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RESEARCH

Algorithmic Decomposition of Geometric Islamic Patterns: A Case Study with Star Polygon Design in the Tombstones of Ahlat

Asli Agirbas[1]

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Abstract
The current practice is to make the mathematical analysis of many Islamic patterns in 2D; however, since such patterns actually have 3D features, the third dimension must also be considered. Indeed, the three-dimensional features of the patterns made by carving on stone are very numerous. In this work, it is proposed to perform a 3D mathematical analysis of patterns of this type by algorithmic decomposition. In the cemetery of Ahlat, which is an existing monument, a tombstone with high three-dimensional features, designed by Asil b. Veys (Uveys), was chosen for algorithmical analysis. The mathematical design rules of the star polygon pattern in the selected monument were determined, as based on the shape grammar theory. The probable rules for the creation of the star polygon pattern in this study were produced simultaneously in the computer environment using a visual programming language and a 3D parametric pattern generator of the pattern was created.

Keywords Design analysis · Geometric islamic patterns · Shape grammars · Design computation · Parametric design

Introduction

It is known that the pattern designers in Anatolia, who lived during the medieval period possessed a good knowledge of applied geometry, and which they reflected in their designs on various different surfaces. However, these designers/masters did not make a comparable effort to theorize the geometric information that they applied, which can be considered as advanced for this particular time.

The patterns, which were processed on the various surfaces by pattern designers/ masters, are usually in the form of tessellations formed from equilateral triangles,

✉ Asli Agirbas
asliagirbas@gmail.com

1 Department of Architecture, Fatih Sultan Mehmet Vakif University, Istanbul, Turkey

Birkhäuser

squares and regular hexagons. The most popular of these are square shaped, because they can be produced more easily than others (Sarhangi 2012). In his book, Broug (2008) focuses specifically on tiles that feature squares and hexagons, and describes their geometric decompositions. Significant studies on the mathematical expansion of Islamic patterns were also performed by Bourgoin (1879), El-Said and Parman (1976), Mulayim (1982), Critchlow (1983), Wilson (1988), Demiriz (2000), Arik and Sancak (2007), Burckhardt (2009), Cromwell (2009), Castera (2011), Bodner (2012), Redondo-Buitrago and Huylebrouck (2015), Makovicky and Makovicky (2017), and Redondo-Buitrago (2018), who have studied and analyzed in wide ranging contexts from different points of view. The Hankin (1925a, 1925b) Method, called "from polygons in contact" was an inspiration for many researchers, for instance Kaplan (2005). In addition, contemporary researchers are working on forming new pattern designs based on those previously obtained. For example, Corcuff (2018) has studied the creation of various variations algorithmically, based on the principles of Bourgoin's pattern decomposition methods. Beatini (2017) has investigated the conversion of such patterns to their kinetic versions.

Necipoglu (1995) and Bonner (2003) emphasize their three-dimensional character by referring to double-layered designs in Islamic patterns. In addition, Rigby (2005) has pointed out the multi-layeredness of these patterns in terms of "interlacing braids or ribbons weaving alternately under and over each other" in Islamic patterns. Furthermore, Bonner (2017) made comprehensive classifications of Islamic pattern tessellations in his book, and referred to dual-level designs in Islamic patterns, along with examples. In respect to the third dimension, he also mentioned the topic of "geometric ornaments on domes" (Bonner 2016), and similarly, Kasraei et al. (2016) focused on geometric ornamentations on curved surfaces within the dome. Moreover, Cenani and Cagdas (2007) explored new methods for the generation of three-dimensional forms, based on Islamic patterns. Also, Alacam et al. (2017) analysed muqarnas by folding principle. In addition, Kaplan and Salesin (2004) have developed a tool that produces new designs by bringing Islamic patterns into the third dimension (by translating the patterns into the hyperbolic plane), and hence, in their attempt to move the patterns into the third dimension, they were able to go one level further up from this. However, in these examples, although 3D was found to be important, the 3D knotting detail of the patterns remained in their two-dimensional representations.

In summary, the majority of the mathematical analysis of patterns on existing monuments and the new pattern generation have so far been made in 2D, even though it can be clearly seen that those patterns produced in the medieval period, especially as carved on stone, have three-dimensional components. As a result of the stone carving process, the three-dimensional character of these patterns extends beyond merely being a multiple-layer, and in some parts transforms into folds. Oney (1978) has stated that stone was the main material used by the Seljuks, who had a large presence in Anatolia, and they developed enriched designs on stone in the thirteenth century. Thus, the reliefs in the examples of the first half of the thirteenth century tended to be low relief, while those in the second half of the thirteenth century were generally high relief, and created multi-layer effects, through compositions that interweave with one another and form larger and raised structures. The Ahlat tombstones, in particular, offer very interesting and rich examples in this regard (Oney 1978). The embellishments in stone ornaments of Anatolian Seljuk architecture can be classified as geometric, vegetal, figurative and writing (Kuban 2002). In the present study, focus was made on geometric ornaments in Ahlat tombstones, and a selected particular 3D star-shape was studied. The geometry of the selected 3D pattern was decomposed algorithmically using visual programming language.

Background

Ahlat Cemetery

The cemetery of Ahlat is located in Ahlat, a district of Bitlis Province in Turkey's Eastern Anatolia Region, which has hosted many different civilizations throughout history. The cemetery, which is included in the tentative list of the World Heritage Convention of the United Nations Educational, Scientific and Cultural Organization (UNESCO) sites, dates back to the twelfth-fifteenth centuries. The historic area, now used as an open-air museum, occupies 200.000 square meters and contains more than 6000 tombstones. These tombstones, which are of high artistic quality, contain carved designs that required great skill to carve them (Fig. 1). Although the tombstones are generally similar to each other in material, form, decoration and writing character, they are all different from each other, especially in the detail of their ornamentation. They are generally in the shape of rectangular, and 40–70 cm wide and 1–5 meters high. The material used in these tombstones is ignimbrite, which is a construction material local to Ahlat, where it is called 'Ahlat Stone' (Isik et al. 2015). Ignimbrite, is volcanic in origin, and is a light and easily shaped stone, which could thus be readily carved by the masters (Baykara and Isik, 2016).

Fig. 1 Tombstones from Ahlat Cemetery (photo by the author)

Designers/Masters of Tombstones in Ahlat Cemetery

The signature of the designer is present on the tombstones of the Ahlat cemetery, and in addition, there are inscriptions in the tombstones which reveal their connection to the designers so that we can follow the development of the style of the designer during his career: for example, some tombstones include soubriquets such as apprentice, headworker and master. According to such inscriptions, it can be seen that the designers from Ahlat first become apprentices, and then advanced to become headworkers, during which process they sign their works of art by mentioning the names of their masters. When they became masters, they signed only with their own names, and were able to train apprentices and headworkers. (Karamagarali, 1992).

On the basis of a study of the inscriptions on the Ahlat tombstones, Karamagarali (1992) managed to connect the following names of many masters and the links between them: Osman b. Hasan, Ibrahim b. Kasim, Hasan b. Yusuf, Muhammed Davud, Ahmed el-Muzeyyin, Veys (Uveys) b. Ahmed, Esed b. Eyyub, Cum'a b. Muhammed, Asil b. Veys (Uveys), Muhammed b. Veys (Uveys), Havend b. Bergi, Esed b. Havend, Haci Yusuf b. Miran, Haci Miran b. Yusuf, Haci Mirce b. Miran, Muhammed b. Miran, Kasim b. Ustad Ali, Ahmed, Buus b. Semsik ed-Darbabi el-Hilati, Kasim b. Muhammed, Ustad Ali, Ustad Hoyeng.

In the present work on the algorithmic decomposition of Islamic patterns in 3D, it is aimed to select and examine those examples in which the third dimension is the dominant feature. According to Karamagarali (1992), Asil b. Veys (Uveys), who worked during the end of the thirteenth century and the beginning of the fourteenth century, created very important and very detailed monuments, and developed a double-layered carving technique, which is of particular interest for the present study. The father of Asil b. Veys (Uveys) was Ustad Veys b. Ahmed, and his master was Esed b. Eyyub, who was also his father's apprentice and headworker.

