Arch. Math. 120 (2023), 195–202

0003-889X/23/020195-8
published online December 11, 2022
https://doi.org/10.1007/s00013-022-01807-0

Archiv der Mathematik

Improved local convergence analysis of the Landweber iteration in Banach spaces

Gaurav Mittal and Ankik Kumar Giri

Abstract. The convergence analysis of the Landweber iteration for solving inverse problems in Banach spaces via Hölder stability estimates is well studied by de Hoop et al. (Inverse Probl 28(4):045001, 2012) in the presence of unperturbed data. For real life problems, it is important to study the convergence analysis in the presence of perturbed data. In this paper, we show that the convergence analysis of the Landweber iteration can also be studied by utilizing the Hölder stability estimates in the presence of perturbed data. Furthermore, as a by-product, we formulate the convergence rates of the Landweber iteration without utilizing any additional smoothness condition. This shows the advantage of Hölder stability estimates over a tangential cone condition in the theory of inverse problems.

Mathematics Subject Classification. 47H17, 65J15, 65J20.

Keywords. Inverse problems, Ill-posed operator equations, Regularization.

1. Introduction and main result.

1.1. Background and main problem. Let $F : D(F) \subset B_1 \to B_2$ be a nonlinear operator between Banach spaces B_1 and B_2. Here $D(F)$ represents the domain of F. In this paper, we are concerned about the solution of following equation:

$$F(u) = v, \quad u \in D(F),\ v \in B_2. \tag{1.1}$$

For practical applications, it is known that v is never available. Instead some perturbed data v^δ fulfilling $\|v^\delta - v\| \le \delta$ is available, where $\delta > 0$. Consequently, (1.1) is ill-posed due to no continuous dependence between the solution and data. We assume that (1.1) has a solution. Let it be represented by $u^\dagger$. To approximately solve (1.1), a number of regularization methods are known in

Hilbert as well as Banach spaces (cf. [2–6,8,10,17]). A well known and classical regularization method is the Landweber iteration [7,17]:

$$J_p(u_{r+1}) = J_p(u_r) - \mu F'(u_r)^* j_p(F(u_r) - v), \quad u_{r+1} = J_q^*(J_p(u_{r+1})),\ r \geq 0. \tag{1.2}$$

Here $F'(u_r)$ is the Fréchet derivative of F at u_r, $F'(u_r)^*$ is the adjoint of $F'(u_r)$, u_0 is an initial guess of the exact solution $u^\dagger$, $p > 1$, p and q are conjugate exponents, and $J_p : B_1 \to 2^{B_1^*}$ defined as $J_p(u) := \{u^* \in B_1^* \mid \langle u, u^* \rangle = \|u\|^p, \|u^*\| = \|u\|^{p-1}\}$ is the duality mapping of B_1 with the gauge function $s \to s^{p-1}$. For a gauge function $s \to s^{q-1}$, the corresponding duality mapping $J_q^* : B_1^* \to B_1$ is the inverse of J_p. The convergence analysis of (1.2) is well studied by utilizing a tangential cone condition [4] in Hilbert as well as Banach spaces [10,17]. In addition, the convergence rates for this method have been obtained by incorporating the source conditions and variational inequalities [10,17]. Recently, de Hoop et al. [7] studied the convergence analysis of (1.2) by utilizing the following Hölder-type stability:

$$\Delta_p(u, \tilde{u}) \leq \mathfrak{A}^p \|F(u) - F(\tilde{u})\|^{\frac{p(1+\epsilon)}{2}}, \quad \forall u,\ \tilde{u} \in \mathcal{B}_\rho(u^\dagger), \tag{1.3}$$

where $\epsilon \in [0, 1]$, $p > 1$, $\mathfrak{A} > 0$, $\Delta_p(u, \bar{u})$ is the Bregman distance of $\bar{u}$ from u and it is given by

$$\Delta_p(u, \bar{u}) := p^{-1}\|\bar{u}\|^p - p^{-1}\|u\|^p - \langle J_p(u), \bar{u} - u \rangle.$$

