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Research

Revisiting the Squinch: From Squaring the Circle to Circling the Square

Abstract. "Squaring the circle," constructing a square that has the same area in a given circle using compass and straightedge, has long been a subject for intellectual investigations among mathematicians and philosophers from antiquity to the pre-modern era. The search for this unattainable ideal articulation found its way into Persian architecture with a different approach: circling the square. This architectonic approach, complementing the philosophical view, started from the square at hand, the chamber, to the circle of the vault. The transformation of the cubic to the domical space is mediated through the squinch, intermediary structural element that unifies the two structures. The two seemingly opposite directions of transforming of one form to another (i.e., square to circle or vice versa) allude to the metaphysical and material attributes involved in this process. This paper discusses the mutual relationship between the intellectual and material transformations and the intermediary role of the squinch.

Introduction

Formation of a domical space over a square plan, by means of geometry, is one of the characteristics of the Irano-Islamic architecture. Pythagorean and Euclidean notions of transcendental geometry, in which forms and figures hold qualitative (and cosmic) attributes, from the ninth century, in Persian architecture, had already made the act of transforming shapes comparable to an alchemic act in which qualities transmute to new ones.[1] Therefore, transforming the lower chamber (square form) into the upper vault (circular form) implied a cosmological act. However, unlike the way this is carried out in actual construction – proceeding from square to circle – in theoretical terms this transformation started from the circle. That is why some scholars believe that "[Persian] traditional architecture can be seen as a development of the fundamental theme of the transformation of the circle through the triangle into the square" [Ardalan and Bakhtiar 1973: 29]. Countless ornaments portraying diverging and converging geometric shapes, with or without structural attributes, substantiate the unique role geometry has played in traditional architecture to express symbolic meanings. Such symbolism associated with the transformation of square to circle led to the arch being conceived as "the beginning of architecture,"[2] and the vault and dome as essential elements, for "without them, architecture is incomplete" [Buzurgmihri 1992: 9]. Classified under practical geometry, architecture, demonstrated the use of the theoretical geometry in practice.[3] The domical space, which that manifested historically intriguing geometrical theorem of squaring the circle, received much architectonic attention.

The Friday Mosque of Isfahan: A millennium of squaring the circle

Located in central Iran, the Friday Mosque of Isfahan (fig. 1) is a prominent architectural expression of Seljuk rule in Persia (1038-1194). What makes this edifice one of astonishing beauty is the vertical elaboration of the structure, its vaulted brick

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structures, and its domed structures from the Seljuk period. The augmented brick vault structures of the Friday mosque in Isfahan represent a level of perfection in Seljuk brick structures (see [Pirniya 2007; Haji Ghasemi 2001]). The mosque has over ten centuries of construction history and features 476 existing cupolas including two major domes, four *iwan*s,[4] and several half vaults. Blunt has described the building in this way:

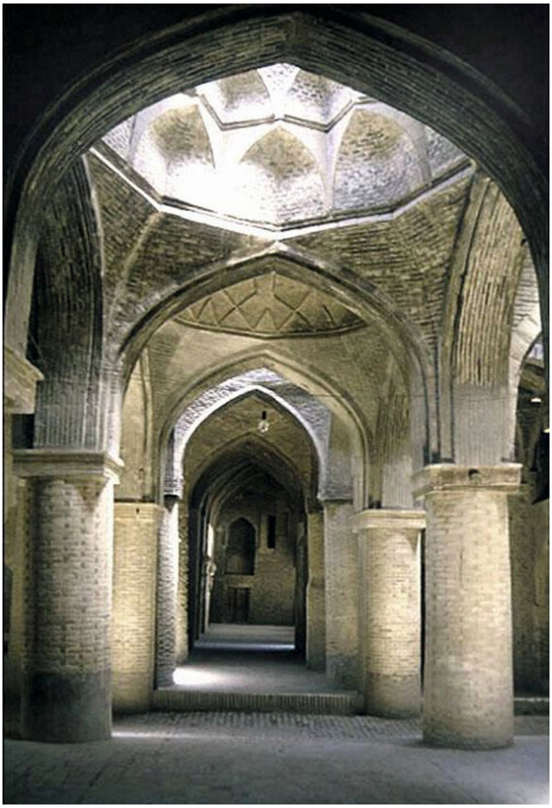

Fig. 1. General view of the shabistan of the Isfahan Friday mosque.
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> The building is constructed in a succession of arches: first a single broad arch in the center of each wall, with pairs of narrow arches cutting the corners; smaller aches in the squinches; and finally, sixteen little arches below the dome itself [Blunt 1966: 33].

