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RESEARCH

Reciprocal Tree-Like Fractal Structures

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Abstract Sometimes the complex structures of nature inspire human constructions. Gothic construction has shown that forces can cross space along intricate paths that may even be arbitrary if correctly dimensioned. In some way, ribbed structures are like trees where the branches conduct forces instead of sap; they operate as branches and trunks descending by fractal ways. Here we discuss reciprocal tree-like fractal structures and the difficulty in their design and erection and solutions for constructive details, as well as the possible analytical questions and automatic generation by means of proper software. The results are shown in the design of the Natural Interpretation Centre in Melilla where we have proposed two connected trees like shown at figures included below.

Keywords Reciprocal structures · Tree structures · Fractals · Variable geometry · Cantilever · Spatial ribs · Umbrellas

Architecture Imitates Nature

Sometimes we are surprised because nature appears to build its structures following our designs, but really we are the imitators. Whatever the sources of our inspiration, there is no doubt that we recall experiences of forests and jungles (Fig. 1). Gothic

F. Escrig Pallarés: deceased.

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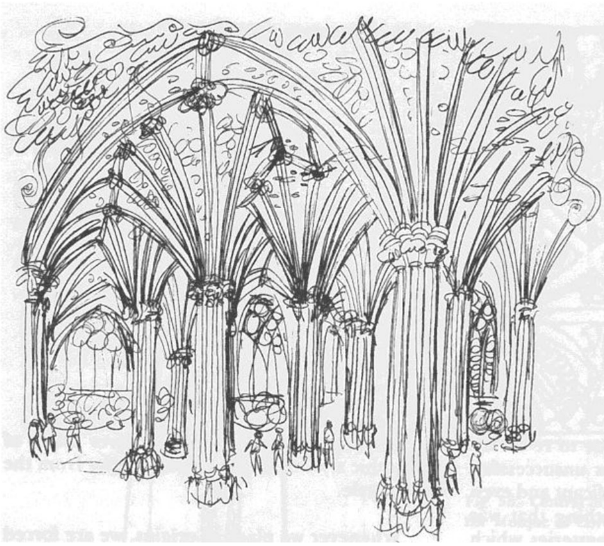

Fig. 1 Lady Chapel in the Wells Cathedral according to Felix Escrig. Image: (Escrig 1998), reproduced by permission

architecture was born in countries where forests were sacred and ribbed structures derived directly from trees and branches (Fig. 2).

Gothic architects made their designs according to nature and, without knowing it, also made use of the theory of fractals. Their first designs were absolutely structural, as in the vaults of Notre Dame in Paris, where some aspects of classical composition were respected, in that ribs are well organized along longitudinal, transversal and diagonal edges. However, in that same structure there appears a new concept, more geometric than structural. The rose windows are illogical structures because they act as pre-stressed stone fabric to load transverse forces. Here the architects introduced an elementary fractal design, where a two-level design appears to "grow" from the centre (Fig. 3).

It took some time to introduce the fractal concept into the design of vaults but then this was done with an unsurpassed mastery, first in wood, as in the vaults of Bath Abbey, and later in stone, as in the vault of the Chapel of Henry VII in Westminster Abbey, directly inspired by the geometry of rose windows.

The complete freedom and imitation of nature arrived later, with masterpieces never before seen. Examples include the vaults of the entrance and hall of Prague Castle, where the stone branches seem to grow and escape off the vault surface. Studies of the fractal components of Gothic architecture have been published by many important researchers and form part of the main studies in architecture design (Goldberger 1996; Bovill 1996).

In our own day there are a great number of proposals that develop fractal and tree-like growth forms, such as the Sagrada Familia by Antony Gaudí and Frei Otto's designs for the tree-like supporting structure of Terminal 1 in the Stuttgart airport (Goldberger 1996). More recent designs include those for the Tote

Banqueting Hall in Mumbai by Serie Architects and the Supertrees in Singapore. The Oriente train station in Lisbon by Santiago Calatrava (1998) and the Mercat Santa Caterina in Barcelona by Enric Miralles (2005) can be also considered as tree-like fractal structures. The most recent designs, such as the metal sculpture by Zaha

