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On Observer Design for a Class of Nonlinear Systems Including Unknown Time-Delay

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Abstract. The observer design for nonlinear systems with unknown, bounded, time-varying delays, on both input and state, is still an open problem for researchers. In this paper, a new observer design for a class of nonlinear system with unknown, bounded, time-varying delay was presented. For the proof of the observer stability, a Lyapunov–Krasovskii function was chosen. Sufficient assumptions are provided to prove the practical stability of the proposed observer. Furthermore, the exponential convergence of the observer was proved in the case of a constant time delay. Simulation results were shown to illustrate the feasibility of the proposed strategy.

Mathematics Subject Classification. Primary 93Dxx; Secondary 34Dxx.

Keywords. Observer, exponential stability, practical stability, Lyapunov–Krasovskii, time delay.

1. Introduction

The observer design for nonlinear systems is a well-known issue in the control theory and it is still an open problem for researchers [21,24,26,28,30]. The analysis of nonlinear systems under time delays is typically more difficult compared to systems without time delays [6,12,15,17,29]. In literature, there are several works on the stability of small delayed systems, for example, the work of [4,22]. Time delay is a property of various dynamical systems, for example, in communications, tele-operation, biological embedded systems, electrical, mechanical, and many other applications [3,7,25]. The observer design for time-delay nonlinear systems has been an attractive alternative for the researchers in recent years. In addition, significant efforts have been made to solve such a problem and many observation approaches have been used. For example, those based on exponential method [10,18], asymptotic approach [11,16], numerical approach [2], sliding mode approach [14,25], algebraic method [9,23], H_∞ method [19,31], and so on. Such methods have been developed for linear [16,27] and for nonlinear [10,18], time-delay systems.

Practically, an exponential convergence of the observation error of a high-gain observer for nonlinear system in triangular form under unknown time delay has been proposed in [10]. Nevertheless, the time delay is still regarded the same for the system and its observer.

In [11], a state observer for single-output single-input nonlinear delay systems has been suggested. Assumptions for exponential observation error with constant known time delay are shown. In [18], an adaptive observer is suggested for nonlinear systems presented in a triangular form. It has been presented that the observer gain has been caused only by updating one parameter of the Riccati equation. Nevertheless, this design is constructed by the fact that the system constant delay is known. Based on the theory of linear systems with time delay, in [23], it has been presented how to design an observer for a class of nonlinear systems under constant time delay. Finally, other notions of parameter identifiability for nonlinear systems under known time delay have been shown in [32]. However, the observation techniques outlined above suppose that the time delay is known [1,10,18,23].

From a practical point of view, dynamics, measurement, noises or disturbances often prevent the error signals from tending to zero. Thus, the property of ultimate boundedness is often established. In this case, all state trajectories are bounded and approach the origin (or some of its sufficiently small neighborhood) in a sufficiently fast manner. This property is referred to as 'practical' stability, which is more suitable in several situations than Lyapunov stability (see [5,8,12]).

In this work, based on the work of [12], a new class of nonlinear systems under unknown small time-varying delay is proposed. A Luenberger observer is suggested, and new criteria are given to insure the practical stability in which the error converges to a small ball. In the case of a constant time delay, the exponential stability of the observer is described. An appropriate choice of the Lyapunov–Krasovskii function (see [20] for more details) is used for the stability analysis of the proposed observer.

The paper is organized as follows. In Sect. 2, the practical stability definition is presented. In Sect. 3, the system description is shown. The observer design and its stability analysis are given in Sect. 4. In Sect. 5, an illustrative example is described and the simulation results show the performances of the suggested observer. Finally, some concluding remarks are given in Sect. 6.

2. Preliminary

We consider the following system:

$$\begin{cases} \dot{x} = f(t, x(t), x(t-\tau(t))), & t \geq 0 \\ x(s) = \varphi(s), & \forall s \in [-\tau^*, 0], \end{cases} \tag{2.1}$$

where $\tau^* = \sup(\tau(t))_{t\in\mathbb{R}_+}$ and $x(t)$ is the system solution with initial function φ verifying:

$$x(s) = \varphi(s), \quad \forall s \in [-\tau^*, 0],$$

φ is a constant function in the Banach space $\mathcal{C}_{n,\tau^*} := \mathcal{C}([-\tau^*, 0], \mathbb{R}^n)$ with norm:

$$\|\varphi\|_{\tau^*} := \max_{s\in[-\tau^*,0]} \|\varphi(s)\|,$$

$f\colon \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a piecewise continuous function in t, and locally Lipschitz in x, and $\tau(t)$ is the time-varying delay.

