Mediterr. J. Math. 12 (2015), 1–7
DOI 10.1007/s00009-014-0401-6
0378-620X/15/010001-7
published online February 25, 2014

Mediterranean Journal of Mathematics

On Generalized α-Biderivations

Ajda Fošner

Abstract. We determine the structure of a generalized α-biderivation of a noncommutative prime ring $\mathcal{R}$. Moreover, we also consider the case when the ring $\mathcal{R}$ is semiprime.

Mathematics Subject Classification (2010). 16W20, 16W25, 16N60.

Keywords. Prime ring, semiprime ring, biderivation, α-biderivation, generalized biderivation, generalized α-biderivation.

1. Introduction and the Main Result

Throughout, $\mathcal{R}$ will represent a ring. For all $x, y \in \mathcal{R}$, the symbol $[x, y]$ will denote the commutator $xy - yx$. Recall that a ring $\mathcal{R}$ is prime if $x\mathcal{R}y = 0$, $x, y \in \mathcal{R}$, implies that $x = 0$ or $y = 0$, and it is semiprime if $x\mathcal{R}x = 0$, $x \in \mathcal{R}$, implies that $x = 0$. As usual, we will denote by C, $\mathcal{Q}_r$, and $\mathcal{Q}_s$ the extended centroid, the right Martindale ring of quotients, and the symmetric Martindale ring of quotients of a semiprime ring $\mathcal{R}$, respectively. The set of all idempotents in C will be denoted by $\mathrm{Idem}(C)$. For the explanation of the extended centroid as well as the right and the symmetric Martindale ring of quotients, we refer the reader to [2].

Let α be an automorphism of a ring $\mathcal{R}$. An additive map $d : \mathcal{R} \to \mathcal{R}$ is called a derivation of $\mathcal{R}$ if $d(xy) = d(x)y + xd(y)$ for all $x, y \in \mathcal{R}$, and is called an α-derivation of $\mathcal{R}$ if $d(xy) = d(x)y + \alpha(x)d(y)$ for all $x, y \in \mathcal{R}$. Moreover, an additive map $g : \mathcal{R} \to \mathcal{R}$ is called a generalized derivation of $\mathcal{R}$ if there exists a derivation d of $\mathcal{R}$ such that $g(xy) = g(x)y + xd(y)$ for all $x, y \in \mathcal{R}$. The concept of generalized derivations has been introduced by Brešar [3]. For results concerning generalized derivations we refer to [7,9,11,12], where further references can be found. Let us also point out that α-derivations and generalized derivations are two natural generalizations of usual derivations.

In the recent years, α-derivations and generalized derivations of associative rings and algebras have been widely investigated by many people in pure algebraic context and operator algebras. The natural question here is whether there exists a unification of both, the definition of a generalized derivation and that of an α-derivation. Based on this idea, we write a definition which is a common generalization of the previous two definitions. An additive map $g : \mathcal{R} \to \mathcal{R}$ is called a generalized α-derivation of $\mathcal{R}$ if there exists an additive map $d : \mathcal{R} \to \mathcal{R}$ such that $g(xy) = g(x)y + \alpha(x)d(y)$ for all $x, y \in \mathcal{R}$. It turns out that the map d is uniquely determined by g and is called an associated additive map of g. Moreover, d is always an α-derivation of $\mathcal{R}$ (see [12,13] for more details).

A biadditive map $D : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is called a biderivation of a ring $\mathcal{R}$ if for all $x, y \in \mathcal{R}$, the maps $x \mapsto D(x, y)$ and $y \mapsto D(x, y)$ are derivations of $\mathcal{R}$. Similarly, a biadditive map $D : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is called an α-biderivation of $\mathcal{R}$ if for all $x, y \in \mathcal{R}$, the maps $x \mapsto D(x, y)$ and $y \mapsto D(x, y)$ are α-derivations of $\mathcal{R}$. In [5] Brešar, Martindale III, and Miers characterized biderivations of a noncommutative prime ring $\mathcal{R}$ and in [4] Brešar determined the structure of an arbitrary α-biderivation of $\mathcal{R}$.

