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RESEARCH

Origin of Irregular Star Polygons in Ground Projection Plans of Muqarnas

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Abstract

Many muqarnas contain star polygons in their ground projection plans. Such star polygons usually have equal edge lengths. In this study, the geometric background of irregular star polygons with unequal edge lengths, observed in some ground projection plans of muqarnas, was investigated. The plan of the muqarnas at the main gate of the Atik Valide Mosque was examined according to the theoretical framework of shape grammars. As a result of the case study, it was concluded that ellipse grids were used to form the layout of the ground projection plan of muqarnas. It was shown that stars placed in the areas formed by the intersections of the ellipse grid formed irregular polygons naturally. In addition, a parametric 3D geometry of muqarnas was obtained by considering a geometric decomposition of the analysed ground projection plan of muqarnas.

Keywords Design analysis · Geometric analysis · Islamic patterns · Shape grammars · Design computation · Parametric muqarnas

Introduction

Muqarnas, a structure that has continually been used from the past to the present, is known as an architectural decorative element. Thus far, the formal properties of the existing muqarnas have been examined by studies such as those of Elkhateeb (2012), Castera (2007), Kharazmi and Sarhangi (2016) and Kashef (2017). In addition, researchers such as Hamekasi et al. (2011) and Senhaji and Benslimane (2019) explored the creation of muqarnas with new techniques.

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It is known that the plan drawings (ground projection plans) of muqarnas have been made since ancient times. For example, one of the oldest examples is the plate found at Takht-i Sulayman (dating to the 1270 s) that contains squares, triangles and rhomboids (Harb 1978). According to Necipoglu (1995), this is the oldest known example of architectural drawing. There are also many muqarnas plans in the Topkapi, Tashkent and Mirza Akbar scrolls (Necipoglu 1995).

Muqarnas have been analysed by various researchers using different classifications. For example, Hamekasi et al. (2011) classifies muqarnas by relationship to construction types. According to this classification, there are three types of muqarnas: corbelled, superimposed and suspended. The cells of corbelled muqarnas are carved from the block face. This process may be performed before or after assembly. In superimposed muqarnas, first the supporting surface (such as a dome or a vault) is made, and muqarnas cells are placed on these surfaces. In suspended muqarnas, the components are attached to the architectural structure after they have been produced on the ground. In this type, space is left between components and the surface where the components are placed (Hamekasi et al. 2011). A fifteenth-century mathematician, Al-Kashi, classified muqarnas according to formal characteristics. Al-Kashi indicated that, in 3D cells of muqarnas, there are vertical facets and roofs (Dold-Samplonius 1992; Gherardini and Leali 2016; Harmsen 2006) (Fig. 1). Al-Kashi's classification is as follows: the first two types are simple muqarnas and clay-plastered muqarnas (or *mutayyan*), both of which have plane facets and roofs. What distinguishes a clay-plastered muqarnas is that the heights of its layers might be different, and some of its layers might not have facets (Dold-Samplonius and Harmsen 2005). The other two types are the curved muqarnas and Shirazi muqarnas with curved roofs (Dold-Samplonius 1992). A curved muqarnas is similar to a simple muqarnas except for its curved roofs. A shirazi muqarnas is similar to a curved muqarnas except for its variety of cells. Moreover, Dold-Samplonius and Harmsen (2005) and Özdural (1990) stated that in the illustrations of Al-Kashi, the geometries in the projection plans of muqarnas cells can be square, rhombus, almond, biped and barley kernels. A biped and an almond make up a rhombus. In addition, as Harmsen (2006) illustrated, there are also half-square, half-rhombus, jug-shaped, and large biped elements in muqarnas plan projections according to Al-Kashi (Fig. 2). However, as Özdural (1990) noted, Al-Kashi explains muqarnas by reducing them to basic elements, however, thus neglecting their design and composition.

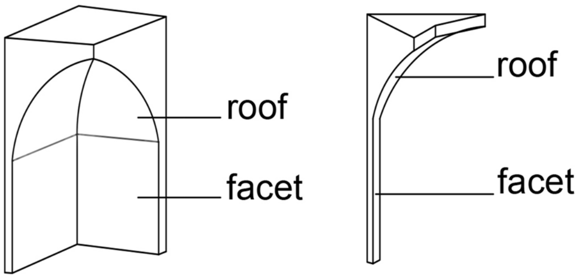
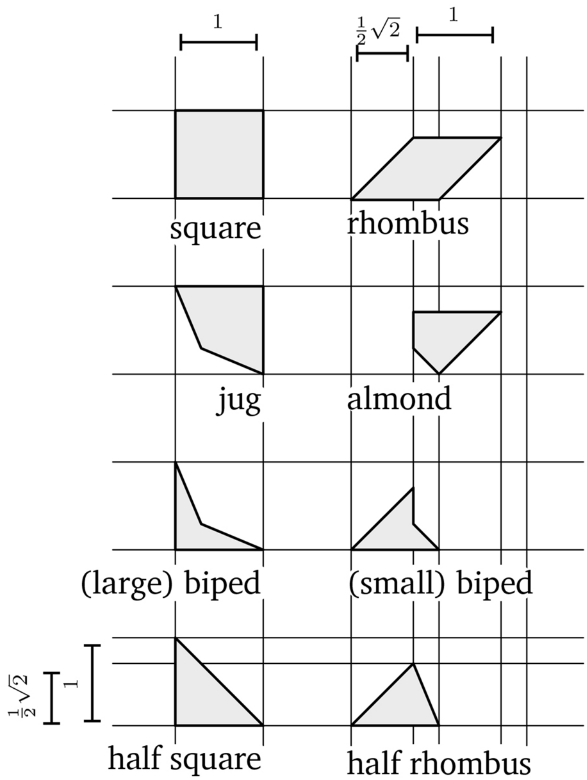


Fig. 1 Cell of a curved muqarnas and an intermediate element: Image: author, redrawn using the drawings of Harmsen (2006) and Dold-Samplonius and Harmsen (2005)

Fig. 2 Muqarnas elements in 2D (Harmsen 2006)



Muqarnas, which appear formally complex, are in fact formed according to simple rules (Dold-Samplonius 1992; Gherardini and Leali 2016). Özdural (1991) and Alacam et al. (2017) agree that muqarnas are created by sequentially raising the plan drawing from 2D to 3D. Uluengin (2018) and Takahashi (2019) also analysed the muqarnas, starting from the ground projection plan. It is observed that in these studies, first, the principle of placing the muqarnas plan on a grid system was applied. This approach is an important step in understanding the design and composition of muqarnas.

Research Aims and Methods

According to these plan analyses, it is observed that most of the plans of muqarnas are composed of various types of star polygons. The latter can be, for example, twelve-, six- or five-point stars. Such stars are also common in Islamic patterns made in two dimensions. The lengths of the edges of various types of stars in these patterns are usually equal within each type. This type of star formation in Islamic patterns is discussed in Bourgoin (1879), Schneider (1980), Critchlow (1983), Hankin (1998), Broug (2008), Cromwell (2009), Bodner (2012), Sarhangi

(2012), Redondo-Buitrago and Huylebrouck (2015) and Bonner (2017). It is known that many muqarnas formed on the basis of a radial grid are created from many overlapping reference circles (Necipoglu 1995). When we examine the Topkapi scroll, it is clear that the stars in many muqarnas plans are produced from reference circles, and therefore the lengths of star polygons' edges are usually equal.

However, in many muqarnas plans, irregular star polygons with unequal edge lengths are observed. In addition, in special cases, such as that of the muqarnas plan in Özdural's study (1991), where some stars remain in the corners of the frame, the lengths of their edges are unequal.

An examination of the the muqarnas plan analysis in Uluengin (2018) and muqarnas drawings in Necipoglu (1995) shows that the edges of many stars are unequal in length. However, neither Uluengin nor Necipoğlu provides a clear mathematical explanation of this phenomenon. Therefore, the present research aimed to answer the following questions: What was the geometry behind the stars in the muqarnas plans with unequal edge lengths? What kind of reference grid was there? Was there a geometric reason behind the unevenness of the edges of stars in these decorations? This study also aimed to determine the reason the edge lengths of various stars in the muqarnas plans are not equal within each star type. In addition, determining the geometric background of this phenomenon can allow an infrastructure to be created for the muqarnas that can be formed by new techniques.

To achieve these aims, the muqarnas at the main gate of the Atik Valide Mosque (Fig. 3) with irregular star polygons in its plan was selected for examination. This is located in the Atik Valide Complex in the Toptasi district of the Uskudar county of Istanbul. The mosque was ordered to be built in 1570 by Nurbanu Sultan, the mother of Murad III, and built by the architect Sinan (Eris et al. 2013).

The hypothesis of this study is that in the plan of the muqarnas of the main gate of the Atik Valide Mosque where the lengths of star polygon edges are unequal, the drawing layout of the muqarnas plan is formed with ellipses. When stars are placed in the ellipse-shaped layout, the edges of some polygonal stars will naturally not be of the same length, but at the same time their order and proportions will be consistent overall.

To determine the exact proportions of the plan, the selected muqarnas was scanned with the Faro Focus 3D laser scanning device. The photos obtained as a result of laser scanning were combined in the Scene programme to form a 3D point cloud model of the muqarnas. The plan view of the muqarnas was obtained in the Scene programme (Fig. 4).

First, the muqarnas plan was drawn on the obtained image using the AutoCAD software. Afterwards, a systematic formation of this plan based on mathematical rules within the framework of the shape grammars theory was studied.

The grid system to be used as a layout was not considered as the boundaries of the muqarnas plan. In contrast, systematic analysis was performed by assuming that the plan of the muqarnas was a certain part of a large grid.

At the same time this system was being created, the resulting pattern in the case of equal edge lengths of star polygons in the plan was explored. This method was used as a test to prove the proposed hypothesis. Since the lengths of the edges of star polygons in Islamic patterns are generally equal, the authors focused on the

Fig. 3 Analysed muqarnas at the main gate of the Atik Valide Mosque (photo by authors)

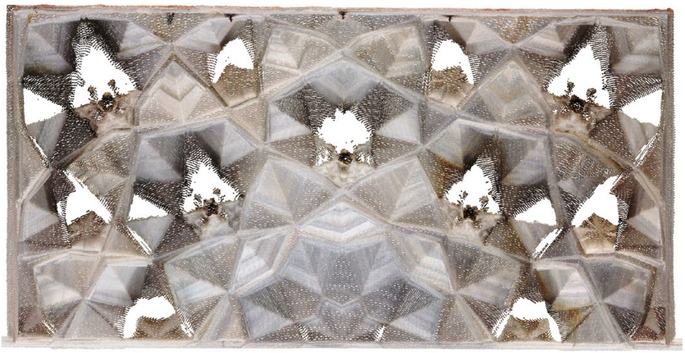


Fig. 4 Plan view of muqarnas obtained by laser scanning

possibility that the edge lengths of star polygons in this muqarnas may be equal in length. However, the analysis revealed that the lengths of edges were unlikely to be equal.

Afterwards, a parametric muqarnas was created using the plan of the muqarnas obtained from the analyses. Grasshopper, a visual programming language, was used for creating parametric muqarnas. Shape grammar theory, used in the analysis of the muqarnas plan, was also used in the creation of parametric muqarnas. With the exception of a few details that would simplify the coding process, the same principles were implemented as software code. The details are described in the following sections.

Analysis of the Muqarnas at the Main Gate of the Atik Valide Mosque

Set of Rules (in 2D)

For the geometric analysis of the selected muqarnas plan, the main axes were formed with the angle of 45° between them (rule 1, subrule 1), and the intermediary axes (rule 1, subrule 2) were formed with the angle of 22.5° between them (in what follows all rules and subrules refer to Table 1).

