Aequationes Math. 58 (1999) 214–222
0001-9054/99/030214-9 $ 1.50+0.20/0



Aequationes Mathematicae

# Monomial selections of set-valued maps

Roman Badora, Zsolt Páles* and László Székelyhidi*

*Dedicated to Professor János Aczél on the occasion of his 75$^{th}$ birthday*

**Summary.** The main result of this paper offers a necessary and sufficient condition for the existence of a multimonomial selection of a set-valued map with weakly compact convex values defined on an amenable semigroup.

**Mathematics Subject Classification (1991).** Primary 39B72.

**Keywords.** Multiadditive function, monomial function selection of set-valued map, invariant mean, amenable semigroup, stability of functional equation.

## 1. Introduction

The original problems concerning stability of linear functional equations lead to different fruitful generalizations of classical concepts. There are different known techniques to prove stability theorems: one of them is based on the use of invariant means introduced by Székelyhidi [18]. It has turned out that this technique can be widely extended via the generalization of scalar-valued invariant means to vector-valued means. The main existence theorems on vector-valued invariant means can be used to derive very general stability theorems concerning different linear functional equations. From this point on the main emphasis is on the existence of vector-valued invariant means instead of stability problems. Existence theorems of this kind can be obtained usually by different functional analytical methods. And at this point a new idea comes into the picture: the so-called selection theorems. The first selection theorems concerned with additive selections: to find necessary and sufficient conditions in order that a set-valued mapping has an additive selection. From additive-type selection theorems one can derive easily quite general

* Research supported by the Hungarian National Foundation for Scientific Research (OTKA), Grant No. T-016846 and by the Hungarian High Educational Research and Development Found (FKFP) Grant No. 0310/1997.

stability theorems for the Cauchy-equation. Going on this line we arrive at the natural problem of finding monomial and multimonomial selections of set-valued mappings under suitable conditions. In this paper we solve this problem using the vector-valued invariant mean technique. We remark that part of these results have been reported in the Problems and Remarks Session of the 35th ISFE in Graz, 1997 (see [16]).

Here we give a short summary of the structure of this paper. In the first part, we recall a recent result of Badora, Ger and Páles [2] on the existence of left (right) invariant means (defined on a subspace of locally convex vector space valued functions over a semigroup). This generalizes the results of Székelyhidi [18], Gajda [5], and Przesławski–Yost [17] who obtained analogous statements for the semi-reflexive locally convex space valued setting.

In the real valued and first order case, such results have recently been obtained by Páles [15]. For the set-valued setting the necessary and sufficient condition for the existence of additive selections has been established in Badora, Ger and Páles [2]. This result offers necessary and sufficient conditions for the existence of additive selections from a set-valued mapping with nonempty weakly compact convex values. The proof uses the vector-valued invariant mean from the previous section and the ideas of the proof of Theorem 2 in Ger [6]. The result obtained answers a more general problem than that of Páles [15], [16] affirmatively. We note that an affirmative answer was also given independently by Jacek Tabor [19].

**Theorem 1.** *Let $(S,\cdot)$ be a left amenable semigroup and let $X$ be a locally convex linear space. Let $\Phi : S \to 2^X$ be a set-valued map such that, for all $s \in S$, $\Phi(s)$ is nonempty, convex and weakly compact. Then $\Phi$ admits an additive selection $A : S \to X$ if and only if there exists a function $f : S \to X$ such that*

$$\Delta_t f(s) := f(ts) - f(s) \in \Phi(t), \quad s, t \in S. \tag{1}$$

In the case when the image space is $\mathbb{R}$ and $F$ is a compact interval valued set-valued map from a commutative semigroup $S$, then this theorem reduces to a recent result of Páles [15] which was proved by a completely different technique based on the sandwich theorems obtained by Nikodem–Páles–Wąsowicz [14].

In the main result of this paper, we derive a necessary and sufficient condition for the existence of a monomial selection of a weakly compact convex set valued map. This result has direct consequences in the stability theory of functional equations.