Star Shapes in the Cemetery of Ahlat

Star shapes occupy an important place in Ahlat tombstones, and there are examples of tombstones in the Ahlat cemetery, dating from the thirteenth century which have sixteen or eighteen-pointed stars. Karamagarali (1992) states that these are the earliest examples of developed star shapes on the stones in period when the Anatolian Seljuks lived there, other than on these tombstones, we only encounter the sixteen-pointed star in the outer portal of Aksaray Sultan Han (1229). She also states that, during this period, as for the tombstones of Ahlat, the eighteen-pointed star is not seen in any other works (Fig. 2).

Karamagarali (1992) states that commencing from the start of the thirteenth century to the end of the third quarter, the stars were sixteen and eighteen-pointed in shape, which began as very large in size, but later became smaller and then began to enter the panel in the form of three rows rather than just one, and knots of ribbons gradually became increasingly like a weaved pattern. In the subsequent years, eight-pointed, ten-pointed, and twelve-pointed stars started to become dominant.

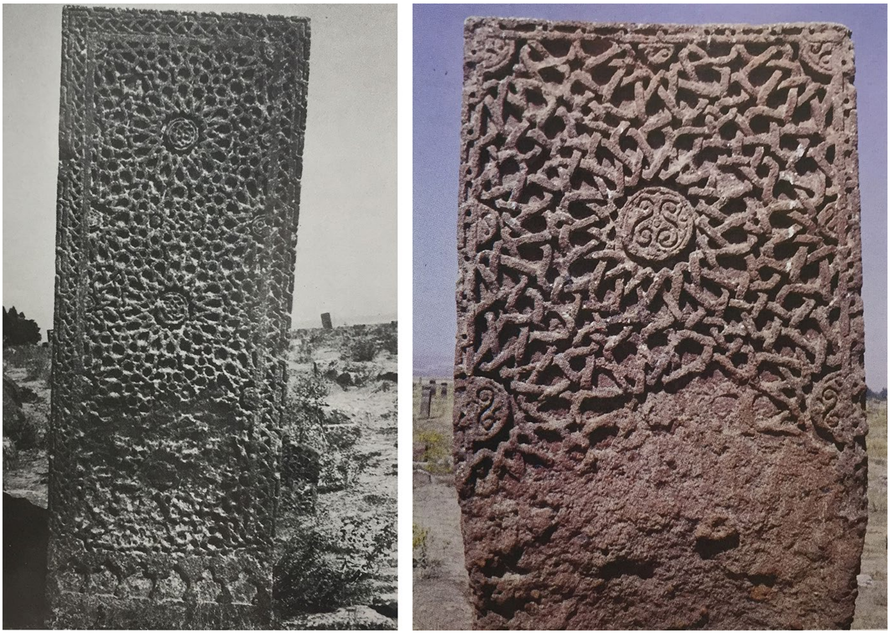


Fig. 2 Eighteen-pointed star shape examples in Ahlat cemetery (Karamagarali 1992)

Geometric Background of the Selected Pattern

The selected double-hexagon based pattern consists of three parts (twelve-pointed star-shape, knots and six-pointed outer star-shape) (Fig. 8). Patterns consisting of twelve-pointed star shape over a layer on double-hexagons can also be found in various forms in other works. The variations are created with using different geometries in the parts. The examples were given in Fig. 3. For instance, the *a* part (a hexagon) of Pattern 1 is the same as the *a* parts of Pattern 3 and Pattern 5, whereas the *b* parts are different (Fig. 3). While the *b* part of Pattern 1 is the same as the *b* part of Pattern 2, the *a* parts are different. While the *a* parts of Patterns 2, 4 and 6 are the same (six-pointed star shape), the geometries of the *b* parts are different. For example, Pattern 4 has a square on the *b* part, while Pattern 6 has an interlaced square on the *b* part. Except the *c* part of Pattern 5, the *c* parts of the other example patterns are the same. The replacement of the *c* part of Pattern 5 in the hexagon is different. When we think of these patterns in 3D, we see that the ribbon passes over another ribbon (Fig. 4). Although the *a* part and the *b* part of the chosen pattern in this study resemble these examples, they are more complex than them.

Pattern 1 in the Niksar Great Mosque (thirteenth century), Pattern 2 and 4 in Ahlat Usta Sagirt Kumbet (thirteenth-fifteenth century), Pattern 3 in Erzurum Cifte Minare Medrese (thirteenth century) and Pattern 5 in one of the Ahlat tombstones (twelfth-fifteenth centuries) can be found (Schneider 1980). The example of Pattern 6 is present in Alcazar of Seville (thirteenth-fifteenth centuries). Other than patterns consisting of twelve-pointed star shape over a layer on double-hexagons, there are also patterns

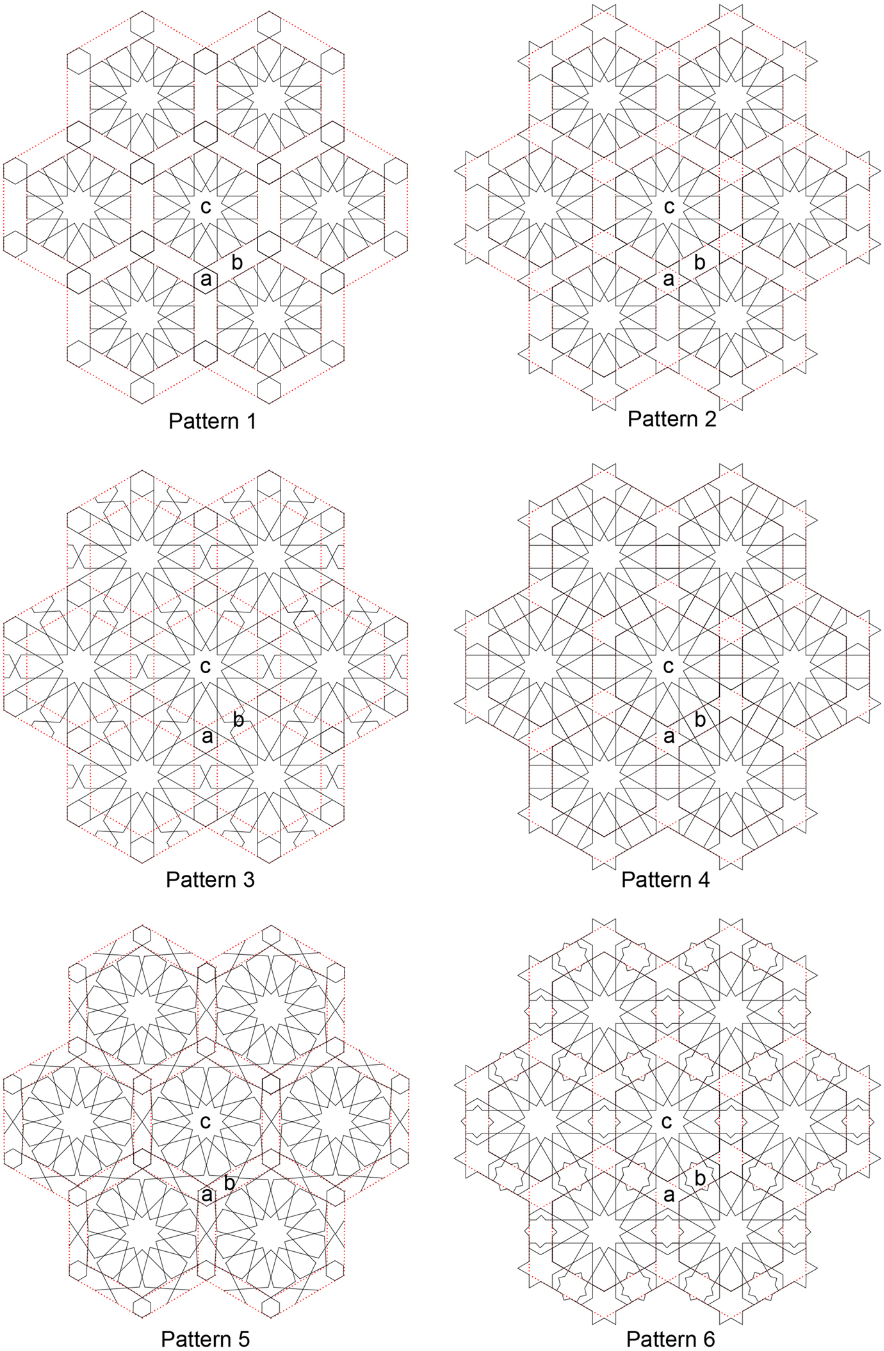


Fig. 3 2D sketches of twelve-pointed star shapes with double hexagon grid [Patterns 1,2,3,4 and 5 were obtained by re-drawing the drawings of Schneider (Schneider 1980)]

