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RESEARCH

New Stereotomic Bond for the Dome in Stone Architecture

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Abstract From a historical analysis of hemispherical domes and from the rigorous study of geometric methods to divide a sphere, this study presents innovative solutions for bonding a hemispherical stereotomic dome in cut stone, solutions that optimize its construction compared to traditional methods. This result is achieved through a five-fold structural geometry of bonding that reduces the number of invariant-ashlars to be produced in relation to maintaining their reduced dimensions and increasing the diameter of the dome they constitute, respecting static laws and improving aesthetic expressivity, and through modern technologies for designing and cutting building elements, thus simplifying the production process.

Keywords Polyhedra · Sphere · Tessellations · Stereotomy · Dome · Stone · Architecture

Introduction: Theme and Purpose of Research

The aim of this study is to update the traditional construction of the domed space in cut stone, which constitutes an important architectural heritage, recognizing in the organic structural morphology of the stone dome, the consubstantiality of the form, structure and symbol.

Starting with a comparative historical analysis of different bonds of hemispherical domes and a rigorous study of the geometric methods used to obtain a structural division of the sphere, this study presents innovative valid alternative solutions for bonding a hemispherical stereotomic dome in cut stone, that optimize its construction compared to traditional methods. This is achieved thanks to the

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particular structural geometry of the bond, one that reduces the number of invariant-ashlars respecting static laws and improving aesthetic expressivity, and thanks to infographic software that allow for the direct transfer from a three-dimensional model to rapid prototyping machines and numerical control machines or to a robot that can accomplish it, simplifying the production process, reaching the unity of idea, project and construction.

Geometry as Foundation of Stereotomic Form-Resistant Structure

Stereotomy, the art of cutting solids, through geometrical knowledge, provides a constructive rationalization of architectonic systems composed of distinct interlocking structural elements, generating a resistant texture that combines aesthetic expressivity with structural strength.

The masonry bonding requires a well-determined geometry to tessellate the spherical surface outlined by the dome, thus generating a form-resistant structure that produces an adornment with structural valence.

"Geometria autem plura praesidia praestat architecturae" (geometry offers many aids to architecture) is written in *Liber I,1,4* of Vitruvio's *De Architectura*, specifying that the art of construction is based on the knowledge of precise geometric rules that allow the control of the shape to be constructed.

Constructed form can be obtained through the *geometria fabrorum*, namely, a knowledge of the empirical rules based on traditional knowledge, or through theoretical geometry in its disciplinary codification, on which the art of stereotomy is based for the control of complex forms.

The knowledge of constructive techniques based on both traditional *modus operandi* and on methods codified by the stereotomic discipline, combined with knowledge of the innovative technology of infographic design and production of structural elements, demonstrates "the potential of load-bearing stone in contemporary architecture when tradition and technological innovation are combined" (D'Amato and Fallacara 2006: 39).

The stereotomic discipline provides a precise geometric definition of the entire architectural system and of each constructive element that composes it, in "strong conceptual affinity with the development of integrated CAD-CAM systems, which ensure the fundamental requisite of stereotomy, specifically design and building precision" (D'Amato and Fallacara 2006: 35). "The role of stereotomy […] may prove strategic because it can help restore theoretical and practical unity to the process of designing and building an architectural work" (D'Amato and Fallacara 2006: 34), a synthesis of the "to know" and "to know-how", of theory and practice.

"The theory—writes Rondelet—is a science that guides all practical operations. This science is the result of experience and reasoning based on the principles of mathematics and physics applied to the different operations of art. It is by the theory that a skilled builder comes to determine the forms and the right dimensions that he must give to each part of the building, according to its position and to the efforts it will have to bear, so it turns out perfection, solidity and economy" (D'Amato 2014: 31).

Analysis of Significant Examples of Historical Stone Domes and Geometric Methods to Divide the Sphere

From the historical analysis of the different types of bonds of hemispherical domes, the present study offers an original examination of the architectural relationship between the configuration, number and dimensions of the ashlars used in relation to the size of span in the dome. This analysis has been done in order to optimize other dome constructions by reducing the number of invariant elements while maintaining the dimensions historically used (from 20 to 50 cm in height of face at the extrados) to achieve a dome diameter close to 10 metres, which is morphologically and typologically useful in architecture.

Bond Obtained Through Cylindrical Projection

This is the most utilized constructive method in history, based on the division of the hemispherical dome into meridians and parallels (latitude–longitude tessellation). In citing two of the most representative cases of stone domes with a diameter between 8 and 10 metres, it is worth mentioning:

- the dome of the Church of S. Corrado (XII–XIII cent.) in Molfetta (Bari, Italy), measuring 8 metres in span (Leonardis 2015: 282) with approximately 34 concentric rows, as I have seen for myself, with a height of face at the extrados of approximately 30 cm, as an example of traditional architecture;
- the dome of Royal Chapel of Château d'Anet (XVI cent.) in Anet (France), projected by Philibert Delorme, measuring 8.21 m in span (Pfnor 1867: 2ème Partie, PL IV), with 28 invariant ashlar types (Potié 1996: 116–117) with a height of face at the extrados from 20 to 50 cm, as an example of architecture codified by stereotomic discipline. This example shows how the stereotomic discipline, compared with tradition, allowed Delorme to control the form definition to optimize the production process of the invariant ashlars, minimizing the number of panneaux needed to cut them.

Bond Obtained Through Conical Projection

Different configurations of bonds of stone domes are found in XVI cent. Spanish architecture and are described in the *Libro de Traças de Cortes de Piedras* by Alonso de Vandelvira, written probably between 1575 and 1591 (Vandelvira 1573). Some of these were made for the sail vaults of the Casa Lonja de Mercaderes de Sevilla, also known as the Archivio General de Indias.

Some of these sail vaults with spherical intrados, derived geometrically from a sphere, were built from the division of the hemispherical space through conical projection, a very used method in stereotomy, as shown in the drawing of the manuscript by Alonso de Vandelvira, illustrated by Amedée François Frézier (Fig. 1) in his *Traité de stéréotomie à l'usage de l'architecture* (Frézier 1980: 331).