Here, we assume that $\mathcal{B}_\rho(u^\dagger) := \{\bar{u} \in B_1 : \Delta_p(\bar{u}, u^\dagger) \leq \rho\} \subset D(F)$ for some $\rho > 0$. However, the convergence analysis of the perturbed version of (1.2), i.e.,

$$J_p(u^\delta_{r+1}) = J_p(u^\delta_r) - \mu F'(u^\delta_r)^* j_p(F(u^\delta_r) - v^\delta), \quad u^\delta_{r+1} = J_q^*(J_p(u^\delta_{r+1})),\ r \geq 0, \tag{1.4}$$

is not yet studied in the literature via the stability estimates (1.3). In this paper, we fill this important gap in the literature. The importance of studying the convergence analysis of an iterative method via stability estimates is that these provide the convergence rates without the requirement of any additional smoothness condition. This is in contrary to the standard analysis. We refer to [11–15] for studying the convergence analysis of several other regularization methods through stability estimates. Very recently, Jin [9] studied the convergence rates for the method (1.4) in Banach spaces for linear ill-posed problems with perturbed data.

To this end, we recall some basic definitions and known results related to our work (see [11,17] for more details). For $u \in B_1$ and $\zeta \in B_1^*$, we write $\langle \zeta, u \rangle = \zeta(u)$ for the duality pairing. In this paper, in order to ensure the well-definedness of the method (1.4), we require the following definitions of the modulus of convexity $\delta_{B_1}(\cdot)$ and the modulus of smoothness $\rho_{B_1}(\cdot)$:

$$\delta_{B_1}(\epsilon) := \inf_{0 \leq \epsilon \leq 2} \big\{ (2 - \|u + \bar{u}\|) :\ u, \bar{u} \in \mathbb{S}, \|u - \bar{u}\| \geq \epsilon \big\},$$

$$\rho_{B_1}(\tau) := \sup_{\tau \geq 0} \big\{ (\|u + \tau\tilde{u}\| + \|u - \tau\tilde{u}\| - 2) :\ u, \tilde{u} \in \mathbb{S} \big\}.$$

Here $\mathbb{S}$ denotes the boundary of the unit sphere in B_1. We say that B_1 is p-convex if $\delta_{B_1}(\epsilon) \geq \mathcal{C}_1\epsilon^p$ for all $\epsilon \in [0,2]$, where $p \geq 0$ and $\mathcal{C}_1 > 0$. Further, we say that B_1 is q-smooth if $\rho_{B_1}(\tau) \leq \mathcal{C}_2\tau^q$ for all $\tau \geq 0$, where $q > 1$ and $\mathcal{C}_2 > 0$. Also, B_1 is uniformly convex if for any $\epsilon \in (0,2]$, $\delta_{B_1}(\epsilon) > 0$ and it is uniformly smooth if $\lim_{\tau\to 0} \rho_{B_1}(\tau)\tau^{-1} = 0$. We note that B_1 is uniformly convex if and only if B_1^* is uniformly smooth. Moreover, any uniformly convex or uniformly smooth Banach space is reflexive. We emphasize that uniform smoothness of B_1 guarantees that $J_p(u)$ is single valued for all $u \in B_1$, i.e., the method (1.4) becomes well-defined.

Finally, we recall a known result that will be utilized in our work.

Lemma 1 ([7,16]). *Let B_1 be a uniformly convex and uniformly smooth Banach space. Then, for all $u, \bar{u} \in B_1$ and $u^*, \bar{u}^* \in B_1^*$, we have:*

1. $\Delta_p(u,\bar{u}) \geq 0$ *and* $\Delta_p(u,\bar{u}) = 0 \iff u = \bar{u}$.
2. *If B_1 is p-convex, then $\Delta_p(u,\bar{u}) \geq \mathcal{C}_3 p^{-1}\|u-\bar{u}\|^p$, where $\mathcal{C}_3 > 0$ is a constant.*
3. *If B_1^* is q-smooth, then $\Delta_q(u^*,\bar{u}^*) \leq \mathcal{C}_4 q^{-1}\|u^*-\bar{u}^*\|^q$, where $\mathcal{C}_4 > 0$ is a constant.*
4. *The following are equivalent*: (a) $\lim_{r\to\infty}\|u_r - u\| = 0$. (b) $\lim_{r\to\infty} \Delta_p(u_r,u) = 0$. (c) $\lim_{r\to\infty}\|u_r\| = \|u\|$ *and* $\lim_{r\to\infty}\langle J_p(u_r),u\rangle = \langle J_p(u),u\rangle$.
5. $\Delta_p(u,\tilde{u}) = p^{-1}\|\tilde{u}\|^p + q^{-1}\|u\|^p - \langle J_p(u),\tilde{u}\rangle = p^{-1}\|\tilde{u}\|^p - p^{-1}\|u\|^p - \langle J_p(u),\tilde{u}\rangle + \|u\|^p$.