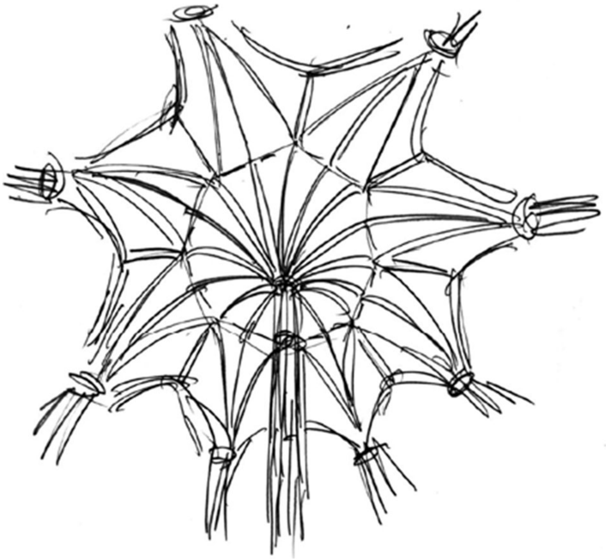

Fig. 2 Westminster Abbey according to F. Escrig. Image: (Escrig and Valcarcel 2004), reproduced by permission

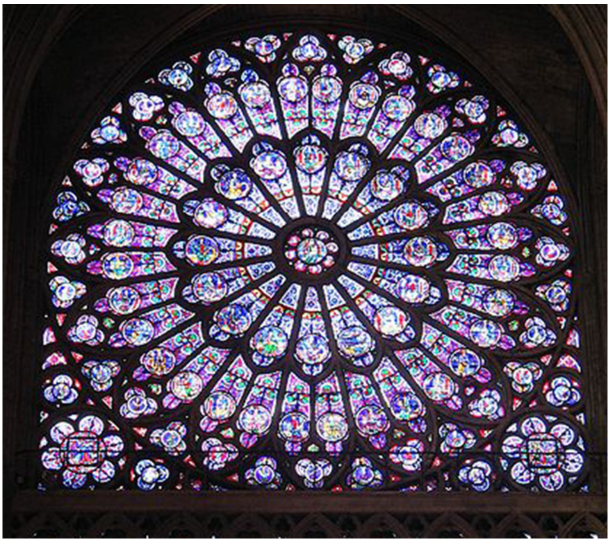

Fig. 3 South rose window, Notre Dame, Paris

Hadid (2012) and the design for the redevelopment of King's Cross Station in London by John McAslan + Partners exhibited at the 2012 Venice Biennale explore other materials and geometries that make the field of this study more and more interesting.

Now we would like to introduce a new design to add to this extensive list, a design which we have called 'reciprocal tree-like fractal structures'.

Reciprocal Fractal Tree-Like Structures

On fol. 899v of the *Codex Atlanticus* Leonardo sketched a few patterns now called 'reciprocal frames' that have been studied in depth. Elsewhere we have demonstrated the great capacity of these designs to act as real structures provided that the joints are properly connected (Sanchez and Escrig 2011). The main characteristic of these structures is that the diameter of bars makes the geometry very complex, with the final form rising out of the plane. The feasibility of these kinds of structures have

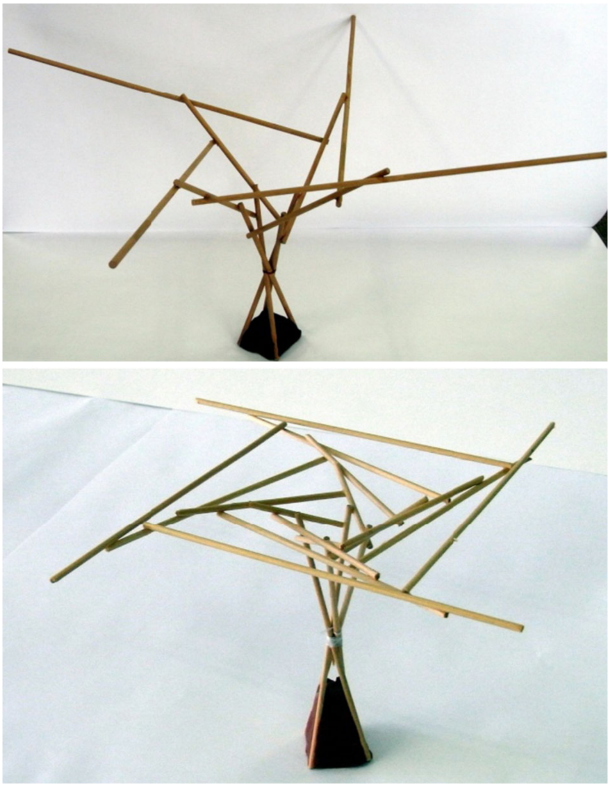

Fig. 4 Umbrella tree-like reciprocal structure model with several levels in two phases of assembling