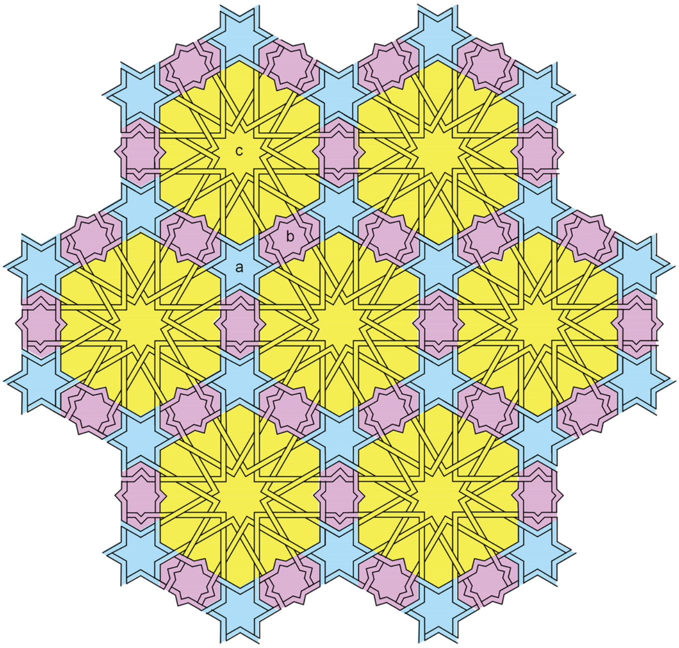


Fig. 4 Pattern 6 with details

consisting of twelve-pointed star shape over a layer on single-hexagons. In addition to this, there are also patterns created by the combination of twelve-pointed star shapes with other star shapes. For example, the pattern consists of the combination of twelve-pointed star shape and sixteen-pointed star shape with interlaced square (the *b* part) can be found in Aksaray Sultan Han (1229).

Methodology

Computer Software

In the present study, the Autocad program was used to provide an initial analysis of a pattern, and then the Grasshopper program, which is a visual programming language program that works as a plug-in in Rhino program, was introduced to create the 3D parametric pattern generator of the pattern, following its algorithmic decomposition. Thus, while decomposing the pattern with codes, the mathematical expansion of the form can be viewed simultaneously on the 3D modeling platform.

Analysis Process

To undertake this process, a technical drawing of the pattern to be analyzed was first made in two dimensions. Thus, if the parts of the historical artifact, are destroyed and lost, they can be revealed as a technical drawing, so to obtain a substructure for the 3D algorithmic analysis to be made in the next step.

In the second step, the pattern is transformed into algorithms. This work is done by examining both the 2D technical drawing of the pattern and the pattern itself. At this stage, it is essential to write the code according to the parts of the pattern determined by the person analyzing it, since the mathematical identification of these elements may differ from one person to another.

Structuring the mathematical background of the code to be written in the theoretical framework of shape grammars has been defined as a methodology. According to the shape grammar theory (Stiny and Gips, 1972; Duarte, 2005; Stiny, 2006; Ozkar, 2011) (in other words rule-based design), forms are created using certain rules. According to this methodology, if the first form is defined as S_0 and the rule to be applied is defined as =>, the computation is described as following: $S_0 => S_1 => S_2 => S_3 => S_4 => \ldots => S_n$ (Knight and Stiny, 2015). By this method, the mathematical background of the pattern's algorithmic decomposition can be presented in a very clear way.

A Case Study

Analyzed Pattern

The most important characteristic of the analyzed pattern in this study is that it is in the form of carving and possesses a knotted structure (Fig. 5). Therefore, when the rules of analysis for this pattern were applied, the pattern was considered three-dimensionally. The examined tombstone was designed by Asil b. Veys (Uveys) who was referred to in the background section.

Initial Analysis (in 2D)

Firstly, the star-shape in the pattern was drawn in 2D. When this had been done, a twelve-pointed inner star was drawn, following which a meticulous focus was made on the parts where a ribbon passes over another ribbon. As a result of this inner star analysis, a system was revealed in which ribbons pile on the top of each other in the third dimension. Thus, one ribbon passes under another at their point of intersection, while at the second intersection this ribbon passes over the top of the other. Then, knots around this inner star drawing were added. It was observed that the third dimensional system of the ribbons also continues on these knots. Although these knots are similar to Solomon's knot (Solomon knot is not a true knot but a link

Fig. 5 Star-shaped pattern for the analysis as described in the text (photo by the author)

diagram), they are preferred to being called as knots in this study, since they are connected to the star shapes in 3D (open-ended/not a closed loop). As a result of this drawing, a unit was formed (Fig. 6).

Twelve-pointed star-shape tessellation of the pattern can easily be created when this unit is placed side by side and repeated (Fig. 7). As was done in other 2D studies, this pattern can be examined by separating it into individual components. For example, this star-shape tessellation can be examined in three parts as twelve-pointed star-shape, knots and six-pointed outer star-shape (Fig. 8). However, it would be problematic to make a 3D algorithmic decomposition of this tessellation in the form of three parts, due to the fact that the coding of the pattern as three separate parts causes them to operate independently of each other. Moreover, overlapping of

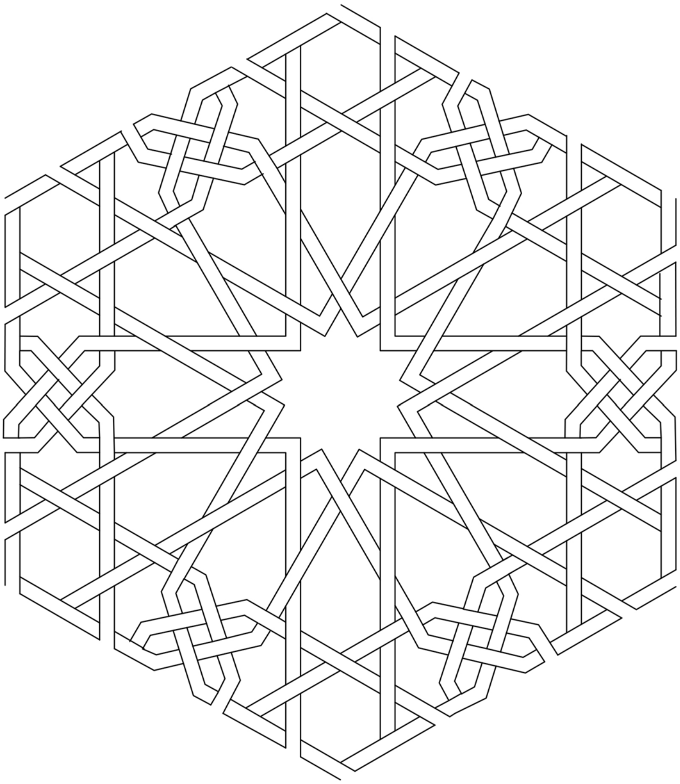


Fig. 6 One unit in 2D drawing

some parts with each other can easily occur, which does not allow the pattern to be parametrized, which is critical for the creation of new derivations can be made from it.