Argaç [1] introduced the following definition. Let $D, G : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ be two biadditive maps. A map $G : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is called a generalized D-biderivation if for every $y \in \mathcal{R}$, the map $x \mapsto G(x, y)$ is a generalized derivation of $\mathcal{R}$ associated with the additive map $x \mapsto D(x, y)$, and for every $x \in \mathcal{R}$, the map $y \mapsto G(x, y)$ is a generalized derivation of $\mathcal{R}$ associated with the additive map $y \mapsto D(x, y)$. Argaç proved that every generalized D-biderivation G of a noncommutative prime ring $\mathcal{R}$ has the form $G(x, y) = \lambda[x, y]$, $x, y \in \mathcal{R}$, for some $\lambda \in C$ [1, Theorem 4.1].

Motivated by the above results we introduce the following definition.

Definition. Let $\mathcal{R}$ be a ring and $D, G : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ two biadditive maps. A map $G : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is called a *generalized α-biderivation* with the associated biadditive map D if for every $y \in \mathcal{R}$, the map $x \mapsto G(x, y)$ is a generalized α-derivation of $\mathcal{R}$ associated with the additive map $x \mapsto D(x, y)$, and for every $x \in \mathcal{R}$, the map $y \mapsto G(x, y)$ is a generalized α-derivation of $\mathcal{R}$ associated with the additive map $y \mapsto D(x, y)$.

The above definition yields that for all $x, y, z \in \mathcal{R}$, we have

$$G(xy, z) = G(x, z)y + \alpha(x)D(y, z)$$

and

$$G(x, yz) = G(x, y)z + \alpha(y)D(x, z).$$

Of course, every α-biderivation is a generalized α-biderivation, but the converse is not generally true.

*Example***.** Let $\mathcal{R}$ be a ring, α an automorphism of $\mathcal{R}$, and D an arbitrary α-biderivation of $\mathcal{R}$. If $\phi : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is any biadditive map such that $\phi(xy, z) = \phi(x, z)y$ and $\phi(x, yz) = \phi(x, y)z$ for all $x, y, z \in \mathcal{R}$, then $D + \phi$ is a generalized α-biderivation of $\mathcal{R}$ with the associated biadditive map D. Namely, for all $x, y, z \in \mathcal{R}$, we have

$$(D + \phi)(xy, z) = (D + \phi)(x, z)y + \alpha(x)D(y, z)$$

and

$$(D + \phi)(x, yz) = (D + \phi)(x, y)z + \alpha(y)D(x, z).$$

The aim of our paper is to determine the structure of a generalized α-biderivation of a noncommutative prime ring and to give the generalization of Theorem 4.1 in [1].

Theorem 1. *Let $\mathcal{R}$ be a noncommutative prime ring and $D : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ a biadditive map. Suppose that $G : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is a nonzero generalized α-biderivation with the associated biadditive map D. Then there exists an invertible $q \in \mathcal{Q}_s$ such that $\alpha(x) = qxq^{-1}$ for all $x \in \mathcal{R}$ and*

$$G(x, y) = D(x, y) = q[x, y], \quad x, y \in \mathcal{R}.$$

Recall that if $\mathcal{R}$ is commutative, then any two α-derivations d_1, d_2 of $\mathcal{R}$ generate a generalized α-biderivation defined by $(x, y) \mapsto d_1(x)d_2(y)$. Thus, the assumption that $\mathcal{R}$ is noncommutative cannot be removed in Theorem 1.

2. The Proof

Throughout this section, $\mathcal{R}$ will represent a noncommutative prime ring and α an automorphism of $\mathcal{R}$. Recall that an automorphism α is $\mathcal{X}$-inner if there exists an invertible $q \in \mathcal{Q}_s$ such that $\alpha(x) = qxq^{-1}$ for all $x \in \mathcal{R}$.