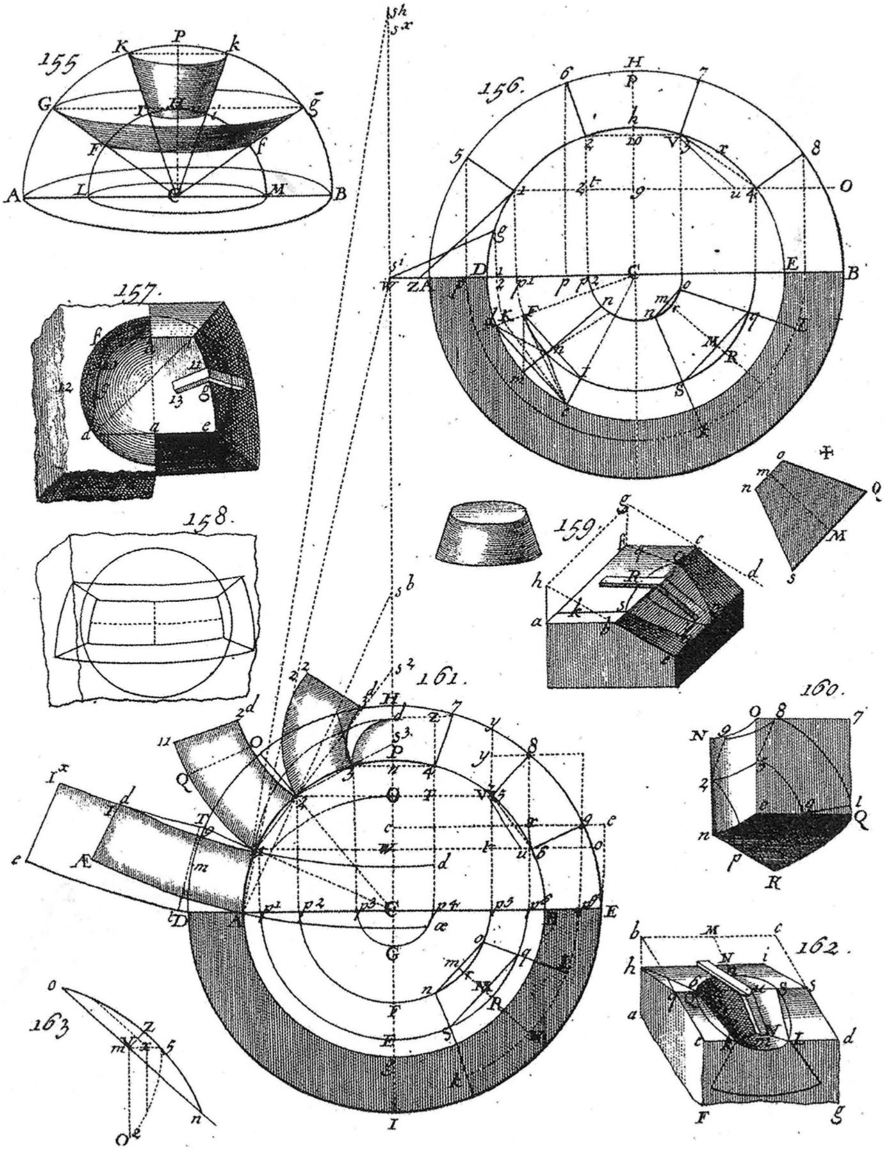

Fig. 1 A. F. Frézier, *Traité de stéréotomie,* 1737–1739, Tome II, Planche 53 (from Frézier 1980:331)

One of the sail vaults of Casa Lonja, constructed using the "hiladas cuadratas" method (Rabasa Díaz 2013: 5–20), measures approximately 8 m in span, with approximately 31 invariant ashlars types.

Bond Obtained According to a Helical Surface

Another type of bond, derived from the conical projection method for circular rows, is the helical dome that A. de Vandelvira calls “en vuelta de capazo” in the *Libro de Traças de Cortes de Piedras*. The term “capazo” refers to a traditional basket twisted into a spiral shape, which highlights the theoretical identity between the art of weaving and masonry weaving. In this bond, given by the division of the sphere according to a helical surface, all elements are different. The difficulty of configuring ashlars in this way has resulted in the rare use of this bond, of which there are six examples built in Spain (Rabasa Díaz 2013: 5–20), the most significant being the vault of the XVI cent. Murcia Cathedral, which covers a span somewhat lower than 6 metres using approximately 80 geometrically different ashlars. The height of the extrados face of the ashlars varies from 20 cm at the base of the dome, to 60 cm nearest the summit (Calvo López et al. 2005: 124–127). These masonry bonds have no practical and economical purpose, since the complexity of defining the ashlars and their diversity in relation to the span to be covered is such, that the construction is not rationalized. One possible justification may be found in the formal expression and in the boldness of the resulting morphological solution, which makes these vaulted constructed spaces uncommon.

The above-mentioned stereotomic methods of conical and cylindrical projection to divide the hemispherical calotte of domes are based on the same geometric methods used since antiquity (Russo 2015: 19–49) in cartography for the planar representation of the terrestrial and celestial spheres. The symbolic value of the dome is expressed by the formal analogy between the architectonical vault and the celestial vault (celestial sphere) through the etymological relationship between the words *stereoma* (celestial vault) and *stereotomy* (Fallacara 2014: 17–21), which has been critically and thoroughly analysed in this study, including some examples of historical literature, architectural literature and constructed forms. For example, the dome in the Church of Santa Maria in Solario (XII cent.) in Brescia, with the intrados decorated with stars on a blue background, proves the coincidence between the constructed form of the dome and the symbol of the celestial sphere, with its character of an absolute platonic form of perfection, as a representation of the Absolute, “sphaera infinita cuius centrum est ubique, circumferentia nusquam” (an infinite sphere, whose centre is everywhere and whose circumference is nowhere), as written in *The Book of the Twenty-Four Philosophers* (XII cent.).

From this, it is possible to conclude that architecture and astronomy contributed to the advancement of the study of the sphere and the representation of the relationships that exist between three-dimensional spherical space and its bi-dimensional planar representation (Puerto 2001:117).

The analogy between cartographic representations and architectural drawings shows, in fact, that the knowledge of geometric rules transcends the individual disciplines in which it is applied, and is an essential requirement for controlling the form to be represented; it also demonstrates how individual disciplines contribute to a greater definition of geometric knowledge and to its development when required and used for the resolution of practical problems that require a geometrically defined response.