1.2. Main result. In order to formulate our main result, we discuss certain assumptions. With the gauge function $s \to s^{p-1}$, we assume that j_p denotes the single valued selection of the duality mapping. For the method (1.4), we engage the following well known discrepancy criterion:

$$\|v^\delta - F(u^\delta_{r_*})\| \leq \tau\delta < \|v^\delta - F(u^\delta_r)\|, \quad 0 \leq r < r_*, \tag{1.5}$$

where $\tau > 1$ satisfies

$$1 - \frac{2}{\tau} - \frac{1}{2\tau^p} > 0 \tag{1.6}$$

and $r_* = r_*(\delta, v^\delta)$ is the stopping index. The utilization of (1.5) yields r_* and $u^\delta_{r_*}$ which is the required approximate solution. Our main result is as follows:

Theorem 1. *Let B_1 be p-convex and q-smooth with conjugate exponents $1 < p, q < \infty$ and let B_2 be an arbitrary Banach space. Moreover, we assume:*

1. *For all $u, \bar{u} \in \mathcal{B}_\rho(u^\dagger)$, it holds that*
$$\|F'(u) - F'(\bar{u})\| \leq \mathcal{C}_5\|u-\bar{u}\|, \quad \textit{where } \mathcal{C}_5 > 0. \tag{1.7}$$
2. *For all $u \in \mathcal{B}_\rho(u^\dagger)$, it holds that $\|F'(u)\| \leq \mathcal{C}_6$, where $\mathcal{C}_6 > 0$.*
3. *$u^\dagger$ is a solution of (1.1) such that $\Delta_p(u_0,u^\dagger) \leq \rho$ for*
$$\rho^{\frac{1}{p}} = 2^{-p\epsilon}\mathcal{C}_6^{-1}(\mathcal{C}_3\mathfrak{A}^2)^{-\frac{1}{\epsilon}}(p^{-1}\mathcal{C}_3)^{(1+\frac{2}{\epsilon})\frac{1}{p}}. \tag{1.8}$$

(4) *μ in (1.4) is such that*

$$\mu < \left(\frac{q}{2\mathcal{C}_4\mathcal{C}_6}\right)^{\frac{1}{q-1}} \text{ and } 4\mathcal{C}_4\mathcal{C}_6^q q^{-1}\mu^{q-1} < 1. \tag{1.9}$$

(5) *The Hölder stability estimate (1.3) and (1.5) hold with τ the same as in (1.6).*

Further, assume that

$$\mathfrak{R} := 1 - \frac{2}{\tau} - \frac{1}{2\tau^p}. \tag{1.10}$$

Then, we have:

(a) *For $0 \le r < r_*$, $\Delta_p(u_{r+1}^\delta, u^\dagger) \le \Delta_p(u_r^\delta, u^\dagger)$.*
(b) *The stopping index r_* is finite.*
(c) *Moreover, for a given $\delta > 0$, if $\rho > 0$ is such that $\rho \le \mathcal{C}_7\delta^p$ for some $\mathcal{C}_7 > 0$, then the following convergence rates can be derived:*

$$\Delta_p(u_{r_*}^\delta, u^\dagger) \le \mathcal{C}_8\delta^p,$$

where $\mathcal{C}_8 = \mathcal{C}_7 - r_*\frac{\mu\mathfrak{R}}{2}$.