## 2. Invariant means for locally convex space-valued functions

Let $X$ be an arbitrary topological vector space over the field of real numbers. If $S$ is a nonempty set, then denote by $\mathcal{B}(S, X)$ the space of all bounded $X$-valued functions defined on $S$. For a set $Y \subseteq X$, $\mathrm{co}(Y)$ denotes the convex hull of $Y$ and $\overline{\mathrm{co}}(Y)$ the closed convex hull of $Y$.

**Definition 1.** Let $L$ be a linear subspace of $\mathcal{B}(S,X)$. A mapping $M : L \to X$ is called a *mean on L* if it satisfies the following properties:
(M1) $M$ is *linear* on $L$;
(M2) for all $f \in L$,

$$M(f) \in \overline{\mathrm{co}}\, f(S).$$

This latter property will be called mean value property.

Now assume that $S$ admits a semigroup structure, that is, $(S,\cdot)$ is a (not necessarily commutative) semigroup. The elements of $S$ induce the notion of *left translation* for functions $f : S \to X$ in the following way. If $t \in S$, then denote

$$\tau_t f(s) := f(ts) \quad (s \in S).$$

The function $\tau_t f$ so defined is called *left translate of f*.

**Definition 2.** The semigroup $S$ is called *left amenable* if there exists a mean $M$ on $\mathcal{B}(S,\mathbb{R})$ which is *invariant with respect to the left translations*, i.e., if it satisfies

$$M(\tau_t f) = M(f)$$

for all $f \in \mathcal{B}(S,\mathbb{R})$ and $t \in S$.

The notions of *right invariant mean* and *right amenability* can analogously be defined. If a both left and right invariant mean exists, then $S$ is called *amenable.*

It is well known that any commutative semigroup is amenable (cf. e.g. Hewitt–Ross [8, Chapter 4, Theorem 17.5] and Day [3]).

The main existence result on invariant means is contained in the following theorem (see Badora–Ger–Páles [2, Theorem 4] and also Tabor [19]). If $X$ is a locally convex space, then denote by $WC(X)$ the family of nonempty weakly compact and convex subsets of $X$ and by $\mathcal{WC}(S,X)$ the space of all functions $f : S \to X$ such that $\overline{\mathrm{co}}\, f(S)$ is weakly compact.

**Theorem 2.** *Let $(S,\cdot)$ be a left amenable semigroup and let $X$ be a locally convex linear space. Then the subspace $\mathcal{WC}(S,X)$ admits a left invariant mean.*

It is not difficult to see that $\mathcal{WC}(S,X)$ is a vector space, moreover it is a subspace of all bounded $X$-valued functions on $S$. In many cases (e.g., when $X$ with the weak topology is a quasi-complete locally convex space), the weak relative compactness of $f(S)$ yields the weak relative compactness of $\mathrm{co}\, f(S)$ (cf. Holmes [9, Theorem 11B, p. 61]). If $X$ is a semi-reflexive locally convex space, then bounded sets are always weakly relatively compact (see Yosida [20, Chapter V, Theorem 3.1, p. 140]). Hence, if $f(S)$ is bounded then $\overline{\mathrm{co}}\, f(S)$ is weakly compact. Therefore, in this case $\mathcal{WC}(S, X)$ is identical with the space of bounded functions, that is, with $\mathcal{B}(S, X)$.

The above theorem is a direct generalization of the following result of Székelyhidi [18] and Gajda [5], cf. also Przesławski–Yost [17, Proposition 1. 3.]

**Corollary 1.** *Let $(S, \cdot)$ be a left amenable semigroup and let $X$ be a semi-reflexive locally convex linear space. Then the space $\mathcal{B}(S, X)$ of all bounded $X$-valued functions admits a left invariant mean.*

## 3. Selection theorems

In this section we formulate and prove the main results of the paper. It will be required to introduce the following notation.