The repetition of increasingly smaller arches plays the central role in the vertical elaboration of the vaults and the domes.

The *shabistan*,[5] a covered arcade space typical of the traditional architecture of mosques, comprises more than 450 vaults in the Friday mosque. Moreover, a remarkable number of cupolas feature markedly different structural and visual patterns. Such variety in the reflective ceiling plan is surprising within almost a consistent grid pattern plan (fig. 2). Asterisk-like geometric patterns, "like blossoms generated by the geometry of the circle" [Ardalan and Bakhtiar 1973: 75], are found in many variations of the reflective ceiling plan, all of which are embedded in or framed by the square transforming the vertical columns to the domical surface of the vault (fig. 3). Sectional studies, from the time of André Godard (1881-1965) to Eugenio Galdieri (1925-2010), who conducted archeological excavations and worked on the restoration of the mosque in the past decades, prove that many of these star patterns are not only decorative; but also play a critical role in bearing weight and in the structure of the vault. Some of the patterns that do not represent the main structure of the vault still adhere to a brick structure, are bound by the rules of construction, and are therefore structural on their own. Because the *shabistans* of the Friday mosque were built in different time periods, they are also representative of the techniques of vault-making of their time, while at the same time demonstrating indelible structural beauty.

This paper will concentrate on the Mosque's sophisticated vaults and dome structures—particularly as they relate to geometry and the transition from the square configuration of the lower chamber to the circle structure of the upper vault. Included herein is a detailed investigation of the geometrical and tectonic implications of such a transition. Two pertinent historical treatises on practical geometry and vault design also facilitate our study in this regard.

Fig. 2. Plan of the Friday mosque of Isafahan showing the reflective ceiling plan. Various asterisk-like geometric patterns of the shabistans make each individual vault space unique. Courtesy of Parsa Beheshti Shirazi