Thus, a secondary analysis was performed on this 2D technical drawing to create a 3D parametric generator of the pattern. In this analysis, a search for a decomposition was begun, in which the parts could operate in a more dependent fashion, rather than in three parts, and so the inner stars and knots were considered as a whole. Hence, the pattern tessellation was actually formed by the two main ribbons, which were used repeatedly, and the six-pointed star-shape was formed by the ends points of these ribbons (Fig. 9). Thus, for example, when the main unit which consists of these ribbons and knots was copied side-by-side, the knots with double-knot were automatically created.

Set of Rules and Algorithmic Decomposition (in 3D)

The possible series of rules for creating this pattern was determined and transferred to the algorithms in the same framework. Accordingly, a 'circle' was

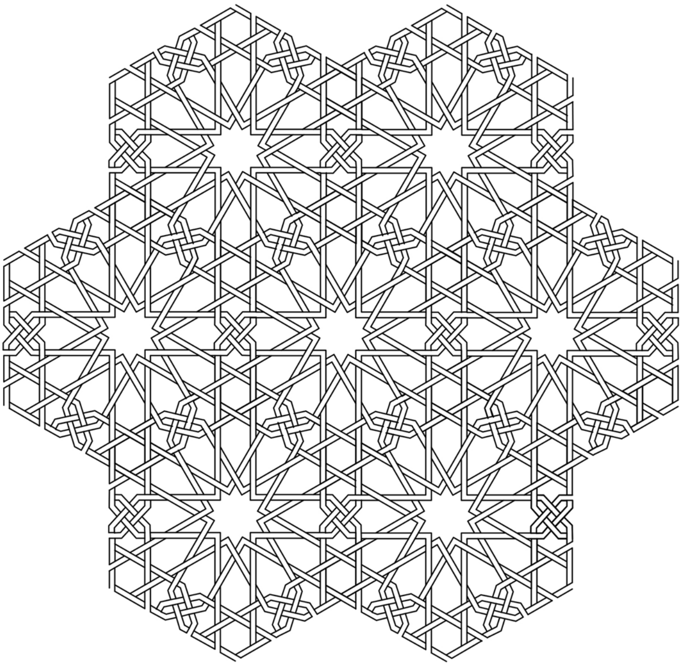


Fig. 7 2D twelve-pointed star-shape tessellation

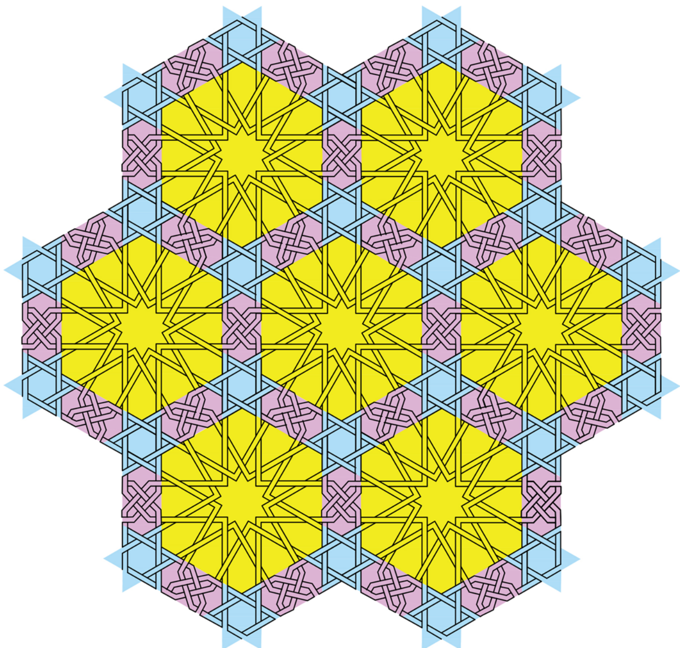


Fig. 8 Three parts in the 2D twelve-pointed star-shape tessellation

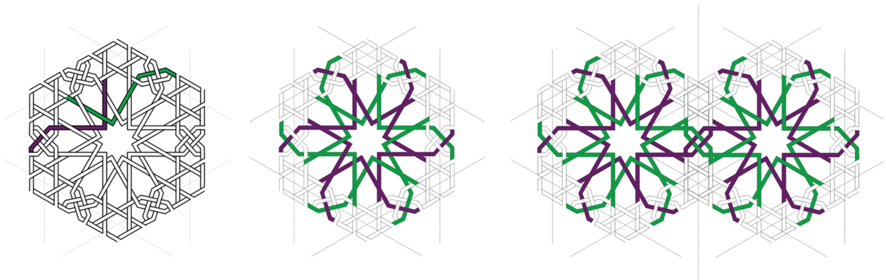


Fig. 9 Initial analysis that forms the basis of algorithmic decomposition

first created and in Grasshopper, the size of which can be parametrized by the ‘number slider’ parameter connected to the radius value of the circle component. The circle was next divided into six equal pieces, and the six point outputs were obtained as a result of this process using ‘divide curve’ component. This value can easily be parametrized by the number slider placed in the ‘number of segments’ input of the divide curve component. A polygon (hexagon) was created by combining these points using ‘list item’ and ‘line’ components (Rule Series 1). Then, a circle was created in the centroid of the hexagon with the help of the ‘area’ and ‘circle’ components. With the aid of a ‘divide curve’ component, this inner circle was divided into twelve equal pieces, and with the aid of the ‘list item’ component, two points were selected from the parts in ¼ of this inner circle (Rule Series 2). Then, with the aid of the ‘evaluate curve’ component, the points were determined at different ratios on the two edges of the hexagon for the first L-shaped line to be created. In addition, these ratios can easily be parametrized by number sliders connected to the ‘evaluate curve’ component. The same process was done to create the second L-shaped line (Rule Series 3). With the aid of the ‘line’ component, two types of L-shaped line were created in two dimensions, combining the two points in the inner circle and the corresponding points on the edges of the hexagon (Rule Series 4) (Fig. 10). Then, the points were defined on the L-shape lines to make these two L-shaped lines three-dimensional. The positions of these points on the Z-axis

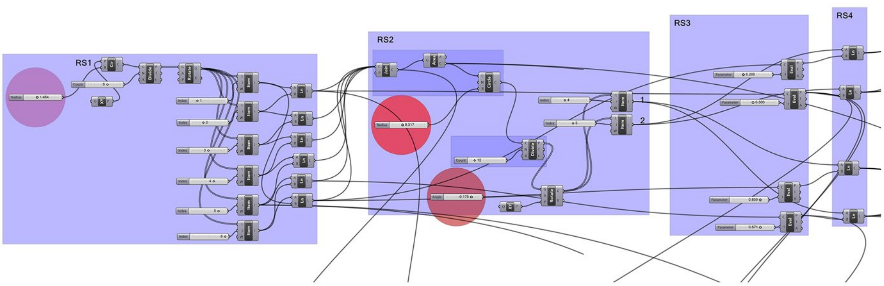


Fig. 10 The part of the script (from Rule Serie 1 to Rule Serie 4)

were determined, and since the ribbons that make up the pattern, extend both up and down in the third dimension (on the Z-axis), they were arranged so that the first and third ones were placed upward, the second and fourth were placed in downward positions. Then, 3D NURBS curves were obtained from these points (Rule Series 5), and the L-shapes, which were now 3D, were offset and made double-curved (Rule Series 6). From the doubled curves, surfaces were created with the help of the ‘ruled surface’ component (Rule Series 7) (Fig. 11).