Lemma 1. ([14, Lemma 12.1]) *If there exist nonzero elements $q_1, q_2, q_3, q_4 \in \mathcal{Q}_r$ such that $q_1xq_2 = q_3\alpha(x)q_4$ for all $x \in \mathcal{R}$, then α is $\mathcal{X}$-inner.*

The next Lemma is a well-known result (see [8, Lemma 1.3.2] and [10, Theorem 1] for the generalization).

Lemma 2. *Let $n > 0$ be an integer, $\mathcal{I}$ a nonzero ideal of $\mathcal{R}$, and $x_i, y_i \in \mathcal{Q}_r$, $i = 1, \ldots, n$. If $\sum_{i=1}^{n} x_izy_i = 0$ for all $z \in \mathcal{I}$, then a_i's are linearly dependent over C and b_i' are linearly dependent over C.*

The following result is a generalization of [1, Lemma 4.2].

Lemma 3. *Let $\mathcal{R}$ be a prime ring, $\mathcal{I}$ a nonzero ideal of $\mathcal{R}$, and $\mathcal{S}$ any set. Suppose that maps $F_1, F_2, F_3 : \mathcal{S} \to \mathcal{Q}_r$ satisfy the identity*

$$F_1(x)zF_2(y) = F_2(x)zF_3(y)$$

for all $x, y \in \mathcal{S}$, $z \in \mathcal{I}$. If $F_2 \neq 0$, then there exists $\lambda \in C$ such that

$$F_1(x) = F_3(x) = \lambda F_2(x)$$

for all $x \in \mathcal{S}$.

Proof. Let $x \in \mathcal{S}$ be an arbitrary element. If $F_2(x) = 0$, then $F_1(x) = F_3(x) = 0$. Namely, $F_1(x)zF_2(y) = F_2(x)zF_3(y) = 0$ for all $y \in \mathcal{S}$, $z \in \mathcal{I}$. Since $F_2 \neq 0$ there exists $y \in \mathcal{S}$ such that $F_2(y) \neq 0$ and since $\mathcal{R}$ is prime, it follows that $F_1(x) = 0$. Similarly, we can show that $F_3(x) = 0$.

Suppose that $F_2(x) \neq 0$ and recall that $F_1(x)zF_2(x) = F_2(x)zF_3(x)$ for all $z \in \mathcal{I}$. By Lemma 2, there exists $\lambda_x \in C$ such that $F_1(x) = \lambda_xF_2(x)$. Thus, $F_2(x)z(\lambda_xF_2(x) - F_3(x)) = 0$ for all $z \in \mathcal{I}$ and $F_3(x) = \lambda_xF_2(x)$, as well. At the end, we have to show that λ_x is independent of an element $x \in \mathcal{S}$. So, assume that $F_2(x) \neq 0$ and $F_2(y) \neq 0$. Then the relation $F_1(x)zF_2(y) = F_2(x)zF_3(y)$ can be written in the form $F_2(x)z(\lambda_x - \lambda_y)F_2(y) = 0$. Since $\mathcal{R}$ is prime, we have $\lambda_x = \lambda_y = \lambda$, as desired. □

In the proof of our main theorem we will need the following partial results.

Lemma 4. *Let $\mathcal{R}$ be a semiprime ring and $D : \mathcal{R}\times\mathcal{R} \to \mathcal{R}$ an additive map. If $G : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is a generalized α-biderivation with the associated biadditive map D, then D is an α-biderivation.*

Proof. For all $x, y, z, w \in \mathcal{R}$, we have

$$\begin{aligned} G(xyz, w) &= G((xy)z, w) = G(xy, w)z + \alpha(xy)D(z, w) \\ &= G(x, w)yz + \alpha(x)D(y, w)z + \alpha(xy)D(z, w). \end{aligned}$$

On the other hand,

$$G(xyz, w) = G(x(yz), w) = G(x, w)yz + \alpha(x)D(yz, w).$$

Comparing the relations, we get

$$\alpha(x)(D(yz, w) - D(y, w)z - \alpha(y)D(z, w)) = 0$$

for all $x, y, z, w \in \mathcal{R}$. Since $\mathcal{R}$ is semiprime, we see that for all $y \in \mathcal{R}$, the map $x \mapsto D(x, y)$ is an α-derivation. With the same idea, we can show that for all $x \in \mathcal{R}$, the map $y \mapsto D(x, y)$ is an α-derivation, as well. The proof is completed. □

Lemma 5. *Let $D : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ be an additive map and $G : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ a generalized α-biderivation with the associated biadditive map D. Then*

$$G(x_1, y_1)z[x_2, y_2] = [\alpha(x_1), \alpha(y_1)]\alpha(z)D(x_2, y_2)$$

for all $x_1, y_1, x_2, y_2, z \in \mathcal{R}$.