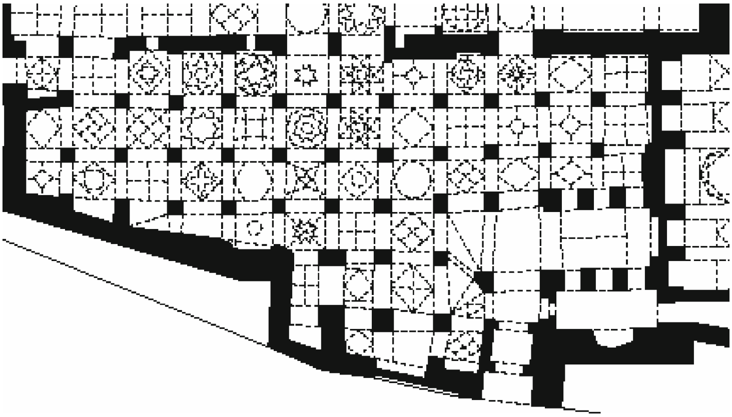


Fig. 2b. Detail of the reflected ceiling plan of the Friday mosque of Isafahan

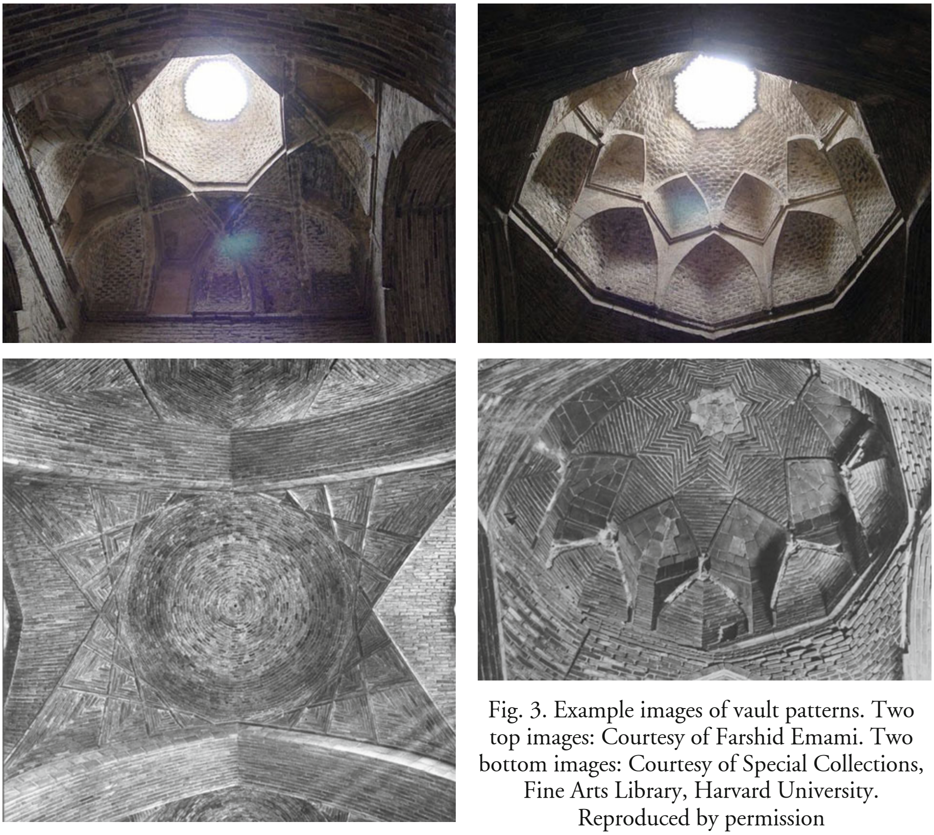


Fig. 3. Example images of vault patterns. Two top images: Courtesy of Farshid Emami. Two bottom images: Courtesy of Special Collections, Fine Arts Library, Harvard University. Reproduced by permission

From Chamber to Vault

The Seljuk use of structural squinches and the supreme transformation of the square into the circle through geometry represents the acme of a very conscious resolution, while the development of a more simplified geometrical resolution integrating square, triangle, and circle through the world of colors and patterns in a super-conscious totality indicates the esoteric blossoming of the Safavid synthesis [Ardalan and Bakhtiar 1973: 29, 31].[6]

Expressing tectonic and structural development, the spatial transition evidenced in the Isfahan Mosque from the cubic chamber to the domical space above is a dominant feature of Irano-Islamic architecture [Pirniya 2007; Haji Ghasemi 2001; Buzurgmihri 1992]. However, the roots of this practice can be traced back to the stone architecture of Sassanid palaces. This spatial paradigm often introduces an intermediate space (i.e., the squinch), which, by means of geometry and the use of triangles, transforms the square into the circle. Highly sophisticated mathematical and geometrical measures underlie this transitional space, which acts as a transition between the cubic and spherical spaces. This transformation can be seen in terms of the two-dimensional (horizontal) plane and the three-dimensional (vertical) volume.

Horizontal plan: interlocking geometrical patterns

The two-dimensional phase of transforming the square into the circle necessitates a geometric investigation of how equilateral polygons are incorporated into a square. Specifically, by dividing a circle's perimeter into equal segments, a series of polygons emerge from the circle. This is made possible through the use of "practical geometry," and is considered to be the foundation for most Persian architectural geometric patterns known as *girih* (lit. knot).

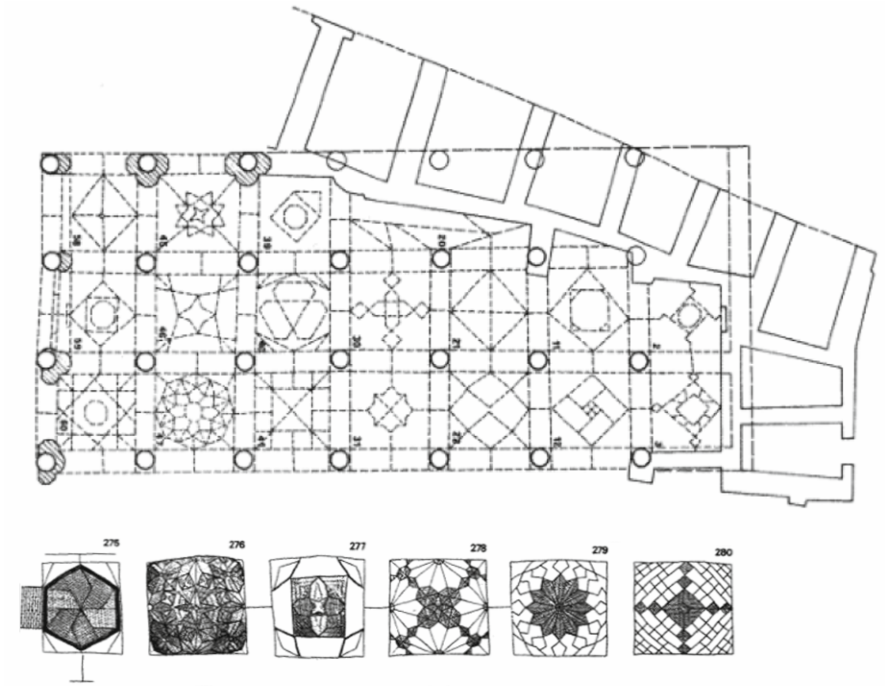


Fig. 4. Examples of the reflective ceilings of the shabistan space (left) and entry corridor vault patterns (right). Source: [Jabal Ameli 1995]