The definition of the practical stability introduced in [3] is presented as follows:

Definition 2.1 [3]. Equation (2.1) is said to be globally uniformly practically exponentially stable if there exists a ball $B_R = \{x \in \mathbb{R}^n \colon \|x\| \leq R\}$ such that B_R is globally uniformly practically exponentially stable, it means that: there exists $R \geq 0$ such that for all $t \geq t_0$ and $\varphi \in \mathcal{C}_{n,\tau^*}$,

$$\|x(t, t_0, \varphi)\| \leq R + \lambda_1 \|\varphi\| \exp(-\lambda_2(t - t_0)),$$

with $\lambda_1 > 0$, $\lambda_2 > 0$.

When $R = 0$, in this case the origin is an equilibrium point, then we point the classical definition of the exponential stability.

Remark 2.2. For stability purpose, we introduce the following assumption: There exist positive constants α_1, α_2, and ζ such that

$$\|f(t, x, y) - f(t, \bar{x}, \bar{y})\| \leq \alpha_1 \|x - \bar{x}\| + \alpha_2 \|y - \bar{y}\|,$$

for all $t \geq 0$, and $x,\ y,\ \bar{x},\ \bar{y} \in \mathbb{R}^n$. Moreover, $\|f(t, 0, 0)\| \leq \zeta$ for all $t \geq 0$.

Note that under this assumption, Eq. (2.1) has a unique solution denoted by $x(t)$. Moreover, the system admits a trivial solution $x(t) = 0$ only when $\zeta = 0$ (see [13]). When $\zeta \neq 0$, then the origin is not necessarily an equilibrium point. It turns out that, under this assumption, a new neighborhood of the origin attracting solution of (2.1) can be estimated.

3. System Description

A class of nonlinear time-delay systems is presented as follows:

$$\begin{cases} \dot{x} = Ax(t) + f(x(t), x_{\tau(t)}, u(t), u_{\tau(t)}), & t \geq 0 \\ y(t) = Cx(t) & \\ x(s) = \varphi(s), & \forall s \in [-\tau^*, 0], \end{cases} \tag{3.1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system, $u(t) \in \mathbb{R}^m$ is the input, $y(t) \in \mathbb{R}^p$ represents the output of the system and $x_{\tau(t)} = x(t - \tau(t))$ and $u_{\tau(t)} = u(t - \tau(t))$ are, respectively, the delayed state and input, and $x = \begin{pmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix}$, $x_{\tau(t)} = \begin{pmatrix} x_{1,\tau(t)} \\ \cdot \\ \cdot \\ \cdot \\ x_{n,\tau(t)} \end{pmatrix}$, $A \in \mathcal{M}_n(\mathbb{R})$, $C \in \mathcal{M}_{p,n}(\mathbb{R})$ where $x_{i,\tau(t)} = x_i(t - \tau(t))$, for $i = 1, \ldots, n$ and $\tau^* > 0$ denotes the known upper bound of $\tau(t)$, and the pair(A, C) is observable.

To complete the description of system (3.1), the following assumptions are considered.

($\mathcal{A}_1$) The state and the input are bounded, that is, $x(t) \in K \subset \mathbb{R}^n$ (that is a compact subset of $\mathbb{R}^n$).

($\mathcal{A}_2$) The function $f(x(t), x_{\tau(t)}, u(t), u_{\tau(t)})$ is globally Lipschitz (on K) with respect to x, $x_{\tau(t)}$ and $u_{\tau(t)}$, uniformly with respect to u.

4. Observer Design

In this section of the paper, we are interested in designing an observer to estimate the states of the time-delay nonlinear system (3.1). Generally, not all the states of a system are available for direct measurement. Then, the unmeasured states must be observed if they are needed for control. The observer is a dynamical system which estimates the states of the system. The main objective in the next section is to design a state estimator for the system (3.1) such that the practical stability convergence of the resulting error system can be guaranteed. The following state observer is proposed:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*}) + L\{Cx(t) - \hat{y}(t)\} \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{4.1}$$

where $\hat{x}(t)$ denotes the estimate of the state $x(t)$. The observation problem consists in finding a gain L so that the system (4.2) is stable.

