Arch. Math. 115 (2020), 457–465

0003-889X/20/040457-9
published online 29 May 2020
https://doi.org/10.1007/s00013-020-01479-8

Archiv der Mathematik

A note on rigidity of Riemannian manifolds with positive scalar curvature

Guangyue Huang and Qianyu Zeng

Abstract. In this short note, we obtain an integral inequality for closed Riemannian manifolds with positive scalar curvature and give some rigidity characterization of the equality case, which generalizes the recent results of Catino which deal with the conformally flat case, and of Huang and Ma which deal with the harmonic curvature case. Moreover, we obtain an integral pinching condition with non-negative constant $\sigma_2(A^\tau)$, which can be seen as a complement to Bo and Sheng who considered conformally flat manifolds with constant quotient curvature of $\sigma_k(A^\tau)$.

Mathematics Subject Classification. Primary 53C20, Secondary 53C21.

Keywords. Einstein, Rigidity, Conformally flat.

1. Introduction. Let (M^n, g) be an n-dimensional Riemannian manifold with $n \geq 3$. It is well-known that for $n \geq 4$, the metric g is conformally flat if and only if its Weyl curvature tensor is zero. If $n = 3$, then it is conformally flat if and only if the Cotton tensor is zero. In the last years, the classifications of conformally flat manifolds under some geometrical or topological assumptions have been paid much attention. For example, in [17], Tani proved that any closed conformally flat manifold with positive Ricci curvature and constant scalar curvature is covered isometrically by $\mathbb{S}^n$ with the round metric. For complete conformally flat manifolds with non-negative Ricci curvature, Carron and Herzlich [2] gave the following classifications: they are either flat, or locally isometric to $\mathbb{R} \times \mathbb{S}^{n-1}$ with the product metric; or are globally conformally equivalent to $\mathbb{R}^n$ or to a spherical space form. For closed conformally flat manifolds satisfying some integral pinching conditions, see [5,7,8,16,18]. On the other hand, for some classifications with point-wise pinching condition on the Ricci curvature, see [4,14,19] and references therein.

The research of the authors is supported by NSFC (Nos. 11971153, 11671121).

Throughout this paper, all the calculations are carried out under the normal coordinates. Denote by R, R_{ij} the scalar curvature and the Ricci curvature respectively. We let $\mathring{R}_{ij} = R_{ij} - \frac{R}{n} g_{ij}$ be the trace-less Ricci curvature. With the help of the properties of the Codazzi tensor, Catino [3] studied closed conformally flat manifolds with positive constant scalar curvature (in this case, the Ricci curvature is a Codazzi tensor) and satisfying an optimal integral pinching condition. He proved the following

Theorem A. *Let (M^n, g) be a closed conformally flat Riemannian manifold with positive constant scalar curvature. Then*

$$\int_M \left(R - \sqrt{n(n-1)}|\mathring{R}_{ij}|\right)|\mathring{R}_{ij}|^{\frac{n-2}{n}} \le 0, \tag{1.1}$$

and equality occurs if and only if (M^n, g) is covered isometrically by either $\mathbb{S}^n$ with the round metric, $\mathbb{S}^1 \times \mathbb{S}^{n-1}$ with the product metric, or $\mathbb{S}^1 \times \mathbb{S}^{n-1}$ with a rotationally symmetric Derdziński metric.

Generalizing the above results of Catino, Huang and Ma [11] studied manifolds with harmonic curvature tensor and positive scalar curvature. They proved

Theorem B. *Let (M^n, g) be a closed Riemannian manifold with harmonic curvature tensor and positive scalar curvature. Then*

$$\int_M \left(R - \sqrt{n(n-1)}|\mathring{R}_{ij}|\right)|\mathring{R}_{ij}|^{\frac{n-2}{n}} \le \sqrt{\frac{(n-1)(n-2)}{2}} \int_M |W||\mathring{R}_{ij}|^{\frac{n-2}{n}}, \tag{1.2}$$

and equality occurs if and only if (M^n, g) is either Einstein or isometrically covered by one of:

(1) *$\mathbb{S}^1 \times \mathbb{S}^{n-1}$ with a product metric;*
(2) *$\mathbb{S}^1 \times \mathbb{S}^{n-1}$ with a rotationally symmetric Derdziński metric.*

The Cotton tensor is defined by

$$C_{ijk} = R_{kj,i} - R_{ki,j} - \frac{1}{2(n-1)}(R_{,i}g_{jk} - R_{,j}g_{ik}), \tag{1.3}$$

where the indices after the comma denotes the covariant derivatives, which is related to the Weyl curvature tensor by

$$-\frac{n-3}{n-2} C_{ijk} = W_{ijkl,l}. \tag{1.4}$$

Thus, it is easy to see that for conformally flat manifolds with constant scalar curvature and $n \ge 4$, the Ricci curvature must be a Codazzi tensor, and hence the curvature tensor is harmonic (since $R_{ij,k} - R_{ik,j} = R_{jkil,l}$, the curvature tensor being harmonic is equivalent to the Ricci curvature being a Codazzi tensor with constant scalar curvature). That is to say, conditions on harmonic curvature are weaker than those on conformal flatness. On the other hand, the key to prove Theorem A and Theorem B is the fact that the Ricci curvature becomes a Codazzi tensor under assumptions.

In this note, we will continue to generalize the above results by removing the conditions on constant scalar curvature and that the Ricci curvature is a Codazzi tensor. Our main results are stated as follows:

Theorem 1.1. *Let (M^n, g) be a closed Riemannian manifold with positive scalar curvature, where $n \geq 3$. Then*

$$\int_M \left[R - \sqrt{n(n-1)}|\mathring{R}_{ij}| - \sqrt{\frac{(n-1)(n-2)}{2}}|W|\right]|\mathring{R}_{ij}|^2 \leq \frac{(n-2)^2}{4n}\int_M |\nabla R|^2 + \frac{n-1}{2}\int_M |C_{ijk}|^2, \tag{1.5}$$

and equality occurs if and only if (M^n, g) is either Einstein or isometrically covered by $\mathbb{S}^1 \times \mathbb{S}^{n-1}$ with the product metric.

The following modified Schouten tensor with a parameter τ was introduced by Gursky and Viaclovsky [6] (see also Li and Li [13]):

$$A^{\tau}_{ij} = R_{ij} - \frac{\tau R}{2(n-1)} g_{ij}, \tag{1.6}$$

where $\tau \in \mathbb{R}$ is a constant. When $\tau = 1$, the tensor A^1_{ij} is exactly the Schouten tensor. We denote by $\sigma_2(A^\tau)$ the 2nd-elementary symmetric function of the eigenvalues of the so-called modified Schouten tensor. Then, for manifolds with non-negative constant $\sigma_2(A^\tau)$, we have the following

Theorem 1.2. *Let (M^n, g) be a closed Riemannian manifold with positive scalar curvature, where $n \geq 3$. If the function $\sigma_2(A^\tau)$ is a non-negative constant, where $\tau < 1$ or $\tau > 3 - \frac{4}{n}$, then*

$$\int_M \left[R - \sqrt{n(n-1)}|\mathring{R}_{ij}| - \sqrt{\frac{(n-1)(n-2)}{2}}|W|\right]|\mathring{R}_{ij}|^2 \leq \frac{n-1}{2}\int_M |C_{ijk}|^2, \tag{1.7}$$

and equality occurs if and only if (M^n, g) is either Einstein or isometrically covered by $\mathbb{S}^1 \times \mathbb{S}^{n-1}$ with the product metric.