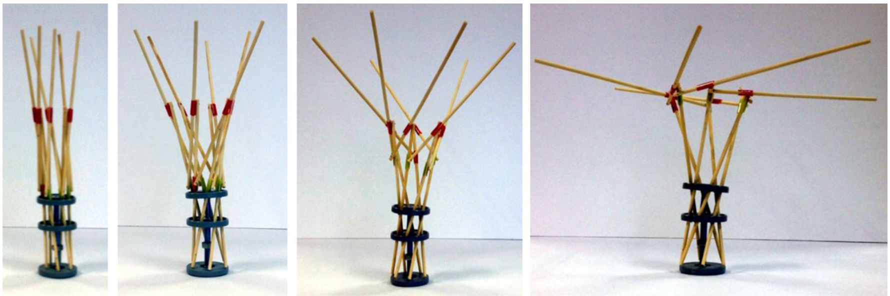

Fig. 5 The same two-level umbrella with sliding joints to make it deployable

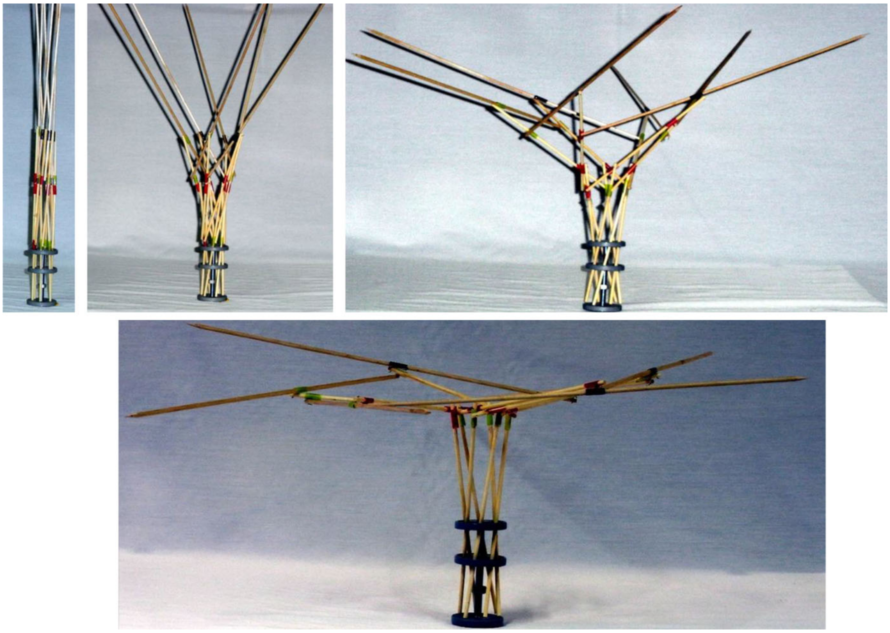

Fig. 6 The three-level umbrella with sliding joints to make it deployable

been shown by Olga Popovic Larsen (2008). Our innovation is that while such designs are usually proposed as roofs supported at their external edges, we designed umbrella-like trees.

Our purpose was thus to study the possibility of combining the concepts of tree-like fractal growth and reciprocity (Fig. 4).

We can increase the levels of growth by either maintaining the same number of branches at each level or by duplicating them each step or new level, as trees does. It may be that, in the strict sense of the term, systems that do not multiply their elements as they grow are not considered fractal, but this is in any case a way of extending their arms based in a defined mathematical form that can be automatized.