Widely prevalent between the tenth and fourteenth centuries, treatises of practical geometry were "how-to" manuals prepared by mathematicians, astronomers, and geometricians of the courts for artisans and architects, providing them with practical instructions for making geometric *girih* patterns. Core to these geometric exercises were the techniques for dealing with circle and embedding polygons in them. Such two-dimensional exercises served as the underpinning intellectual practice for the transformation of the square plan of the bay into the circular form of the vault. Mastery of this procedure would eventually lead to the production of numerous geometric patterns of the vaults in the orthographic projection. The ceilings of the *shabistan* spaces of the Friday mosque represent a variety of such formal explorations in planar view, as shown in Fig. 4.

Abū'l-Wafā al-Būzjānī was a Persian mathematician, geometer, and astronomer of the tenth century who described various methods of drawing polygons within a circle. In his seminal practical treatise, *Kitab fima Yahtaju Ilaihi al-Kuttab wa al-Ummal min 'Ilm al-Hisab Kitāb fīmā yahtā ju ilayhi al-sāni' min a'māl al-handasa* (*A book on those geometric constructions which are necessary for a craftsman*), he devoted several chapters to this issue. The book comprises twelve chapters, as well as an introduction entitled "On Understanding the Straightedge, Compass, and Square," which discusses the importance of using accurate tools for facilitating the truthfulness of geometric practices. Chapters 3, 4, and 5 discuss in detail how to draw polygons within a circle and vice versa (fig. 5).

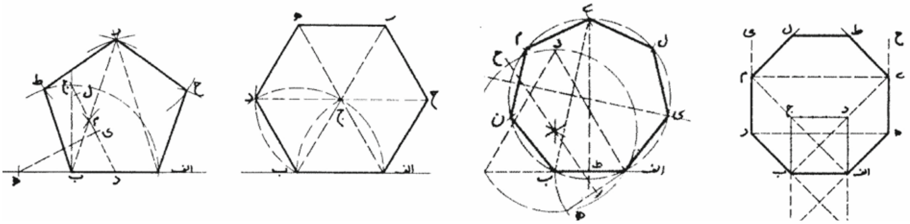


Fig. 5. Al-Būzjānī's drawing equilateral polygons using a compass. From [Jazbi 2005]

What is important to note is that al-Būzjānī's treatise is contemporaneous with the emergence of the new brick and masonry *shabistan* structure of the Friday Mosque.[7] Moreover, numerous decorative two-dimensional geometric patterns associated with that period suggest that the master mason and his apprentices were well aware of the principles of practical geometry detailed in al-Būzjānī's treatise.[8] In fact, at the beginning of his third chapter, al-Būzjānī disapproves of experimental methods of dividing a circle's perimeter to equilateral shapes. Instead, he argues for an accurate and methodic calculation, as articulated below:

> It is prevalent amongst craftsmen that when they want to draw a polygon in or on a circle, they experiment with the leg distance of a compass and mark the circle's perimeter several times in order to find the numbers of divisions on the circle. But this way of dividing [a circle] is not acceptable for architects-masons, prudent individuals, and master craftsmen. Dividing the circle using the above method is not only a very difficult task, but the points of division are also approximate and are not accurate. Therefore, the preferred act for architect-masons and masters of craft is to conceive it in a way that ensures that the length of the polygon is identified first. …[9]

Al-Būzjānī's instructions for drawing polygons and shapes within other shapes can be considered one of the earliest documents used by architects and craftsmen in Iran after the Arab conquests. Strong parallels can be drawn between solutions devised for patterns in the vaults and dome of the Friday mosque and geometric investigations in al-Būzjānī's treatise.

For example, one such parallel is found between drawing a pentagon within a circle and the North Dome of the Friday Mosque, known as the Khaki Dome,[10] in which a pentagonal pattern comprises a five-pointed star that is integrated as ribs to the dome structure:

> [the dome's] supports are articulated so as to reflect the structure of its zone of transition. The latter, with its richly outlined muqarnas,[11] appears like the bejeweled base for an astounding dome whose ribs have formed a complex geometric pattern generated by a pentagon around a (probably) open oculus [Grabar 1990: 39].