Remark 1.1. If we add the conditions $W_{ijkl} = 0$ and that the scalar curvature is constant in Theorem 1.1, then (1.5) becomes

$$\int_M (R - \sqrt{n(n-1)}|\mathring{R}_{ij}|)|\mathring{R}_{ij}|^2 \leq 0. \tag{1.8}$$

On the other hand, if we add the condition that the curvature tensor is harmonic in Theorem 1.1, then (1.5) becomes

$$\int_M \left[R - \sqrt{n(n-1)}|\mathring{R}_{ij}| - \sqrt{\frac{(n-1)(n-2)}{2}}|W|\right]|\mathring{R}_{ij}|^2 \leq 0. \tag{1.9}$$

Comparing (1.8) and (1.9) with (1.1) and (1.2), respectively, our Theorem 1.1 gives a new estimate.

Remark 1.2. Since $R = \sqrt{n(n-1)}|\mathring{R}_{ij}|$ on $\mathbb{S}^1 \times \mathbb{S}^{n-1}$ with the product metric, Theorem 1.1 can also be interpreted as an interpolation curvature estimate.

Remark 1.3. In [1], Bo and Sheng gave some rigidity characterization for conformally flat manifolds with constant quotient curvature of $\sigma_k(A^\tau)$. Our Theorem 1.2 gives an integral pinching condition with the $\sigma_2(A^\tau)$ assumption, which can be seen as a complement.

2. Proof of the results.

2.1. Proof of Theorem 1.1. Recall that the Riemannian curvature tensor and Weyl curvature tensor are related by

$$\begin{aligned} W_{ijkl} &= R_{ijkl} - \frac{1}{n-2}(R_{ik}g_{jl} - R_{il}g_{jk} + R_{jl}g_{ik} - R_{jk}g_{il}) \\ &\quad + \frac{R}{(n-1)(n-2)}(g_{ik}g_{jl} - g_{il}g_{jk}) \\ &= R_{ijkl} - \frac{1}{n-2}(\mathring{R}_{ik}g_{jl} - \mathring{R}_{il}g_{jk} + \mathring{R}_{jl}g_{ik} - \mathring{R}_{jk}g_{il}) \\ &\quad - \frac{R}{n(n-1)}(g_{ik}g_{jl} - g_{il}g_{jk}). \end{aligned} \tag{2.1}$$

Using formula (2.1), it is easy to check that

$$\mathring{R}_{kl}R_{ikjl} = \mathring{R}_{kl}W_{ikjl} + \frac{1}{n-2}(|\mathring{R}_{ij}|^2 g_{ij} - 2\mathring{R}_{ik}\mathring{R}_{jk}) - \frac{1}{n(n-1)}R\mathring{R}_{ij}. \tag{2.2}$$

It follows that

$$\begin{aligned} \mathring{R}_{kj,ik} &= \mathring{R}_{kj,ki} + \mathring{R}_{lj}R_{lkik} + \mathring{R}_{kl}R_{ljik} \\ &= \frac{n-2}{2n}R_{,ij} + \mathring{R}_{ik}\mathring{R}_{jk} + \frac{1}{n}R\mathring{R}_{ij} - \Big[\mathring{R}_{kl}W_{ikjl} \\ &\quad + \frac{1}{n-2}(|\mathring{R}_{ij}|^2 g_{ij} - 2\mathring{R}_{ik}\mathring{R}_{jk}) - \frac{1}{n(n-1)}R\mathring{R}_{ij}\Big] \\ &= \frac{n-2}{2n}R_{,ij} + \frac{n}{n-2}\mathring{R}_{ik}\mathring{R}_{jk} + \frac{1}{n-1}R\mathring{R}_{ij} \\ &\quad - \mathring{R}_{kl}W_{ikjl} - \frac{1}{n-2}|\mathring{R}_{ij}|^2 g_{ij}. \end{aligned} \tag{2.3}$$

On the other hand, formula (1.3) is equivalent to

$$C_{ijk} = \mathring{R}_{kj,i} - \mathring{R}_{ki,j} + \frac{n-2}{2n(n-1)}(R_{,i}g_{jk} - R_{,j}g_{ik}). \tag{2.4}$$