Proof. We engage the fundamental theorem of the Fréchet derivative along with (1.7) to deduce that

$$\|F(u_r^\delta) - v^\delta - F'(u_{r+1}^\delta)(u_{r+1}^\delta - u^\dagger)\| \le \delta + \frac{\mathcal{C}_5}{2}\|u_{r+1}^\delta - u^\dagger\|^2. \tag{1.11}$$

It is known that $\Delta_p(u_0, u^\dagger) \le \rho$. Suppose by the induction principle that

$$\Delta_p(u_s^\delta, u^\dagger) \le \rho \quad \text{for } s = 0, 1, \ldots, r.$$

We claim that $\Delta_p(u_{r+1}^\delta, u^\dagger) \le \rho$. Using induction, the mean value inequality, (2) of Theorem 1 and (2) of Lemma 1, we obtain

$$\|F(u_s^\delta) - v\| \le \mathcal{C}_6(p\mathcal{C}_3^{-1})^{\frac{1}{p}}\Delta_p(u_s^\delta, u^\dagger)^{\frac{1}{p}} \le \mathcal{C}_6(p\mathcal{C}_3^{-1})^{\frac{1}{p}}\rho^{\frac{1}{p}}, \tag{1.12}$$

where $s = 0, 1, \ldots, r$. Next, by taking $\bar{u}^* = J_p(u_{r+1}^\delta)$ and $u^* = J_p(u_r^\delta)$ in (3) of Lemma 1, we derive that

$$\Delta_q(J_p(u_r^\delta), J_p(u_{r+1}^\delta)) \le \mathcal{C}_4 q^{-1}\|J_p(u_{r+1}^\delta) - J_p(u_r^\delta)\|^q. \tag{1.13}$$

After applying (5) of Lemma 1 and the result that $J_p^{-1}(u^*) = J_q^*(u^*)$ along with the definition of the duality mapping, we note that

$$\begin{aligned}
&\Delta_q(J_p(u_r^\delta), J_p(u_{r+1}^\delta)) \\
&\quad = q^{-1}\|J_p(u_{r+1}^\delta)\|^q - q^{-1}\|J_p(u_r^\delta)\|^q - \langle J_q^*(J_p(u_r^\delta)), J_p(u_{r+1}^\delta)\rangle + \|J_p(u_r^\delta)\|^q \\
&\quad = q^{-1}\|J_p(u_{r+1}^\delta)\|^q - q^{-1}\|J_p(u_r^\delta)\|^q - \langle J_q^*(J_p(u_r^\delta)), J_p(u_{r+1}^\delta)\rangle + \|u_r^\delta\|^p \\
&\quad = q^{-1}\|J_p(u_{r+1}^\delta)\|^q - q^{-1}\|J_p(u_r^\delta)\|^q - \langle J_p(u_{r+1}^\delta) - J_p(u_r^\delta), u_r^\delta\rangle.
\end{aligned}$$

Plugging (1.13) in the last estimate we deduce that

$$\begin{aligned}
&q^{-1}(\|J_p(u_{r+1}^\delta)\|^q - \|J_p(u_r^\delta)\|^q) \\
&\quad \le \Delta_q(J_p(u_r^\delta), J_p(u_{r+1}^\delta)) + \langle J_p(u_{r+1}^\delta) - J_p(u_r^\delta), u_r^\delta\rangle \\
&\quad \le \mathcal{C}_4 q^{-1}\|J_p(u_{r+1}^\delta) - J_p(u_r^\delta)\|^q + \langle J_p(u_{r+1}^\delta) - J_p(u_r^\delta), u_r^\delta\rangle.
\end{aligned} \tag{1.14}$$

Again we note from (5) of Lemma 1 that

$$\Delta_p(u_{r+1}^\delta, u^\dagger) - \Delta_p(u_r^\delta, u^\dagger) = q^{-1}(\|J_p(u_{r+1}^\delta)\|^q - \|J_p(u_r^\delta)\|^q) \\ -\langle J_p(u_{r+1}^\delta) - J_p(u_r^\delta), u^\dagger\rangle. \tag{1.15}$$

By combining (1.14) and (1.15), we note that

$$\Delta_p(u_{r+1}^\delta, u^\dagger) - \Delta_p(u_r^\delta, u^\dagger) \leq \mathcal{C}_4 q^{-1}\|J_p(u_{r+1}^\delta) - J_p(u_r^\delta)\|^q \\ +\langle J_p(u_{r+1}^\delta) - J_p(u_r^\delta), u_r^\delta - u^\dagger\rangle.$$