The dynamics of the observer error is expressed as follows:

$$\dot{e} = \dot{x} - \dot{\hat{x}} = (A - LC)e + f(x(t), x_{\tau(t)}, u(t), u_{\tau(t)}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*}). \tag{4.2}$$

Remark 4.1. For the nominal part of the system (3.1) which is linear, since (A, C) is an observable canonical form, there exists a gain matrix L such that for all positive definite symmetric matrix Q, there exists a positive definite symmetric matrix P which satisfies.

$$PA_c + A_c^T P = -Q, \tag{4.3}$$

where $A_c = A - LC$.

Remark 4.2. To invoke assumptions ($\mathcal{A}_1$) and ($\mathcal{A}_2$), the term $f(x(t), x_{\tau(t)}, u(t), u_{\tau(t)}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*})$ is rewritten as follows by adding and subtracting $f(x(t), x_{\tau^*}, u(t), u_{\tau^*})$:

$$\begin{aligned} &f(x(t), x_{\tau(t)}, u(t), u_{\tau(t)}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*}) \\ &\quad = f(x(t), x_{\tau^*}, u(t), u_{\tau^*}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*}) + \bar{f}, \\ &\bar{f} = f(x(t), x_{\tau(t)}, u(t), u_{\tau(t)}) - f(x(t), x_{\tau^*}, u(t), u_{\tau^*}). \end{aligned}$$

The following inequalities hold globally (on K) thanks to assumption ($\mathcal{A}_2$),

$$\begin{aligned} \|f(x(t), x_{\tau^*}, u, u_{\tau^*}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*})\| &\leq \nu\|(x - \hat{x})\| + \nu\|(x_{\tau^*} - \hat{x}_{\tau^*})\| \\ &\leq \nu\|e\| + \nu\|e_{\tau^*}\| \end{aligned} \tag{4.4}$$

$$\|\bar{f}\| \leq \nu_0 \|x_{\tau^*} - x_{\tau(t)}\| + \nu_0 \|u_{\tau^*} - u_{\tau(t)}\|, \tag{4.5}$$

where ν is a Lipschitz constant in (4.4), and $\nu_0 > \nu_{\bar{f}}$, with $\nu_{\bar{f}}$, is a Lipschitz constant of $\bar{f}$, in (4.5).

From assumption $(\mathcal{A}_1)$, there exists a bounded constant $\nu_1 > \nu_0 \nu_{xu}$ such that (4.5) can be written as follows:

$$\|\bar{f}\| \leq \nu_1, \tag{4.6}$$

where ν_{xu} is a positive constant which refers to the boundedness of $\|x_{\tau^*} - x_{\tau(t)}\| + \|u_{\tau^*} - u_{\tau(t)}\|$.

Theorem 4.3. *Suppose that assumptions* $(\mathcal{A}_1)$–$(\mathcal{A}_2)$ *are fulfilled, and there exist two matrices* Q *and* P *which verify Eq.* (4.3), *such that*

$$\lambda_{\min}(Q) > \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \left(2\lambda_{\max}(P)\nu + \frac{5}{4} + \lambda_{\max}(P)^2 \nu^2 \right), \tag{4.7}$$

with $\lambda_{\min}$ (*respectively,* $\lambda_{\max}$) *represents the minimum* (*respectively, the maximum*) *eigenvalue.*

Then, the error dynamics (4.2) *is globally* (*on* K) *practically exponentially stable.*

Proof. Define the Lyapunov–Krasovskii candidate function:

$$V(e) = V_1(e) + V_2(e), \tag{4.8}$$

with $V_1(e) = e^T P e$ which satisfies the following inequality:

$$\lambda_{\min}(P)\|e\|^2 \leq V_1(e) \leq \lambda_{\max}(P)\|e\|^2$$

and

$$V_2(e) = \int_{t-\tau*}^{t} e^{-\frac{\alpha}{2\tau*}(t-\sigma)} e^T(\sigma) e(\sigma) \mathrm{d}\sigma$$

with α a positive constant defined thereafter. Taking the time derivative of (4.8) along the trajectories of (4.2), and making use of (4.3), we have

$$\begin{aligned} \dot{V}(e) \leq & -\eta V_1 + 2e^T P \bar{f} \\ & + 2e^T P\{f(x(t), x_{\tau^*}, u(t), u_{\tau^*}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*})\} \\ & + e^T e - e_{\tau^*}^T e_{\tau^*} e^{-\frac{\alpha}{2}} - \frac{\alpha}{2\tau^*} V_2, \end{aligned} \tag{4.9}$$

where $\eta = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$. Inequality (4.9) can be written as:

$$\begin{aligned} \dot{V}(e) + \frac{\alpha}{2\tau^*} V(e) \leq & -\left(\eta - \frac{\alpha}{2\tau^*}\right) e^T P e + 2e^T P \bar{f} \\ & + 2e^T P\{f(x(t), x_{\tau^*}, u(t), u_{\tau^*}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*})\} \\ & + e^T e - e_{\tau^*}^T e_{\tau^*} e^{-\frac{\alpha}{2}}. \end{aligned} \tag{4.10}$$