An external hexagon was created to form the knots. In order to do this, the internal hexagon was offset (Rule Series 8). Subsequently, the positions of the points in the inner hexagon and the outer hexagon that would be used to create knots were determined, using the ‘list item’ and ‘evaluate curve’ components. The end points of two L-shapes were also taken into account when determining the positions of these points, because, the end point of each part must be included so that this pattern can operate parametrically as a whole (for working the parts dependently upon each other). (Rule Series 9). Next, a line was created from these points with the help of the ‘line’ component. This process was also carried out for the two groups of points (Rule Series 10), and as was done in Rule Series 5, the positions of points were determined on the knot lines. The positions of the points on the Z-axis were determined. NURBS curves were formed from these points (Rule Series 11), according to the form of the knot on the third dimension. Following the procedure used in Rule Series 6, the knot curves, which are now three-dimensional, were doubled by offsetting (Rule Series 12), and surfaces were also created from the knot curves that were double curved by offsetting (Rule Series 13) (Fig. 12).

Knot surfaces and L-shape surfaces to be in the form of six-fold rotational symmetry were arrayed with the help of the ‘polar array’ component, and hence a unit was determined (Rule Series 14), which was copied to create three units. At this stage, it is clear that the double knots (interweaving knots) were formed by placing two units side by side (Rule Series 15) (Fig. 13).

In order to construct the inner star, the end points towards the inner part of the three units were determined with the help of the ‘end points’ component (Rule Series 16). From these points, lines were created using the ‘line’ component (Rule Series 17). As was done in Rule Series 5, the points were defined on the lines. According to the form of interweaving ribbons of the inner star in third dimension, the positions of the points on the Z-axis were determined. NURBS curves were created from these points (Rule Series 18). As was done in Rule Series 6, the inner star curves, which were now three-dimensional, were doubled by the offsetting procedure (Rule Series 19) (Fig. 14). As was done in Rule Series 7, the surfaces were formed from the inner-star curves. Thus, when the pattern was parametrized, the ribbons of the inner star could also be moved in coordination with the entire pattern. The parametrically generated inner star was also copied to create necessary parts of the tessellation (Rule Series 20). Finally, L-shapes surfaces that form the main star, inner star surfaces and knots surfaces were extruded together to give thickness to the pattern (Rule Series 21).