We incorporate (1.4) and assumption (2) of Theorem 1 in the last inequality to derive that

$$\begin{aligned}\Delta_p(u_{r+1}^\delta, u^\dagger) - \Delta_p(u_r^\delta, u^\dagger) &\leq \mathcal{C}_4(\mathcal{C}_6\mu)^q q^{-1}\|\mathcal{A}_r^\delta\|^p - \mu\langle j_p(\mathcal{A}_r^\delta), F'(u_r^\delta)(u_r^\delta - u^\dagger)\rangle \\ &= \mathcal{C}_4(\mathcal{C}_6\mu)^q q^{-1}\|\mathcal{A}_r^\delta\|^p - \mu\langle j_p(\mathcal{A}_r^\delta), \mathcal{A}_r^\delta\rangle + \mu\langle j_p(\mathcal{A}_r^\delta), \mathcal{A}_r^\delta - F'(u_r^\delta)(u_r^\delta - u^\dagger)\rangle \\ &\leq \mathcal{C}_4(\mathcal{C}_6\mu)^q q^{-1}\|\mathcal{A}_r^\delta\|^p - \mu\|\mathcal{A}_r^\delta\|^p + \mu\|\mathcal{A}_r^\delta\|^{p-1}\|\mathcal{A}_r^\delta - F'(u_r^\delta)(u_r^\delta - u^\dagger)\| \\ &\leq \mathcal{C}_4(\mathcal{C}_6\mu)^q q^{-1}\|\mathcal{A}_r^\delta\|^p - \mu\|\mathcal{A}_r^\delta\|^p + \mu\|\mathcal{A}_r^\delta\|^{p-1}\left(\delta + \frac{\mathcal{C}_5}{2}\|u_{r+1}^\delta - u^\dagger\|^2\right),\end{aligned} \tag{1.16}$$

where $\mathcal{A}_r^\delta = F(u_r^\delta) - v^\delta$ and the last inequality holds due to (1.11) and the definition of the duality mapping. We plug the Hölder stability estimate (1.3) and (2) of Lemma 1 in (1.16) to further write it as

$$\Delta_p(u_{r+1}^\delta, u^\dagger) - \Delta_p(u_r^\delta, u^\dagger) \leq \frac{1}{2}(2\mathcal{C}_4(\mathcal{C}_6\mu)^q q^{-1} - \mu)\|\mathcal{A}_r^\delta\|^p - \frac{\mu}{2}\|\mathcal{A}_r^\delta\|^p \\ +\mu\delta\|\mathcal{A}_r^\delta\|^{p-1} + \frac{\mu}{2}\mathcal{C}_5\mathfrak{A}^2(p\mathcal{C}_3^{-1})^{\frac{2}{p}}\|F(u_r^\delta) - v\|^{p+\epsilon}.$$

Inserting (1.12) in the last estimate, we derive that

$$\Delta_p(u_{r+1}^\delta, u^\dagger) - \Delta_p(u_r^\delta, u^\dagger) \leq \frac{1}{2}(2\mathcal{C}_4(\mathcal{C}_6\mu)^q q^{-1} - \mu)\|\mathcal{A}_r^\delta\|^p - \frac{\mu}{2}\|\mathcal{A}_r^\delta\|^p \\ +\mu\delta\|\mathcal{A}_r^\delta\|^{p-1} + \frac{\mu}{2}\mathcal{C}_5\mathfrak{A}^2(p\mathcal{C}_3^{-1})^{\frac{2}{p}}(\mathcal{C}_6(p\mathcal{C}_3^{-1})^{\frac{1}{p}}\rho^{\frac{1}{p}})^\epsilon\|F(u_r^\delta) - v\|^p. \tag{1.17}$$

Using (1.9) and the estimate

$$(\alpha_1 + \alpha_2)^p \leq 2^{p-1}(\alpha_1^p + \alpha_2^p) \quad \text{for } \alpha_1, \alpha_2 \geq 0,\ p \geq 1 \tag{1.18}$$

in (1.17), we get

$$\Delta_p(u_{r+1}^\delta, u^\dagger) - \Delta_p(u_r^\delta, u^\dagger) \leq -\frac{\mu}{2}\|\mathcal{A}_r^\delta\|^p + \mu\delta\|\mathcal{A}_r^\delta\|^{p-1} + \frac{\mu}{4}\delta^p \\ +\frac{1}{2}\left(2\mathcal{C}_4(\mathcal{C}_6\mu)^q q^{-1} - \mu + \frac{\mu}{2}\right)\|\mathcal{A}_r^\delta\|^p. \tag{1.19}$$

