are only a mental help, a tool of thought….[52]

A model that describes what nature really *is* would be a "true model," but no true model can be formulated on the basis of our "large-scale experience," because

> we find nature behaving so entirely differently from what we observe in visible and palpable bodies of our surroundings.... A completely satisfactory model *of this type* is not only practically inaccessible, but not even thinkable. Or, to be precise, we can, of course, think it, but however we think it, it is wrong; not perhaps quite as meaningless as a "triangular circle," but much more so than a "winged lion."[53]

Schrödinger thus claims that there is an essential difference between the microphysical and macrophysical worlds. He admits that one can imagine models in the microphysical world, but he asserts that it would be a mistake to believe that they are true models.[54]

We see that Schrödinger distinguished between our capacity to envision an exact model and our grounds for believing that such a model is a true model. He soon argued that the danger in seeking a true model could be traced to a philosophical mistake: An absolutely precise model could be misinterpreted as a true model, one that "exists so to speak in the Platonic realm of ideas – that we approach to it gradually, without perhaps ever reaching it, owing to human imperfections."[55] Schrödinger thus implies that searching for a true Platonic model would be fatal to the success of a theory.*

Schrödinger does not claim, of course, that models are useless: A research program can bear fruit only if a scientist pursues it by searching for a clear model or *Bild*. Its clarity thus will coincide with its adequacy. To Schrödinger, a good theory could be represented by a precise model in a non-Platonic sense. We see that Schrödinger is an outstanding example of a physicist whose science and philosophy were strongly coupled in his mind.

Conclusions

Helmholtz's thought stood on the cusp between two influential developments in modern physics. On the one hand, through his attention to perception and psychology, he paved the way for Ernst Mach's phenomenology; on the other hand, through his attention to Kant's *a priori* forms of intuition, he paved the way for Hertz's *Bild* conception of physical theory. Among philosophers, Schlick's famous view of truth as unambiguous correlation between structures in cognition and in the world also owes a debt to Helmholtz and led him to a view of indeterminism similar to Hertz's.

Hertz's explicit introduction and justification of hidden quantities in physical theory at the close of the nineteenth century was a turning point in the development of theoretical physics. Since hidden quantities cannot be observed in principle, they belong to a purely theoretical realm, which lent scientific acceptability to the idea that this realm has exigencies of its own that are distinct from those of the empirical realm. With

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* It is noteworthy that Bas van Fraassen's widely discussed program of "constructive empiricism" presents a similar point of view to Schrödinger's contention that we are unable to construct a true model. I thank Don Howard for calling this to my attention.

his conception of *Bild*, Hertz implied that physical theory should meet intellectual standards of its own. Thus, if a physical theory is not necessarily constructed by starting from empirical observations, it possesses a sort of independence from them. When Hertz introduced hidden quantities into physical theory along side visible ones, he implied that a one-to-one correspondence between theoretical concepts and empirical observations was unnecessary.[56]

Schrödinger stretched this independence to its extreme when he argued that theoretical concepts and empirical observations might even be in opposition to each other, that there was a dichotomy between a pure theory and an observation language. Although this would require a radical revision in the ontology of microphysics, he affirmed that such a revision would fall entirely within the rational tradition of physics and of Western thought.[57] To him, the continuity of description at the theoretical level should not be abandoned.[58] His conception of *Bild* thus represents a continuation of Hertz's conception of *Bild* and one that Boltzmann also discussed.[59]

The main features of the *Bild* conception were an outspoken anti-inductivism and an affirmation of a partial separation of theory and observation. Once accepted, this loosened the bonds that still justified attempts, such as those by Helmholtz and Hertz, to unify physics through a generalization of mechanics.* Such attempts soon were seen to be implausible,[60] and were rejected entirely with the advent of the theories of relativity and quantum mechanics. In the absence of Hertz's conception of *Bild* and the discussions it stimulated, it would have been difficult to accept Einstein's later notion of a "theory of principle."

Schrödinger's extreme view that theory and observation are independent was not accepted by the majority of his contemporaries, and not by Einstein in particular. Causal gaps, even if limited to the level of observation, were unacceptable to Einstein and others who situated the principle of continuity and causality on the same conceptual level. Einstein's insistence on completeness in quantum theory rested on a one-to-one correspondence between theoretical concepts and empirical observations.[61] If Schrödinger's wave function did not yield a complete description of observables, it had to refer, according to Einstein, to a statistical ensemble of particles and not to an individual particle.[62]

Schrödinger, by contrast, believed that incompleteness in description rested on the indistinguishability of Bose-Einstein particles and hence on an illegitimate attribution of the individuality of classical particles in the microphysical realm. At the same time, he could not accept Heisenberg's and Bohr's Copenhagen interpretation of quantum mechanics, because to him that interpretation implied that causal gaps and discontinuities on the observational level would forbid the formulation of a continuous and complete theory or model. One thus can argue that Schrödinger considered the fundamental defect of the Copenhagen interpretation to be its ignoring of his distinction between the language of the theoretical level and that of the observational level.

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* These forms of generalized mechanics are then to be considered as transitional forms of theories preceding more (axiomatically) audacious theoretical constructions, such as H.A. Lorentz's and Max Abraham's electromagnetic theories of matter.

Heisenberg and Bohr and their followers, however, would have argued that a continuous descriptive language on the observational level is impossible to achieve.[63] We must remember that, in a different ontological context, Bohr postulated the necessity of employing two complementary languages to describe waves and particles when he found that one alone was inadequate to do so.[64]

Acknowledgments

I thank Don Howard for his insightful comments on my paper and Roger H. Stuewer for his careful editorial work on it.

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