If $(S, \cdot)$ is a semigroup and $X$ is a vector space. For $t \in S$, we define the difference operator $\Delta_t$ by $\Delta_t := \tau_t - I$. Then, for any function $f : S \to X$, we have

$$\Delta_t f(s) := \tau_t f(s) - f(s) = f(ts) - f(s).$$

For $n$ variable functions, we define the $i$-th partial translation and difference operators $\underset{(i)}{\tau}$ and $\underset{(i)}{\Delta}$ by

$$\underset{(i)}{\tau}{}_t F(s_1, \dots, s_n) := F(s_1, \dots, ts_i, \dots, s_n),$$

$$\underset{(i)}{\Delta}{}_t F(s_1, \dots, s_n) := F(s_1, \dots, ts_i, \dots, s_n) - F(s_1, \dots, s_i, \dots, s_n),$$

where $F : S^n \to X$ is an arbitrary function.

**Theorem 3.** *Let $(S, \cdot)$ be a commutative semigroup and $X$ be a locally convex space. Let $\Phi : S^n \to WC(X)$ and assume that there exists a function $f : S \to X$ such that*

$$\frac{1}{k_1! \cdots k_n!} \Delta_{t_1}^{k_1} \cdots \Delta_{t_n}^{k_n} f(s) \in \Phi(t_1, \dots, t_n), \quad s, t_1, \dots, t_n \in S. \tag{2}$$

*Then there exists a function $F : S^n \to X$ such that $F$ is a selection of $\Phi$, i.e.,*

$$F(t_1, \dots, t_n) \in \Phi(t_1, \dots, t_n), \quad t_1, \dots, t_n \in S \tag{3}$$

*and, for all $i = 1, \dots, n$, $F$ satisfies the functional equation*

$$\frac{1}{k_i!} \underset{(i)}{\Delta}{}_u^{k_i} F(t_1, \dots, t_n) = F(t_1, \dots, u, \dots, t_n), \quad u, t_1, \dots, t_n \in S. \tag{4}$$

*Proof.* The semigroup $S$ is commutative and hence it is amenable (cf. Hewitt–Ross [8, Chapter 4, Theorem 17.5] and Day [3]). Thus, by Theorem 2, the space $\mathcal{WC}(S,X)$ of all $X$-valued functions whose range has a weakly compact closed convex hull admits a left invariant mean. Denote by $M$ such an invariant mean.

On the other hand, for all fixed $t_1,\dots,t_n$, the function

$$s \mapsto \varphi_{t_1,\dots,t_n}(s) := \frac{1}{k_1!\cdots k_n!}\Delta_{t_1}^{k_1}\cdots\Delta_{t_n}^{k_n} f(s), \quad s\in S$$

belongs to $\mathcal{WC}(S,X)$, because, by (2), the range of this function is contained in $\Phi(t_1,\dots,t_n)$ and this latter set is convex and weakly compact, and hence $\overline{\mathrm{co}}\,\varphi_{t_1,\dots,t_n}(S)$ is also weakly compact.

Thus, we may apply $M$ to $\varphi_{t_1,\dots,t_n}$. Define

$$F(t_1,\dots,t_n) := M_s[\varphi_{t_1,\dots,t_n}(s)] := M[\varphi_{t_1,\dots,t_n}].$$

By the mean value property of $M$, and by (2) again, we have at once that (3) is valid.

In the rest of the proof, we show that $F$ satisfies the functional equation (4) as well.

Let $i\in\{1,\dots,n\}$ and $u,t_1,\dots,t_n\in S$ be fixed. We start with computing the left hand side of (4).

$$\begin{aligned}
&k_1!\cdots k_n! \underset{(i)}{\Delta}{}_u^{k_i} F(t_1,\dots,t_n)\\
&\quad = k_1!\cdots k_n! \sum_{j=0}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}F(t_1,\dots,u^j t_i,\dots,t_n)\\
&\quad = \sum_{j=0}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}M_s\left[\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\Delta_{u^j t_i}^{k_i} f(s)\right]\\
&\quad = M_s\left[\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\left(\sum_{j=0}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}\Delta_{u^j t_i}^{k_i}\right)f(s)\right]\\
&\quad = M_s\left[\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\sum_{j=0}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}\sum_{\mu=0}^{k_i}\binom{k_i}{\mu}(-1)^{k_i-\mu}f((u^j t_i)^\mu s)\right]\\
&\quad = M_s\left[\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\sum_{\mu=0}^{k_i}\binom{k_i}{\mu}(-1)^{k_i-\mu}\sum_{j=0}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}f(u^{j\mu}t_i^\mu s)\right]\\
&\quad = M_s\left[\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\sum_{\mu=1}^{k_i}\binom{k_i}{\mu}(-1)^{k_i-\mu}\sum_{j=0}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}f(u^{j\mu}t_i^\mu s)\right]
\end{aligned}$$

