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Vertex-edge domination in graphs

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Abstract. We establish that for any connected graph G of order $n \geq 6$, a minimum vertex-edge dominating set of G has at most $n/3$ vertices, thus affirmatively answering the open question posed by Boutrig et al. (Aequ Math 90(2):355–366, 2016).

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1. Introduction

Let $G = (V_G, E_G)$ be a graph. A vertex $u \in V_G$ is said to *ve-dominate* an edge $xy \in E_G$ if (1) $u = x$ or $u = y$, that is, u is incident to xy, or (2) ux or uy is an edge in G. A set $D \subseteq V_G$ is a *vertex-edge dominating* set (or simply, a *ve-dominating* set) of G if for every edge $e \in E_G$, there exists a vertex $u \in D$ such that u *ve*-dominates e. The *vertex-edge domination number* of G, denoted $\gamma_{ve}(G)$, is the minimum cardinality of a vertex-edge dominating set of G. Herein, our main result is the following theorem.

Theorem 1.1. *If G is a connected graph of order $n \geq 6$, then $\gamma_{ve}(G) \leq \lfloor \frac{n}{3} \rfloor$.*

In other words, we affirmatively answer the question posed by Boutrig et al. [3]. So far, only a partial answer has been known, that is, the aforementioned (tight) upper bound holds for any C_5-free connected graph [3] (and so for any tree as established also in [14]).

Background. The concept of vertex-edge domination in graphs was introduced by Peters [17], and then investigated by several authors, in particular, lower and upper bounds on the vertex-edge domination number in different graph classes were studied in [3,14–16,19], vertex-edge degrees and vertex-edge

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domination polynomials of graphs were considered for example in [5,20,21], while [3,6,15,16] focused on relations between some ve-domination parameters, and algorithmic aspects were discussed in [15]. Finally, some other variants—total, global, etc.—of ve-domination were studied in [2,7,12,13,18].

From a practical point of view, the problem contributes to a bunch of applications related to graph searching/guarding problems, see for example [1,9,11]. In particular, the concept of ve-domination may be thought of as a variation on the searchlight guarding problem [22] or—when restricted to connected plane graphs with particular embeddings—on the k-periscope guarding problem in grids [10]. In addition, ve-domination is applicable in chemical graph theory [5,8].

Notation. Let $G = (V_G, E_G)$ be a connected graph of order $|V_G| = n$. The neighborhood of a vertex v in G is denoted by $N_G(v)$, while its degree is denoted by $\deg_G(v)$. For a set $S \subseteq V_G$, the set of all un-ve-dominated edges (by any element of S) in G is denoted by $un_G(S)$. A P_3-*packing* of G is a set of vertex-disjoint 3-vertex paths in G, and a *maximum* P_3-packing of G is a P_3-packing of G covering the maximum number of vertices in G. Finally, for a maximum P_3-matching $\mathbb{P}_3$ of G, the set of all vertices of paths in $\mathbb{P}_3$ is denoted by $V(\mathbb{P}_3)$, while the set of all degree two vertices (so-called *centers*) of all paths in $\mathbb{P}_3$ is denoted by $C(\mathbb{P}_3)$. All the other graph theory terminology not presented here can be found for example in [4].

2. Proof of Theorem 1.1

Let $\mathbb{P}_3$ be a maximum P_3-matching of a connected simple graph $G = (V_G, E_G)$ of order $n \geq 6$ that minimizes the cardinality of $un_G(C(\mathbb{P}_3))$ over all maximum P_3-matchings of G. Clearly, we have the following observation.

Observation. *If* $un_G(C(\mathbb{P}_3)) = \emptyset$ *then* $\gamma_{ve}(G) \leq |C(\mathbb{P}_3)|$, *and hence* $\gamma_{ve}(G) \leq \lfloor \frac{n}{3} \rfloor$.

Therefore, all we need is to argue that indeed $un_G(C(\mathbb{P}_3)) = \emptyset$. Our proof is based on a sequence of claims (some of their simple proofs could be omitted, however, we present all of them, repetitively, for the convenience of the reader).

Suppose to the contrary that $|un_G(C(\mathbb{P}_3))| > 0$ and let xy be an edge that belongs to $un_G(C(\mathbb{P}_3))$. For $v \in \{x, y\}$, let $\bar{v}$ denote the unique vertex in $\{x, y\} \backslash \{v\}$.

Claim 2.1. *If* $\deg_G(v) \geq 2$ *then* $N_G(v) \backslash \{\bar{v}\} \subseteq V(\mathbb{P}_3) \backslash C(\mathbb{P}_3)$.