An ellipse was then formed with its two focal points on a formed intermediary axis (rule 2, subrule 1). This ellipse can be formed on any intermediary axis. This ellipse was then replicated into eight in a circular formation. The two focal points of each ellipse correspond to the intermediary axes (rule 2, subrule 2). Thus, a circle was defined at the intersection of ellipses. The centre of this circle is the intersection of the axes.

To create the first star formation, the intersection points between the circle and the main axes were first determined. Then, based on the centre of the circle, another circle with a smaller radius was created by scaling the circle. The intersection of the smaller circle with the intermediary axes was determined (rule 3, subrule 1). Then, the lines between these points were formed to create the first star formation (rule 3, subrule 2).

To create the second star formation, another circle was formed based on the same centre by scaling the smaller circle. The intersection points between the last circle and the main axes were determined (rule 4, subrule 1). By forming lines between these points and the points on the intermediary axes determined in the first star formation, the second star formation was created (rule 4, subrule 2).

For the third star formation, the intersection points of the related ellipses were first determined. These intersection points are on a single circle, assuming that the intersection point of the axes (the origin) is the centre (rule 5, subrule 1). The lines were formed between the points defined on the main axes for the first star formation and the intersection points of these ellipses, and thus the third star formation was created (rule 5, subrule 2).

For the fourth star formation, the intersection points of the related ellipses were first determined. Again, these intersection points are on a single circle, assuming that the intersection point of the axes (the origin) is the centre (rule 6, subrule 1). The lines were formed between the points on the last formed star and the intersection points of these ellipses, and the fourth star formation was created (rule 6, subrule 2).

In Rule 7, a frame was first created through the boundaries of the ellipse grid (rule 7, subrule 1). Then, the corresponding ellipse intersection points were determined on the ellipse grid to create a secondary ellipse grid (rule 7, subrule 2). The ellipse grid was then copied with reference to these points (rule 7, subrule 3). The obtained

Table 1 Set of rules (in 2D)

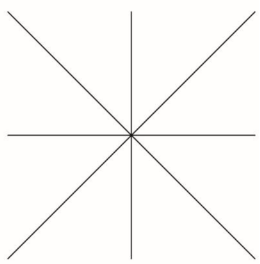
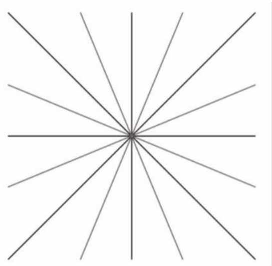
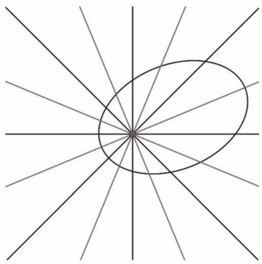
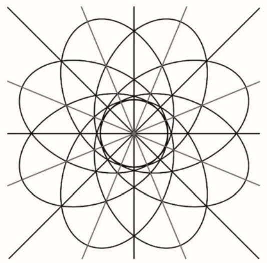
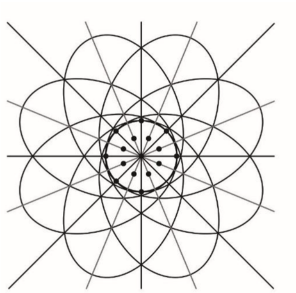
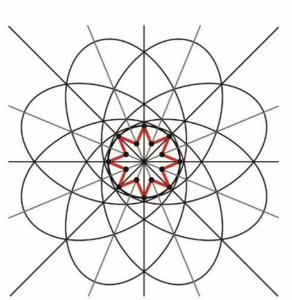
Rules	Process	
Rule 1 (Determining the reference axes)	 Subrule 1 The main axes were formed with the angle of 45° between them.	 Subrule 2 The intermediary axes were formed with the angle of 22.5° between them.
Rule 2 (Determining the reference ellipse grid)	 Subrule 1 An ellipse was formed with its two focal points on a formed intermediary axis.	 Subrule 2 This ellipse was replicated into eight arranged in a circular formation. Thus, a circle was defined at the intersection of ellipses.
Rule 3 (The first star formation)	 Subrule 1 Another circle with a smaller radius was created by scaling the circle. The intersection points were determined.	 Subrule 2 The lines between these points were formed to create the first star formation.

Table 1 (continued)

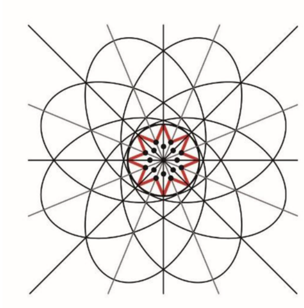
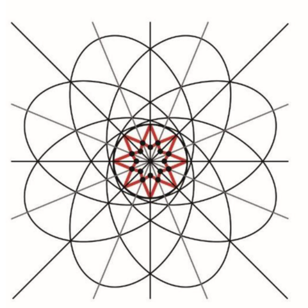
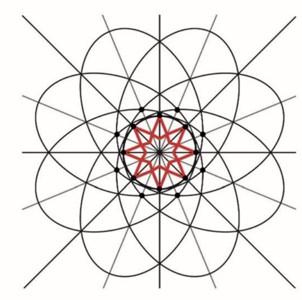
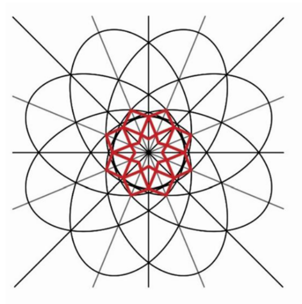
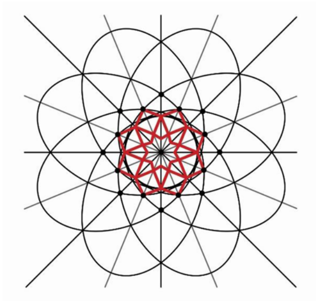
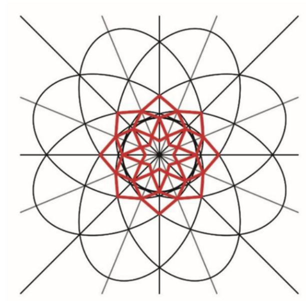
Rule 4 (The second star formation)	 Subrule 1 Another circle was formed by scaling the smaller circle. The intersection points were determined.	 Subrule 2 By forming lines between the points, the second star formation was created.
Rule 5 (The third star formation)	 Subrule 1 The intersection points of the related ellipses were determined.	 Subrule 2 The lines were formed between the points, and thus the third star formation was created.
Rule 6 (The fourth star formation)	 Subrule 1 The intersection points of the related ellipses were determined.	 Subrule 2 The lines were formed between the points, and the fourth star formation was created.

Table 1 (continued)

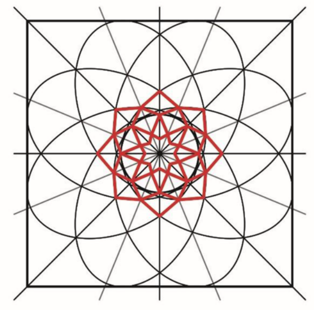
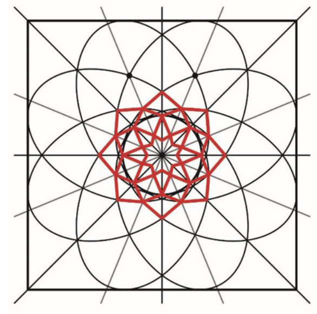
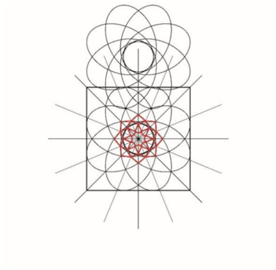
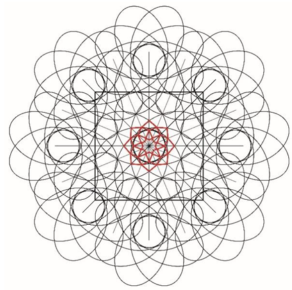
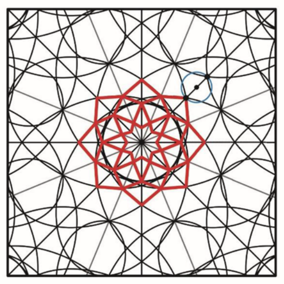
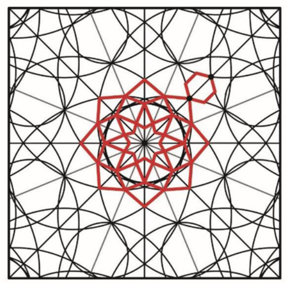
Rule 7 (The secondary ellipse grid formation)	 Subrule 1 A frame was created through the boundaries of the ellipse grid.	 Subrule 2 The corresponding ellipse intersection points were determined on the ellipse grid.
	 Subrule 3 The ellipse grid was then copied with reference to these points.	 Subrule 4 The obtained grid was replicated into eight arranged in a circular formation. Thus, the secondary ellipse grid was created.
Rule 8 (The first six-point irregular star formation)	 Subrule 1 The distance between the determined points was regarded as a diameter, and a circle was created.	 Subrule 2 A hexagon was formed inside this circle.

Table 1 (continued)

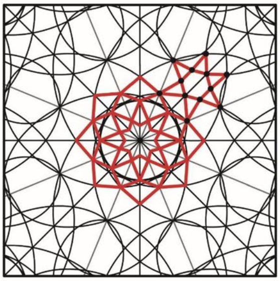
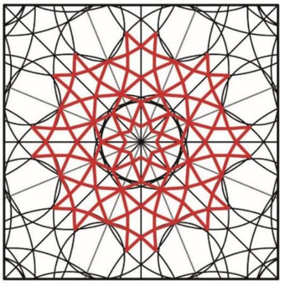
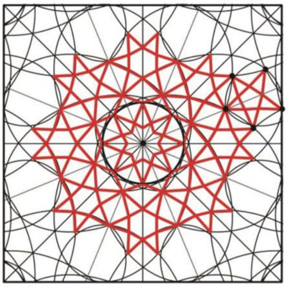
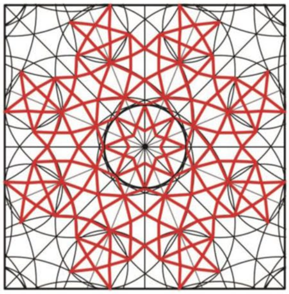
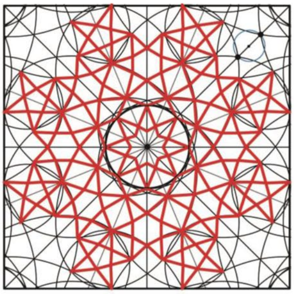
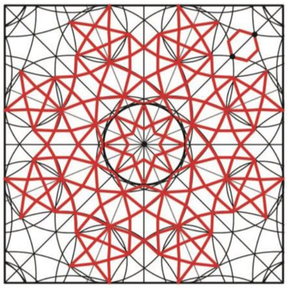
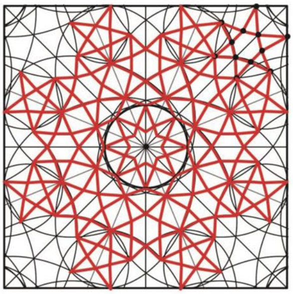
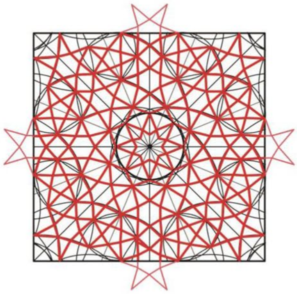
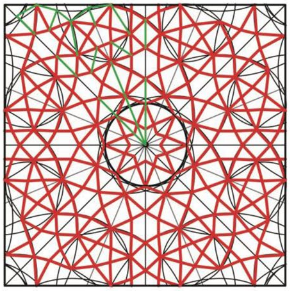
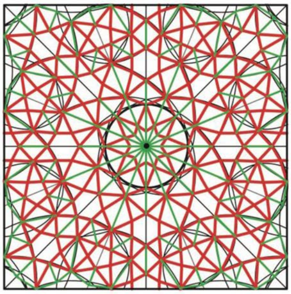
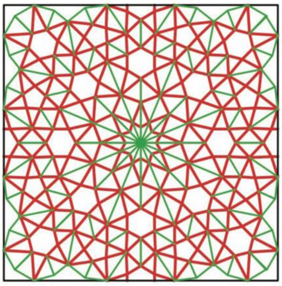
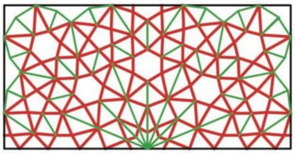
	 Subrule 3 The corner points of this hexagon and the ellipse grid's intersection points closest to this hexagon were determined. The lines were formed between the related points.	 Subrule 4 The resulting six-point irregular star polygon was replicated into eight arranged in a circular formation.
Rule 9 (Five-point irregular star formation)	 Subrule 1 The corresponding intersection points of the ellipse grid and a pentagon were determined. The lines were formed between the related points and the pentagon.	 Subrule 2 The resulting five-point irregular star polygon was replicated into eight arranged in a circular formation.
Rule 10 (The second six-point irregular star formation)	 Subrule 1 The distance between the determined points was regarded as a diameter, and a circle was created.	 Subrule 2 A hexagon was formed inside this circle.