Division of the Sphere Through Polyhedron

For the purposes of the present study, it is necessary to divide the dome into smaller and equal portions as much as possible in order to reduce the number of repeatable invariant ashlars. The method chosen is to subdivide the sphere according to the spherical polyhedron obtained through the projection of the edges of a polyhedron from its centre onto the sphere in which it is inscribed. The subdivision of the sphere resulting from a spherical polyhedron is based on the rules of projective geometry, fundamental to stereotomy. From the treatise by Frézier of 1737–1739, it follows that stereotomy, understood as the art *de la coupe des pierres* (of stone cutting) is the direct consequence of the rigorous method *du trait géométrique* (of the geometric drawing), originally used in carpentry for the cut-off of structural elements, based on the projective method analysed and codified by Girard Desargues, who generalized the Euclidean geometry to infinite space and to homogeneous coordinates.

Knowledge of classical geometry is indispensable to the stereotomic architectural project. In fact, the projective geometric method has been known from classical antiquity up until the codification of projective geometry by G. Desargues. In his work entitled *Elements*, Euclid shows a surface discretization by pyramids with their apices in the centre of a sphere and bases onto its surface (Book XII, Proposition XVII, as cited by Commandino 1575: 244) and describes the geometric relationships (Book XIII, Proposition XVI, as cited by Commandino 1575: 259–260) between the five Platonic polyhedra and the sphere in which they are inscribed, observing also the presence of the irrational number.

This irrational number, known as the *golden ratio* and called *divina proportione* by Luca Pacioli (Pacioli 1509), is found in the geometric duality of Platonic polyhedra and is highlighted in a drawing by Johannes Kepler in his *Mysterium Cosmographicum* published in 1596 (Fig. 2), in which each polyhedron can be inscribed in a sphere and can circumscribe another sphere (Kepler 1621). Classical antiquity studied the geometry of the sphere and formulated methods for calculating its volume and surface by inscribing in it geometric solids with known rules of

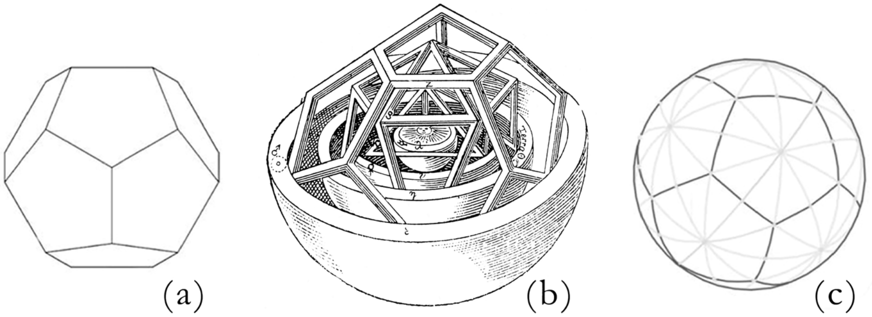

Fig. 2 From Dodecahedron (**a**) to Spherical Dodecahedron (**c**), according to geometric relation between polyhedron and sphere as shown illustration by J. Kepler in *Mysterium Cosmographicum*, 1596 (**b**) from Rolt-Wheeler, F. 1910. *The Science-History of the Universe,* 113. New York: The Current Literature Publishing Company. Image elaboration: author

calculation and which therefore can approximate the geometry of the sphere, as demonstrated in *Proposition 17* of Euclid's *Elements* and in *On the Sphere and Cylinder* by Archimede di Siracusa in 225 B.C. (Heath 1897: 29–34), where the sphere is subdivided into cones and trunks of the cone, which are inscribed in it.

The geometrical method of projection was well known in classical antiquity, as evidenced by the gnomonic projection studied by Talete di Mileto (640–547 B.C.) considered the oldest cartographic projection, as well as the Euclid's *Ottica,* which formulates the eye view and "psycho-physiological" (Panofsky 2007: 13) vision of space as a projection of straight-looking visual rays that, starting from the eye, intercept the shapes in space and acquire the perception of their morphology and position. It follows that classical antiquity knew the radial projection of polyhedron edges from the centre of the sphere in which the polyhedron is inscribed, onto the spherical surface, thus obtaining the spherical polyhedron. This deduction is confirmed by Socrates' speech in Plato's *Fedone, 110b*: "this true Earth, to those looking at it from above, looks similar to one of those colourful spheres of leather divided into twelve segments" (Burnet 1903), referring to a spherical subdivision of the ancient leather balls used in the game called σφαίρισ (sfairis) or "ball game". In 1958, near the Sanctuary of the Great Gods at Samothrace, a terracotta model (Vanderpool 1958: 325) (dated between 275 and 250 B.C.) was found; it representing an ancient leather ball divided into twelve polygonal portions engraved on its surface, a ball divided by a spherical dodecahedron whose faces were painted with many bright colours to differentiate them, just as Plato's *Fedone* describes. This leather ball, similar to modern soccer balls outlined by a spherical truncated icosahedron, shows that spherical polyhedra are very useful to the practice of subdividing spherical surfaces into equal parts.

Polyhedral Geometry to Optimize the Construction of Vaulted Spaces

From studies to approximate and subdivide the sphere through polyhedral geometry are derived geodesic structures used in architecture to discretise the sphere and construct covers, thus minimizing structural elements.

Stereometry drawings by Leonardo da Vinci demonstrate that he studied the octant projection (Tyler 2017) to subdivide the sphere through a spherical octahedron, which, Leonardo used as a cartographic projection in 1508–1514, as shown in the *fol. 521r* of his *Codex Atlanticus* (1478–1519) (Fig. 3b) which is saved in Royal Library of Windsor Castle. A larger division of the sphere is represented by Leonardo in *fol. 67r* of *Paris Manuscript G* (Leonardo 1989) entitled *Del risolvere in piramide il corpo sperico* (On solving the spherical body within the pyramid), where he subdivides the sphere into pyramids with triangular bases onto spherical surfaces and with their apex at the centre of the sphere, according to what he wrote near his drawing that is similar to a geodesic sphere (Fig. 3c).