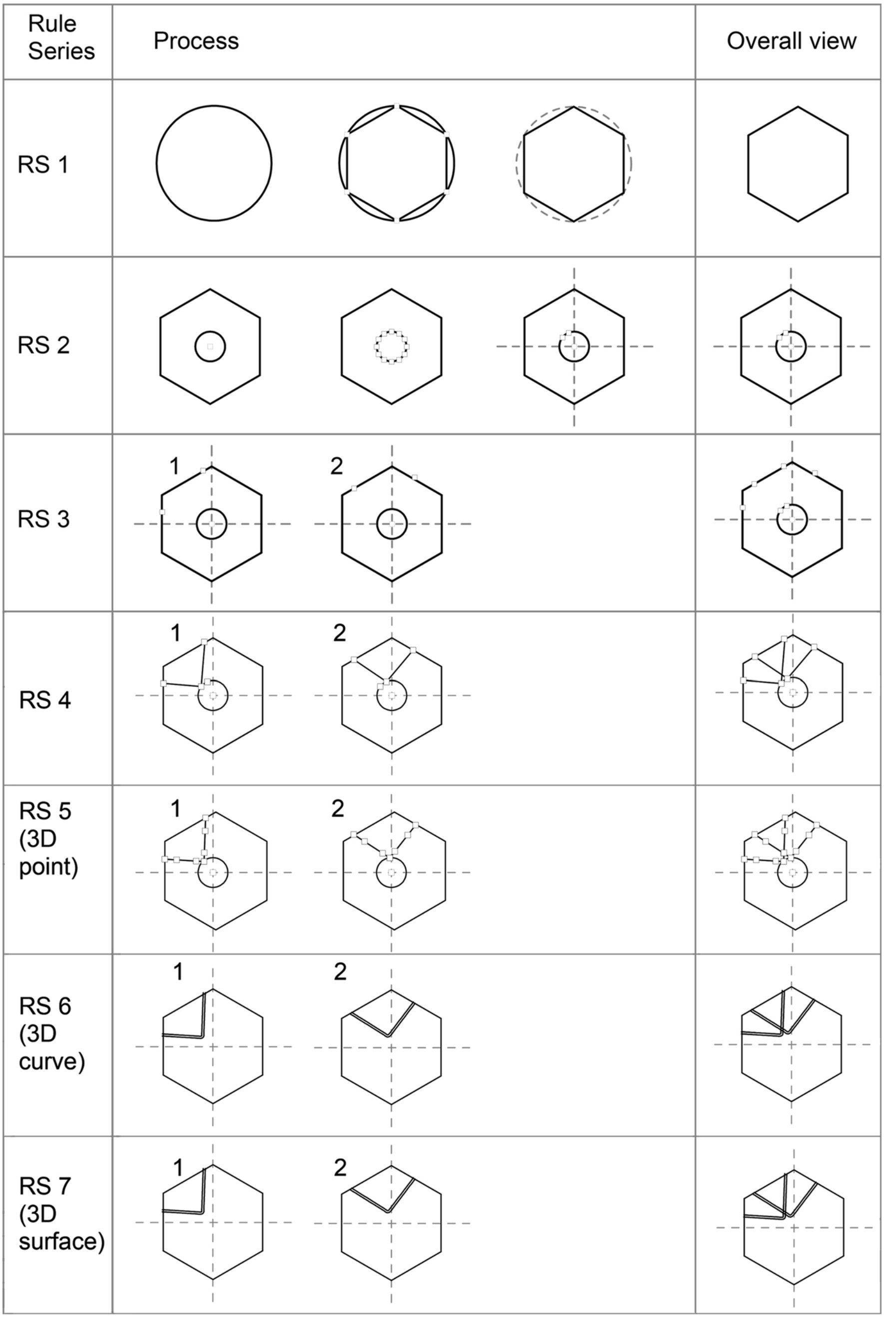


Fig. 11 Shape rules of the creation of L-Shapes

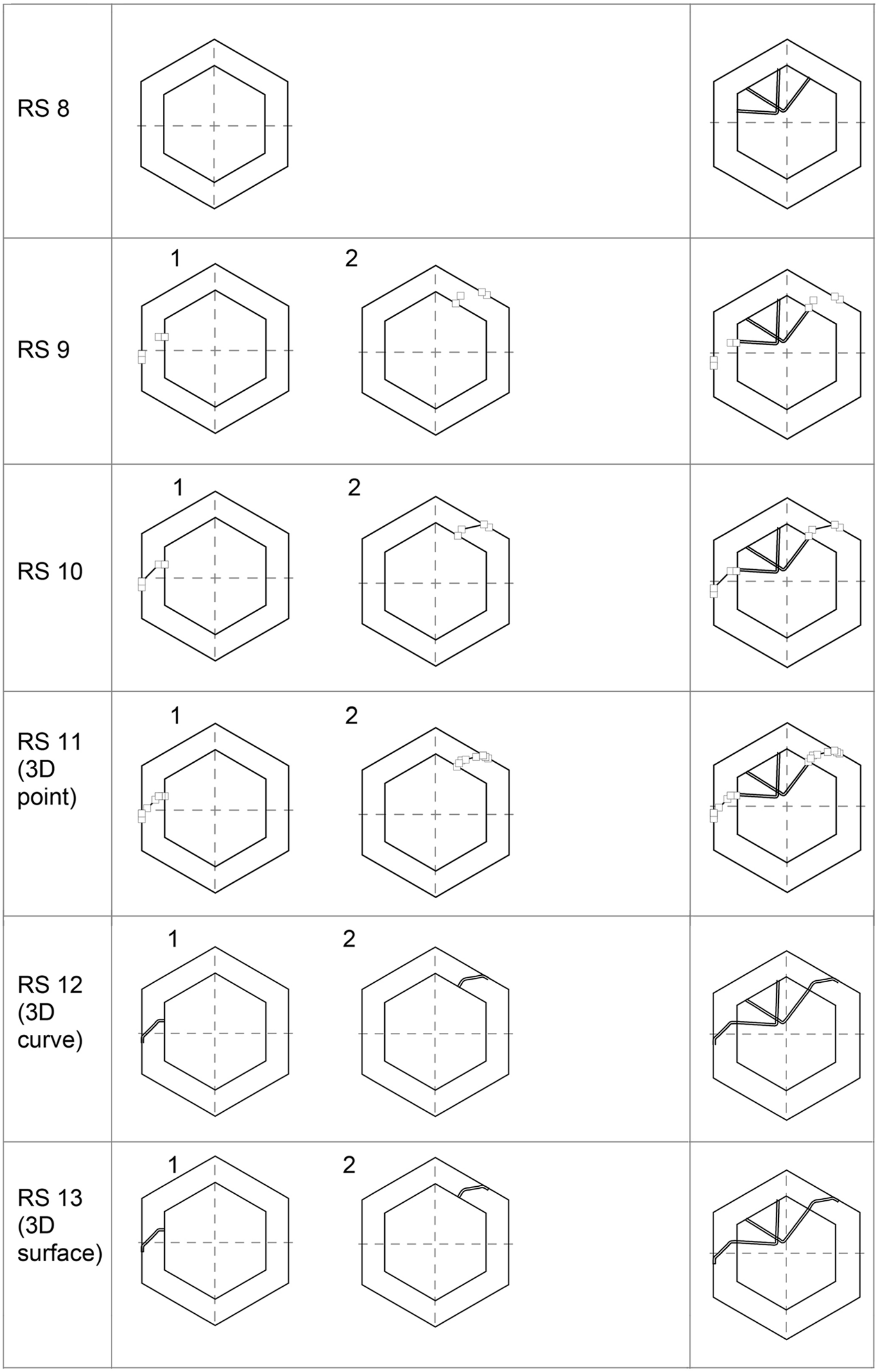


Fig. 12 Shape rules of the creation of knots

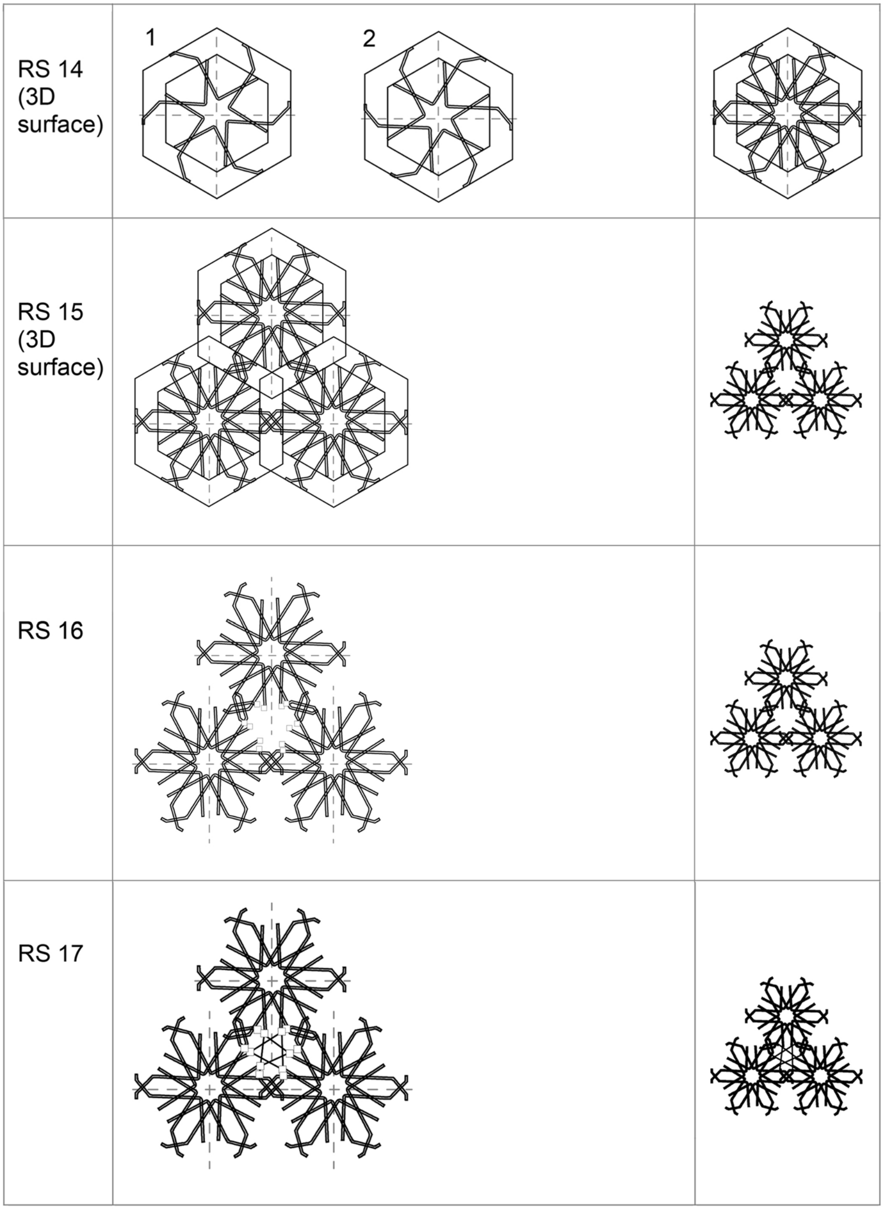


Fig. 13 Shape rules of the creation of one unit and inner star-shape

Results

In the present study, the tessellation resembles a network system in three dimensions. The ribbons that make up the network in this system are very regularly connected to