Table 1 (continued)

	 Subrule 3 The corner points of this hexagon and the ellipse grid's intersection points closest to this hexagon were determined. The lines were formed between the related points.	 Subrule 4 The resulting six-point irregular star polygon was replicated into eight arranged in a circular formation.
Rule 11 (Almond creation)	 Subrule 1 Almonds were created in the smallest module in the frame by forming the lines in the spaces between the irregular star polygons.	 Subrule 2 These almonds were replicated to cover the whole pattern.
Rule 12 Elimination	 Subrule 1 The grids and axes were eliminated.	 Subrule 2 The geometry of the plan of muqarnas was obtained by eliminating a half of the pattern.

grid was replicated into eight arranged in a circular formation with respect to the centre of the circle. Thus, the secondary ellipse grid was created (rule 7, subrule 4).

For the first six-point irregular star formation, one point on the last-formed star and one point at the intersection of the secondary ellipse grid were determined first. The distance between these points was regarded as a diameter, and a circle was created (rule 8, subrule 1). A hexagon was formed inside this circle (rule 8, subrule 2). Then, the corner points of this hexagon and the ellipse grid's intersection points closest to this hexagon were determined. The lines were formed between the related points. As a result, the new star, a six-point irregular star polygon, was constructed (rule 8, subrule 3). The latter was replicated into eight arranged in a circular formation with respect to the centre of the circle (rule 8, subrule 4).

The corresponding intersection points of the ellipse grid and a pentagon were determined for the five-point irregular star polygon formation. Three of these points were also the corresponding corner points of six-point irregular star polygons. The lines were formed between the related points and the pentagon (rule 9, subrule 1). Thus, the five-point irregular star polygon appeared automatically. The resulting five-point irregular star polygon was replicated into eight arranged in a circular formation with respect to the centre of the circle (rule 9, subrule 2).

For the second six-point irregular star formation, two points were determined at the intersection of the ellipse grid. The distance between these points was regarded as a diameter, and a circle was created (rule 10, subrule 1). A hexagon was formed inside this circle (rule 10, subrule 2). Then, the corner points of this hexagon and the ellipse grid's intersection points closest to this hexagon were determined. The ellipse grid's points were actually the corners of the first six-point irregular star and five-point irregular star. The lines were formed between the related points. As a result, an irregular star polygon appeared automatically (rule 10, subrule 3). The resulting six-point irregular star polygon was replicated

Table 2 Formation of regular six- and five-point stars

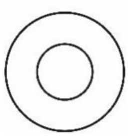
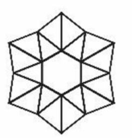
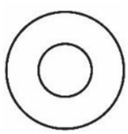
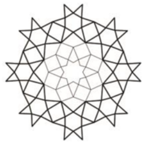
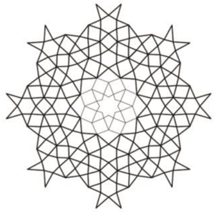
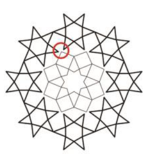
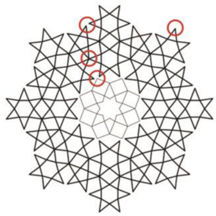
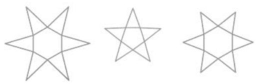
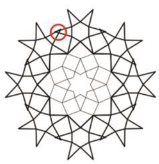
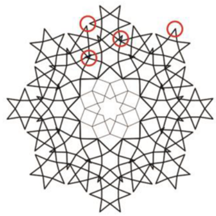
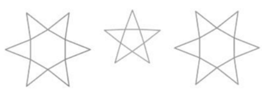
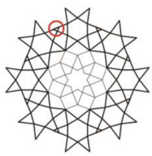
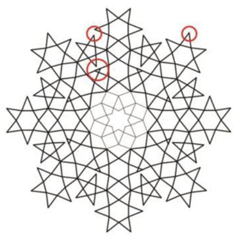
Star type	Formation process			
Regular six-point star				
Regular five-point Star				

Table 3 Problems in the use of six- and five-point regular star polygons with equal edge lengths

Type	Details	The case of the first-row six-point star	Composition
The obtained plan of the muqarnas (the plan with irregular stars)	The obtained plan had first-row six- and five-point irregular stars and a second-row six-point irregular star. 		
The corresponding stars with equal edge lengths	-A test was performed with the outer arms of six-point regular stars touching each other. -The six-point star ornamental base size was kept constant. -The size of the eight-point star (the inner star) was kept constant. 	 a	
	-A test was performed with the inner arms of six-point regular stars touching each other. -The six-point star ornamental base size was kept constant. -The size of the eight-point star (the inner star) was kept constant. 	 b	
	-A test was performed with the inner arms of six-point regular stars touching each other. -The six-point star ornamental base size was changed. -The size of the eight-point star (the inner star) was kept constant. 	 c	

into eight arranged in a circular formation with respect to the centre of the circle (rule 10, subrule 4).

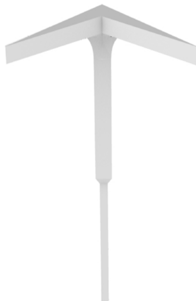
In Rule 11, almonds were created in the smallest module in the frame by forming the lines in the spaces between the irregular star polygons (rule 11, subrule 1). These almonds were replicated to cover the whole pattern (rule 11, subrule 2).

In Rule 12, the grids and axes were eliminated (rule 12, subrule 1). The geometry of the plan of muqarnas was obtained by eliminating a half of the pattern (rule 12, subrule 2).

Proof/verification

In the muqarnas plan analysis, it was observed that star polygons did not have edges of the same length. However, since it was known that the edges of star polygons were known to have equal lengths in many muqarnas plans and Islamic pattern samples, the idea emerged to verify that the edge lengths of star polygons in the analysed muqarnas plan were unequal. In performing this verification, the edges of star polygons were assumed to have equal lengths. The relevant research question was as follows: if the edge lengths of star polygons in this selected muqarnas plan were equal, would the same or similar plan emerge?

Table 4 Types of cells in the muqarnas at the main gate of the Atik Valide Sultan Mosque

Main cells	 a Triangle cell	 b Deltoid cell	 c Almond cell
Intermediate Cells	 d Intermediate triangle cell	 e Intermediate deltoid cell	 f Biped cell
Ornamental Bases	 g Pentagon (the centre of a five-point irregular star polygon)	 h Six-point star (the centre of a six-point irregular star polygon)	

To this end, regular star polygons (star polygons with equal edge lengths) were created first. To create a six-point star, two circles were first formed and two hexagons were placed inside the circles. The corner points of hexagons were joined to form the lines. Thus, a six-point star was created. If desired, the almond alignments could be added by directing them towards the centre of the circles (Table 2). To create a five-point star, two circles were formed and two pentagons were placed inside the circles. The lines were created by using the corner points of the pentagons. Thus, a five-point star was created. If desired, almond alignments could be added by directing them towards the centre of the circles (Table 2).

During the verification, the inner star pattern of the muqarnas obtained from the previous analysis was retained, and it turned out that star polygons with edges of equal lengths could not, in any case, form the original plan of the selected muqarnas. As shown in Table 3a, when the outer arms of the regular six-point stars touch each other, a gap appears between the inner arms of the regular six-point stars and the inner circle. When the whole composition was examined, it was observed that the arms of the stars did not merge and remained exposed in many points. In addition, as shown in Table 3b, when the regular six-point stars were placed so that they touched the endpoints of the inner star pattern, their outer arms overlapped each other. In other words, if the regular six-point stars were positioned so that their arms closest to the centre were touching each other, their outer arms overlapped. The same result was obtained when the scale was changed (Table 3c).

The Third Dimension

A muqarnas is created by combining various types of cells. It is observed that the muqarnas examined in this study is created with six types of cells (Table 4a–f). Apart from the main and intermediate cells, there are parts in pentagon-shape and six-point star geometry, which can be described as two types of ornamental bases (Table 4g, h). There are detailed ornaments on these base parts that are placed at the star centres in the muqarnas plan.