These studies derive from the excellent knowledge of polyhedra that Leonardo demonstrated so well by drawing their wooden structures in the *Divina proportione* by Luca Pacioli, where each polyhedron is represented as "vacuus", that is as a framed structure that points out its edges and allows us to understand its supporting

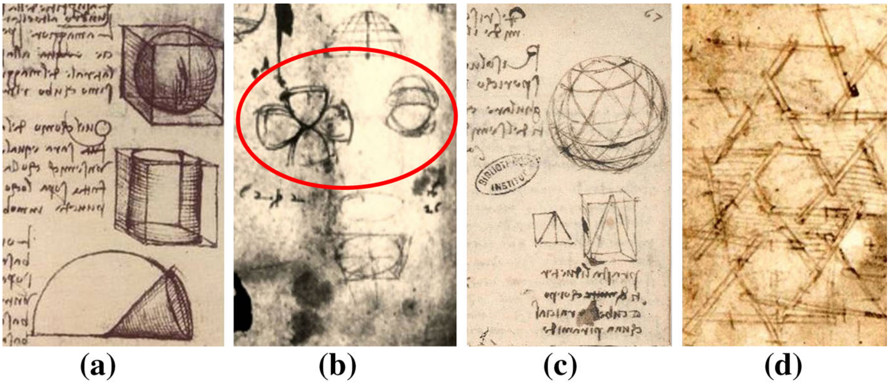

Fig. 3 **a** Leonardo da Vinci, Conical and cylindrical projections from *Paris Manuscript G*, fol. 61v (Leonardo 1989). **b** L. da Vinci, octant projection, *Codex Atlanticus*, fol. 521r, with ellipse that identify this particular projection (Leonardo 2000). Image elaboration: author. **c** L. da Vinci, *Paris Manuscript G*, fol. 67r. (Leonardo 1989) (**d**) L. Da Vinci, *Codex Atlanticus*, fol. 899v (da Vinci 2000)

structure. It is assumed that Leonardo used wooden models to draw the "empty" structure of the solids, because "some coherent documents of the city of Florence indicate that a series of wooden polyhedrons belonging to Pacioli were purchased from the common for a public exhibition" (Livio 2007: 203). It is most probably these studies that inspired Leonardo to build coverings with the reciprocal structures illustrated in *fol. 899v* (Fig. 3d) of the *Codex Atlanticus* (Leonardo 2000), based on the mutual support of equal beams and favouring a fast and economical construction, even for military purposes.

Several centuries later, in 1954, in order to resolve post-war housing crisis (Tebala 2010: 870), Richard Buckminster Fuller obtained the patent for the construction of large diameter geodesic domes through the frequent replication of a certain number of invariant structural elements consisting of small aluminium rods (Fuller 1954), optimized in number and size, compared to what would be needed to build a traditional aluminium dome with the same diameter.

By "rotating each side of the tessellation by the same angle around its midpoint" (Brocato, Mondardini 2011: 1945), a reciprocal structure or "nexorade" (Baverel 2000) can be obtained, starting from a geodetic one, and vice versa. For the purposes of this study, one must note that Leonardo's reciprocal structures were made up of equal elements, thus achieving the maximum optimization of the wooden cover. The complete structure had many short elements and a slight curvature, factors for which the uniqueness of the invariant structural element can be preserved. This uniqueness can be preserved in the case of the reciprocal arrangement of rods derived from regular spherical polyhedra, where the curved element repeats itself onto the entire spherical surface. The reciprocal sphere generated by the regular icosahedron, for example, comprises 30 elements equal to each other. In reciprocal spheres of greater diameter, which are made up of many structural elements resulting from more complex polyhedra, several different invariant elements are required, as in geodetic spheres and their duals (Goldberg polyhedra).

The constructive system that transposes the logic of the wooden structure to cutting stone structure is represented by the patents of Joseph Abeille and Jean Truchet, approved in 1699 and published in *Machines et inventions approuvées par l'Académie Royale des Sciences* (Gallon 1735: 159–164) which present stone bonds for a "flat vault" with a single ashlar configured by stereotomic techniques. The studies conducted by professor Claudio D'Amato Guerrieri and by professor Giuseppe Fallacara allowed them to theorize and experiment with the topologic deformation of this planar reciprocal structure on a barrel vault called Portale Abeille, accomplished by repeating two types of reciprocally interconnected invariant ashlars, presented at the Tenth International Architecture Exhibition held in Venice in 2006. The study and the extension of the stereotomic reciprocal tessellation to the spherical surface is the topic of a doctoral thesis by engineer L. Mondardini (Mondardini 2015) directed by M. Brocato and co-tutored by G. Fallacara (Fallacara 2012).

These studies bring to the Bin Jassim Dome, constructed in Qatar in 2012, a covering hammam designed by architect J. Caspari. The dome was conceived by a team comprising architect G. Fallacara, engineer M. Brocato, the construction company Mecastone by L. Tamboréro, the SNBR company for the realization of stonecutting by digital computer-machines, EDM Projects and Protostyle (Fallacara 2012: 120). The dome measures approximately 6 m in extrados diameter and consists of 110 stone trapezoidal ashlars, of which 8 are element-type invariants, whose face at the extrados measures approximately one metre in length.

In the present study, the knowledge and comparison of the number and morphology of invariant structural elements, as well as the size and configuration of historical constructed forms with the study of the possible geometrical methods of spherical division, have allowed me to achieve the design of a new stereotomic bond solution optimized for domes in cut stone.

Definition of a New Stereotomic Bond for the Dome in Cut Stone and Results of the Research

This study analyses the possibility of making a new bond for the construction of the stone domes, according to a particular geodesic tessellation with five-fold symmetry. The bond configured in this study is obtained according to the geometry given by a suitable spherical polyhedron that, in dividing the dome into parts that are repeated equal to themselves throughout the hemispherical surface, reduces the number of invariant ashlars. The polyhedron chosen is a spherical disdyakis triacontahedron or a spherical regular dodecahedron whose regular pentagonal faces (Fig. 4d) were divided into ten triangles through the 5 lines of symmetry of the regular pentagon (Fig. 4f). Each of these triangles is the minimum portion in which the sphere is divided into equal and symmetrical parts (Wenninger 1999: 3–15), and, through analysis of the various possible tessellations of the sphere, a configuration is chosen into which it can be divided, and which may prove effective for its