Proof. Using Lemma 4, we obtain

$$\begin{aligned} G(x_1x_2, y_1y_2) &= G(x_1, y_1y_2)x_2 + \alpha(x_1)D(x_2, y_1y_2) \\ &= G(x_1, y_1)y_2x_2 + \alpha(y_1)D(x_1, y_2)x_2 + \alpha(x_1)D(x_2, y_1)y_2 \\ &\quad +\alpha(x_1y_1)D(x_2, y_2). \end{aligned}$$

On the other hand,

$$\begin{aligned} G(x_1x_2, y_1y_2) &= G(x_1x_2, y_1)y_2 + \alpha(y_1)D(x_1x_2, y_2) \\ &= G(x_1, y_1)x_2y_2 + \alpha(x_1)D(x_2, y_1)y_2 + \alpha(y_1)D(x_1, y_2)x_2 \\ &\quad +\alpha(y_1x_1)D(x_2, y_2). \end{aligned}$$

Comparing the above relations, we get

$$G(x_1, y_1)[x_2, y_2] = [\alpha(x_1), \alpha(y_1)]D(x_2, y_2)$$

for all $x_1, y_1, x_2, y_2 \in \mathcal{R}$. Replacing x_2 by zx_2 and using Lemma 4 we get the desired identity. □

Note that in the proof of Lemma 4 and in the proof of Lemma 5 we did not use the primeness of a ring $\mathcal{R}$.

We are now in a position to prove Theorem 1. The main idea of the proof comes from [4].

Proof of Theorem 1. According to Lemma 5, we have

$$G(x_1, y_1)z[x_2, y_2] = [\alpha(x_1), \alpha(y_1)]\alpha(z)D(x_2, y_2)$$

for all $x_1, y_1, x_2, y_2, z \in \mathcal{R}$. Since G is nonzero and $\mathcal{R}$ is noncommutative we can find $x_1, y_1, x_2, y_2 \in \mathcal{R}$ such that $G(x_1, y_1) = q_1 \neq 0$, $[x_2, y_2] = q_2 \neq 0$, $[\alpha(x_1), \alpha(y_1)] = q_3 \neq 0$, and $D(x_2, y_2) = q_4 \neq 0$. Namely, it is easy to see that D is also nonzero. Moreover, $[\alpha(x_1), \alpha(y_1)] \neq 0$ if and only if $G(x_1, y_1) \neq 0$ and $[x_2, y_2] \neq 0$ if and only if $D(x_2, y_2) \neq 0$.

So, we have $q_1 z q_2 = q_3 \alpha(z) q_4$ for all $z \in \mathcal{R}$ and, according to Lemma 1, α is $\mathcal{X}$-inner. Thus, there exists an invertible $q \in \mathcal{Q}_s$ such that $\alpha(x) = qxq^{-1}$ for all $x \in \mathcal{R}$. It follows that

$$G(x_1, y_1)z[x_2, y_2] = q[x_1, y_1]zq^{-1}D(x_2, y_2)$$

for all $x_1, y_1, x_2, y_2, z \in \mathcal{R}$. Multiplying the above identity from the left by q^{-1}, we obtain

$$q^{-1}G(x_1, y_1)z[x_2, y_2] = [x_1, y_1]zq^{-1}D(x_2, y_2).$$

Let us define maps $F_1, F_2, F_3 : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ by $F_1(x, y) = q^{-1}G(x, y)$, $F_2(x, y) = [x, y]$, and $F_3(x, y) = q^{-1}D(x, y)$. Then