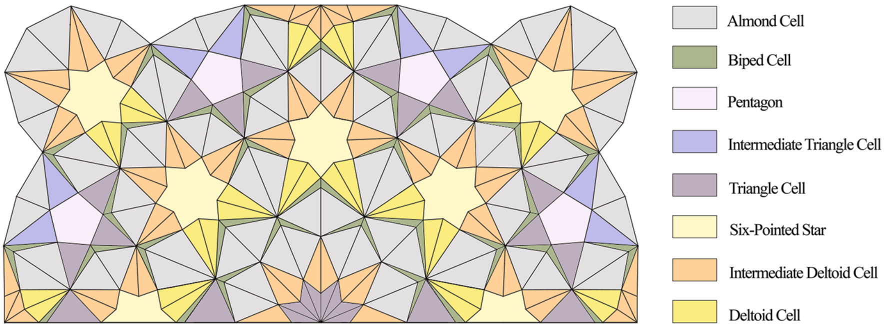


Fig. 5 Composition of cells in the muqarnas plan

Table 5 Analysis of locations and sizes of cells in the muqarnas plan

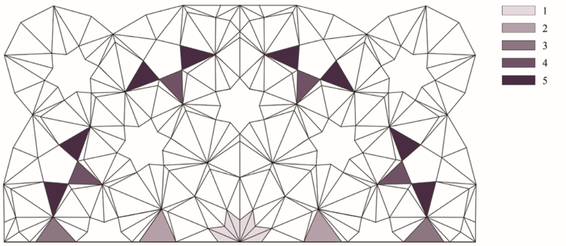
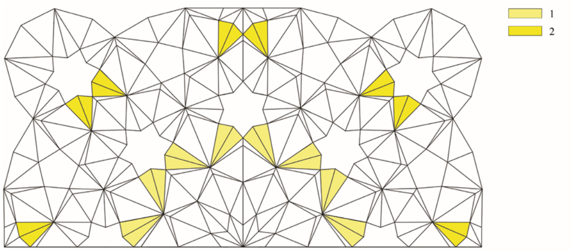
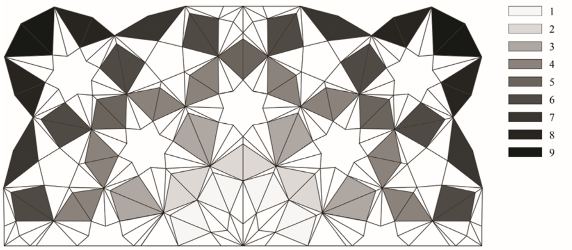
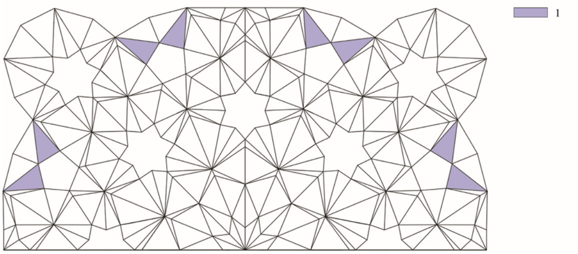
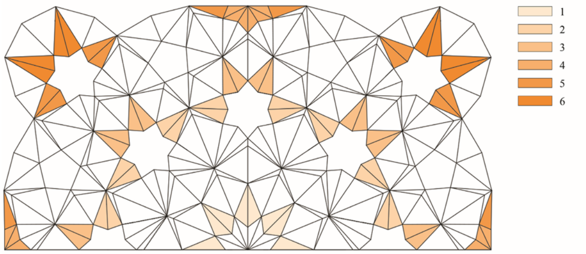
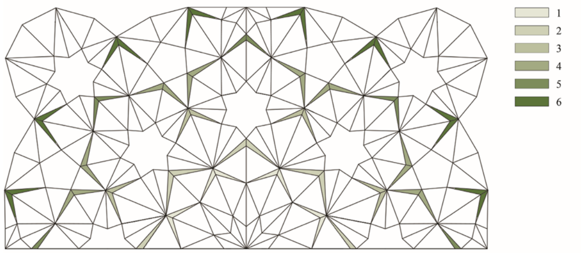
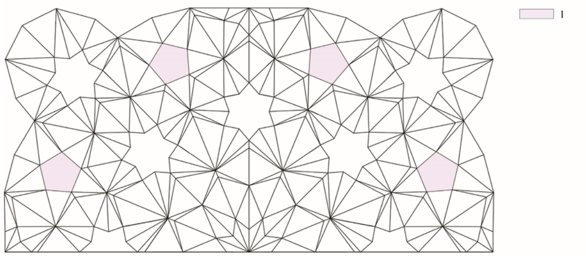
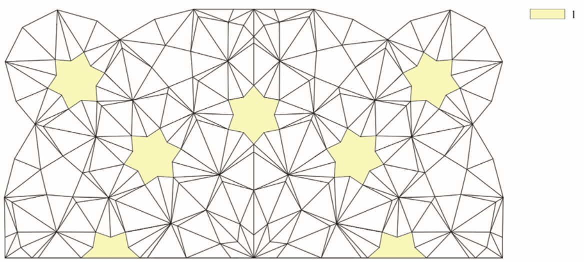
No.	Explanation	Image
a	Triangle cells of various sizes (parts of five-point irregular star polygons). *Although the cells in the edge (labelled as 2 and 3 in the legend) look similar to almond cells in the plan, it is determined that those are triangle cells, as can be seen in 3D.	
b	Deltoid cells of various sizes (parts of six-point irregular star polygons).	
c	Almond cells of various sizes.	
d	Intermediate triangle cells (parts of five-point irregular star polygons).	

Table 5 (continued)

e	Intermediate deltoid cells of various sizes (parts of six-point irregular star polygons).	
f	Biped cells of various sizes.	
g	Pentagons (centres of five-point irregular star polygons).	
h	Six-point stars (centres of six-point irregular star polygons).	

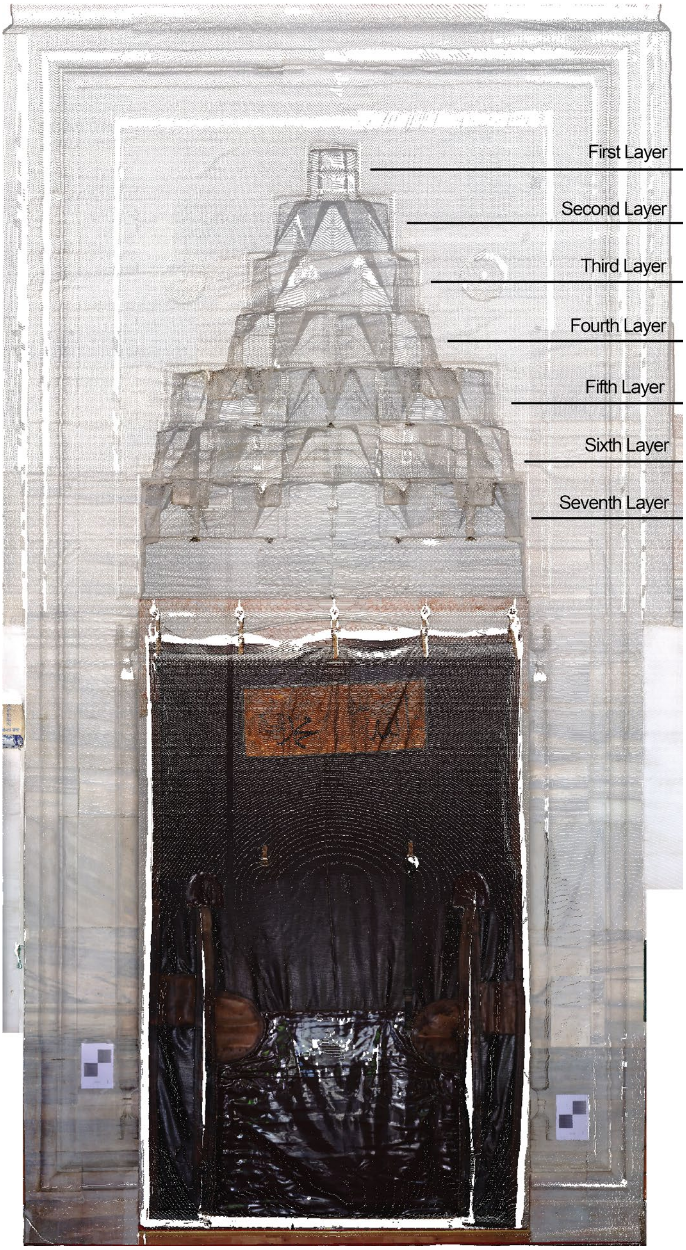


Fig. 6 Front view of the muqarnas

The main cells that constitute the muqarnas can be defined as triangle cells (Table 4a), deltoid cells (Table 4b) and almond cells (Table 4c). These main cells have two stages. For example, there is a grading in the middle part of the body of

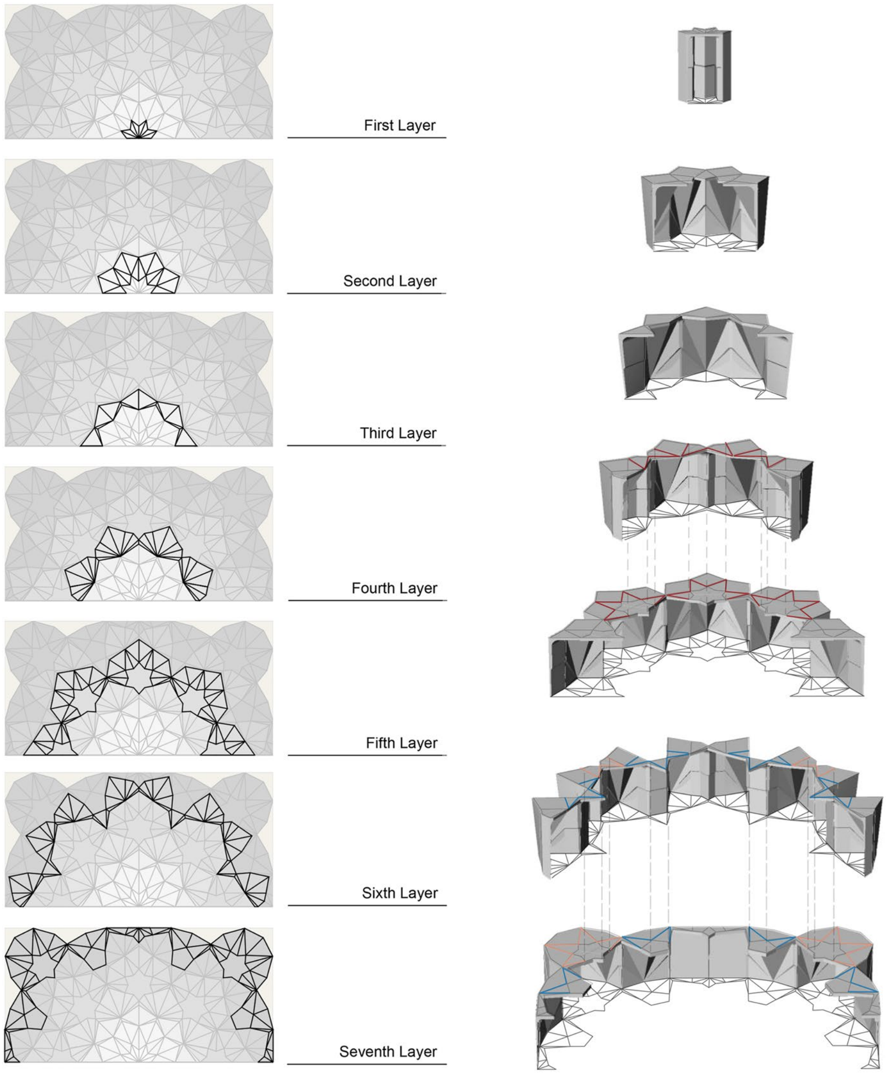


Fig. 7 Layer representation of the muqarnas

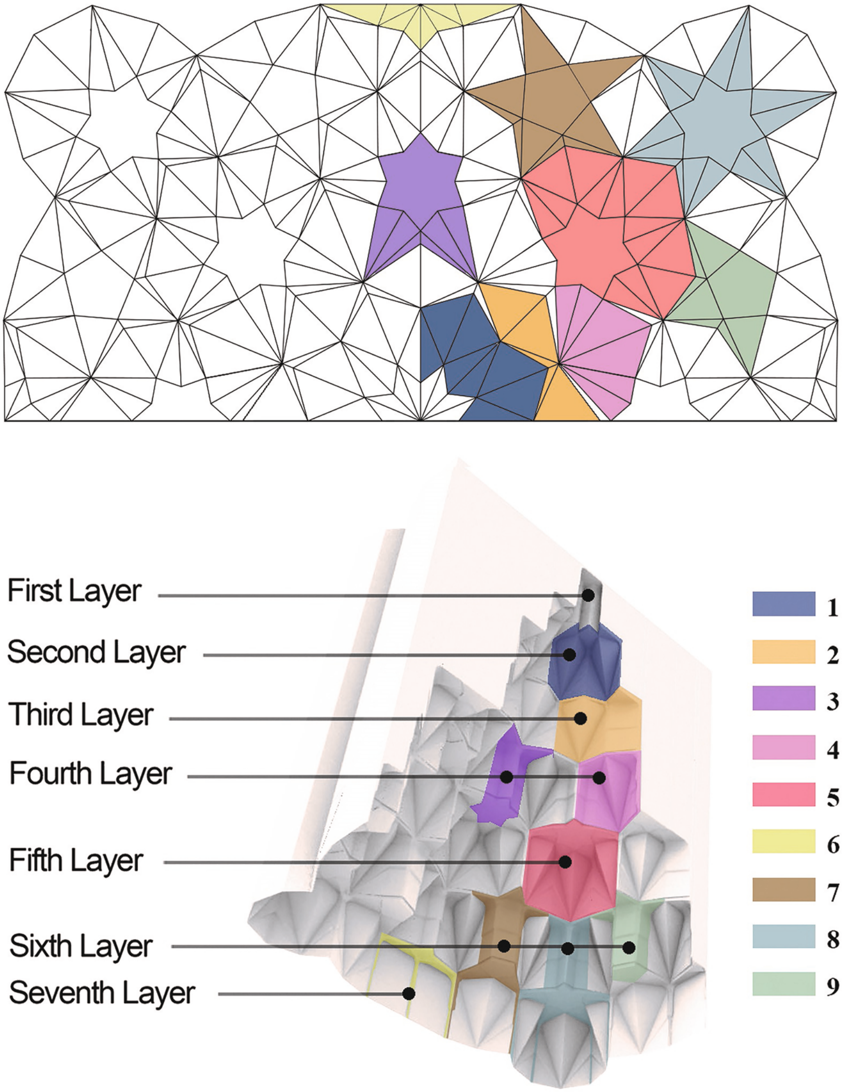


Fig. 8 Representation of various cell combination compositions

a triangle cell and a deltoid cell. There is a radial grading in an almond cell from the bottom to the wings. A triangle cell has a plain body, while a deltoid cell has a double-winged body. In the examined muqarnas, triangle cells are used in the arms of five-point irregular star polygons. Deltoid cells are used in the arms of six-point irregular star polygons.