Therefore, from (2.3) and (2.4), we have

$$\begin{aligned} C_{kij,k} &= \Delta\mathring{R}_{ij} - \mathring{R}_{kj,ik} + \frac{n-2}{2n(n-1)}(g_{ij}\Delta R - R_{,ij}) \\ &= \Delta\mathring{R}_{ij} - \Big(\frac{n-2}{2n}R_{,ij} + \frac{n}{n-2}\mathring{R}_{ik}\mathring{R}_{jk} + \frac{1}{n-1}R\mathring{R}_{ij} - \mathring{R}_{kl}W_{ikjl} \\ &\quad - \frac{1}{n-2}|\mathring{R}_{ij}|^2 g_{ij}\Big) + \frac{n-2}{2n(n-1)}(g_{ij}\Delta R - R_{,ij}), \end{aligned} \tag{2.5}$$

which shows

$$\begin{aligned}\Delta\mathring{R}_{ij} =&\frac{n-2}{2n}R_{,ij}+\frac{n}{n-2}\mathring{R}_{ik}\mathring{R}_{jk}+\frac{1}{n-1}R\mathring{R}_{ij}-\mathring{R}_{kl}W_{ikjl}\\ &-\frac{1}{n-2}|\mathring{R}_{ij}|^2g_{ij}-\frac{n-2}{2n(n-1)}(g_{ij}\Delta R-R_{,ij})+C_{kij,k},\end{aligned}\tag{2.6}$$

and hence

$$\begin{aligned}\frac{1}{2}\Delta|\mathring{R}_{ij}|^2 =&|\nabla\mathring{R}_{ij}|^2+\mathring{R}_{ij}\Delta\mathring{R}_{ij}\\ =&|\nabla\mathring{R}_{ij}|^2+\frac{n}{n-2}\mathring{R}_{ij}\mathring{R}_{ik}\mathring{R}_{jk}-W_{ikjl}\mathring{R}_{ij}\mathring{R}_{kl}+\frac{1}{n-1}R|\mathring{R}_{ij}|^2\\ &+\frac{n-2}{2(n-1)}R_{,ij}\mathring{R}_{ij}+C_{kij,k}\mathring{R}_{ij}.\end{aligned}\tag{2.7}$$

We recall the following inequalies (cf. [12, Lemma 3.4]):

$$|W_{ikjl}\mathring{R}_{ij}\mathring{R}_{kl}|\leq\sqrt{\frac{n-2}{2(n-1)}}|W||\mathring{R}_{ij}|^2,\tag{2.8}$$

and

$$\mathring{R}_{ij}\mathring{R}_{ik}\mathring{R}_{jk}\geq-\frac{n-2}{\sqrt{n(n-1)}}|\mathring{R}_{ij}|^3,\tag{2.9}$$

with equality in (2.9) at some point $p\in M$ if and only if $\mathring{R}_{ij}$ can be diagonalized at p and the eigenvalue multiplicity of $\mathring{R}_{ij}$ is at least $n-1$ (see also [9,10], or [15]). Thus, (2.7) becomes

$$\begin{aligned}\frac{1}{2}\Delta|\mathring{R}_{ij}|^2 \geq&|\nabla\mathring{R}_{ij}|^2-\sqrt{\frac{n}{n-1}}|\mathring{R}_{ij}|^3-\sqrt{\frac{n-2}{2(n-1)}}|W||\mathring{R}_{ij}|^2+\frac{1}{n-1}R|\mathring{R}_{ij}|^2\\ &+\frac{n-2}{2(n-1)}R_{,ij}\mathring{R}_{ij}+C_{kij,k}\mathring{R}_{ij}.\end{aligned}\tag{2.10}$$