$$= M_s\left[\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\sum_{\mu=1}^{k_i}\binom{k_i}{\mu}(-1)^{k_i-\mu}\Delta_{u^\mu}^{k_i}f(t_i^\mu s)\right]$$
$$= M_s\left[\sum_{\mu=1}^{k_i}\binom{k_i}{\mu}(-1)^{k_i-\mu}\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\Delta_{u^\mu}^{k_i}f(t_i^\mu s)\right]$$
$$= k_1!\cdots k_n!M_s\left[\sum_{\mu=1}^{k_i}\binom{k_i}{\mu}(-1)^{k_i-\mu}\varphi_{t_1,\ldots,u^\mu,\ldots,t_n}(t_i^\mu s)\right]$$
$$= k_1!\cdots k_n!M_s\left[\sum_{\mu=1}^{k_i}\binom{k_i}{\mu}(-1)^{k_i-\mu}\varphi_{t_1,\ldots,u^\mu,\ldots,t_n}(s)\right].$$

Here, the last equality follows from the translation invariance of the mean $M$. Thus, we get equality in the above chain of equations if we omit $t_i$ everywhere. Hence

$$k_1!\cdots k_n!\underset{(i)}{\Delta_u^{k_i}}F(t_1,\ldots,t_n) = \sum_{j=0}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}M_s\left[\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\Delta_{u^j}^{k_i}f(s)\right]$$
$$= \sum_{j=1}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}M_s\left[\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\Delta_{u^j}^{k_i}f(s)\right].$$

Using the identity

$$\Delta_{u^j}^{k_i} = (\tau_{u^j}-I)^{k_i} = (\tau_u^j - I)^{k_i}$$
$$= (\tau_u - I)^{k_i}(\tau_u^{j-1}+\cdots+I)^{k_i} = \Delta_u^{k_i}(\tau_u^{j-1}+\cdots+I)^{k_i},$$

we get

$$k_1!\cdots k_n!\underset{(i)}{\Delta_u^{k_i}}F(t_1,\ldots,t_n)$$
$$= \sum_{j=1}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}M_s\left[\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\Delta_{u^j}^{k_i}f(s)\right]$$
$$= \sum_{j=1}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}M_s\left[(\tau_u^{j-1}+\cdots+I)^{k_i}\left(\prod_{\nu\neq i}\Delta_{t_\nu}^{k_\nu}\right)\cdot\Delta_u^{k_i}f(s)\right]$$
$$= k_1!\cdots k_n!\sum_{j=1}^{k_i}\binom{k_i}{j}(-1)^{k_i-j}M_s\left[(\tau_u^{j-1}+\cdots+I)^{k_i}\varphi_{t_1,\ldots,u,\ldots,t_n}(s)\right]$$

$$= k_1! \cdots k_n! \sum_{j=1}^{k_i} \binom{k_i}{j} (-1)^{k_i - j} M_s \left[ j^{k_i} \varphi_{t_1,\ldots,u,\ldots,t_n}(s) \right]$$
$$= k_1! \cdots k_n! \left( \sum_{j=1}^{k_i} \binom{k_i}{j} (-1)^{k_i - j} j^{k_i} \right) F(t_1, \ldots, u, \ldots, t_n).$$

In order to complete the proof of the theorem, it suffices to show that

$$\sum_{j=1}^{k_i} \binom{k_i}{j} (-1)^{k_i - j} j^{k_i} = k_i!.$$

Observe that the $k_i$-th difference operator applied to the power function $x \to x^{k_i}$ results the constant function $k_i!$. One can observe that the left hand side of the above relation can be written as $\Delta_1^{k_i} x^k|_{x=0}$. Hence, the equality follows. □

In the case $n = 1$, we immediately obtain the following corollary.