The intermediate cells of muqarnas can be defined as intermediate triangle cells (Table 4d), intermediate deltoid cells (Table 4e) and biped cells (Table 4f). Intermediate cells are those that act as a binding in the joints of the main cells. As

Table 6 Cell combinations

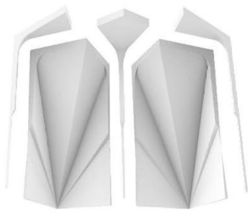
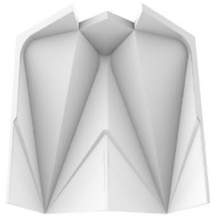
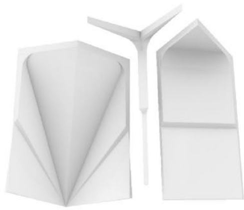
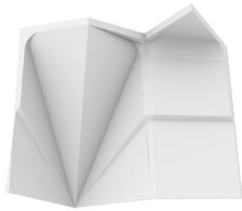
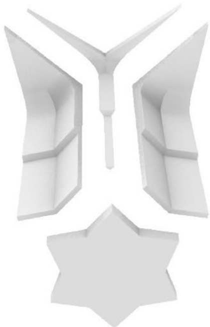
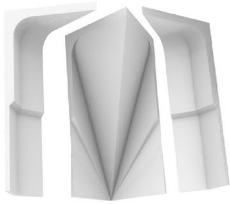
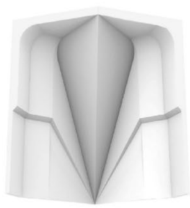
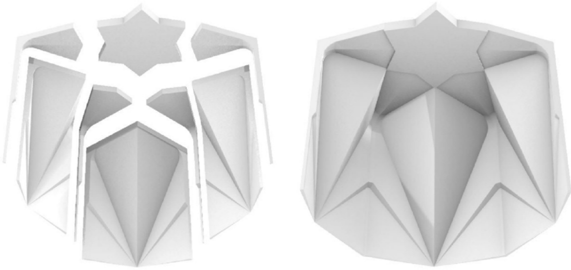
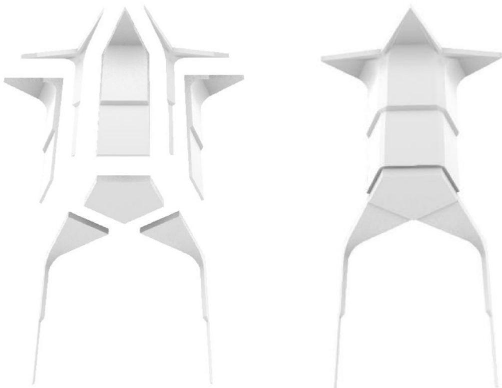
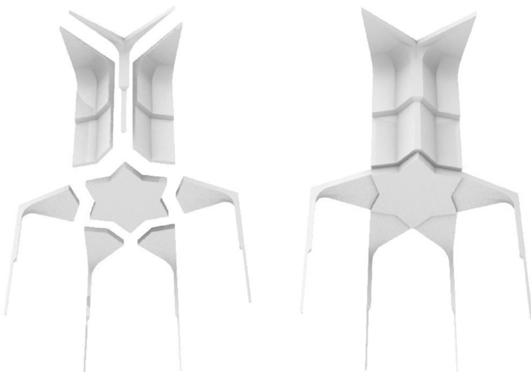
No	Explanation	Decomposed State	Composition State
a	Combination example in the second layer Almond cells (two pieces) + intermediate deltoid cell (middle) + half-deltoid intermediate cells (two pieces)		
b	Combination example in the third layer Almond cell (left) + biped cell (middle) + triangle cell (right)		
c	Combination example in the fourth layer Deltoid cells (two pieces) + biped cell (middle) + six-point star ornamental base (In other words, a combination of two arms of a six-point irregular star polygon and its centre)		
d	Combination example in the fourth layer Almond cell (centre) + deltoid cells (two pieces)		

Table 6 (continued)

e	Combination example in the fifth layer Intermediate deltoid cells (four pieces) + almond cells (three pieces) + six-point star ornamental base. (In other words, a combination of four arms of a six-point irregular star polygon and its centre)	
f	Combination example in the sixth layer Triangle cells (three pieces) + biped cells (two pieces) + intermediate triangle cells (two pieces) + pentagon. (In other words, a five-point irregular star polygon composition)	
g	Combination example in the sixth layer Deltoid cells (two pieces) + biped cell + intermediate deltoid cells (four pieces) + six-point star ornamental base. (In other words, a six-point irregular star polygon composition)	

in the case of the main cells, intermediate triangle cells are used in the arms of five-point irregular star polygons, while intermediate deltoid cells are used in the arms of six-point irregular star polygons.

In order to understand muqarnas composition, three types of main cells, three types of intermediate cells and two types of ornamental bases are defined in various colours on the muqarnas plan. As the study of the colour muqarnas plan shows, intermediate triangle cells and triangle cells create a five-point irregular

Table 6 (continued)

h	Combination example in the sixth layer Triangle cells (three pieces) + biped cells (two pieces) + pentagon. (In other words, a combination of three arms of a five-point irregular star polygon and its centre)	
i	Combination example in the seventh layer Intermediate deltoid cells (three pieces) + half-deltoid intermediate cells (two pieces). (In other words, a partially formed six-point irregular star polygon)	

star polygon. In addition, intermediate deltoid cells and deltoid cells create a six-point irregular star polygon (Fig. 5).

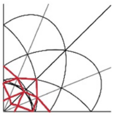
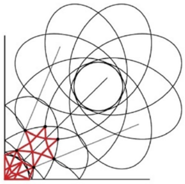
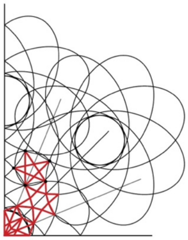
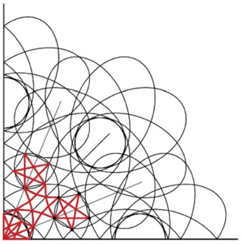
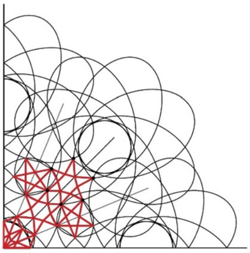
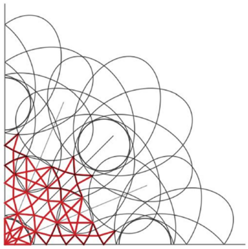
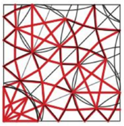
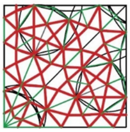
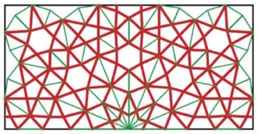
An examination of the muqarnas plan shows that cells vary by dimension within their types. It is determined that triangle cells are of five different sizes (Table 5a), deltoid cells are of two different sizes (Table 5b), almond cells are of nine different sizes (Table 5c), intermediate triangle cells are of a single size (Table 5d), intermediate deltoid cells are of six different sizes (Table 5e) and biped cells have six different sizes (Table 5f). In addition, the pentagon (the centre of a five-point irregular star polygon) type has one size (Table 5g), and the six-point star (the centre of a six-point irregular star polygon) type has one size (Table 5h).

Cell Combinations

The holistic form of the muqarnas is created with a combination of various types of cells. The cells are arranged into different layers in the muqarnas. As an image obtained by laser scanning shows (Fig. 6), the muqarnas consists of seven layers. In Fig. 7, the layers are separated by colour tones on the muqarnas plan. As five- and six-point irregular stars in the plan rise between the layers, their arms correspond to different layers. Each irregular star has two layer transitions.

One cell combination at the second layer, one cell combination at the third layer, two cell combinations at the fourth layer, one cell combination at the fifth layer, three cell combinations at the sixth layer and one cell combination at the

Table 7 Creation process of the muqarnas plan of the Atik Valide Mosque's main gate in the Grasshopper programme

		
a Formation of the first, second, third and fourth stars	b First six-point irregular star formation	c Five-point irregular star formation
		
d Second five-point irregular star formation	e Second six-point irregular star formation	f Formation of halves of six-point irregular stars remaining on the edges
		
g Determination of the boundaries of the shape	h Formation of almonds	i Elimination of the axes and grids, and obtaining the entire shape by mirroring

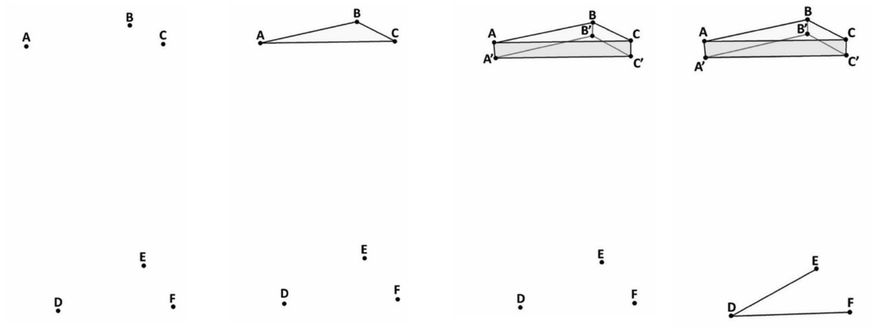
seventh layer are examined. The combinations of cells discussed are shown on the muqarnas plan and a 3D model in different colours (Fig. 8).

The cells and their compositions are as follows.

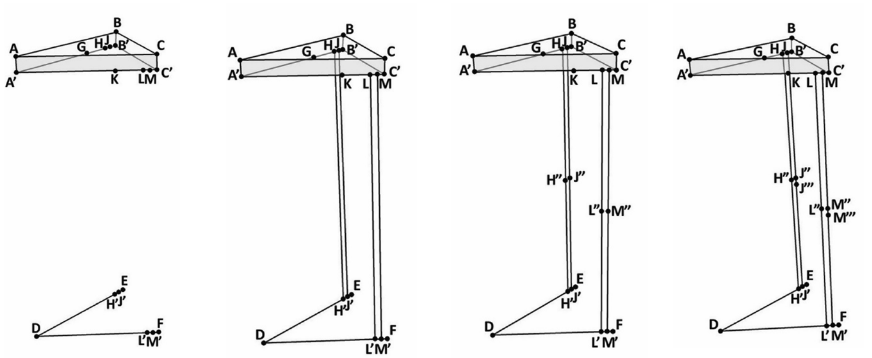
- In the second layer, the composition obtained by combining almond cells, intermediate deltoid cell and half-deltoid intermediate cells is exemplified (Table 6a).
- In the third layer, the composition obtained by combining an almond cell, a biped cell and a triangle cell is exemplified (Table 6b).
- In the first example in the fourth layer, the composition obtained by combining deltoid cells, a biped cell and a six-point star ornamental base (the centre of a six-point irregular star polygon) is exemplified (Table 6c). In other words, a

Table 8 Creation of triangle cell

a	b	c	d
First, points A, B, C, D, E and F in two different planes were determined by reference to the intersection points in the muqarnas plan drawing.	Lines were created by using points A, B and C, and a surface was then defined.	Afterwards, points A’, B’ and C’ on the Z axis were determined. A volume was created by using points A, B, C and points A’, B’, C’.	Lines were formed between points A’ and B’, points A’ and C’, points D and E, and points D and F.