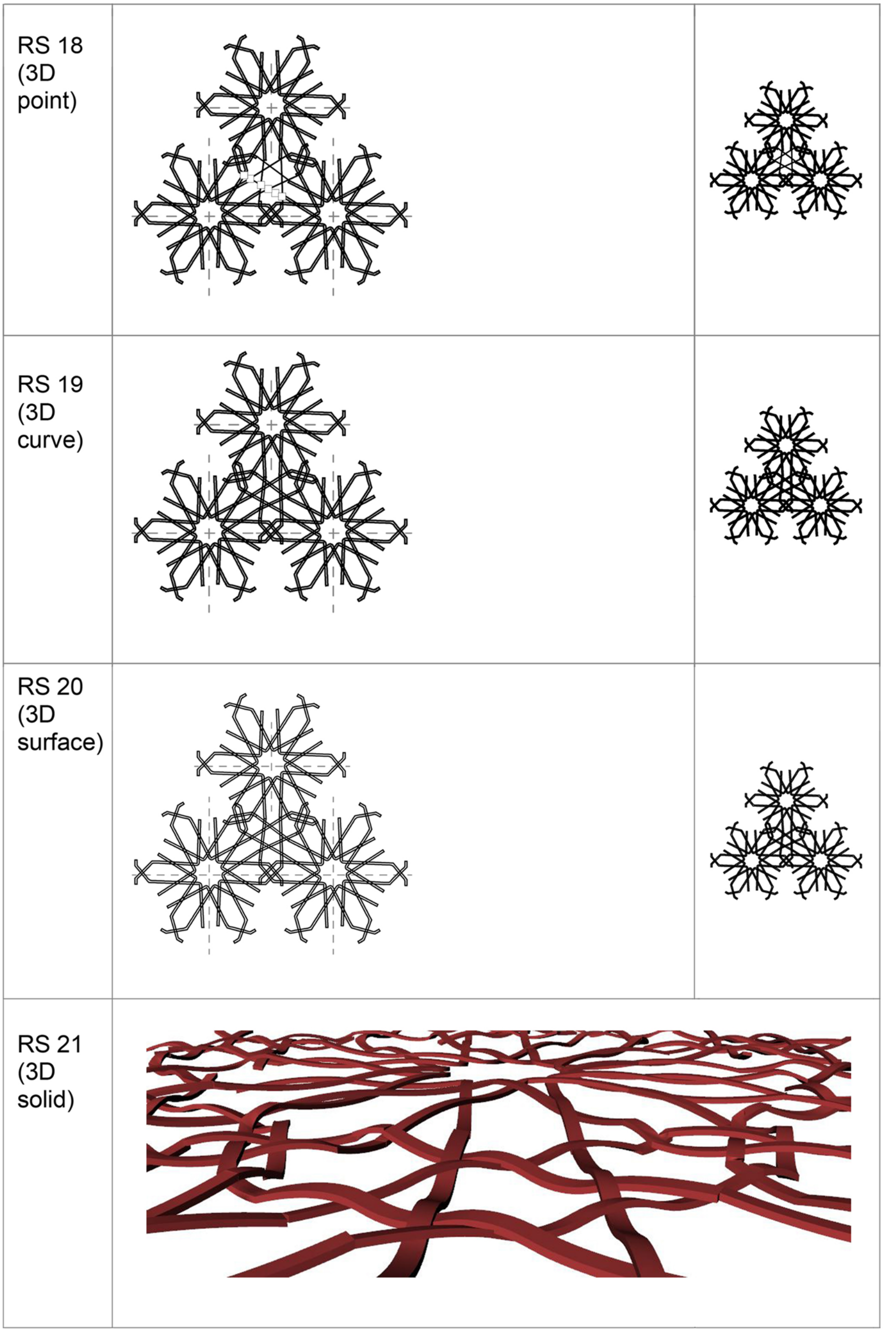


Fig. 14 Shape rules of the creation of inner star-shape

each other in star-shapes and knots. Thus, for example, if one ribbon passes over the other in its first encounter with another ribbon, it will pass under it in its second encounter, then pass over it in its third encounter, and so on.

For the parametrization of the pattern, all the tessellation must be considered as a whole, otherwise, if the tessellation is considered to be composed of separate parts, the latter function independently of each other in the parametrization, and the integrity of the tessellation is lost. Therefore, in the coding exercise, the interrelation between parts should be established, but in order to achieve this, the pattern must be interpreted differently. However, the interpretation as required, also reveals very different geometric expansions for the creation of tessellation. Thus, for example, when the twelve-pointed star-shape and the knots are considered together to provide a 3D form, as in the example of the present work, the formation of a large part of the tessellation in two main ribbons was evidenced. The repetition of these ribbons creates both twelve-pointed star-shapes and knots, and also reveals double knots with the placement of the ribbons side by side.

Based on this pattern, a 3D parametric pattern generator was created, and by means of this script, new 3D pattern designs can be produced. The script variables that can be parametrized, are: the size of the twelve-pointed star-shape (the size of the inner hexagon), the size of the diameter of the inner circle of the twelve-pointed star-shape, the angle of connection between the ribbons of the twelve-pointed star-shape and the inner circle, the size of the knots (the size of the outer hexagon), ascent and descent values of the ribbons as they passing over/under one other in 3D, the width of the ribbons, and the thickness of the ribbons in third dimension (Fig. 15). By varying the values of these variables parametrically, different pattern variations can be achieved. In the parametrization process, each variable is set with minimum and maximum value limits so that the system in the pattern is not corrupted. This prevents one part from overtaking another part. In Tables 1 and 2 the minimum and maximum values that variables can take are shown. The range of values that can be taken by the variables includes decimals, and in addition, Tables 1 and 2 also show the various pattern variations that occur in the parametrization process. While the value of one variable is changed in the pattern variations, the values of the other variables were kept constant, in order that the change in the pattern can be seen more clearly. Thus, for example, the value of "the size of the diameter of the inner circle of the twelve-pointed star-shape" (V2) variable was increased from 0.247 to 0.598, while the values of other variables were kept constant, with the result that the corresponding change in the pattern can be identified distinctly.

Limitations and Future Work

In this study, the star-shape design of Asil b. Veys (Uveys) was examined algorithmically in the third dimension. It will be interesting to study the star-shapes of Asil b. Veys (Uveys)'s other works, especially to study the knots of the stars in the third dimension. Thus, comparison between the various works of the designer can be made, and the progress of his work can be observed during his career. Furthermore, it may prove worthwhile to undertake a more comprehensive analysis of the patterns

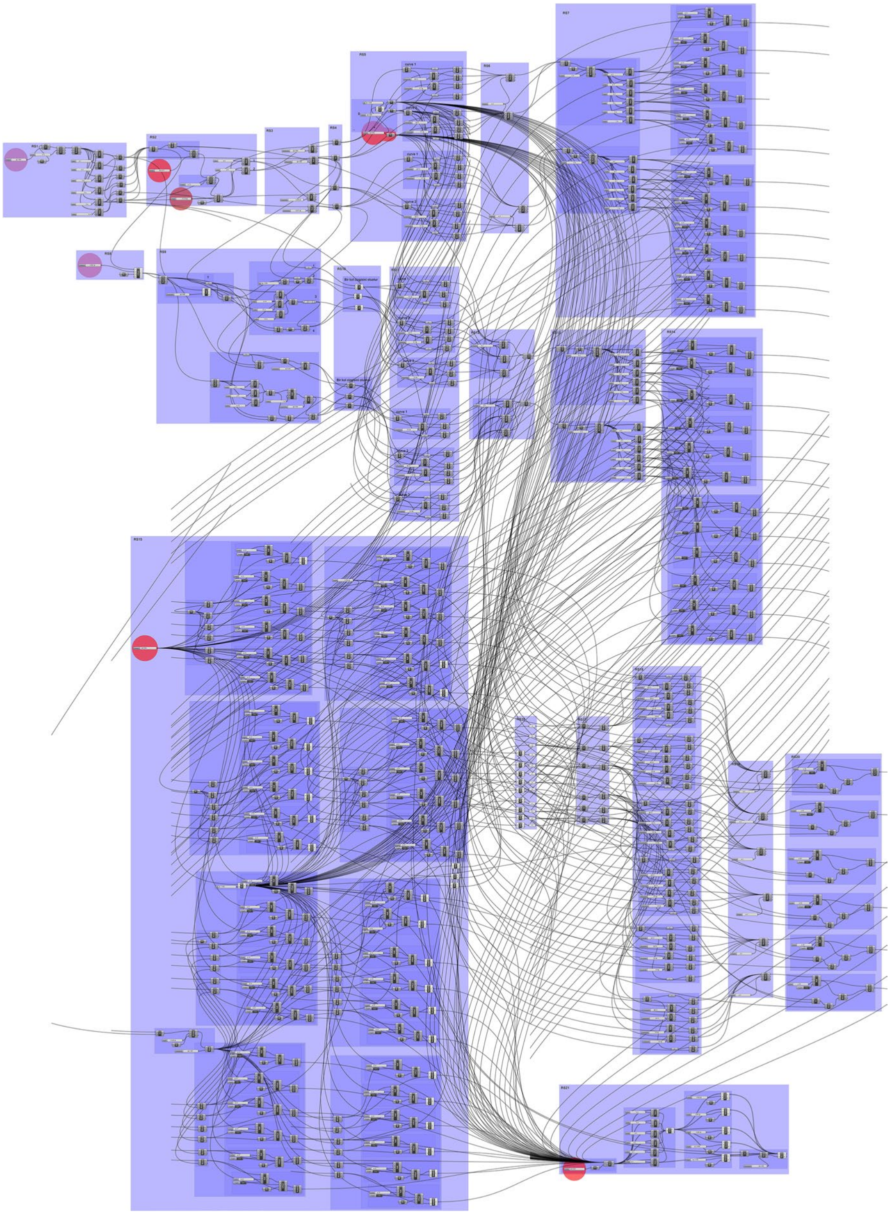


Fig. 15 General view of the script developed by 21 rules (the places with red color show 7 variables)

in the Ahlat cemetery. It is likely that algorithmic decomposition of the various 3D patterns in these historical artifacts will be useful for suggesting and illuminating possible future designs.