We have $2e^TP\bar{f} \leq 2\|e\|\|P\|\|\bar{f}\|$ (by Cauchy–Schwarz), $\|P\| = \lambda_{\max}(P)$, and $\|\bar{f}\| \leq \nu_1$ (inequality (4.6)). So $2e^TP\bar{f} \leq 2\lambda_{\max}(P)\nu_1\|e\|$. We have

$$\begin{aligned}&2e^TP\{f(x(t), x_{\tau^*}, u(t), u_{\tau^*}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*})\}\\ &\quad\leq 2\|e\|\|P\|\|f(x(t), x_{\tau^*}, u(t), u_{\tau^*}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*})\|\\ &\quad\leq 2\lambda_{\max}(P)\|e\|\|f(x(t), x_{\tau^*}, u(t), u_{\tau^*}) - f(\hat{x}(t), \hat{x}_{\tau^*}, u(t), u_{\tau^*})\|\\ &\quad\leq 2\lambda_{\max}(P)\|e\|\big(\nu\|e\| + \nu\|e_{\tau^*}\|\big).\end{aligned}$$

So,

$$\begin{aligned}\dot{V}(e) + \frac{\alpha}{2\tau^*}V(e) \leq& -\lambda_{\min}(P)\left(\eta - \frac{\alpha}{2\tau^*}\right)\|e\|^2 + 2\lambda_{\max}(P)\nu_1\|e\|\\ &+ 2\lambda_{\max}(P)\nu\|e\|\|e_{\tau^*}\| + 2\lambda_{\max}(P)\nu\|e\|^2 + \|e\|^2\\ &- \|e_{\tau^*}\|^2e^{-\frac{\alpha}{2}}.\end{aligned} \tag{4.11}$$

Inequality (4.11) can be expressed as:

$$\begin{aligned}\dot{V}(e) + \frac{\alpha}{2\tau^*}V(e) \leq& \left(-\lambda_{\min}(P)\left(\eta - \frac{\alpha}{2\tau^*}\right) + 2\lambda_{\max}(P)\nu + 1\right)\|e\|^2\\ &+ 2\lambda_{\max}(P)\nu_1\|e\| + 2\lambda_{\max}(P)\nu\|e\|\|e_{\tau^*}\|\\ &- \|e_{\tau^*}\|^2e^{-\frac{\alpha}{2}}.\end{aligned}$$

So, we have

$$\begin{aligned}\dot{V}(e) + \frac{\alpha}{2\tau^*}V(e) \leq& -\xi_1(\alpha, \tau^*)\|e\|^2 + \theta\|e\|\\ &+ \xi_2\|e\|\|e_{\tau^*}\| - \|e_{\tau^*}\|^2e^{-\frac{\alpha}{2}}\end{aligned} \tag{4.12}$$

with $\xi_1(\alpha, \tau^*) = \big(\lambda_{\min}(P)(\eta - \frac{\alpha}{2\tau^*}) - (2\lambda_{\max}(P)\nu + 1)\big)$, $\xi_2 = 2\lambda_{\max}(P)\nu$ and $\theta = 2\lambda_{\max}(P)\nu_1$.

The following inequality is verified: $2xy \leq x^2 + y^2$; $x, y \in \mathbb{R}$. Replacing x by $\frac{1}{2}\|e\|$ and y by θ, we find

$$\theta\|e\| \leq \frac{1}{4}\|e\|^2 + \theta^2. \tag{4.13}$$

Using Eq. (4.13), inequality (4.12) can be written as:

$$\begin{aligned}\dot{V}(e) + \frac{\alpha}{2\tau^*}V(e) - \theta^2 \leq& -\left(\xi_1(\alpha, \tau^*) - \frac{1}{4}\right)\|e\|^2\\ &+ \xi_2\|e\|\|e_{\tau^*}\| - \|e_{\tau^*}\|^2e^{-\frac{\alpha}{2}}\end{aligned} \tag{4.14}$$