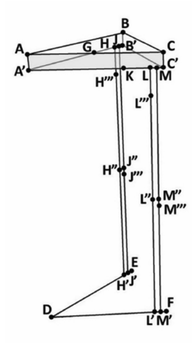
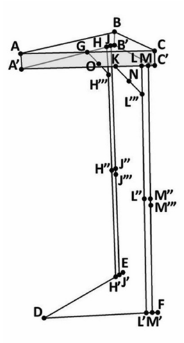
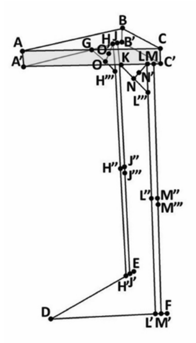
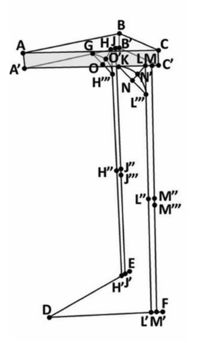
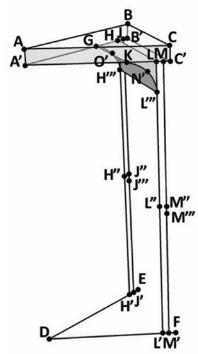
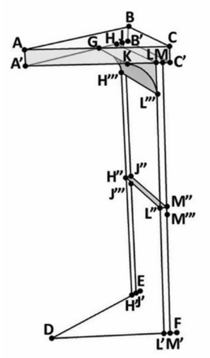
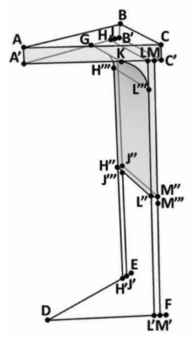
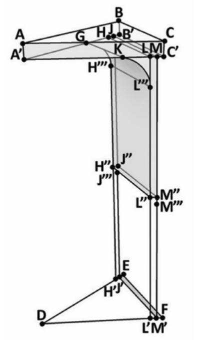
e	f	g	h
Points G, H and J on the A’B’ line, points K, L and M on the A’C’ line, points H’ and J’ on the DE line, and points L’ and M’ on the DF line were created.	Lines were formed between points H and H’, J and J’, L and L’, and M and M’.	Points H’’, J’’, L’’ and M’’ were defined in the middle of each of the four created lines.	Point J’’’ was defined between points J’’ and J’, and point M’’’ was defined between points M’’ and M’.



combination of two arms of a six-point irregular star polygon and its centre is created.

- In the second example in the fourth layer, the composition obtained by the combination of an almond cell and deltoid cells is exemplified (Table 6d).

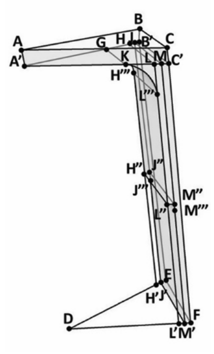
Table 8 (continued)

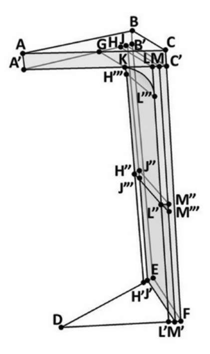
i	j	k	l
Point H''' was defined between points H and H'', and point L''' was defined between points L and L''.	Point O was placed between points G and H''', and point N was placed between points K and L'''.	Point O' was placed between points O and H, and point N' was placed between points N and L.	An arc was created, connecting points G, O' and H''', and another arc was created to connect points K, N' and L'''.
			
m	**n**	**o**	**p**
A plane was created between the arc KN'L''' and points K, L and L'''. Similarly, a plane was created between the arc GO'H''' and points G, H and H'''. A volume between these two planes was then created.	To create the graded surface, lines were first created between points H'' and J'' and between points L'' and M''. A surface was then identified between these lines.	A volume was created between this defined surface and the surface created using points H, J, L and M. Thus, the stage of the cell was created.	Lines were first formed between points J', E, F and M', and then a surface was created from these lines.
			

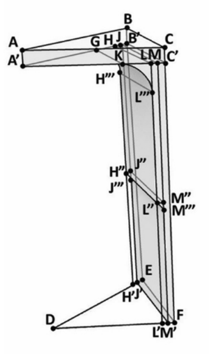
- In the fifth layer, the composition obtained by the combination of intermediate deltoid cells, almond cells and a six-point star ornamental base (the centre of a six-point irregular star polygon) is exemplified. In other words, the combination of four arms of a six-point irregular star polygon and its centre is created (Table 6e).

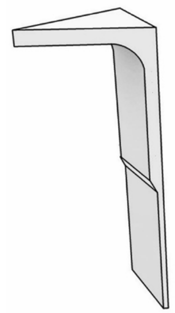
Table 8 (continued)

r	s	t	u
A volume was created between this defined surface and the surface created using points J, B', C' and M.	To create a bevel, lines were formed between points H'', J''', M''' and L'', and then a surface was defined.	To cover the side surface gaps, surfaces were created between points H'', J'' and J''' and between points L'', M'' and M'''.	The formation of a triangle cell was completed by hiding the points and lines.









- In the first example in the sixth layer, the composition obtained by the combination of triangle cells, biped cells, intermediate triangle cells and a pentagon (the centre of a five-point irregular star polygon) is exemplified (Table 6f). In other words, a combination with a five-point irregular star polygon composition is created.
- In the second example in the sixth layer, the composition obtained by the combination of deltoid cells, a biped cell, intermediate deltoid cells and a six-point star ornamental base (the centre of a six-point irregular star polygon) is exemplified (Table 6g). In other words, a six-point irregular star polygon composition is created.
- In the third example in the sixth layer, the composition obtained by the combination of triangle cells, biped cells and a pentagon (the centre of a five-point irregular star polygon) is exemplified (Table 6h). In other words, a combination of three arms of a five-point irregular star polygon and its centre is created.
- In the seventh layer, the composition obtained by the combination of intermediate deltoid cells and half-deltoid intermediate cells is exemplified (Table 6i). In other words, a partial formation of six-point irregular star polygons is created.

Parameterization

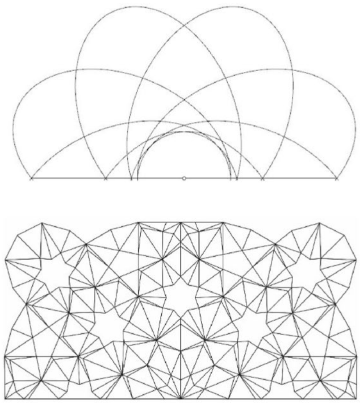
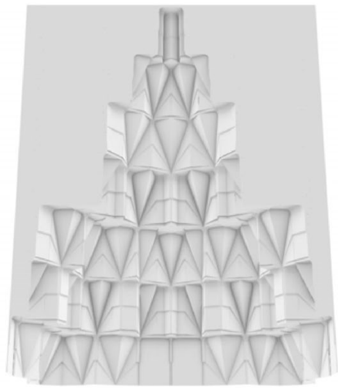
Creation of Parametric Geometry

To parameterize the selected muqarnas, the plan was first prepared in 2D in Grasshopper. While this plan was being prepared, the rules in Table 1 were applied.

Table 9 The variables and value ranges for parametric muqarnas

Variable	Explanation	Value range of the related parameter (includes decimal values)		
		Min	Max	Unit
V1	The value range of the small radius of ellipses in the primary ellipse grid	22	35	cm
V2	The value range of the large radius of ellipses in the primary ellipse grid	33.15	38.50	cm
V3	The value range of the distance from the ellipse grid to the origin	19.4	24.9	cm
V4	The value range of the distance from the points (close to the centre) on the second star to the origin (the first layer in a 3D representation)	1	12.76	cm
V5	The value range of the distance from the points (away from the centre) on the second star to the origin (the first layer in a 3D representation)	0.25	15	cm
V6	The value range of the distance from the secondary ellipse grid to the origin	75.5	94.3	cm
V7	The value range of the angle of rotation of the hexagon in the six-point irregular star	0	90	Degree
V8	The value range of the thickness of a biped cell	−1	1	cm
V9	The proximity value of the points of a six-point star ornamental base to the centre	0	1	cm
V10	The value range of the height of layer 7	0	20	cm
V11	The value range of the height of layer 6	20	40	cm
V12	The value range of the height of layer 5	40	60	cm
V13	The value range of the height of layer 4	60	80	cm
V14	The value range of the height of layer 3	80	100	cm
V15	The value range of the height of layer 2	100	120	cm
V16	The value range in the Z axis of a six-point star ornamental base	−10	0	cm

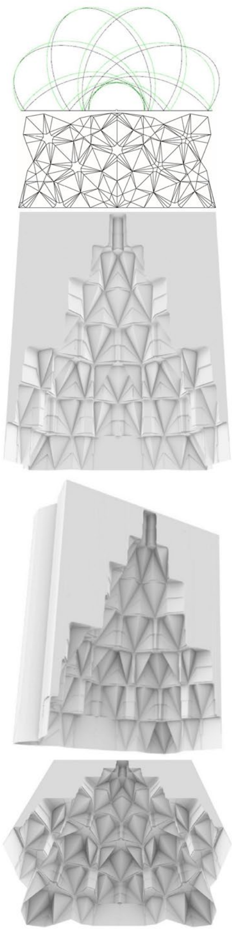
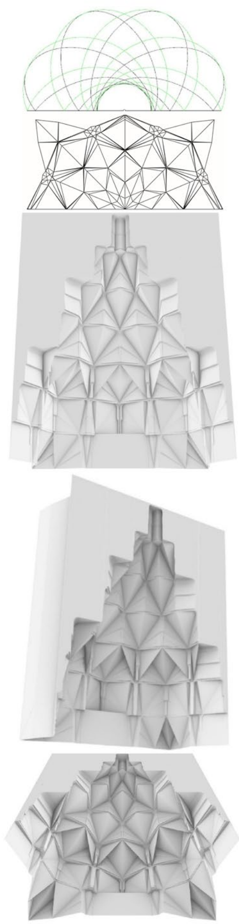
Table 10 The combination of variable values that lead to the proportions of the muqarnas analysed at the main gate of the Atik Valide Mosque

Proportional/ Regular Values	Plan	Perspective
V1: 25 V2: 37 V3: 22 V4: 10 V5: 7 V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25		

The only difference is that a quarter of the pattern was formed to generate a shorter code. Additionally, to obtain the final shape, a mirroring operation was performed.

As Table 7 shows, the first, second, third and fourth stars were formed in relation to the ellipse grid (Table 7a). The six-point irregular star was then formed in relation to the secondary ellipse grid. Thus, the star could be parameterized (Table 7b). Later, a five-point irregular star was created in relation to the points on the arms of the six-point irregular star (Table 7c). Similarly, another five-point irregular star was formed in relation to the points on the arms of the six-point irregular star (Table 7d). When the second six-point irregular star was formed, the related points of previously formed other irregular stars were considered. Thus, when a star was parameterized, if there were other stars associated with that star, they were also parameterized at the same time (Table 7e). Later, halves of six-point irregular stars were formed at the edges of a quarter-slice part (Table 7f). The boundaries of the shape were then determined, and almonds were formed (Table 7g, h). Finally, the axes and grids were eliminated, and the created pattern was mirrored (Table 7i).