Proof. The fact that $N_G(v) \backslash \{\bar{v}\} \subseteq V(P_3)$ follows directly from the maximality of $\mathbb{P}_3$, while the fact that $N_G(v) \backslash \{\bar{v}\} \cap C(P_3) = \emptyset$—from the assumption that $xy \in un_G(C(\mathbb{P}_3))$. □

So let $\Pi = v\bar{v}v_1^1v_2^1v_3^1v_1^2v_2^2v_3^2 \ldots v_1^kv_2^kv_3^k$ be <u>any</u> of the longest paths in G, taken over $v \in \{x, y\}$, such that the 3-vertex path $v_1^jv_2^jv_3^j$ belongs to $\mathbb{P}_3$, $j =$

$1, \ldots, k$ (notice that such a path Π exists as G is connected); let V_Π denote the vertex set of Π.

Claim 2.2. *$N_G(v_3^j) \subseteq V(\mathbb{P}_3) \cup \{x, y\}$ for any $j = 1, \ldots, k$.*

Proof. Suppose to the contrary that there exists $z \in N_G(v_3^j) \backslash (V(\mathbb{P}_3) \cup \{x, y\})$ for some $1 \leq j \leq k$. Then

$$\mathbb{P}_3' = \left(\mathbb{P}_3 \backslash \bigcup_{i=1}^{j} \{v_1^i v_2^i v_3^i\}\right) \cup \left\{v\bar{v}v_1^1, v_2^1 v_3^1 v_1^2, v_2^2 v_3^2 v_1^3, \ldots, v_2^{j-1} v_3^{j-1} v_1^j, v_2^j v_3^j z\right\}$$

is a P_3-packing of G with $|V(\mathbb{P}_3')| > |V(\mathbb{P}_3)|$, which contradicts the maximality of $\mathbb{P}_3$. □

Claim 2.3. *$N_G(v_2^j) \subseteq V(\mathbb{P}_3)$ for any $j = 1, \ldots, k$.*

Proof. Suppose to the contrary that there exists $z \in N_G(v_2^j) \backslash V(\mathbb{P}_3)$ for some $1 \leq j \leq k$. If $z \in \{x, y\}$, then edge xy is ve-dominated by $v_2^j \in C(\mathbb{P}_3)$, a contradiction with $xy \in un_G(C(\mathbb{P}_3))$. Next, if $z \notin \{x, y\}$, then

$$\mathbb{P}_3' = \left(\mathbb{P}_3 \backslash \bigcup_{i=1}^{j} \{v_1^i v_2^i v_3^i\}\right) \cup \left\{v\bar{v}v_1^1, v_2^1 v_3^1 v_1^2, v_2^2 v_3^2 v_1^3, \ldots, v_2^{j-1} v_3^{j-1} v_1^j, z v_2^j v_3^j\right\}$$

is a P_3-packing of G with $|V(\mathbb{P}_3')| > |V(\mathbb{P}_3)|$, which contradicts the maximality of $\mathbb{P}_3$. □

Claim 2.4. *$\left(N_G(v_3^j) \backslash \{v_2^1, v_2^2, \ldots, v_2^{j-1}\}\right) \cap C(\mathbb{P}_3) = \emptyset$ for any $j = 2, \ldots, k$.*

Proof. Suppose that there exists $z \in \left(N_G(v_3^j) \backslash \{v_2^1, \ldots, v_2^{j-1}\}\right) \cap C(\mathbb{P}_3)$ for some $2 \leq j \leq k$. Observe that edge xy is ve-dominated by v_1^1 and any edge incident to v_3^j is ve-dominated by z. Next, it follows from Claims 2.2 and 2.3 that all the other edges that are ve-dominated <u>only</u> by the vertices $v_2^1, v_2^2, \ldots, v_2^j \in C(\mathbb{P}_3)$ can be also ve-dominated by the vertices $v_1^1, v_1^2, \ldots, v_1^j$. Therefore

$$\mathbb{P}_3' = \left(\mathbb{P}_3 \backslash \bigcup_{i=1}^{j} \{v_1^i v_2^i v_3^i\}\right) \cup \left\{\bar{v}v_1^1 v_2^1, v_3^1 v_1^2 v_2^2, \ldots, v_3^{j-1} v_1^j v_2^j\right\}$$

is another maximum P_3-packing of G and $|un_G(C(\mathbb{P}_3'))| < |un_G(C(\mathbb{P}_3))|$, which contradicts the choice of $\mathbb{P}_3$. □

Claim 2.5. *$N_G(v_3^k) \subseteq V_\Pi$, and moreover, $v \in N_G(v_3^k)$ (and hence $\deg_G(v_3^k) \geq 2$).*

Proof. The fact that $N_G(v_3^k) \subseteq V_\Pi$ follows immediately from the maximality of k and Claims 2.2 and 2.4. Next, suppose to the contrary that $v \notin N_G(v_3^k)$.