Now, the right site of the above inequality can be written as follows:

$$\begin{aligned}&-\left(\xi_1(\alpha, \tau^*) - \frac{1}{4}\right)\|e\|^2 + \xi_2\|e\|\|e_{\tau^*}\| - \|e_{\tau^*}\|^2e^{-\frac{\alpha}{2}}\\ &\quad= -\left(\xi_1(\alpha, \tau^*) - \frac{1}{4} - \frac{\xi_2^2}{4}e^{\frac{\alpha}{2}}\right)\|e\|^2 - \left(\frac{\xi_2}{2e^{-\frac{\alpha}{4}}}\|e\| - \|e_{\tau^*}\|e^{-\frac{\alpha}{4}}\right)^2.\end{aligned}$$

To satisfy inequality (4.14), all we need to do is to choose α such that

$$\left(\xi_1(\alpha,\tau^*) - \frac{1}{4} - \frac{\xi_2^2}{4}e^{\frac{\alpha}{2}}\right) > 0,$$

which is equivalent to

$$\lambda_{\min}(P)\frac{\alpha}{2\tau^*} + \frac{\xi_2^2}{4}(e^{\frac{\alpha}{2}} - 1) < \lambda_{\min}(Q)\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)} - \left(2\lambda_{\max}(P)\nu + \frac{5}{4} + \frac{\xi_2^2}{4}\right). \tag{4.15}$$

Let

$$\omega(x) = \lambda_{\min}(P)\frac{x}{2\tau^*} + \frac{\xi_2^2}{4}(e^{\frac{x}{2}} - 1),$$

we have $\omega(x) > 0 \ \forall x > 0$ and $\omega(0) = 0$. Let $\gamma = \lambda_{\min}(Q)\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)} - \left(2\lambda_{\max}(P)\nu + \frac{5}{4} + \frac{\xi_2^2}{4}\right)$.

Since ω is continuous at 0, there exists $\delta > 0$ such that $\forall x \in]0,\delta[$, $0 < \omega(x) < \frac{\gamma}{2}$.

We choose $\delta_1 = \min(\delta, 2\eta\tau^*)$. Let $\alpha \in]0,\delta_1[$; then inequality (4.15) is verified and $\eta - \frac{\alpha}{2\tau^*} > 0$.

Inequality (4.14) becomes

$$\dot{V}(e) \le -\frac{\alpha}{2\tau^*}V + \theta^2.$$

It follows that

$$V(e(t)) \le e^{-\frac{\alpha}{2\tau^*}t}V(e(0)) + 2\theta^2\frac{\tau^*}{\alpha}. \tag{4.16}$$

According to (4.8), we have

$$\begin{aligned} V(e(0)) &\le \lambda_{\max}(P)\|e(0)\|^2 - \int_{-\tau^*}^{0} e^{\frac{\alpha}{2\tau^*}\sigma}\|e(\sigma)\|^2 \mathrm{d}\sigma \\ &\le (\lambda_{\max}(P) + \tau^*)\mathrm{sup}_{s\in[-\tau^*,0]}\|e(s)\|^2, \end{aligned} \tag{4.17}$$

and

$$\lambda_{\min}(P)\|e(t)\|^2 \le V(e(t)), \tag{4.18}$$

then

$$\|e(t)\| \le \lambda_1 e^{-\frac{\alpha}{4\tau^*}t}\mathrm{sup}_{s\in[-\tau^*,0]}\|e(s)\| + \lambda_2 \tag{4.19}$$

where $\lambda_1 = \sqrt{\frac{\lambda_{\max}(P)+\tau^*}{\lambda_{\min}(P)}}$ and $\lambda_2 = \sqrt{\frac{2\theta^2\tau^*}{\alpha\lambda_{\min}(P)}}$.
So the error dynamics (4.2) is globally (on K) practically exponentially stable.
□

The following theorem provides the stability result in the case of a constant time delay in which we state the exponential stability behavior of the system (4.2).

Theorem 4.4. *Let τ_c be a known constant time delay. Consider system* (3.1) *with $\tau(t) = \tau_c$. If the function $f(x(t), x_{\tau_c}, u(t), u_{\tau_c})$ is globally Lipschitz (on the compact $K \subset \mathbb{R}^n$) with respect to x and x_{τ_c}, and uniformly with respect to u and u_{τ_c}. Then, system* (4.1) *is a globally (on K) exponential observer for system* (3.1).