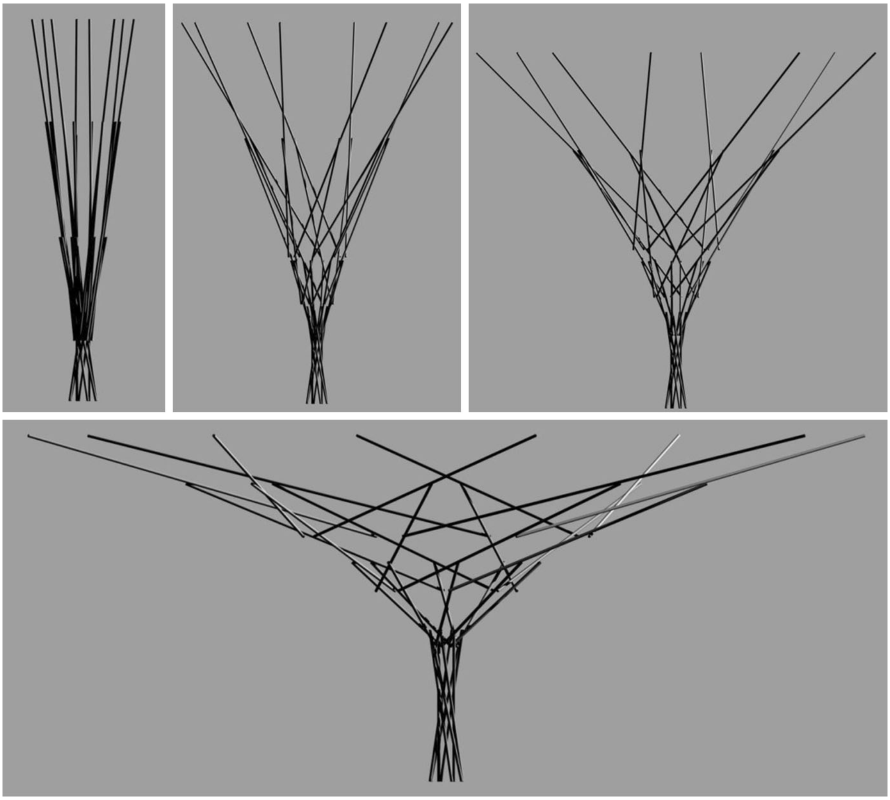

Fig. 7 The four-level umbrellas obtained by a computer program. Image: authors

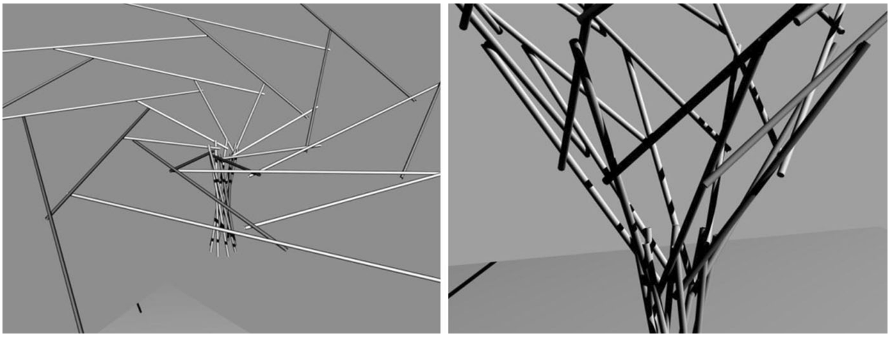

Fig. 8 Details of joints for the sliding and twisting bars to make possible the deployment of reciprocal structure umbrella

Tree-form umbrellas that are reciprocal grids can be studied from the point of view of fractal theory.

Similar works have been investigated by others, but such studies are not abundant and have been undertaken from other point of view (Sieder et al. 2012).

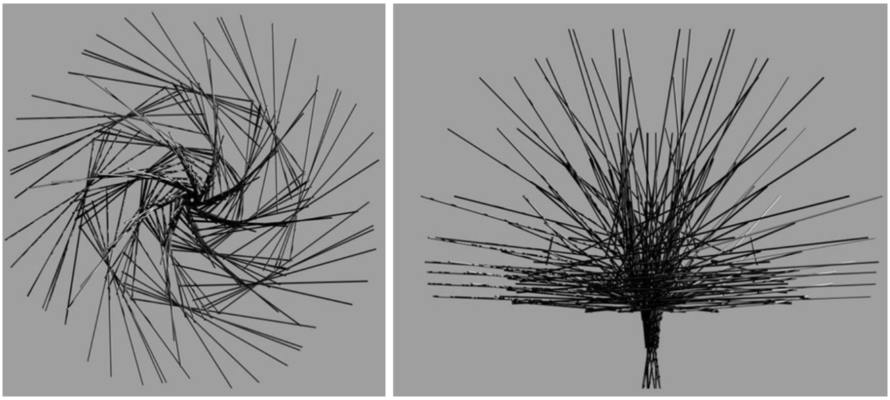

Fig. 9 The four level umbrella deploying obtained by a computer program

Fig. 10 Initial design for a Centre for Nature Interpretation in Melilla, North Africa