Integrating both sides of (2.10) gives

$$\begin{aligned}0\geq&\int_M|\nabla\mathring{R}_{ij}|^2+\int_M\Big[-\sqrt{\frac{n}{n-1}}|\mathring{R}_{ij}|^3-\sqrt{\frac{n-2}{2(n-1)}}|W||\mathring{R}_{ij}|^2+\frac{1}{n-1}R|\mathring{R}_{ij}|^2\Big]\\ &+\frac{n-2}{2(n-1)}\int_M R_{,ij}\mathring{R}_{ij}+\int_M C_{kij,k}\mathring{R}_{ij}.\end{aligned}\tag{2.11}$$

Using the definition of the Cotton tensor given by (2.4) and the fact that the Cotton tensor is trace-less in any two indices, we obtain

$$\int_M C_{kij,k}\mathring{\mathrm{R}}_{ij}=-\int_M C_{kij}\mathring{\mathrm{R}}_{ij,k}=-\frac{1}{2}\int_M|C_{ijk}|^2.$$

On the other hand,

$$\int_M R_{,ij}\mathring{R}_{ij}=-\int_M R_{,i}\mathring{R}_{ij,j}=-\frac{n-2}{2n}\int_M|\nabla R|^2,$$

where we used the second Bianchi identity $\mathring{R}_{ij,j} = \frac{n-2}{2n} R_{,i}$. Hence, (2.11) becomes

$$\begin{aligned} 0 \geq & \int_M |\nabla \mathring{R}_{ij}|^2 + \int_M \Big[-\sqrt{\frac{n}{n-1}} |\mathring{R}_{ij}| - \sqrt{\frac{n-2}{2(n-1)}} |W| + \frac{1}{n-1} R \Big] |\mathring{R}_{ij}|^2 \\ & - \frac{(n-2)^2}{4n(n-1)} \int_M |\nabla R|^2 - \frac{1}{2} \int_M |C_{ijk}|^2 \\ \geq & \int_M \Big[-\sqrt{\frac{n}{n-1}} |\mathring{R}_{ij}| - \sqrt{\frac{n-2}{2(n-1)}} |W| + \frac{1}{n-1} R \Big] |\mathring{R}_{ij}|^2 \\ & - \frac{(n-2)^2}{4n(n-1)} \int_M |\nabla R|^2 - \frac{1}{2} \int_M |C_{ijk}|^2, \end{aligned} \tag{2.12}$$

which yields the desired estimate (1.5).

Now, we consider the case of equality in (1.5), that is

$$\begin{aligned} \int_M \left[R - \sqrt{n(n-1)} |\mathring{R}_{ij}| - \sqrt{\frac{(n-1)(n-2)}{2}} |W| \right] |\mathring{R}_{ij}|^2 \\ = \frac{(n-2)^2}{4n} \int_M |\nabla R|^2 + \frac{n-1}{2} \int_M |C_{ijk}|^2. \end{aligned} \tag{2.13}$$

In this case, the inequalities (2.8), (2.9), and (2.12) become equalities. In particular, the second equality in (2.12) implies

$$\nabla \mathring{R}_{ij} = 0, \tag{2.14}$$

which shows that $\mathring{R}_{ij,j} = 0$ and hence the scalar curvature is constant. Furthermore, the Ricci curvature is parallel and hence the metric g has harmonic curvature. Thus, (2.13) becomes

$$\int_M \left[R - \sqrt{n(n-1)} |\mathring{R}_{ij}| - \sqrt{\frac{(n-1)(n-2)}{2}} |W| \right] |\mathring{R}_{ij}|^2 = 0. \tag{2.15}$$

As stated in the lines following (2.9), $\mathring{R}_{ij}$ has, at each point p, an eigenvalue of multiplicity $n-1$ or n. Writing $\mathring{R}_{ij} = a v_i v_j + b g_{ij}$ at p, with some scalars a, b and a vector v, we see that the left-hand side of (2.8) is zero at every point p. As (2.8) is an equality, the metric g must be conformally flat or Einstein. Our claim about the equality case now follows from Theorem B of Huang and Ma (or see the proof of Catino's Theorem A) since the Ricci curvature is parallel.