Proof. The time delay is constant and known ($\tau(t) = \tau_c$). As a matter of the fact, we have $\theta = 0$ and, consequently, we have $\lambda_2 = 0$. Then, the error dynamics (4.2) is globally (on K) exponentially stable. □

5. Example

Let us consider the nonlinear system under unknown time-variable delay:

$$\begin{cases} \dot{x}_1 = -3x_1 + x_2 + \frac{1}{2}(u_{\tau(t)}x_{2,\tau(t)} + \cos(u_{\tau(t)}x_{1,\tau(t)})) \\ \dot{x}_2 = -4x_2 + x_1 + \frac{1}{2}u_{\tau(t)}x_1 \\ y = x_1, \end{cases} \tag{5.1}$$

where $A = \begin{pmatrix} -3 & 1 \\ 1 & -4 \end{pmatrix}$,

$$f(x(t), x_{\tau(t)}, u(t), u_{\tau(t)}) = \frac{1}{2}\begin{pmatrix} u_{\tau(t)}x_{2,\tau(t)} + \cos(u_{\tau(t)}x_{1,\tau(t)}) \\ u_{\tau(t)}x_1 \end{pmatrix},$$

the input $u = \sin(2\pi f t)$ with $f = 50Hz$; the function $\tau(t)$ is defined as follows: $\tau(t) = 0.001\frac{\sin^2(t)}{2}$ and the initial conditions for the system are $x(0) = [1 \ \ 2]^T$, for the observer $\hat{x}(0) = [-1 - 1]^T$.

For this example, it is clear that the pair (A, C) is observable; we also have the Lipchitz constant defined in (4.4) equal to $\frac{\sqrt{2}}{2}$. In this case, the input and the states are bounded. The gain L is chosen such that $A_c = A - LC$ is stable; we choose $L = [1 \ \ 1]^T$ and it implies that $A_c = \begin{pmatrix} -4 & 1 \\ 0 & -4 \end{pmatrix}$, we also

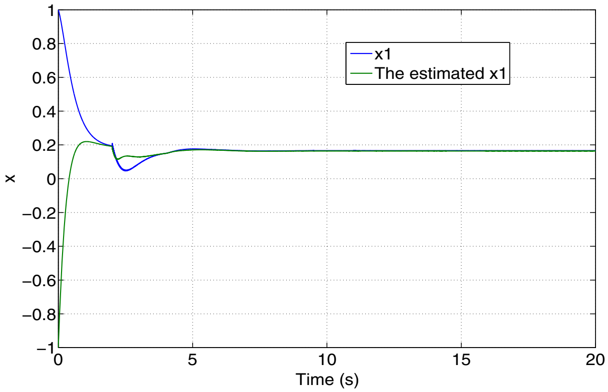

Figure 1. The evolution of x_1 and its estimate $\hat{x}_1$

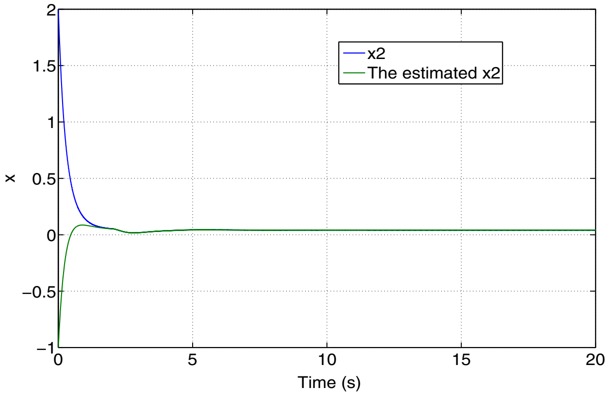

Figure 2. The evolution of x_2 and its estimate $\hat{x}_2$

choose the matrix Q as follows: $Q = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ and by solving the Lyapunov equation defined in (4.3), the matrix P is given by $P = \begin{pmatrix} 1 & 0.125 \\ 0.125 & 1.0313 \end{pmatrix}$. So, the assumptions of the theorem (4.3) are satisfied . It is clear from Figs. 1 and 2 that the estimated magnitudes converges practically to the real one.

6. Conclusion

In this paper, an observer design for a class of nonlinear systems under time-varying delay was proposed. Sufficient assumptions were given to guarantee a practical stability of the suggested observer. Furthermore, the exponential convergence of the observer was proved in the case of a constant time delay. Simulation results were shown to illustrate the good performances of the suggested observer. As a perspective, a new observer design for the same system taking into account the reduction of the number of assumptions will be a future work.

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