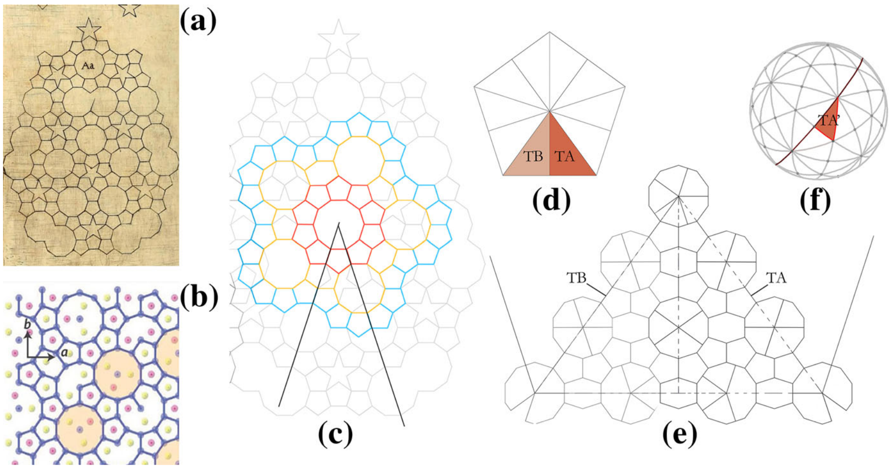

Fig. 4 **a** Johannes Kepler, *Harmonices Mundi*, 1619, tessellation *Aa*. Illustration from: http://www.gramunion.com/twoofswordstarot-blog.tumblr.com/114264474323. **b** Atomic lattice of quasicrystal with icosahedral symmetry (from Takakura et al. 2007: 58–63). **c** Study for definition of stereotomic bond from Kepler's tessellation *Aa*. Image by the author. **d** Division of the regular pentagon in minimum triangular parts that correspond to the spherical division of faces of the regular dodecahedron. Image by the author. **e** Division of minimum triangular part TA and its symmetrical TB according to stereotomic design derived from Kepler's *Aa*. Image by the author. **f** Division of the sphere in minimal equal and symmetrical parts, according to the spherical dodecahedron. Image by the author

construction in cut stone. With radial projection was obtained the spherical dodecahedron, and the tessellation of this minimum triangle is repeated symmetrically onto the whole sphere surface, determining the bond of the stone dome according to precise and interdependent static, geometric, formal and typological requirements (Fig. 4c). The configuration chosen for the subdivision of the minimum triangle TA and its symmetrical triangle TB (Fig. 4), which determines the stereotomic bond of the dome in cut stone, is based on the 5-fold symmetry, respecting the geometric properties of the polyhedron from which it originated, and is derived from J. Kepler's "Aa" tessellation (Fig. 4a) found in his *Harmonices Mundi* (Kepler 1619: 58–59). It is the same geometry that characterizes the atomic structure of quasicrystals (Fig. 4b), which has never been applied to the architectural construction of domed spaces in cut stone. Indeed this 5-fold symmetry is present in nature in the atomic lattice of quasicrystals with icosahedral symmetry (Takakura et al. 2007: 58–63) discovered in 1984 by Dan Schechtman, who received the Nobel Prize for Chemistry in 2011 for this discovery, and it is proportionate according to the golden ratio because it is constituted by the juxtaposition of regular pentagons from which the tessellations coded by Roger Penrose derive.

This well-connected and well-proportioned configuration is harmonic, deriving etymologically from the ancient greek ἁρμονία (harmony) (Folicaldi 2005: 29), meaning "connection, concord, proportionate structure, agreement" (Montanari 2003: 324), and increases the stability of the atomic lattice; in addition, the interweaving, according to five-fold symmetry, hinders the process of dislocation due to the sliding of its sections, thus preventing the deformation of its materials, and resulting in greater hardness and structural strength at break.

The transposition of this type of five-fold tessellation in stereotomic dome architecture has considerable advantages in structural terms, as it improves the interlocking of ashlar tiles and generates a highly resistant texture, and in formal terms, as this particular harmonic geometric composition, involving the golden ratio, determines a strongly expressive embroidery.

In the choice of the tessellation and the design of the bond, where it is possible to calculate the number of element-type invariants, one must consider their deformation when projecting on the sphere and making architectural choices that simultaneously involve its planar and spherical design, topologically equivalents or homoeomorphic ones. In fact, what is equal in the plan is not equal on the sphere: pentagons in Fig. 4e are equal in the planar pentagon (Fig. 4d) but all different in the spherical pentagon. Instead, the ashlars composing nonagons and decagons respond to the symmetry rules of the triangle, optimizing their number.

The control of this complex geometry is made simpler by modern three-dimensional infographic modelling technology tools that, compared to the traditional geometric tracing method, allow us to visualize the geometric division on the plane and the corresponding structural tessellation in space, and to verify the equivalences and symmetries of the parts in which it is subdivided with visible security of the actual accuracy of the design, without resorting to mathematical calculations and more complex geometric construction rules.

The bond defined according to this tessellation (Fig. 5a) is particularly effective for the geometry that optimizes the number of invariant ashlars, for the new strong aesthetic definition and for its static interlocking. The ashlar invariants of this dome (Fig. 5a) are 34 and are from 25 cm to 40 cm long. The diameter of the dome is 10.06 metres long at the extrados, and the thickness of ashlars is 22 cm. This research study describes the method by which the stereotomic definition of the dome and the construction of a *maquette* (Fig. 5c-5d) is possible, as well as the method allowing me to determine the phases of bond assembly and to take its static efficiency into consideration.

Compared to the methods used in past, existing infographic CAD/CAM software allow the direct transfer from ideal to real, from a three-dimensional model to rapid prototyping machines and a numerical control machine and robot, simplifying the production process. From the infographic model, in fact, it was possible to prototype ashlars in PLA material with the *Ultramaker*^{2} machine for the construction of a *maquette* at a scale of 1:14.37, with a dome diameter of 70 cm at the extrados. The height of the face of ashlars at the extrados ranges from a minimum of 1.5 cm to a maximum of 4 cm. After manually numbering and classifying all the prototyped ashlars (Fig. 5b), the *maquette* was constructed in five working days and the assembly process was simplified by construction drawings prepared for that purpose in which same-type invariant ashlars have the same colour, and the same number and belong to the same layer in infographic software.