$$F_1(x_1, y_1)zF_2(x_2, y_2) = F_2(x_1, y_1)zF_3(x_2, y_2)$$

and, by Lemma 3, there exists $\lambda \in C$ such that

$$F_1(x, y) = F_3(x, y) = \lambda[x, y]$$

for all $x, y \in \mathcal{R}$. If we denote $q_0 = \lambda q$, then $G(x, y) = D(x, y) = q_0[x, y]$. Note that $q_0 \neq 0$ is invertible and $\alpha(x) = q_0 x q_0^{-1}$, $x \in \mathcal{R}$. The proof is completed. □

3. The Semiprime Case

Let $\mathcal{R}$ be a semiprime ring and α an automorphism of $\mathcal{R}$. In [6], Eremita observed the structure of α-biderivations $D : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$. More precisely, he considered the identity $F_1(x)zF_2(y) = F_3(x)\alpha(z)F_4(y)$, $x, y \in \mathcal{S}$, $z \in \mathcal{R}$, where $\mathcal{S}$ is an arbitrary set and $F_1, F_2, F_3, F_4 : \mathcal{S} \to \mathcal{R}$ are any maps.

Theorem 2. ([6, Theorem 3.1]) *Let $\mathcal{R}$ be a semiprime ring and $\mathcal{S}$ any set. Suppose that maps $F_1, F_2, F_3, F_4 : \mathcal{S} \to \mathcal{R}$ satisfy the identity*

$$F_1(x)zF_2(y) = F_3(x)\alpha(z)F_4(y)$$

for all $x, y \in \mathcal{S}$, $z \in \mathcal{R}$. Then there exist $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5 \in \mathrm{Idem}(C)$ and an invertible $q \in \mathcal{Q}_s$ such that $\epsilon_i \epsilon_j = 0$, $i \neq j$, $\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + \epsilon_5 = 1$, $\epsilon_1 \alpha(z) = \epsilon_1 q z q^{-1}$ for all $z \in \mathcal{R}$, and

$$\epsilon_1 F_1(x) = \epsilon_1 F_3(x) q, \quad \epsilon_1 F_2(y) = \epsilon_1 q^{-1} F_4(y), \quad x, y \in \mathcal{S}.$$

Moreover, $\epsilon_2 F_2(x) = \epsilon_2 F_4(x) = 0$, $\epsilon_3 F_2(x) = \epsilon_3 F_3(x) = 0$, $\epsilon_4 F_1(x) = \epsilon_4 F_4(x) = 0$, $\epsilon_5 F_1(x) = \epsilon_5 F_3(x) = 0$ for all $x \in \mathcal{S}$.

As an application of the above theorem, Eremita generalized the result of Brešar [4] concerning the structure of α-biderivations. The natural question here is, whether the analogue result holds true also for generalized α-biderivations. Corollary 1 answers this question in the affirmative. Namely, if $D : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is a biadditive map and $G : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ a generalized α-biderivation with the associated biadditive map D, then, by Lemma 5, we have $G(x_1, y_1)z[x_2, y_2] = [\alpha(x_1), \alpha(y_1)]\alpha(z)D(x_2, y_2)$ for all $x_1, y_1, x_2, y_2, z \in \mathcal{R}$. Let us define maps $F_1, F_2, F_3, F_4 : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ by $F_1(x, y) = G(x, y)$, $F_2(x, y) = [x, y]$, $F_3(x, y) = [\alpha(x), \alpha(y)]$, and $F_4(x, y) = D(x, y)$, $x, y \in \mathcal{R}$. Then

