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ERRATUM

Erratum to: Top local cohomology modules and Gorenstein injectivity with respect to a semidualizing module

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Abstract. This note makes a correction to the paper "Top local cohomology modules and Gorenstein injectivity with respect to a semidualizing module".

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Keywords. Local cohomology, Semidualizing, Dualizing, G_C-injective, C-injective.

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Throughout this paper, R is a commutative Noetherian ring and all modules are unital. In the proof of (iv) $\Rightarrow$ (i) of Theorem 3.1 in the original paper [4], we used explicitly the assumption that R is Cohen–Macaulay. More precisely, the statement (i) in Theorem 3.1 of the original paper implies that R is Cohen–Macaulay and $\mathrm{H}^d_{\mathfrak{m}}(R)$ is a C-injective R-module. Therefore the corrected version of Theorem 3.1 in the original paper is as follows. (Note that the rest of the paper is unaffected because in all appeals to Theorem 3.1, its new hypothesis that R is Cohen–Macaulay is automatically satisfied.)

Theorem 3.1. (new) *Let $(R, \mathfrak{m})$ be a complete local ring of Krull dimension d, and let C be a semidualizing R-module. Then the following statements are equivalent.*

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(i) *C is a dualizing R-module.*
(ii) *R is Cohen–Macaulay, and $\mathrm{H}^d_{\mathfrak{m}}(R)$ is a C-injective R-module.*
(iii) *R is Cohen–Macaulay, and $\mathrm{H}^d_{\mathfrak{m}}(R)$ is a G_C-injective R-module.*
(iv) *R is Cohen–Macaulay, and $\mathrm{H}^d_{\mathfrak{m}}(R)$ has finite G_C-injective dimension.*

Proof. (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (iv) follows from the proof of Theorem 3.1.

(iv) $\Rightarrow$ (i) Suppose that $\mathrm{H}^d_{\mathfrak{m}}(R)$ has finite G_C-injective dimension. Since $\mathrm{H}^d_{\mathfrak{m}}(R)$ is an Artinian R-module, we have that $\mathrm{H}^d_{\mathfrak{m}}(R)^\vee$ is a finitely generated R-module (see [2, Theorem 3.4.7]). By [3, Theorem 2.16], $\mathrm{Gid}_{R\ltimes C}(\mathrm{H}^d_{\mathfrak{m}}(R)) < \infty$. Since R is complete, $\mathrm{Gfd}_{R\ltimes C}(\mathrm{H}^d_{\mathfrak{m}}(R)^\vee) < \infty$ by [1, Theorem 4.25]. So $\mathrm{Gpd}_{R\ltimes C}(\mathrm{H}^d_{\mathfrak{m}}(R)^\vee) < \infty$, by [1, Theorem 4.23]. In addition, $\mathcal{GP}_C\text{-}\mathrm{pd}_R(\mathrm{H}^d_{\mathfrak{m}}(R)^\vee) = \mathrm{Gpd}_{R\ltimes C}(\mathrm{H}^d_{\mathfrak{m}}(R)^\vee) < \infty$, by [3, Theorem 2.16]. The Local Duality Theorem implies that $\mathrm{H}^d_{\mathfrak{m}}(R)^\vee \cong \Omega$, where Ω is a canonical module of R. By definition, this module is non-zero and finitely generated, so it has finite depth, and it has finite injective dimension because R is Cohen–Macaulay. Since it has finite G_C-projective dimension, the fact that C is dualizing follows from [5, Corollary 2.9]. □

Recently, Zargar [6] emphasizes the above mistake.

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