Mounting took place without centring through the traditional construction technique using a trammel that was obtained through a small wooden rod equal in length to the radius of the sphere inscribed in the dome, with one end positioned in

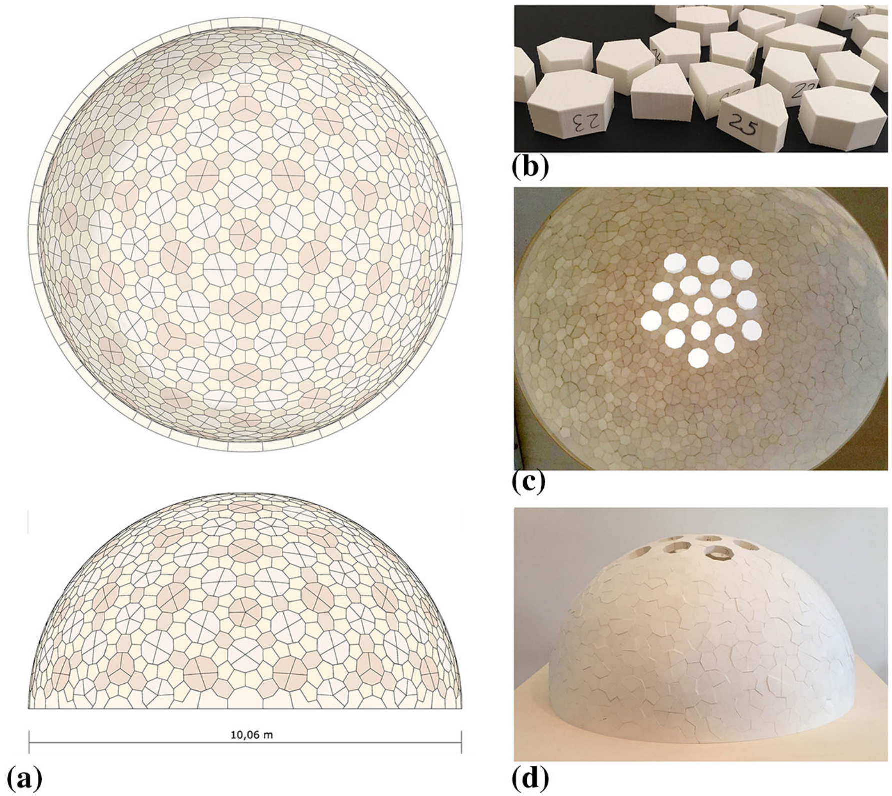

Fig. 5 **a** New stereotomic bond for the dome in stone architecture. Image by the author. **b** Numbering and classifying all the prototyped ashlars. Photo by the author. Intrados (**c**) and lateral view (**d**) of *maquette* constructed by the author. Extrados diameter: 70 cm. Scale of *maquette*: 1:14,37. Photos by the author

the centre of the dome and the other in the faces of the ashlars, which allows for the definition of perfect sphericity of the intrados. The construction of the *maquette* is very useful to confirm the interlocking achieved by the bond and to understand the most effective laying rules.

This research analysed the static behaviour of this new bond and its holing limits through the subtraction of ashlars that compose nonagons and decagons. Static analysis was done through the computer modelling, showing that the structural form is verified, also through the appropriate subtraction of particular ashlars, thereby unloading the structural form, as shown in the *maquette*. The forces' distribution is carried out along the discharge arches composed of the pentagonal and hexagonal structural elements, as illustrated in Fig. 6.

To achieve a dome configured in the way that is presented in this study, but with a smaller diameter and the same size of ashlars, it is necessary to repeat the design process with few tiles in the minimum triangle, the smallest part in which it is possible to divide the sphere into equal and symmetrical parts.

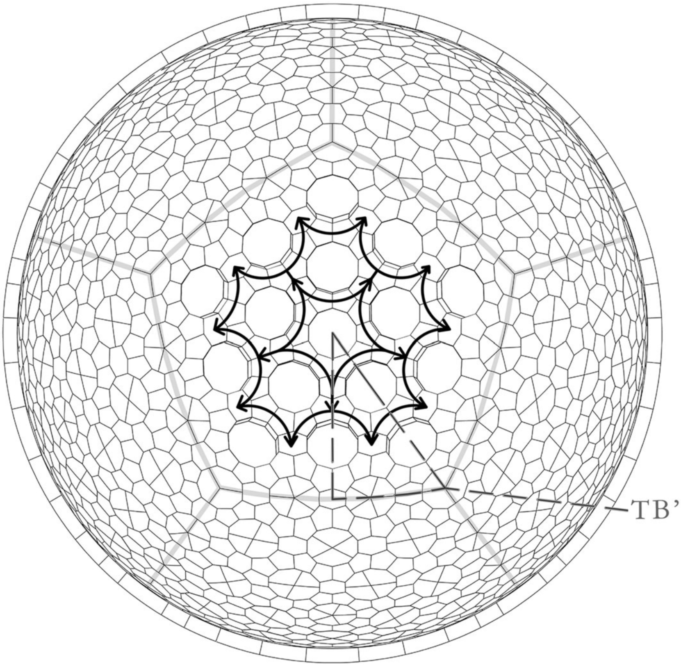

Fig. 6 View of intrados of new stereotomic dome, in which are highlighted: the geometry of the spherical dodecahedron (with grey lines), the distribution of static forces (with black curved lines and arrows) and minimum triangular part TB' (with dotted lines). Image by the author

Another dome is configured in this study (Fig. 7a), with the diameter of the extrados of 7.12 m, and 17 or 18 types of ashlar invariants. To verify the feasibility of the cutting work and the assembly of the bond, a stone prototype (Fig. 7c, d) of this smaller dome was made by the société SNBR at Sainte-Savine-Troyes, in France, using Robot ABB. The scale of the stone prototype is 1:5.74, and the extrados diameter measures 1.24 m. The prototype was further simplified since the decagons and the nonagons are made up of only one ashlar, because they are smaller than at real scale; in fact, they measure approximately 10 cm in length and are 4 cm thick (Fig. 7b). Ashlars were made in *pierre de Lens*, cut by the robot in sixty hours; they were numbered and then placed on polyurethane centring, which was prepared in four hours by the robot and was transferred by engraving the design of the bonding project. The execution of this design is not indispensable to dome construction, as demonstrated by the PLA *maquette*, but it was helpful in speeding up assembly time, which took place on two working days by two people, including cleaning of the prototype after assembly.