Again, it follows from Claims 2.2 and 2.3 that all the edges that are *ve*-dominated <u>only</u> by the vertices $v_2^1, v_2^2 \ldots, v_2^k \in C(\mathbb{P}_3)$ can be also *ve*-dominated by the vertices $v_1^1, v_1^2 \ldots, v_1^k$. Therefore

$$\mathbb{P}_3' = \left(\mathbb{P}_3 \backslash \bigcup_{i=1}^{k} \{v_1^i v_2^i v_3^i\}\right) \cup \{\bar{v}v_1^1 v_2^1, v_3^1 v_1^2 v_2^2, \ldots, v_3^{k-1} v_1^k v_2^k\}$$

is a P_3-packing of G with $V(\mathbb{P}_3') = V(\mathbb{P}_3)$ and $|un_G(C(\mathbb{P}_3'))| < |un_G(C(\mathbb{P}_3))|$ as v_1^1 also *ve*-dominates xy, which contradicts the choice of $\mathbb{P}_3$. □

Claim 2.6. *$N_G(v_1^j) \subseteq V(\mathbb{P}_3) \cup \{x, y\}$ for any $j = 1, \ldots, k$.*

Proof. Recall that we have assumed that Π is <u>any</u> of the longest paths taken over $v \in \{x, y\}$. Therefore, since $v \in N_G(v_3^k)$ (by Claim 2.5), the reversed path $\bar{v}vv_3^k v_2^k v_1^k \ldots v_3^1 v_2^1 v_1^1$ is of the same length as Π and so it can also play the role of Π in Claim 2.2, which results in the desired property. □

Now, the crucial observation is that we must have $|V_\Pi| = 5$, that is, $k = 1$. Indeed, suppose to the contrary that $k \geq 2$.

Claim 2.7. *If $k \geq 2$ then the set $\{v, \bar{v}, v_1^k, v_2^k, v_3^k\}$ induces a 5-vertex cycle in G.*

Proof. Suppose that $\{v, \bar{v}, v_1^k, v_2^k, v_3^k\}$ does not induce a 5-vertex cycle in G (recall that $vv_2^k, \bar{v}v_2^k \notin E_G$ by Claim 2.3). Let $O_G(v_2^k) \subseteq E_G$ be the set of edges that are *ve*-dominated <u>only</u> by $v_2^k \in C(\mathbb{P}_3)$ (and no other vertex in $C(\mathbb{P}_3)$). We consider two cases.

Case 1: $\bar{v}v_1^k \notin E_G$. It follows then from Claims 2.2, 2.3 and 2.6 that v_3^k dominates any edge in $O_G(v_2^k)$, and therefore

$$\mathbb{P}_3' = \left(\mathbb{P}_3 \backslash \{v_1^k v_2^k v_3^k\}\right) \cup \{v_2^k v_3^k v\}$$

is another maximum P_3-packing of G and $|un_G(C(\mathbb{P}_3'))| < |un_G(C(\mathbb{P}_3))|$ as v_3^k also *ve*-dominates xy, which contradicts the choice of $\mathbb{P}_3$.

Case 2: $\bar{v}v_1^k \in E_G$. Then $vv_1^k \in E_G$ or $\bar{v}v_3^k \in E_G$, or $v_1^k v_3^k \in E_G$. If $vv_1^k \in E_G$ then it follows from Claims 2.2, 2.3 and 2.6 that v dominates any edge in $O_G(v_2^k)$. Therefore

$$\mathbb{P}_3' = \left(\mathbb{P}_3 \backslash \{v_1^k v_2^k v_3^k\}\right) \cup \{v_3^k v\bar{v}\}$$

is another maximum P_3-packing of G and $|un_G(C(\mathbb{P}_3'))| < |un_G(C(\mathbb{P}_3))|$ as v also *ve*-dominates xy, which contradicts the choice of $\mathbb{P}_3$.