This completes the proof of Theorem 1.1.

2.2. Proof of Theorem 1.2. By virtue of the definition of $\sigma_2(A^\tau)$, we have

$$\begin{aligned}\sigma_2(A^\tau) =&\frac{1}{2}[(\mathrm{tr}A^\tau)^2 - |A^\tau_{ij}|^2]\\ =&\frac{1}{2}\Big[\frac{4(n-1)(1-\tau)+n\tau^2}{4(n-1)}R^2 - |R_{ij}|^2\Big]\\ =&\frac{1}{2}\Big[\frac{[2(n-1)-n\tau]^2}{4n(n-1)}R^2 - |\mathring{R}_{ij}|^2\Big].\end{aligned} \tag{2.16}$$

Since $\sigma_2(A^\tau)$ is a non-negative constant, we have (see [1, Proposition 2.11],

$$|\nabla A^\tau_{ij}|^2 \geq |\nabla(\mathrm{tr}A^\tau)|^2. \tag{2.17}$$

By a direct calculation, we have

$$\begin{aligned}|\nabla A^\tau_{ij}|^2 =&|\nabla R_{ij}|^2 + \frac{n\tau^2-4(n-1)\tau}{4(n-1)^2}|\nabla R|^2\\ =&|\nabla \mathring{R}_{ij}|^2 + \frac{[2(n-1)-n\tau]^2}{4n(n-1)^2}|\nabla R|^2,\end{aligned} \tag{2.18}$$

and

$$|\nabla(\mathrm{tr}A^\tau)|^2 = \frac{[2(n-1)-n\tau]^2}{4(n-1)^2}|\nabla R|^2, \tag{2.19}$$

then (2.17) is equivalent to

$$|\nabla \mathring{R}_{ij}|^2 \geq \frac{[2(n-1)-n\tau]^2}{4n(n-1)}|\nabla R|^2. \tag{2.20}$$

Inserting

$$|\nabla R|^2 \leq \frac{4n(n-1)}{[2(n-1)-n\tau]^2}|\nabla \mathring{R}_{ij}|^2$$

with $\tau \neq 2 - \frac{2}{n}$ into (2.12) yields

$$\begin{aligned}0 \geq&\left(1 - \frac{(n-2)^2}{[2(n-1)-n\tau]^2}\right)\int_M |\nabla \mathring{R}_{ij}|^2 + \int_M \Big[-\sqrt{\frac{n}{n-1}}|\mathring{R}_{ij}|\\ &-\sqrt{\frac{n-2}{2(n-1)}}|W| + \frac{1}{n-1}R\Big]|\mathring{R}_{ij}|^2 - \frac{1}{2}\int_M |C_{ijk}|^2.\end{aligned} \tag{2.21}$$

It is easy to check that

$$\frac{(n-2)^2}{[2(n-1)-n\tau]^2} < 1$$

is equivalent to $\tau < 1$ or $\tau > 3 - \frac{4}{n}$. In this case, we have from (2.21),

$$0 \geq \int_M \Big[-\sqrt{\frac{n}{n-1}}|\mathring{R}_{ij}| - \sqrt{\frac{n-2}{2(n-1)}}|W| + \frac{1}{n-1}R\Big]|\mathring{R}_{ij}|^2 - \frac{1}{2}\int_M |C_{ijk}|^2, \tag{2.22}$$

and the desired estimate (1.7) is attained.

If the equality in (1.7) holds, then

$$\int_M \left[R - \sqrt{n(n-1)}|\mathring{R}_{ij}| - \sqrt{\frac{(n-1)(n-2)}{2}}|W|\right]|\mathring{R}_{ij}|^2 = \frac{n-1}{2}\int_M |C_{ijk}|^2. \tag{2.23}$$

In this case, the inequalities (2.8), (2.9), and (2.22) become equalities, which also shows that

$$\nabla \mathring{R}_{ij} = 0, \tag{2.24}$$

and the Ricci curvature is parallel. Since the rest of proof is the same as that of Theorem 1.1, we omit it here.