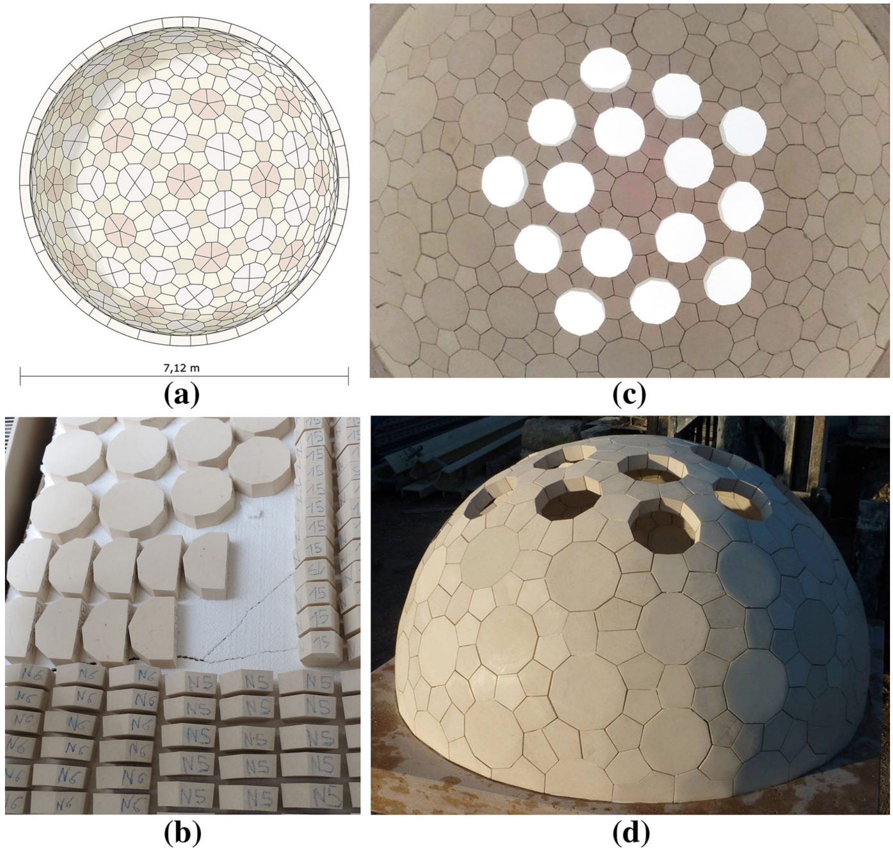

Fig. 7 **a** Another dome configured with smaller diameter. Image by the author. **b** Ashlars cut by SNBR for this research at Sainte-Savine-Troyes (France) for construction stone prototype. Photo by SNBR. Intrados (**c**) and lateral view (**d**) of stone prototype constructed by SNBR and projected by author. Extrados diameter: 1,24 m; Scale of prototype: 1:5,74. Photo by SNBR

Conclusions

This study presents a new valid alternative bond for a hemispherical stone dome and reports important results in optimizing the construction process. The number of stone invariant-type ashlars is reduced while maintaining their normal dimensions and increasing the diameter of dome that they constitute. This new bonding determines a good static and aesthetic configuration, in the coincidence of form and structure.

I have deposited a Patent for In-dustrial Invention at the Ufficio Italiano Brevetti e Marchi (UIBM) of the Ministry of Economic Development for the new stereotomic bond for domes that I designed in this research study.

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Roma Tre) and is in continuity with research on the upgrading of traditional construction techniques with cut stone, which for many years have constituted the cultural identity of the Faculty of Architecture in Bari and, at a later time, of the DICAR, and which was conducted by its founder professor Claudio D'Amato Guerrieri, and by professor Giuseppe Fallacara. Static analysis through computer modelling was performed by engineer Daniele Malomo, a PhD student in the DICAR Department at the University of Pavia and a collaborator at EUCENTRE *(European Centre for Training and Research in Earthquake Engineering).*

References

Baverel, O. 2000. Nexorades: a family of interwoven space structures. Ph.D. thesis, Department of Civil Engineering, University of Surrey.

Brocato, Maurizio and Mondardini, Lucia. 2011. Un sistema voltato di nuova concezione basato sull'apparecchiatura della volta piana di Joseph Abeille. In *Atti del Primo Congresso Internazionale di Rete Vitruvio, Rete Interuniversitaria Italiana di Architettura*, ed. Claudio D'Amato, vol. IV, 1945. Bari: PolibaPress/Arti Grafiche Favia.

Burnet, J. 1903. *Platonis Opera,* tomo I, 110b. Oxford: Oxford University Press.

Calvo López, J., Alonso Rodríguez, M. A., Rabasa Díaz, E., López-Mozo, A. 2005. *Cantería renacentista en la Catedral de Murcia,*124-127. Murcia: Colegio Oficial de Arquitectos de Murcia.

Commandino, Federico. 1575. *De gli Elementi d'Euclide, libro duodecimo et de solidi libro II. Con gli scholii antichi et commentarii*, 244. *De gli Elementi d'Euclide, libro decimoterzo et de solidi terzo. Con gli scholii antichi et commentarii*, 259-260. Urbino: Domenico Frisolino.

D'Amato, C. 2014. *Studiare l'architettura. Un vademecum e un dialogo,* 31. Roma: Gangemi Editore.

D'Amato Guerrieri, Claudio and Fallacara, Giuseppe. 2006. Archetipi costruttivi dell'architettura in pietra. In: *Città di Pietra/Cities of Stone. Pietre di Puglia. Costruire in pietra portante/Stones of Apulia. To Build load-bearing Stone*, ed. Claudio D'Amato Guerrieri, 39. Venezia: Marsilio.

Fallacara, G. 2012. *Stereotomy. Stone Architecture and New Research,* 120-123. Paris: Presses des Ponts et Chaussées.

Fallacara, G. 2014. Stereotomia e rappresentazione del mondo. In: *Stereotomic design,* eds. Giuseppe Fallacara, Vincenzo Minenna, 17–21. Maglie (LE): Edizioni Gioffreda.

Folicaldi, F. 2005. *Il numero e le sue forme. Storie di poliedri da Platone a Poinsot passando per Luca Pacioli*, 29. Firenze: Nuova Grafica Fiorentina, per conto di Nardini Editore.