Otherwise, if $vv_1^k \notin E_G$ then we must have $\bar{v}v_3^k \in E_G$ or $v_1^k v_3^k \in E_G$. It follows from Claims 2.2, 2.3 and 2.6 that v_3^k dominates any edge in $O_G(v_2^k)$, and therefore

$$\mathbb{P}_3' = \left(\mathbb{P}_3 \backslash \{v_1^k v_2^k v_3^k\}\right) \cup \{v_2^k v_3^k v\}$$

is another maximum P_3-packing of G and $V(\mathbb{P}_3') = V(\mathbb{P}_3)$ and $|un_G(C(\mathbb{P}_3'))| < |un_G(C(\mathbb{P}_3))|$, which is again a contradiction with the choice of $\mathbb{P}_3$. □

Claim 2.8. *If $k \geq 2$ then the set $\{v, \bar{v}, v_1^1, v_2^1, v_3^1\}$ induces a 5-vertex cycle in G.*

Proof. Analogously as in the proof of Claim 2.6, by reversing the path Π and following the arguments in the proof of Claim 2.7, we obtain that $\{v, \bar{v}, v_1^1, v_2^1, v_3^1\}$ also induces a 5-vertex cycle in G. □

Next, in order to argue $k = 1$, we consider two cases.

Case 1: $v_2^1 v_2^k \notin E_G$. Observe that in this case, keeping in mind Claim 2.3, each edge $v_2^1 z \in E_G$, where $z \neq v_1^1, v_3^1$ (recall that $v_2^1 v, v_2^1 \bar{v} \notin E_G$), is *ve*-dominated by some $v_2^i \in C(\mathbb{P}_3)$, $2 \leq i \leq k-1$. By the same argument, each edge $v_2^k z \in E_G$, where $z \neq v_1^k, v_3^k$ (again recall that $v_2^k v, v_2^k \bar{v} \notin E_G$), is *ve*-dominated by some $v_2^j \in C(\mathbb{P}_3)$, $2 \leq j \leq k-1$. Therefore, by exchanging two paths $v_1^1 v_2^1 v_3^1$ and $v_1^k v_2^k v_3^k$ in $\mathbb{P}_3$ with paths $v_1^1 \bar{v} v_1^k$ and $v_3^k v v_3^1$, we obtain another maximum P_3-packing $\mathbb{P}_3'$, but with $|un_G(C(\mathbb{P}_3'))| < |un_G(C(\mathbb{P}_3))|$ (by Claim 2.3) as now xy is also *ve*-dominated, which contradicts the choice of $\mathbb{P}_3$.

Case 2: $v_2^1 v_2^k \in E_G$. Similarly as above, keeping in mind Claim 2.3, observe that each edge $v_2^k z \in E_G$, where $z \neq v_1^k, v_3^k$, in particular, edge $v_2^1 v_2^k$, is *ve*-dominated by $v_2^1 \in C(\mathbb{P}_3)$. Also, keeping Claims 2.2 and 2.7 in mind, each edge $v_3^k z \in E_G$, where $z \neq v, v_2^k$, is *ve*-dominated by some $v_2^j \in C(\mathbb{P}_3)$, $1 \leq j \leq k-1$. Consequently, exchanging path $v_1^k v_2^k v_3^k$ in $\mathbb{P}_3$ with $v_3^k v \bar{v}$ results in another maximum P_3-packing $\mathbb{P}_3'$, but with $|un_G(C(\mathbb{P}_3'))| < |un_G(C(\mathbb{P}_3))|$ (by Claim 2.3) as now xy is also *ve*-dominated, which contradicts the choice of $\mathbb{P}_3$, and ultimately $k > 1$.

We now continue with a sequence of claims for $k = 1$.

Claim 2.9. *The set $\{v, \bar{v}, v_1^1, v_2^1, v_3^1\}$ induces a 5-vertex cycle in G.*

Proof. The arguments are similar to those in the proof of Claim 2.7, see Case 2. Namely, suppose to the contrary that vv_1^1 or $\bar{v}v_3^1$, or $v_1^1 v_3^1$ belongs to E_G (recall $v_2^1 v, v_2^1 \bar{v} \notin E_G$). If $vv_1^1 \in E_G$, then exchange $v_1^1 v_2^1 v_3^1$ in $\mathbb{P}_3$ with $v_3^1 v \bar{v}$, otherwise, exchange path $v_1^1 v_2^1 v_3^1$ in $\mathbb{P}_3$ with $v_2^1 v_3^1 v$. It follows from Claims 2.2, 2.3 and 2.6 that the resulting 3-vertex path set $\mathbb{P}_3'$ is another maximum P_3-packing $\mathbb{P}_3'$ with $|un_G(C(\mathbb{P}_3'))| < |un_G(C(\mathbb{P}_3))|$ as now xy is also *ve*-dominated, which contradicts the choice of $\mathbb{P}_3$. □

Claim 2.10. $\deg_G(v_1^1) = \deg_G(v_2^1) = \deg_G(v_3^1) = 2$.