By incorporating the discrepancy principle (1.5) and (1.9) in (1.19), we obtain

$$\Delta_p(u_{r+1}^\delta, u^\dagger) - \Delta_p(u_r^\delta, u^\dagger) \leq -\frac{1}{2}\left(1 - \frac{2}{\tau} - \frac{1}{2\tau^p}\right)\mu\|\mathcal{A}_r^\delta\|^p, \tag{1.20}$$

where $r+1 \leq r_*$. This and the choice of τ mentioned in (1.6) together with the induction hypothesis guarantee that $\Delta_p(u_{r+1}^\delta, u^\dagger) < \rho$. Therefore, our claim holds which completes the proof of assertion (a).

Next, we show that the stopping index $r_* < \infty$. For this, we incorporate (1.9) and (1.20) to write

$$\frac{\mathfrak{R}}{2}\mu\|\mathcal{A}_r^\delta\|^p \leq \Delta_p(u_r^\delta, u^\dagger) - \Delta_p(u_{r+1}^\delta, u^\dagger).$$

Summing this from $r = 0$ to $r^* - 1$, we deduce that

$$\sum_{r=0}^{r^*-1} \|\mathcal{A}_r^\delta\|^p \leq \frac{2}{\mu\mathfrak{R}}\Delta_p(u_0, u^\dagger).$$

This, the choice of u_0, and (1.5) yield

$$r_*(\tau\delta)^p \leq \sum_{r=0}^{r^*-1} \|\mathcal{A}_r^\delta\|^p \leq \frac{2}{\mu\mathfrak{R}}\Delta_p(u_0, u^\dagger) \leq \frac{2\rho}{\mu\mathfrak{R}}.$$

We note that as $\frac{2\rho}{\mu\mathfrak{R}} < \infty$ and both τ, δ are positive quantities, r_* can never be infinite. This proves assertion (b).

To this end, we deduce the convergence rates for the method (1.4). It follows from the Hölder stability estimate (1.3) that

$$\Delta_p(u_r^\delta, u^\dagger) \leq \mathfrak{A}^p\|F(u_r^\delta) - F(u^\dagger)\|^{\frac{p(1+\epsilon)}{2}}.$$

This with a slightly modified version of (1.18) (i.e., $(\alpha_1 + \alpha_2)^p \leq 2^p(\alpha_1^p + \alpha_2^p)$ for $\alpha_1, \alpha_2 \geq 0,\ p \geq 0$) implies that

$$\Delta_p(u_r^\delta, u^\dagger) \leq 2^{p_1}\mathfrak{A}^p(\|\mathcal{A}_r^\delta\|^{p_1} + \delta^{p_1}) \implies -\|\mathcal{A}_r^\delta\|^{p_1} \leq -\frac{\Delta_p(u_r^\delta, u^\dagger)}{2^{p_1}\mathfrak{A}^p} + \delta^{p_1},$$

where $p_1 = \frac{p(1+\epsilon)}{2}$. Inserting the last estimate in (1.20), we obtain

$$\Delta_p(u_{r+1}^\delta, u^\dagger) - \Delta_p(u_r^\delta, u^\dagger) \leq -\frac{1}{2}\mu\mathfrak{R}\left(\frac{\Delta_p(u_r^\delta, u^\dagger)}{2^{p_1}\mathfrak{A}^p} + \delta^{p_1}\right)^{\frac{2}{1+\epsilon}}. \quad (1.21)$$

With some minor rearrangements, (1.21) leads to

$$\Delta_p(u_{r+1}^\delta, u^\dagger) \leq \Delta_p(u_r^\delta, u^\dagger) - \frac{\mu\mathfrak{R}}{2}\delta^p.$$

Consequently, by the induction hypothesis, we derive that

$$\Delta_p(u_{r_*}^\delta, u^\dagger) \leq \Delta_p(u_0^\delta, u^\dagger) - \frac{\mu\mathfrak{R}}{2}r_*\delta^p.$$

From the last inequality, we can deduce the convergence rates in assertion (c) which completes the proof. □

Remark 1. The assumptions considered in our work are standard and similar to [7]. Consequently, our results are applicable on a severely ill-posed inverse conductivity problem related to electrical impedance tomography (EIT) [1]. de Hoop et al. [7] showed that the inverse conductivity problem fulfills a Hölder stability estimate (1.3) for $p = 2$ and $\epsilon = 1$. In addition to this, it is also known that the operator associated with this inverse conductivity problem fulfills (1) and (2) of Theorem 1. Therefore, by carefully choosing the other parameters such as τ, μ etc., one can apply our results on the inverse conductivity problem.