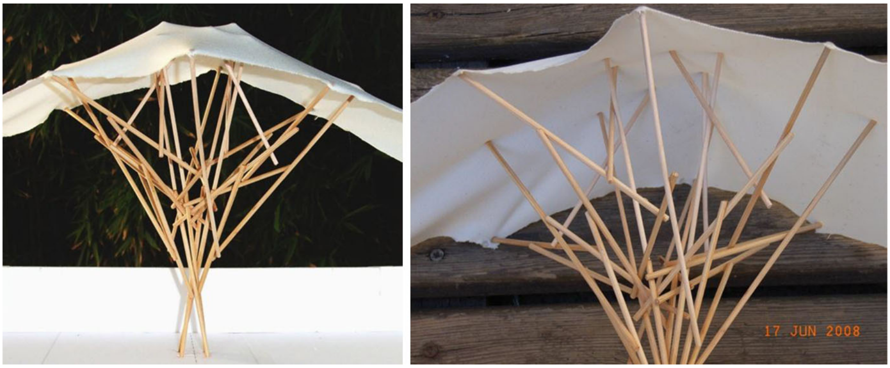

Fig. 11 Preliminary model to define a tree-like structure for the Melilla design

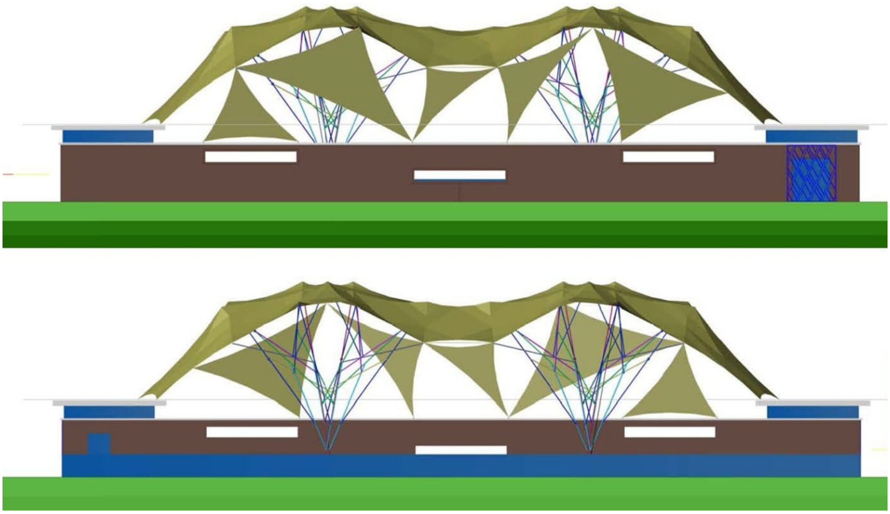

Fig. 12 Front (*below*) and rear (*above*) elevations for the final design proposal

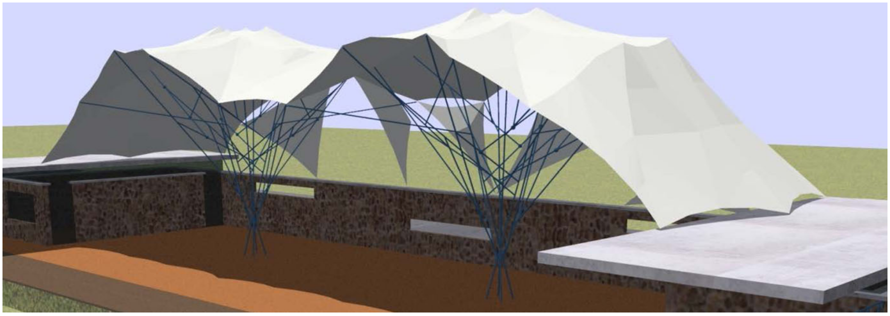

Fig. 13 General view of the final proposal for the Melilla Visitors Centre for Nature Interpretation

Deployable Reciprocal Tree-Like Fractal Structures

Another aspect that we have introduced in our studies is the mobility. This is one of our main objectives in the design of meshes composed with bars. This is because not only can they change in form over time, but also because they greatly facilitate assembly, since each component is carried from the fabrication plant to the worksite in a compact parcel (Escrig 2012).

Figure 5 shows our first attempts by means of a model that simultaneously fulfills the conditions of reciprocal quality, fractal growth and deployability by twisting and sliding joints. Solving all requirements at the same time is very complicated, not only because of the need for the design of a proper joint but because of the simultaneous movement that is required. Our proposal consists in twisting the main supports around a hyperbolic surface by means of two rings placed at equal distances from the hyperboloid centre.

Figure 6 shows the growth process, as well as the addition of other levels of bars in the deploying process.

If we build this system by computer methods it is possible to check the model in real time, as seen in Escrig (2013) and Fig. 7. Figure 8 shows details of joints that have to twist and slide during the deployment process. The design of these joints in a built example will be seen below.

If we superpose several steps of deployment in the same graphic we can obtain a figure that relatively simple but appears complex. We can profit from this kind of image to design a real structure that will be shown in the next section (Fig. 9).