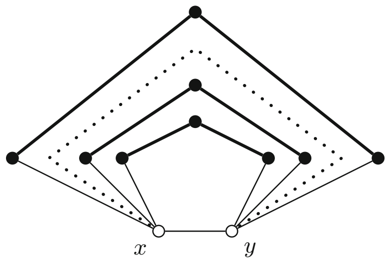

Figure 1. The only possible left case; 3-vertex paths in $\mathbb{P}_3$ are marked with bold lines

Proof. The equality $\deg_G(v_1^1) = \deg_G(v_3^1) = 2$ follows from Claims 2.2, 2.6 and 2.9. Suppose now that $\deg_G(v_2^1) \geq 3$. Let $z \in N_G(v_2^1)\backslash\{v_1^1, v_3^1\}$ (recall that $v_2^1 v, v_2^1 \bar{v} \notin E_G$). If $z \in C(\mathbb{P}_3)$ then, considering Claim 2.3, exchanging path $v_1^1 v_2^1 v_3^1$ in $\mathbb{P}_3$ with $v_3^1 v \bar{v}$ results in another maximum P_3-packing with $|un_G(C(\mathbb{P}_3'))| < |un_G(C(\mathbb{P}_3))|$ as xy is also ve-dominated, which contradicts the choice of $\mathbb{P}_3$.

Otherwise, if $z \notin C(\mathbb{P}_3)$, then exchange path $v_1^1 v_2^1 v_3^1$ in $\mathbb{P}_3$ with $v_2^1 v_3^1 v$. It follows from Claim 2.3, the choice of xy, and $\deg_G(v_1^1) = 2$ that all the other edges that have been ve-dominated by elements in $C(\mathbb{P}_3)$ remain ve-dominated by elements in $C(\mathbb{P}_3')$. Therefore, the resulting P_3-packing $\mathbb{P}_3'$ is a maximum P_3-packing, with $|un_G(C(\mathbb{P}_3'))| \leq |un_G(C(\mathbb{P}_3))|$: the edge xy is now ve-dominated, but edge $\bar{v}v_1^1$ is not. Moreover, it follows from the choice of $\mathbb{P}_3$ that $|un_G(C(\mathbb{P}_3'))| = |un_G(C(\mathbb{P}_3))|$ must hold. Now, for the un-ve-dominated edge $\bar{v}v_1^1 \in un_G(C(\mathbb{P}_3'))$, by repeating all the aforementioned arguments applied to edge xy, we obtain that $\deg_G(v_2^1) = 2$ (see the discussion above when discussing the equality $k = 1$), which is a contradiction. □

Concluding, we are driven to the case where the only possibility for G is to consist of a number of (at least two as $n \geq 6$) 5-vertex cycles, all of them, pairwisely, sharing only the edge xy (see Fig. 1 for an illustration). But then, by replacing two 3-vertex paths in $\mathbb{P}_3$ with two vertex-disjoint 3-vertex paths with centers at x and y, respectively, we obtain another maximum P_3-packing $\mathbb{P}_3'$ with $un_G(C(\mathbb{P}_3')) = \emptyset$, and therefore, we must also have $un_G(\mathbb{P}_3) = \emptyset$, a final contradiction.

3. γ_{ve}-extremal graphs

The P_2-*corona* of a graph $G = (V_G, E_G)$ is the graph of order $3|V_G|$ obtained from G by attaching a distinct path P_2 to each vertex $v \in V_G$ by adding an edge between v and a leaf of its corresponding path P_2, while the *corona* $G \circ H$ of G and another graph H is the graph formed from one copy of G and $|V_G|$

copies of H, where the i-th vertex of G is adjacent to every vertex in the i-th copy of H.

A graph G of order n is called γ_{ve}*-extremal* if $\gamma_{ve}(G) = \lfloor n/3 \rfloor$. The complete characterization of all ve-extremal trees was given in [3,14]: a tree T is γ_{ve}-extremal if and only if T is a P_2-corona of some tree. As regards arbitrary graphs, one can easily observe that if G is a P_2-corona or $G = H \circ P_2$ for some graph H, then G is γ_{ve}-extremal—however, to the best of our knowledge, the complete characterization of all γ_{ve}-extremal graphs remains an open problem.

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