**Corollary 2.** *Let $(S, \cdot)$ be a commutative semigroup and $X$ be a locally convex space. Let $\Phi : S \to WC(X)$ and assume that there exists a function $f : S \to X$ such that*

$$\frac{1}{k!} \Delta_t^k f(s) \in \Phi(t), \quad s, t \in S. \tag{5}$$

*Then there exists a function $F : S \to X$ such that $F$ is a selection of $\Phi$ and $F$ satisfies the functional equation*

$$\frac{1}{k!} \Delta_u^k F(t) = F(u), \quad u, t \in S. \tag{6}$$

However, in the case $k_1 = \cdots = k_n$, we can prove a stronger result than that of following from Theorem 3 directly, because the commutativity need not be assumed here.

**Theorem 4.** *Let $(S, \cdot)$ be a left amenable semigroup and $X$ be a locally convex space. Let $\Phi : S^n \to WC(X)$ and assume that there exists a function $f : S \to X$ such that*

$$\Delta_{t_1} \cdots \Delta_{t_n} f(s) \in \Phi(t_1, \ldots, t_n), \quad s, t_1, \ldots, t_n \in S. \tag{7}$$

*Then there exists a function $F : S^n \to X$ such that $F$ is a selection of $\Phi$, and, for all $i = 1, \ldots, n$, $F$ satisfies the functional equation*

$$\underset{(i)}{\Delta_u} F(t_1, \ldots, t_n) = F(t_1, \ldots, u, \ldots, t_n), \quad u, t_1, \ldots, t_n \in S \tag{8}$$

*or, in other words, $F$ is $n$-additive.*

*Proof.* By Theorem 2, the space $\mathcal{WC}(S,X)$ admits a left invariant mean. Denote by $M$ such an invariant mean. Arguing similarly as in the proof of Theorem 3, define

$$F(t_1,\ldots,t_n) := M_s[\Delta_{t_1}\cdots\Delta_{t_n}f(s)].$$

By the mean value property of $M$, and by (7), we have that $F$ is a selection of $\Phi$.

We show that $F$ is additive in the $i$th variable. Let, for $j \neq i$, $t_j \in S$ and $u, v \in S$. Then we have

$$\begin{aligned}
&F(t_1,\ldots,uv,\ldots,t_n) - F(t_1,\ldots,u,\ldots,t_n) - F(t_1,\ldots,v,\ldots,t_n)\\
&\quad = M_s\Big[\Big(\Delta_{t_1}\cdots\Delta_{uv}\cdots\Delta_{t_n} - \Delta_{t_1}\cdots\Delta_{u}\cdots\Delta_{t_n} - \Delta_{t_1}\cdots\Delta_{v}\cdots\Delta_{t_n}\Big)f(s)\Big]\\
&\quad = M_s\Big[\Delta_{t_1}\cdots(\Delta_{uv}-\Delta_u-\Delta_v)\cdots\Delta_{t_n}f(s)\Big]\\
&\quad = M_s\Big[\Delta_{t_1}\cdots(\tau_{uv}-I-\tau_u+I-\tau_v+I)\cdots\Delta_{t_n}f(s)\Big]\\
&\quad = M_s\Big[\Delta_{t_1}\cdots(\tau_u-I)(\tau_v-I)\cdots\Delta_{t_n}f(s)\Big]\\
&\quad = M_s\Big[\Delta_{t_1}\cdots\Delta_u\Delta_v\cdots\Delta_{t_n}f(s)\Big]\\
&\quad = M_s\Big[(\tau_{t_1}-I)\Delta_{t_2}\cdots\Delta_u\Delta_v\cdots\Delta_{t_n}f(s)\Big]\\
&\quad = M_s\Big[\tau_{t_1}(\Delta_{t_2}\cdots\Delta_u\Delta_v\cdots\Delta_{t_n})f(s)\Big] - M_s\Big[\Delta_{t_2}\cdots\Delta_u\Delta_v\cdots\Delta_{t_n}f(s)\Big] = 0.
\end{aligned}$$

□

## References


[1] R. Badora, *On some generalized invariant means and their applications to the stability of the Hyers–Ulam type*, Ann. Polon. Math. *58* (1993), 147–159.