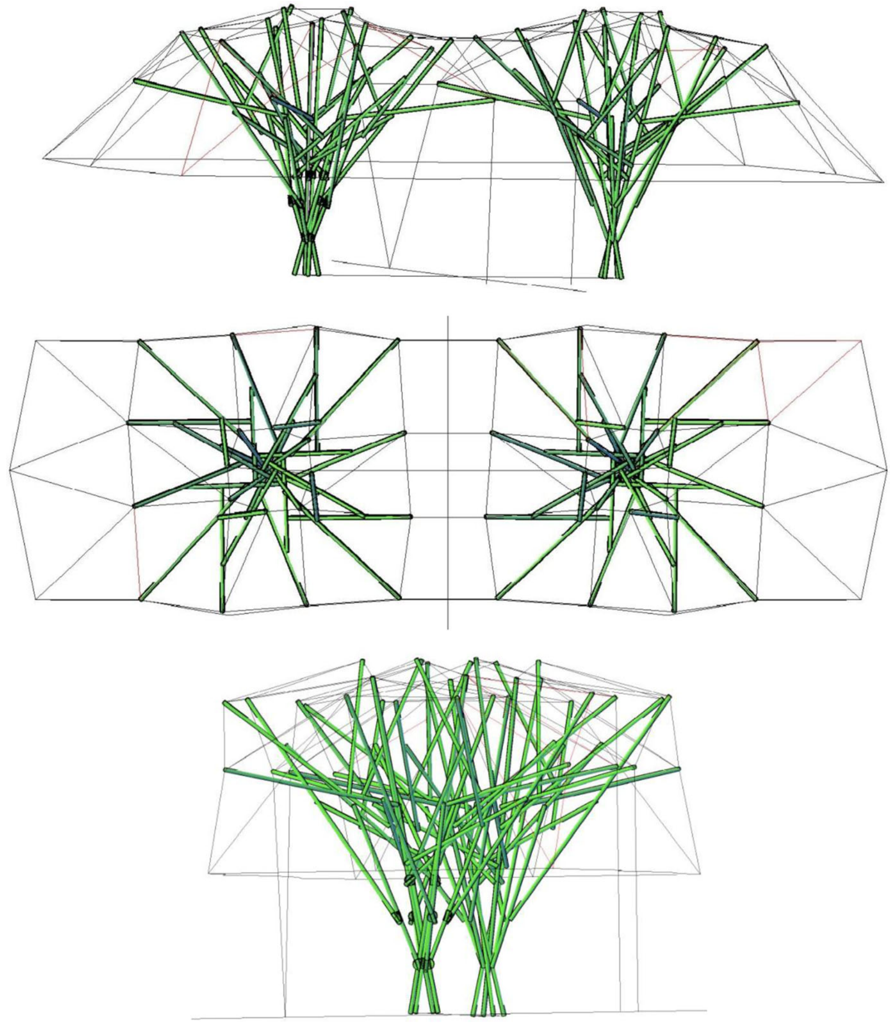

Fig. 14 The design considering the size of pipes

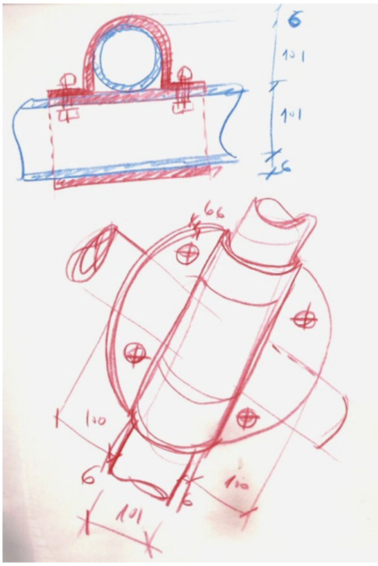

Fig. 15 Preliminary sketch of the joint design

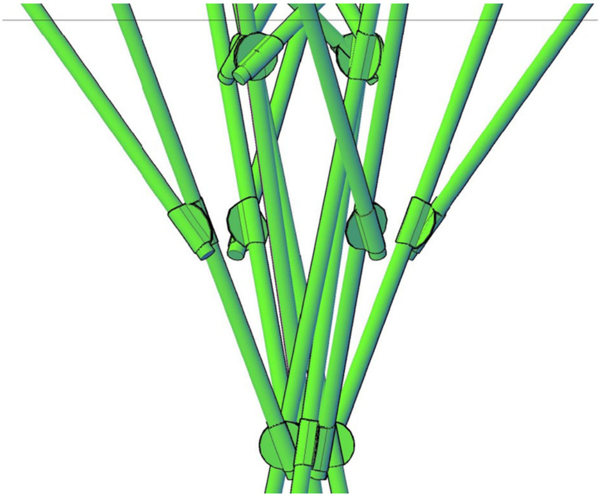

Fig. 16 Joints can twist and slide in every position in space to orientate bars

An Application for a Reciprocal Frame Fractal Tree Like Structure

Melilla is located in the north of Africa and it has a dry climate with almost desert vegetation. With the project proposed in 2008 we intended to adapt a form inspired by both Muslim tents and local trees. We profited from our previous studies to define a complex bunch of branches connected in such way that every bar is linked to others by means of only two points and each supports the origin of another. The initial site was a desolate place that would be revitalized (Fig. 10), but in the end the