$$F_1(x_1, y_1)zF_2(x_2, y_2) = F_3(x_1, y_1)\alpha(z)F_3(x_2, y_2)$$

and, by Theorem 2, there exist $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5 \in \mathrm{Idem}(C)$ and an invertible $q \in \mathcal{Q}_s$ such that $\epsilon_i\epsilon_j = 0$, $i \neq j$, $\epsilon_1+\epsilon_2+\epsilon_3+\epsilon_4+\epsilon_5 = 1$, $\epsilon_1\alpha(z) = \epsilon_1 qzq^{-1}$ for all $z \in \mathcal{R}$, and $\epsilon_1 G(x, y) = \epsilon_1 D(x, y) = \epsilon_1 q[x, y]$ for all $x, y \in \mathcal{R}$. Moreover, $\epsilon_i[\mathcal{R}, \mathcal{R}] = 0$, $i = 2, 3, 5$, and $\epsilon_j D(\mathcal{R}, \mathcal{R}) = \epsilon_j G(\mathcal{R}, \mathcal{R}) = 0$, $j = 2, 4, 5$. Setting $\gamma_1 = \epsilon_1$, $\gamma_2 = \epsilon_2 + \epsilon_4 + \epsilon_5$, and $\gamma_3 = \epsilon_3$, we have the next corollary.

Corollary 1. *Let $\mathcal{R}$ be a semiprime ring and $D : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ a biadditive map. Suppose that $G : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is a generalized α-biderivation with the associated biadditive map D. Then there exist $\gamma_1, \gamma_2, \gamma_3 \in \mathrm{Idem}(C)$ and an invertible $q \in \mathcal{Q}_s$ such that $\gamma_i\gamma_j = 0$, $i \neq j$, $\gamma_1 + \gamma_2 + \gamma_3 = 1$, and*

(1) $\gamma_1\alpha(x) = \gamma_1 qxq^{-1}$ *for all* $x \in \mathcal{R}$,
(2) $\gamma_1 G(x, y) = \gamma_1 D(x, y) = \gamma_1 q[x, y]$ *for all* $x, y \in \mathcal{R}$,
(3) $\gamma_2 G(x, y) = \gamma_2 D(x, y) = 0$ *for all* $x, y \in \mathcal{R}$,
(4) *the ring* $\gamma_3\mathcal{R}$ *is commutative.*

References

[1] Argaç, N.: On prime and semiprime rings with derivations. Algebra Colloquium **13**, 371–380 (2006)

[2] Beidar, K.I., Martindale, W.S. III., Mikhalev, A.V.: Rings with Generalized Identities. Marcel Dekker, Inc., New York (1996)

[3] Brešar, M.: The distance of the compositum of derivations to the generalized derivations. Glasgow Math. J. **33**, 89–93 (1991)

[4] Brešar, M.: On generalized biderivations and related maps. J. Algebra **172**, 764–786 (1995)

[5] Brešar, M., Martindale, W.S. III., Miers, C.R.: Centralizing maps in prime rings with involution. J. Algebra **161**, 342–357 (1993)

[6] Eremita, D.: A functional identity with an automorphism in semiprime rings. Algebra Colloquium **8**, 301–306 (2001)

[7] Fošner, M., Vukman, J.: Identities with generalized derivations in prime rings. Mediterranean J. Math. **9**, 847–863 (2012)

[8] Herstein, I.N.: Rings with Involution. The University of Chicago Press, Chicago (1976)

[9] Hvala, B.: Generalized derivations in rings. Comm. Algebra **26**, 1147–1166 (1998)

[10] Lanski, C.: A note on GPIs and their coefficients. Proc. Am. Math. Soc. **98**, 17–19 (1986)

[11] Lee, T.-K.: Generalized derivations of left faithful rings. Comm. Algebra **27**, 4057–4073 (1999)

[12] Lee, T.-K.: Generalized skew derivations characterized by acting on zero products. Pac. J. Math. **216**, 293–301 (2004)

[13] Liu, K.-S.: Differential identities and constants of algebraic automorphisms in prime rings, Ph.D. Thesis, National Taiwan University, Taipei (2006)

[14] Passman, D.: Infinite Crossed Products. Academic Press, San Diego (1989)

Ajda Fošner
Faculty of Management
University of Primorska
Cankarjeva 5
6104 Koper
Slovenia
e-mail: ajda.fosner@fm-kp.si

Received: May 10, 2013.
Revised: February 9, 2014.
Accepted: February 12, 2014.