[2] R. Badora, R. Ger and Zs. Páles, *Additive selections and the stability of the Cauchy functional equation*, preprint.

[3] M. M. Day, *Amenable semigroups*, Illinois J. Math. *1* (1957), 509–544.

[4] G.-L. Forti, *Hyers–Ulam stability of functional equations in several variables*, Aequationes Math. *50* (1995), 143–190.

[5] Z. Gajda, *Invariant means and representation of semigroups in the theory of functional equations*, Pr. Nauk. Uniw. Śl. Katow. *1273*, 1992.

[6] R. Ger, *The singular case in the stability behaviour of linear mappings*, Grazer Math. Ber., *316* (1992), 59–70.

[7] R. Ger, *A survey of recent results on stability of functional equations,* Proc. of the 4th International Conference on Functional Equations and Inequalities, Pedagogical University in Cracow, 1994, 5–36.

[8] E. Hewitt and K. A. Ross, *Abstract Harmonic Analysis*, Die Grundlehren der Mathematischen Wissenschaften, Vol. 115, Springer-Verlag, Berlin–Göttingen–Heidelberg, 1963.

[9] R. B. Holmes, *Geometric Functional Analysis and its Applications*, Graduate Texts in Mathematics, Vol. 24, Springer-Verlag, New York–Heidelberg–Berlin, 1975.

[10] D. H. Hyers, *On the stability of linear functional equation*, Proc. Nat. Acad. Sci. U.S.A. *27* (1941), 222–224.

[11] D. H. Hyers and Th. M. Rassias, *Approximate homomorphisms*, Aequationes Math. *44* (1992), 125–153.
[12] M. Kuczma, *An Introduction to the Theory of Functional Equations and Inequalities*, Państwowe Wydawnictwo Naukowe, Warszawa–Krakow–Katowice, 1985.
[13] K. Nikodem and Zs. Páles, *A characterization of midpoint-quasiaffine functions*, Publ. Math. Debrecen *52* (1998), 575–595.
[14] K. Nikodem, Zs. Páles and Sz. Wąsowicz, *Abstract separation theorems of Rodé type and their applications*, Ann. Polonici Math., submitted.
[15] Zs. Páles, *Generalized stability of the Cauchy functional equation*, Aequationes Math., to appear.
[16] Zs. Páles, *11. Problem* in Report of Meeting, Aequationes Math. *55* (1998), 301.
[17] K. Przesławski and D. Yost, *Continuity properties of selectors and Michael's theorem*, Michigan Math. J. *36* (1989), 113–134.
[18] L. Székelyhidi, *A note on Hyers's theorem*, C. R. Math. Rep. Acad. Sci. Canada *VIII* (1986), 127–129.
[19] Jacek Tabor, *Monomial selections of set-valued functions*, Publ. Math. Debrecen, submitted.
[20] K. Yosida, *Functional Analysis*, Grundlehren der Mathematischen Wissenschaften, Vol. 123, Springer-Verlag, New York–Berlin–Heidelberg, 1980.

R. Badora
Institute of Mathematics
Silesian University
ul. Bankowa 14
PL-40-007 Katowice
Poland
e-mail: robadora@gate.math.us.edu.pl

Zs. Páles
Institute of Mathematics and Informatics
Lajos Kossuth University
Pf. 12
H–4010 Debrecen
Hungary
e-mail: pales@math.klte.hu

L. Székelyhidi
Institute of Mathematics and Informatics
Lajos Kossuth University
Pf. 12
H–4010 Debrecen
Hungary
e-mail: szekely@math.klte.hu

Manuscript received: June 30, 1998 and, in final form, April 26, 1999.