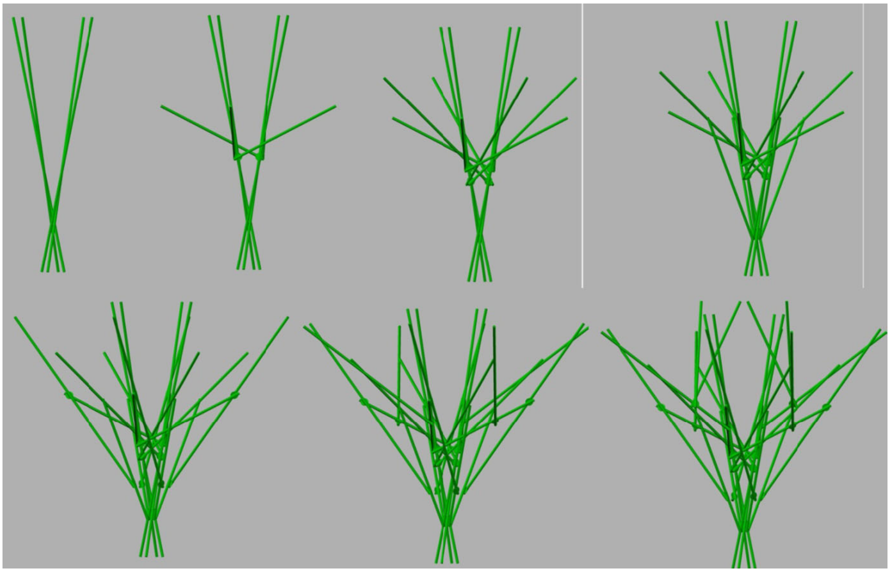

Fig. 17 Sequence of mounting bars with predefined angles

Fig. 18 Group of joints solved with disc plates to connect bars

structure was built in an urban park with many other buildings whose designs were also interpretations of nature. To complete the design we checked some models, such as those shown in Fig. 11, and put together a global proposal (Figs. 12, 13).

If we consider the size of bars and the support system that reciprocity imposes, the solution increases in complexity. It then becomes necessary to draw each bar with the correct diameter and to define its position in a precise way (Fig. 14).

Another problem was the design of a kind of joint capable of rotating, sliding and being fixed when arriving at the correct position. For this we needed to invent a special solution that permits us to orient the bars in every direction in space

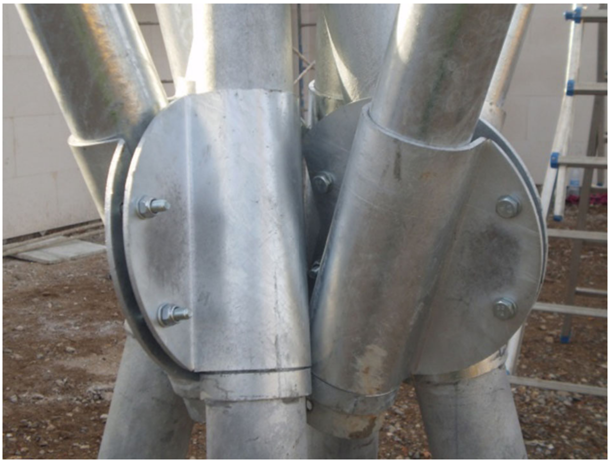

Fig. 19 With the correct position of bars indicated by means of a ring around the pipes