Therefore, we complete the proof of Theorem 1.2.

Funding Funding was provided by National Natural Science Foundation of China (Grant No. 11971153).

Acknowledgements. We would like to thank the referee for suggestions which make the paper more readable.

References

[1] Bo, L., Sheng, W.M.: Some rigidity properties for manifolds with constant k-curvature of modified Schouten tensor. J. Geom. Anal. **29**, 2862–2887 (2019)

[2] Carron, G., Herzlich, M.: Conformally flat manifolds with nonnegative Ricci curvature. Compos. Math. **142**, 798–810 (2006)

[3] Catino, G.: On conformally flat manifolds with constant positive scalar curvature. Proc. Amer. Math. Soc. **144**, 2627–2634 (2016)

[4] Cheng, Q.-M.: Compact locally conformally flat Riemannian manifolds. Bull. Lond. Math. Soc. **33**, 459–465 (2001)

[5] Gursky, M.J.: Locally conformally flat four- and six-manifolds of positive scalar curvature and positive Euler characteristic. Indiana Univ. Math. J. **43**, 747–774 (1994)

[6] Gursky, M.J., Viaclovsky, J.A.: Fully nonlinear equations on Riemannian manifolds with negative curvature. Indiana Univ. Math. J. **52**, 399–419 (2003)

[7] Hebey, E., Vaugon, M.: Un théorème de pincement intégral sur la courbure concirculaire en géométrie conforme. C. R. Acad. Sci. Paris Sér. I Math. **316**, 483–488 (1993)

[8] Hebey, E., Vaugon, M.: Effective L_p pinching for the concircular curvature. J. Geom. Anal. **6**, 531–553 (1996)

[9] Huang, G.: Some rigidity characterizations on critical metrics for quadratic curvature functionals. Anal. Math. Phys. **10**, 1–14 (2020)

[10] Huang, G., Chen, L.: Some characterizations on critical metrics for quadratic curvature functions. Proc. Amer. Math. Soc. **146**, 385–395 (2018)

[11] Huang, G., Ma, B.: Riemannian manifolds with harmonic curvature. Colloq. Math. **145**, 251–257 (2016)

[12] Huisken, G.: Ricci deformation of the metric on a Riemannian manifold. J. Differ. Geom. **21**, 47–62 (1985)

[13] Li, A., Li, Y.Y.: On some conformally invariant fully nonlinear equations II. Liouville, Harnack and Yamabe. Acta Math. **195**, 117–154 (2005)

[14] Noronha, M.: Some compact conformally flat manifolds with nonnegative scalar curvature. Geom. Dedicata **47**, 255–268 (1993)

[15] Okumura, M.: Hypersurfaces and a pinching problem on the second fundamental tensor. Am. J. Math. **96**, 207–213 (1974)

[16] Pigola, S., Rigoli, M., Setti, A.G.: Some characterizations of space-forms. Trans. Amer. Math. Soc. **359**, 1817–1828 (2007)

[17] Tani, M.: On a conformally flat Riemannian manifold. Tohoku Math. **19**, 205–214 (1967)

[18] Xu, H.-W., Zhao, E.-T.: L_p Ricci curvature pinching theorems for conformally flat Riemannian manifolds. Pacific J. Math. **245**, 381–396 (2010)

[19] Zhu, S.-H.: The classification of complete locally conformally flat manifolds of nonnegative Ricci curvature. Pacific J. Math. **163**, 189–199 (1994)

Guangyue Huang and Qianyu Zeng
Department of Mathematics
Henan Normal University
Xinxiang 453007
People's Republic of China
e-mail: hgy@henannu.edu.cn

Qianyu Zeng
e-mail: qyzeng06@126.com

Received: 27 February 2020