Table 11 Various types of muqarnas formed by changing the variable values affecting the 2D formation

No.	Variables' values	Alternative 1	Variables' values	Alternative 2
1	**V1: 28** V2: 37 V3: 22 V4: 10 V5: 7 V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 V1: The value range of the small radius of ellipses in the primary ellipse grid		**V1: 32** V2: 37 V3: 22 V4: 10 V5: 7 V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	

As stated in the above sections, muqarnas are traditionally formed by combining 3D elements. In this parametric muqarnas creation study, the method of forming muqarnas with such 3D elements was used. As an example, the method of modelling a triangle cell in Grasshopper is shown in Table 8.

Table 11 (continued)

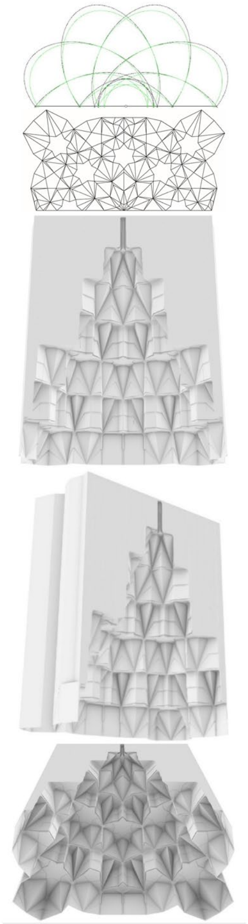
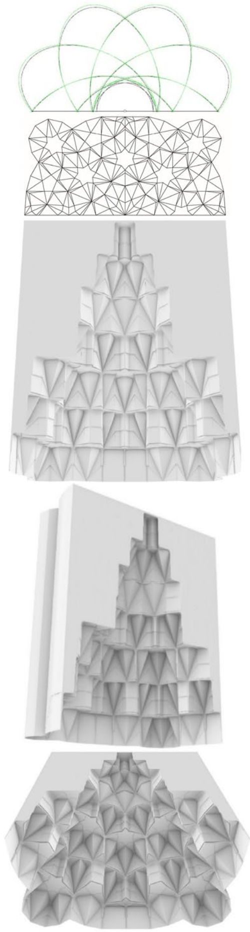
2	V1: 25 **V2: 33.5** V3: 22 V4: 10 V5: 7 V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 V2: The value range of the large radius of ellipses in the primary ellipse grid		V1: 25 **V2: 38.5** V3: 22 V4: 10 V5: 7 V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	

Table 11 (continued)

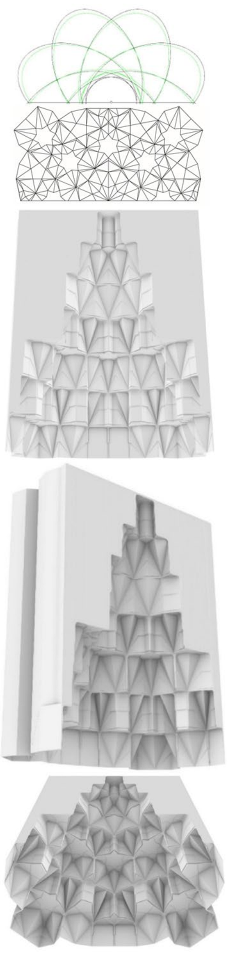
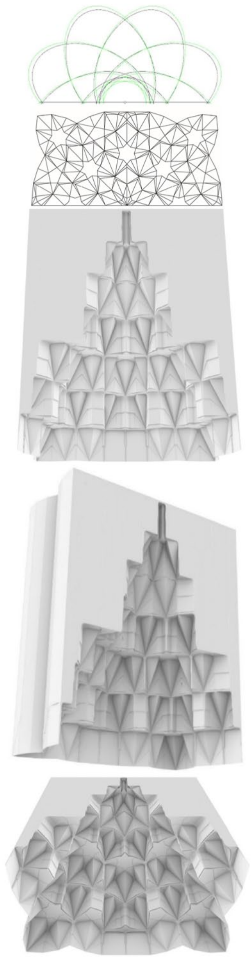
3	V1: 25 V2: 37 **V3: 19.4** V4: 10 V5: 7 V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 V3: The value range of the distance from the ellipse grid to the origin		V1: 25 V2: 37 **V3: 24.4** V4: 10 V5: 7 V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	

Table 11 (continued)

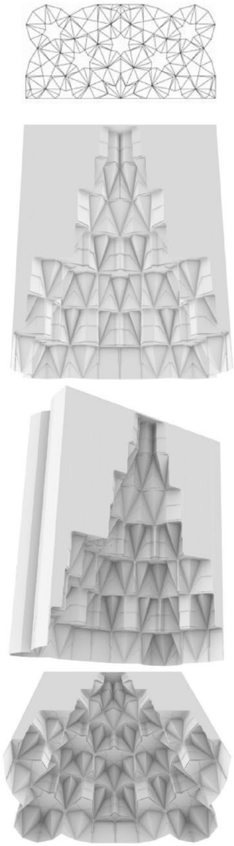
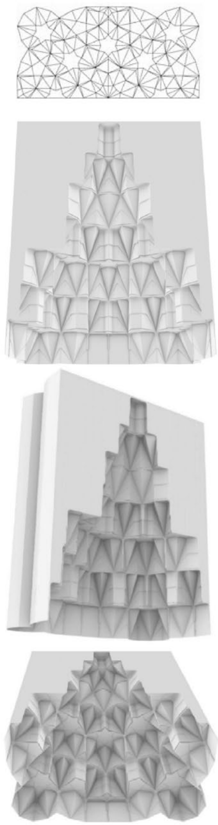
4	V1: 25 V2: 37 V3: 22 **V4: 2.25** V5: 7 V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 V4: The value range of the distance from the points (close to the centre) on the second star to the origin (the first layer in a 3D representation)		V1: 25 V2: 37 V3: 22 **V4: 8** V5: 7 V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	

Table 11 (continued)

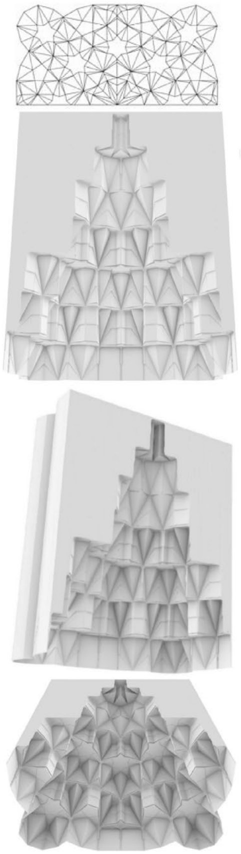
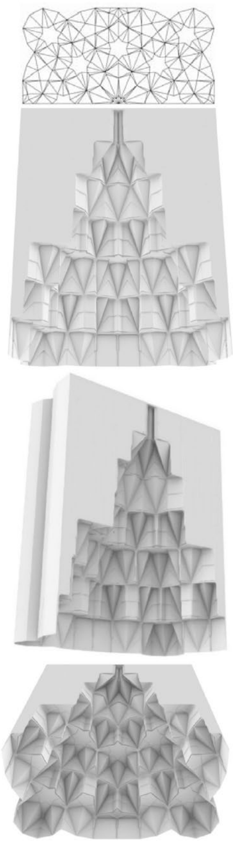
5	V1: 25 V2: 37 V3: 22 V4: 10 **V5: 0.45** V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 V5: The value range of the distance from the points (away from the centre) on the second star to the origin (the first layer in a 3D representation)		V1: 25 V2: 37 V3: 22 V4: 10 **V5: 12** V6: 88.536 V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	

Table 11 (continued)

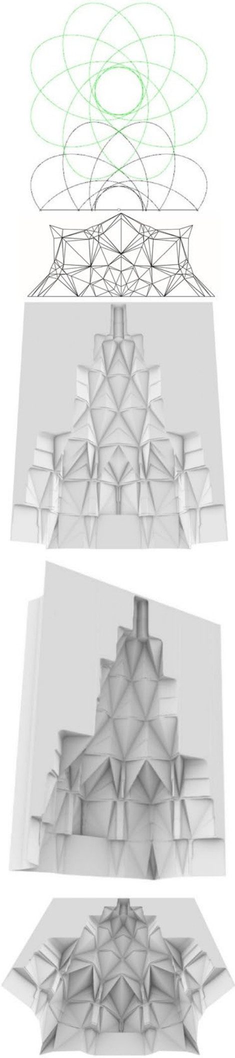
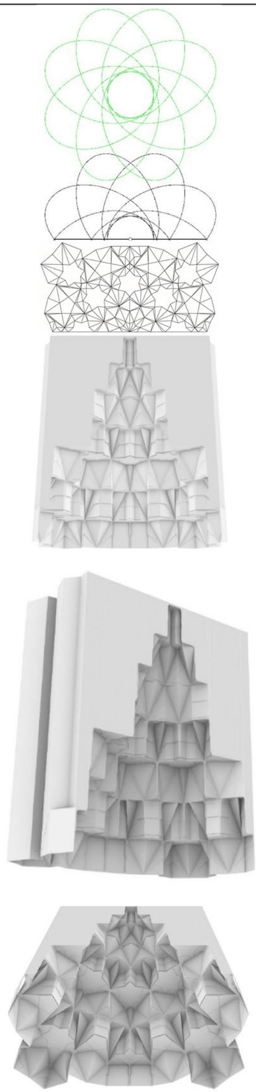
6	V1: 25 V2: 37 V3: 22 V4: 10 V5: 7 **V6: 75.5** V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 V6: The value range of the distance from the secondary ellipse grid to the origin		V1: 25 V2: 37 V3: 22 V4: 10 V5: 7 **V6: 94.25** V7: 45 V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	

Table 11 (continued)

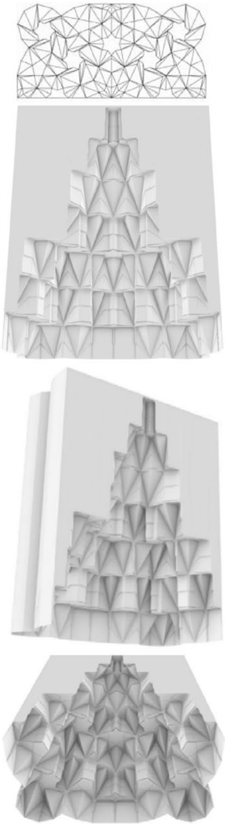
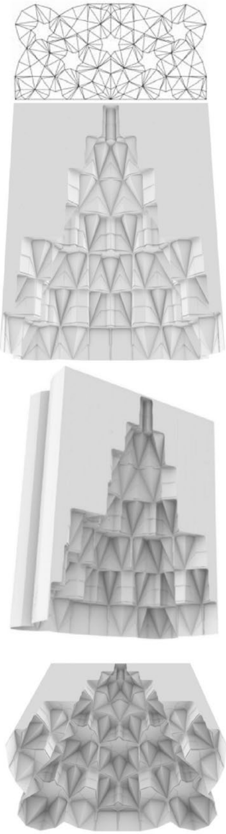
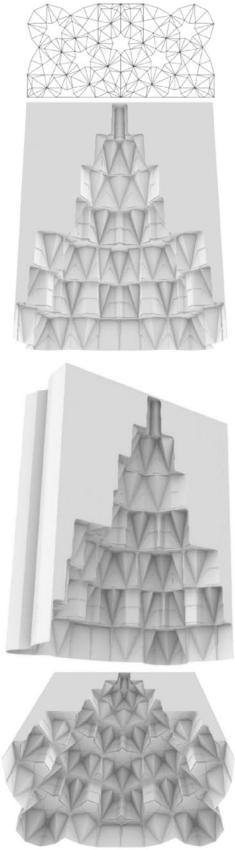
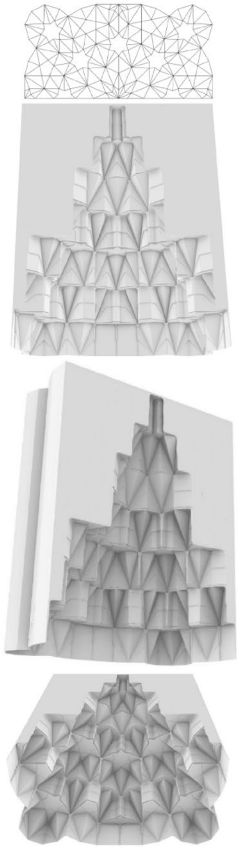
7	V1: 25 V2: 37 V3: 22 V4: 10 V5: 7 V6: 88.536 **V7: 0** V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 V7: The value range of the angle of rotation of the hexagon in the six-point irregular star		V1: 25 V2: 37 V3: 22 V4: 10 V5: 7 V6: 88.536 **V7: 68** V8: 0.15 V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	