In the next step of this study, various 3D form exploration experiments can be done via transferring Islamic patterns with 3D knotting details to the hyperbolic

Table 1 Five variables and the effects of the variable values on the pattern are shown with examples (parametrization)

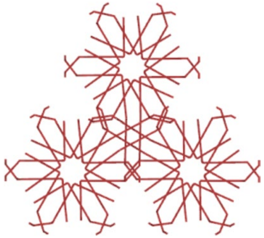
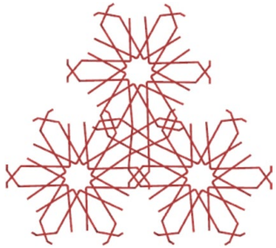
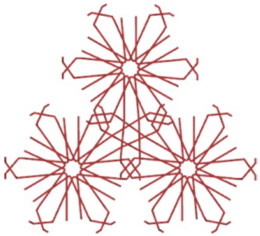
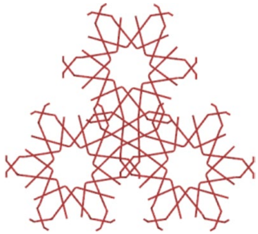
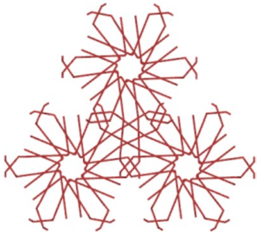
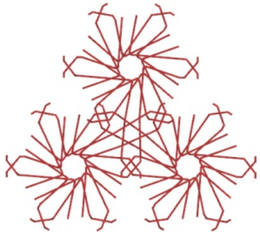
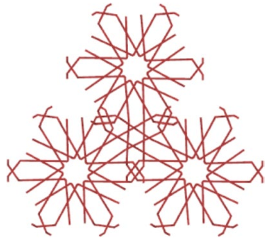
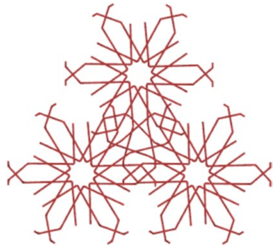
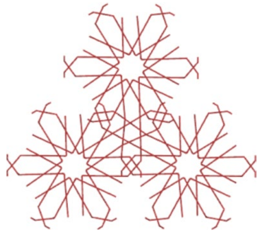
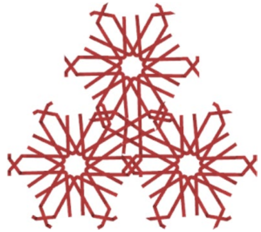
Variable		Value range of the related parameter (type: including decimal values) Min	Max	Unit[a]	Parametrization sample			
V1	The size of the twelve-pointed star-shape (the size of the inner hexagon)	1.400	1.700	cm		**V1: 1.475** V2: 0.382 V3: − 0.369 V4: 0.538 V5: 0.054 V6: 0.020 V7: 0.040		**V1: 1.558** V2: 0.382 V3: − 0.369 V4: 0.538 V5: 0.054 V6: 0.020 V7: 0.040
V2	The size of the diameter of the inner circle of the twelve-pointed star-shape	0.100	1	cm		V1: 1.558 **V2: 0.247** V3: − 0.369 V4: 0.538 V5: 0.054 V6: 0.020 V7: 0.040		V1: 1.558 **V2: 0.598** V3: − 0.369 V4: 0.538 V5: 0.054 V6: 0.020 V7: 0.040
V3	The angle of connection between the ribbons of the twelve-pointed star-shape and the inner circle	− 1.800	1.800	radians		V1: 1.558 V2: 0.382 **V3: − 0.708** V4: 0.538 V5: 0.054 V6: 0.020 V7: 0.040		V1: 1.558 V2: 0.382 **V3: 0.560** V4: 0.538 V5: 0.054 V6: 0.020 V7: 0.040

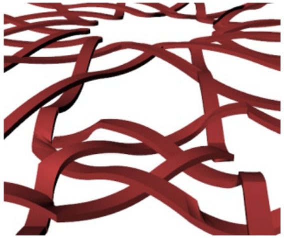
Table 1 (continued)

Variable		Value range of the related parameter (type: including decimal values)			Parametrization sample			
		Min	Max	Unit[a]				
V4	The size of the knots (the size of the outer hexagon)	0.500	0.800	cm		V1: 1.558 V2: 0.382 V3: − 0.369 **V4: 0.500** V5: 0.054 V6: 0.020 V7: 0.040		V1: 1.558 V2: 0.382 V3: − 0.369 **V4: 0.717** V5: 0.054 V6: 0.020 V7: 0.040
V5	The width of the ribbons	0.010	0.145	cm		V1: 1.558 V2: 0.382 V3: − 0.369 V4: 0.538 **V5: 0.041** V6: 0.020 V7: 0.040		V1: 1.558 V2: 0.382 V3: − 0.369 V4: 0.538 **V5: 0.100** V6: 0.020 V7: 0.040

Bold values represent the changed parameters

[a]If desired, the units (such as m, mm) in Rhino can be changed. In this case, the script can also work properly

Table 2 Two variables and the effects of the variable values on the pattern are shown with examples (parametrization)

Variable		Value range of the related parameter (type: including decimal values)			Parametrization sample			
		Min	Max	Unit[a]				
V6	The thickness of the ribbons in third dimension	0	0.3	Cm		V1: 1.558 V2: 0.382 V3: – 0.369 V4: 0.538 V5: 0.054 **V6: 0.020** V7: 0.040		V1: 1.558 V2: 0.382 V3: – 0.369 V4: 0.538 V5: 0.054 **V6: 0.040** V7: 0.040
V7	Ascent and descent values of the ribbons	0	0.3	cm		V1: 1.558 V2: 0.382 V3: – 0.369 V4: 0.538 V5: 0.054 V6: 0.020 **V7: 0.030**		V1: 1.558 V2: 0.382 V3: – 0.369 V4: 0.538 V5: 0.054 V6: 0.020 **V7: 0.060**

Bold values represent the changed parameters

[a]If desired, the units (such as m, mm) in Rhino can be changed. In this case, the script can also work properly

plane. Moreover, the principle in 3D knotting detail resembles the principle of reciprocal structures. On this basis, the patterns can be considered as three-dimensional and experiments can be made on them with the aim to create self-supporting structures.

Conclusion

In this study, a mathematical analysis was made of patterns with 3D features in existing monuments, through algorithms based on shape grammar theory. In particular, this approach offers the advantage that various complex 3D patterns containing knots can be examined. Thus, for example, as in this study, a pattern, which was transferred into algorithms via parametric design tools, can be easily modified by changing the parameters in the algorithm and this can create an infrastructure for the future 3D designs. This method can also be applied to those artifacts which contain various patterns that cannot be read, and for which many alternative 3D variations can be produced in a short time.

A substructure has been created to examine 3D patterns algorithmically for which there is no single mathematical set of rules that can be used to create any one particular form; rather, it is possible to create a given form by the application of different sets of rules. However, it is important that the pattern analyzer, who defines the parts, must provide the parts algorithmically such that they operating dependently on each other. In this way, a suitable 3D parametric pattern generator can be created that is specific for the particular pattern being investigated.

The use of a constructivist methodology with well-defined steps could be one of the solutions to the current problem of the designers' reasoning on dataflow programming, because the rules become break points in the relevant definitions, and these break points form the backbone of the designs. Therefore, within a parametric definition, desired changes can easily be made by focusing on a specific part. In addition, the important benefit of parametric design tools is that the numerical values specified within a parametric definition can be changed very easily. Thus, many different variations, which can be done with the same rules series, can be created easily.

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Asli Agirbas is an assistant professor in the Department of Architecture at Fatih Sultan Mehmet Vakif University. She received her M.S. Arch from Pratt Institute, New York and her Ph.D. from Mimar Sinan Fine Arts University, Istanbul. Her research focuses on computer aided architectural design and contemporary macro-scale designs.