Fig. 20 The installation of tree structures

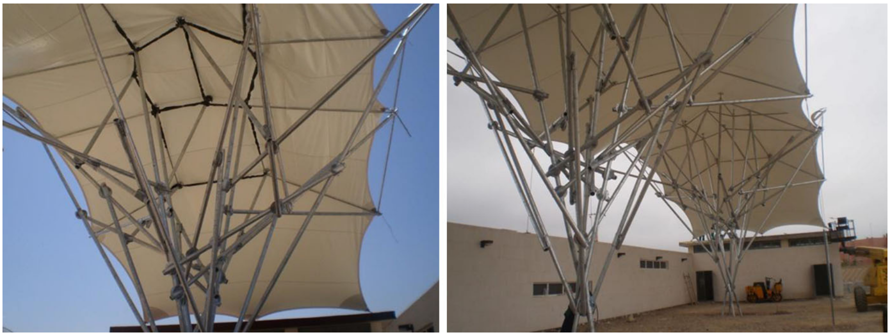

Fig. 21 Mounting the fabric roof

Fig. 22 Artificial tree-like structures in front of and behind natural trees

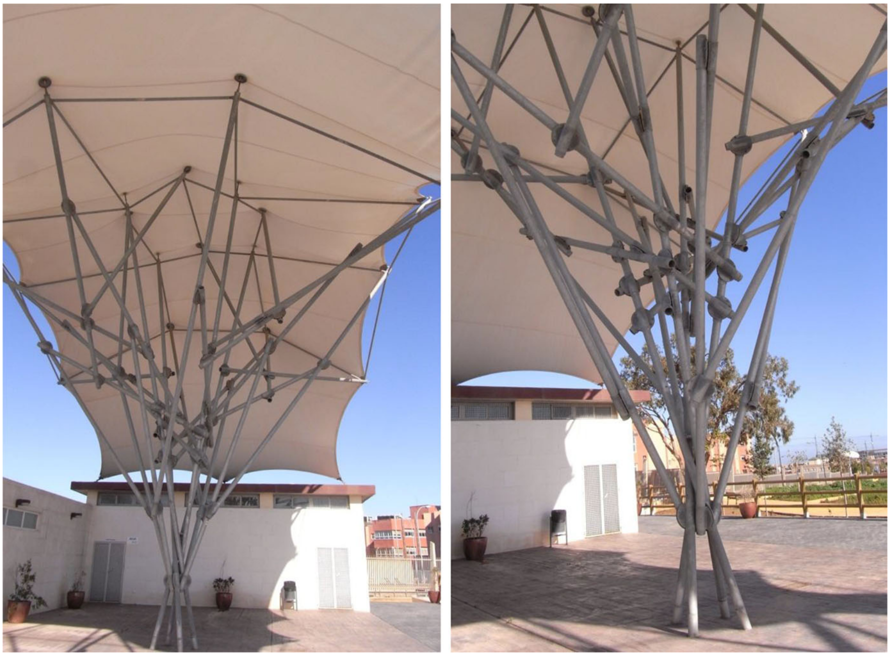

Fig. 23 Mounting the fabric roof

(Figs. 15, 16). In Fig. 17 we show the sequence of assembly and the angles to complete the construction.

Figure 18 shows the actual construction, where we can see that the solution proposed is correct. To facilitate assembly we placed a ring around the pipes to situate the geometry correctly (Fig. 19).

Fig. 24 The finished roof in January 2011

Fig. 25 Lateral view of the finished roof

The Building in Progress

After having checked the proposed solution and completed the main building, we proceeded to install our structure, which consisted in two symmetrical tree-like structures (Fig. 20). To cover the enceinte we decided to use a tensile fabric roof supported by the extremities of the highest branches, as shown in Fig. 21. Figure 22 shows the similarity of this structure with natural trees, and in Figs. 23, 24 and 25 we show the final built structure.

Acknowledgments The authors belong to the research group "Architectural Technology" of the School of Architecture at the University of Seville, and over last 20 years have developed different works related to modular, lightweight and transformable architecture. This paper shows one of the last applications of reciprocal structures developed by the authors, and is dedicated to the memory of Professor Félix Escrig, who passed away in August 2013. All images are by the authors unless otherwise noted.

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Félix Escrig Pallarés (1950–2013) earned his Ph.D. in architecture from the University of Seville. He was a Fulbright Fellow at Berkeley and Professor of Structures at the University of Seville. He specialized in Design of Structures for architecture, and created many important designs for Expo 92 in Seville and Expo 2008 in Zaragoza, as well as many other projects in the field of lightweight structures. Among other awards and distinctions, he won the IFAI Award in the USA in 1995 and 2009, the IASS TSUBOY Award, the Pioner Award from the Space Research Centre of Surrey, the Alluprogecto First Prize in 2005 and the Tubular Construction Institute First Prize in 2006. He organized many symposia in his speciality field and was a regular participant in many others. He was Editor of Advances in Architecture for WIT Press in Southampton, editor of *STAR Structural Architecture* at the Universidad de Sevilla, and editor of the book series "Textos de Arquitectura". He was the author of more than a 100 technical papers, 20 technical disclosure papers and 7 books, 2 of them in English. He is also a novelist and a playwright, with several works published. He was director of the School of Architecture for 11 years, and Director of Department of Building Structures and Soil Engineering for 6 years.

Maria Teresa Rodríguez-León has been an architect since 2007 and is currently working on her Ph.D. at the University of Seville. She has been assistant professor at the University of Seville since 2009, in the Department of Building Structures and Soil Engineering, and has participated in several master's degrees about structures rehabilitation and innovation in architecture. She has collaborated with the office of Félix Escrig and José Sánchez since 2007, participating in many projects, with a specialization in tubular steel

structures. She is the author of several chapters in books and few technical papers, in addition to many presentations in technical conferences. She was the Secretariat of the Conference Transformables 2013. She is a member of the research group TEP114, together with Félix Escrig and José Sánchez.