Frézier, Amedée François. 1980. *La théorie et la pratique de la coupe des pierres et des bois, pour la construction des voutes et autres parties des bâtimens civils & militaires, ou Traité de stereotomie à l'usage de l'architecture* (1737-1739). Tome second, Planche 53, 331. Nogent-le-Roi: Jacques LAGET-L.A.M.E.

Fuller, R. B. (1954) Building construction. US Patent 2682235 A, 29 June 1954.

Gallon, Jean-Gaffin. 1735. *Machines et inventions approuvées par l'Académie Royale des Sciences depuis son établissement jusqu'à present; avec leur Description. Dessinées & publiées du consentement de l'Académie, par M. Gallon. Tome Premier. Voute Plate inventée par M. Abeille, 1699, N°. 50,* 159-160-161, Planche N° 50. *Voute Plate inventée par Le Pere Sebastien, de l'Academie Royale des Sciences, 1699, N°. 51,* 163-164, Planche N° 51. Paris: Chez Gabriel Martin, Jean-Baptiste Coignard, Fils, Hippolyte-Louis Guerin, Ruë S. Jacques. Avec Privilege du Roy.

Heath, T. L. 1897. *The works of Archimedes*, 29–34. Cambridge: at the University Press.

Kepler, Johannes. 1619. *Harmonices Mundi.* Liber II, Fig. Aa, between 58-59. Frankfurt: Lincii Austriae, Sumptibus Godofredi Tampachii Bibl. Francof. Excudebat Ioannes Plancus.

Kepler, Johannes. 1621. *Prodromus dissertationum cosmographicarum, continens Mysterium Cosmographicum, de admirabili proportione orbium coelestium, deque causis coelorum numeri, magnitudinis, motuumque periodicorum genuinis & propriis, demonstratum, per quinque regularia corpora geometrica Libellus primum Tubingae in lucem datus MDXCXI.* Francofurti, recusus typis Erasmi Kempferi, sumptibus Godefridi Tampachii. Uppsala Universitet.

Leonardis, E. 2015. *Architettura romanica pugliese. Il progetto e la costruzione in pietra portante dell'edificio per il culto,* 282. Roma: Gangemi Editore.

Leonardo da Vinci. 2000. *Il Codice Atlantico della Biblioteca Ambrosiana di Milano*. Firenze: Giunti.

Leonardo da Vinci. 1989. *I manoscritti dell'Institut de France. Il manoscritto G*, ed. Augusto Marinoni. Florence: Giunti-Barbera. Illustration also present on http://www.progetti.iisleviponti.it/Leonardo/html/architettura.html.

Livio, M. 2007. *La sezione aurea. Storia di un numero e di un mistero che dura da tremila anni*, 203. Ariccia (RM): Biblioteca Universale Rizzoli.

Mondardini, L. 2015. Contribution au développement des structures en pierre de taille: modélisation, optimisation et outils de conception. Ph.D. thesis, Paris Est en cotutelle avec Politecnico di Bari, Dipartimento di scienze dell'ingegneria civile e dell'architettura, École doctorale Ville, Transports et Territoires (Champs-sur-Marne, Seine-et-Marne; 2015-….), en partenariat avec Laboratoire Géométrie- Structure et Architecture/GSA (Paris).

Montanari, F. 2003. *Vocabolario della lingua greca*, 324. Milano: Loescher Editore.

Pacioli, Luca. 1509. *Divina Proportione.*Venezia: Paganino Paganini. Biblioteca Ambrosiana di Milano.

Panofsky, E. 2007. *La prospettiva come "forma simbolica"*, 13. Milano: Abscondita.

Pfnor, R. 1867. *Monographie du Chateau d'Anet construit par Philibert Delorme en MDXLVIII, dessinée, gravée, et accompagnée d'un text historique & descriptif,* Le Château d'Anet. 2ème Partie: La Chapelle Pl. IV. Coupe longitudinale. Paris: Chez l'Auteur, 146 rue de Vaugirard et chez les principaux libraires.

Pinto Puerto, F. 2001. *Las esferas de piedra. Sevilla como lugar de encuentro entre arte y ciencia en el Renacimiento*, 117. Sevilla: Diputación de Sevilla, Servicio de Publicaciones, Sección Arte, RC Impresores.

Potié, P. 1996. *Philibert de l'Orme. Figures de la pensée constructive,* 116-117. Marseille: Editions Parenthèses.

Rabasa Díaz, E. 2013. Estereotomía: teoría y práctica, justificación y alarde. *Informes de la construcción,* (65): 5–20.

Russo, L. 2015. *Stelle, atomi, velieri. Percorsi di storia della scienza*, 19–49. Milano: Mondadori.

Takakura, H., Pay Gómez, C., Yamamoto, A., De Boissieu, M., Tsai, A. P. 2007. Atomic structure of the binary icosahedral Yb–Cd quasicrystal. In: *Nature Materials*, n. 6, 58–63.

Tebala, Gianni. 2010. Geodesic domes: doing more with less. In *Convegno Internazionale AED. Disegnare il tempo e l'armonia. Il disegno di architettura osservatorio nell'universo. Atti del Convegno*, ed. Emma Manelli, Gaia Lavoratti, 870. Firenze: ALINEA.

Tyler, C. W. 2017. Leonardo da Vinci's World Map. *Cosmos and History: The Journal of Natural and Social Philosophy* (13): 261–280. Available online at http://christophertyler.org/CWTyler/Art%20Investigations/ART%20PDFs/LeonardoWorldMap2014.pdf

Vandelvira, Alonso de. 1573. *Exposición y declaración sobre el tratado de Cortes de Fábricas que escribió Alonso de Valdelvira por el excellente e insigne architecto, maestro mayor de obras de la Santa Iglesia de Toledo* (Libro de Traças de Cortes de Piedras). Titulo 74. Ms. Biblioteca Universitaria de la Escuela Técnica Superior de Arquitectura (E.T.S.), Universidad Politécnica de Madrid.

Vanderpool, E. News Letter from Greece.1958. In: *American Journal of Archaeology*, vol. 62, n. 3, 325. Boston: Archaeological Institute of America.

Wenninger, M. J. 1999. *Spherical Models*, 3-15. Mineola, New York: Dover Publications, Inc.

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