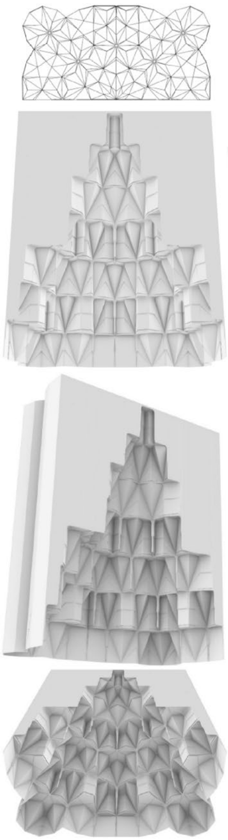
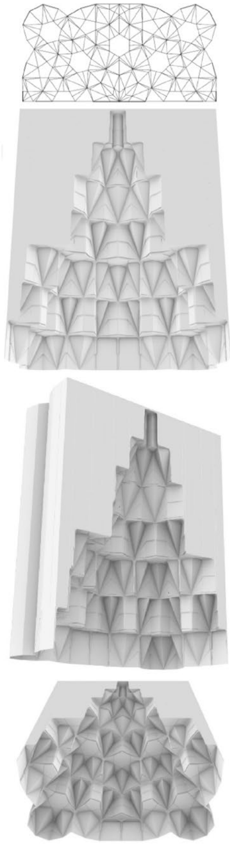
Table 11 (continued)

8	V1: 25 V2: 37 V3: 22 V4: 10 V5: 7 V6: 88.536 V7: 45 **V8: -0.5** V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 V8: The value range of the thickness of a biped cell 		V1: 25 V2: 37 V3: 22 V4: 10 V5: 7 V6: 88.536 V7: 45 **V8: 1** V9: 0.8 V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 	

Variables

There are nine variables in the parameterization of the muqarnas plan in two dimensions. These are the value ranges of: the small radius of ellipses in the primary ellipse grid (V1); the large radius of ellipses in the primary ellipse grid (V2); the distance from the ellipse grid to the origin (V3); the distance from the points (close to the centre) on the second star to the origin (the first layer in a 3D representation) (V4); the distance from the points (away from the centre) on the second star to the origin (the first layer in a 3D representation) (V5); the distance from the secondary ellipse grid to the origin (V6); the angle of rotation of the hexagon in the six-point irregular star (V7); and the thickness of a biped cell (V8); as well as the proximity value of the points of a six-point star ornamental base to the centre (V9). When the minimum and maximum values of variables were determined, the maximum

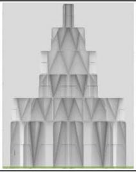
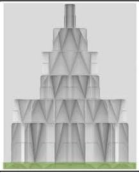
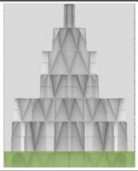
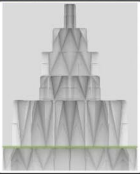
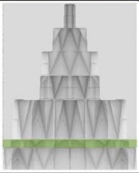
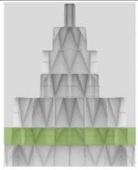
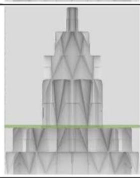
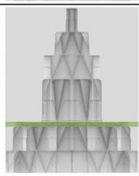
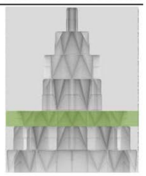
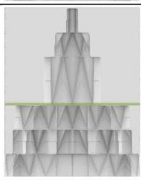
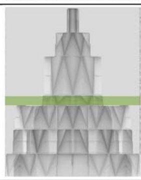
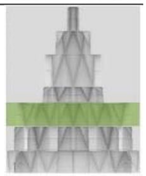
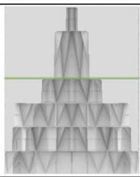
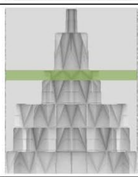
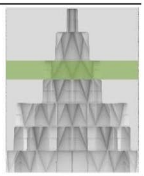
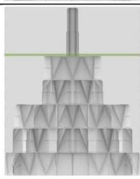
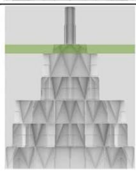
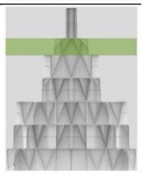
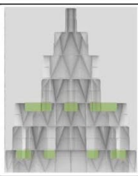
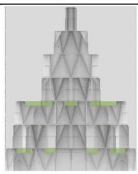
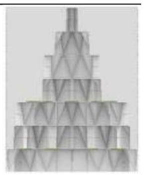
Table 11 (continued)

9	V1: 25 V2: 37 V3: 22 V4: 10 V5: 7 V6: 88.536 V7: 45 V8: 0.15 **V9: 0.1** V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25 V9: The proximity value of the points of a six-point star ornamental base to the centre		V1: 25 V2: 37 V3: 22 V4: 10 V5: 7 V6: 88.536 V7: 45 V8: 0.15 **V9: 1** V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	

value range in which the muqarnas plan was not degraded or did not disintegrate was considered (Table 9). Table 10 shows the combination of variable values that results in the proportions of the analysed muqarnas at the main gate of the Atik Valide Mosque.

In addition, various types of muqarnas formed by changing the variable values in 2D are shown in Table 11. For variables related to the ellipse grid, how the ellipse grid was parameterized in the background is indicated by the green colour. For example, two different muqarnas are shown for the first parameterization, where the values of V1 were 28 and 32 (while other variable values were kept constant). In the examples, only variables related to the muqarnas plan were changed. However, as Table 11 shows, when a 2D plan is parameterized, the 3D version of the muqarnas changes indirectly.

Table 12 Various types of muqarnas formed by changing the variable values affecting the 3D formation

No.	Variables' values	Alternative 1	Variables' values	Alternative 2	Variables' values	Alternative 3
10	**V10: 0** V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25		**V10: 6** V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25		**V10: 14** V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	
11	V10: 20 **V11: 20** V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25		V10: 20 **V11: 26** V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25		V10: 20 **V11: 34** V12: 60 V13: 80 V14: 100 V15: 120 V16: -1.25	
12	V10: 20 V11: 40 **V12: 40** V13: 80 V14: 100 V15: 120 V16: -1.25		V10: 20 V11: 40 **V12: 44** V13: 80 V14: 100 V15: 120 V16: -1.25		V10: 20 V11: 40 **V12: 53** V13: 80 V14: 100 V15: 120 V16: -1.25	
13	V10: 20 V11: 40 V12: 60 **V13: 60** V14: 100 V15: 120 V16: -1.25		V10: 20 V11: 40 V12: 60 **V13: 68** V14: 100 V15: 120 V16: -1.25		V10: 20 V11: 40 V12: 60 **V13: 73** V14: 100 V15: 120 V16: -1.25	
14	V10: 20 V11: 40 V12: 60 V13: 80 **V14: 80** V15: 120 V16: -1.25		V10: 20 V11: 40 V12: 60 V13: 80 **V14: 88** V15: 120 V16: -1.25		V10: 20 V11: 40 V12: 60 V13: 80 **V14: 96** V15: 120 V16: -1.25	
15	V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 **V15: 100** V16: -1.25		V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 **V15: 108** V16: -1.25		V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 **V15: 114** V16: -1.25	
16	V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 **V16: -8**		V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 **V16: -4.5**		V10: 20 V11: 40 V12: 60 V13: 80 V14: 100 V15: 120 **V16: 0**	

There are seven variables in the parameterization of the muqarnas in three dimensions. These are the value ranges of: the height of layer 7 (V10); the height of layer 6 (V11); the height of layer 5 (V12); the height of layer 4 (V13); the height of

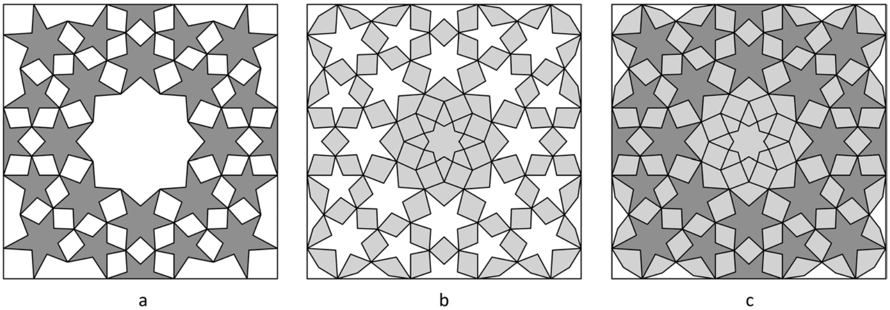


Fig. 9 Elements in the muqarnas plan: **a** five- and six-point irregular stars; **b** almonds; **c** stars and almonds

layer 3 (V14); the height of layer 2 (V15); and the value range in the Z axis of a six-point star ornamental base (V16) (Table 9). The different types of muqarnas formed by changing these variables' values are given in Table 12. For example, using parameterization 8, three different muqarnas were shown, where the corresponding values of V8 were 0, 6 and 14 while other variable values were kept constant.

Conclusions

As a result of the design analysis of the muqarnas, it has been observed that there are two main elements in the composition: a five-point irregular star and a six-point irregular star. In addition, almond elements are used in parts other than five- and six-point irregular stars in the muqarnas plan composition (Fig. 9).

In this study, the selected muqarnas plan was formed with a potential ellipse grid within the framework of shape grammars. As a result, irregular star polygons were automatically generated. This is the proof of the hypothesis put forward.

The most important feature of the muqarnas is their plans. These plans are produced based on geometric rules. Based on these plans, 3D elements of the muqarnas can be produced and combined by traditional methods to create muqarnas. Alternatively, again, muqarnas can be created from the plans using new techniques. For example, in this study a parametric muqarnas was obtained from the plan.

Within the framework of the shape grammar theory, muqarnas can be analysed through the codes. The latter help form parametric muqarnas. For example, it is possible to parameterize the geometric structure in the plan of muqarnas by creating it through the codes. Thus, different plan structures can be obtained, and muqarnas can be differentiated in 3D.

Using parametric muqarnas, different variations of muqarnas can be created easily and quickly. In addition, alternatives for unreadable portions of damaged historical muqarnas can be generated. Furthermore, the written parametric muqarnas script can be used and developed by other designers and researchers.

Today, there is a tendency to use traditional elements (such as muqarnas) in contemporary buildings in the Middle East. These elements, which have the potential to be developed with modern techniques, can present interesting ideas to the designers due to the underlying geometric compositions. Therefore, analysing how muqarnas were made in ancient times would be an important basis for new studies of muqarnas.

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