2. Conclusion and future scope. In this paper, we have shown that one can obtain the convergence rates of the Landweber iteration method through stability estimates in the presence of perturbed data without the utilization of any additional smoothness concept. This paper fills an important gap in the literature. With this paper, the study of convergence analysis of the Landweber method for perturbed as well as unperturbed data via stability estimates is complete. One of the most important future tasks in the direction of studying the convergence analysis via stability estimates is to derive the optimal convergence rates. In this direction, the optimality conditions discussed in [4,17] can be used as a reference.

References

[1] Alessandrini, G., Vessella, S.: Lipschitz stability for the inverse conductivity problem. Adv. Appl. Math. **35**(2), 207–241 (2005)

[2] Argyros, I.K., George, S.: Unified convergence analysis of frozen Newton-like methods under generalized conditions. J. Comput. Appl. Math. **347**, 95–107 (2019)

[3] Bakushinsky, A.B., Kokurin, M.: Iterative Methods for Approximate Solution of Inverse Problems. Mathematics and its Applications (New York), 577. Springer, Dordrecht (2004)

[4] Engl, H.W., Hanke, M., Neubauer, A.: Regularization of Inverse Problems. Springer, Berlin (2000)

[5] George, S., Sabari, M.: Numerical approximation of a Tikhonov type regularizer by a discretized frozen steepest descent method. J. Comput. Appl. Math. **330**, 488–498 (2018)

[6] Hanke, M., Neubauer, A., Scherzer, O.: A convergence analysis of the Landweber iteration for nonlinear ill-posed problems. Numer. Math. **72**, 21–37 (1995)

[7] de Hoop, M.V., Qiu, L., Scherzer, O.: Local analysis of inverse problems: Hölder stability and iterative reconstruction. Inverse Problems **28**(4), 045001 (2012)

[8] Jin, Q.: Inexact Newton–Landweber iteration for solving nonlinear inverse problems in Banach spaces. Inverse Problems **28**, 065002 (2012)

[9] Jin, Q.: Convergence rate of a dual gradient method for constrained linear ill-posed problems. Numer. Math. **151**, 841–877 (2022)

[10] Kaltenbacher, B., Neubauer, A., Scherzer, O.: Iterative Regularization Methods for Nonlinear Ill-posed Problems. De Gruyter, Berlin (2008)

[11] Mittal, G., Giri, A.K.: Novel multi-level projected iteration to solve inverse problems with nearly optimal accuracy. J. Optim. Theory Appl. **194**, 643–680 (2022)

[12] Mittal, G., Giri, A.K.: A novel two-point gradient method for regularization of inverse problems in Banach spaces. Appl. Anal. **101**(18), 6596–6622 (2022)

[13] Mittal, G., Giri, A.K.: On variational regularization: finite dimension and Hölder stability. J. Inverse Ill-Posed Probl. **29**(2), 283–294 (2021)

[14] Mittal, G., Giri, A.K.: Convergence rates for iteratively regularized Gauss–Newton method subject to stability constraints. J. Comput. Appl. Math. **400**, 113744 (2022)

[15] Mittal, G., Giri, A.K.: Nonstationary iterated Tikhonov regularization: convergence analysis via Hölder stability. Inverse Problems **38**(12), 125008 (2022)

[16] Schöpfer, F., Louis, A.K., Schuster, T.: Nonlinear iterative methods for linear ill-posed problems in Banach spaces. Inverse Problems **22**, 311–329 (2006)

[17] Schuster, T., Kaltenbacher, B., Hofmann, B., Kazimierski, K.S.: Regularization Methods in Banach Spaces. De Gruyter, Berlin (2012)

Gaurav Mittal and Ankik Kumar Giri
Department of Mathematics
Indian Institute of Technology Roorkee
Roorkee 247667
India
e-mail: gmittal@ma.iitr.ac.in

Ankik Kumar Giri
e-mail: ankik.giri@ma.iitr.ac.in

Received: 22 September 2022

Revised: 24 October 2022

Accepted: 7 November 2022