Al-Būzjānī introduced three different methods to accomplish this. The following is a translation of his first method for this problem:

> If we wish to embed an equilateral pentagon in a circle, we first draw the diameter ADG, and from point D which is the center, we draw the perpendicular line of DB. Then we divide radius AD at point H in half, and draw an arc centering H and with the radius of HB until it intersects diameter AG in point R. Then centered on B and with radius BR, we draw arc RT to cut arc BT. This arc is one fifth of the circumference. Now, we find out arcs TY, YK, KE, and EB equal to BT and draw these chords to find the equilateral pentagon of BTYKE.[12]

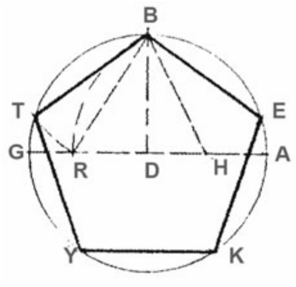
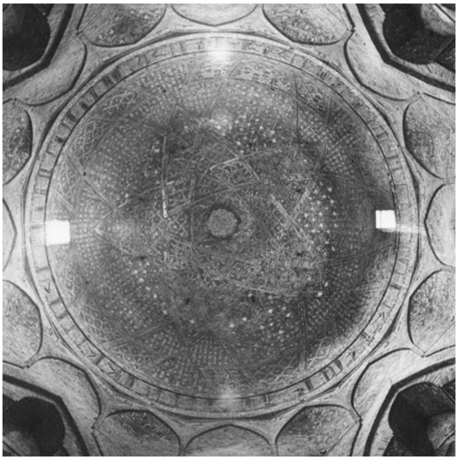
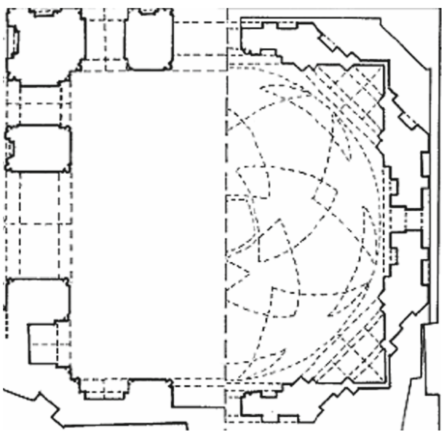


Fig. 6. Upper left) Drawing of an equilateral pentagon embedded within a circle according to al-Būzjānī [Jazbi 2005];
upper right) Khaki dome of Friday mosque: Courtesy of Special Collections, Fine Arts Library, Harvard University. Reproduced by permission;
lower right) Khaki dome, the reflective ceiling drawing [Jabal Ameli 1995]





Due to the necessity of construction, in which the lower square is constructed first, the architect needed to perform the transition starting from the square shape after which he embedded other polygons within the square to transform it into a circle.

Bujzani discussed the notion of drawing polygons in or on other polygons in chapter 6, "On Drawing Shapes in and on Shapes." There he explained the methodology of how to locate various polygons within a square – a practice that masons needed in transforming the plan of the base geometry into the vault of the Friday Mosque. Such geometrical exercises required changing the tools of drawing into the tools of building, such as a mason's square and ropes or string for drawing lines to perform like a compass in construction. Contemporary master mason Asghar Sharbaf, whose father and grandfather were also master masons, documented this practical tradition and indicated that the first stage in making a muqarnas is to draw it in planar form [Sharbaf 2006]. Additionally, Persian architectural drawings of the sixteenth century, known as the *Topkapi Scroll*, contain many interlacing geometric *girih* patterns intended for execution in three-dimensional forms of ribbed vaults and muqarnas, which often required making a full-scale drawing on the floor and projecting it to the upper levels by means of plumb lines. The abstract nature of *girih* patterns, while inevitably governed by strict laws of geometry (as seen above), also required creativity in its process of conception. While many of the interlocking patterns featured vegetal motifs that originated from the natural world, the architect-engineer "deliberately reworked naturalistic motifs into unreal forms that gave free reign to the artistic imagination" [Necipoğlu 1995: 75], expressing a two-dimensional space, the drawing of which reflects a three-dimensional space.

Vertical elevation: the arch, the beginning of architecture

Structurally, the vertical transformation is made possible through the use of arches. The arch is considered as the first step in the birth of the cupola, an essential element in this architectural progression [Buzurgmihri 1992]. The first stage in the construction process is to build four arches on the columns of the lower structure, uniting them. In plan form, these arches are consistently projected on the geometry of the square. Developing a process for constructing the arch provides an essential basis for further transformation of the structure with the goal of completing a vault. A considerable number of the vaults in the Friday Mosque were built after the fifteenth century, a period from which a valuable treatise on vault and cradle vault construction has survived.

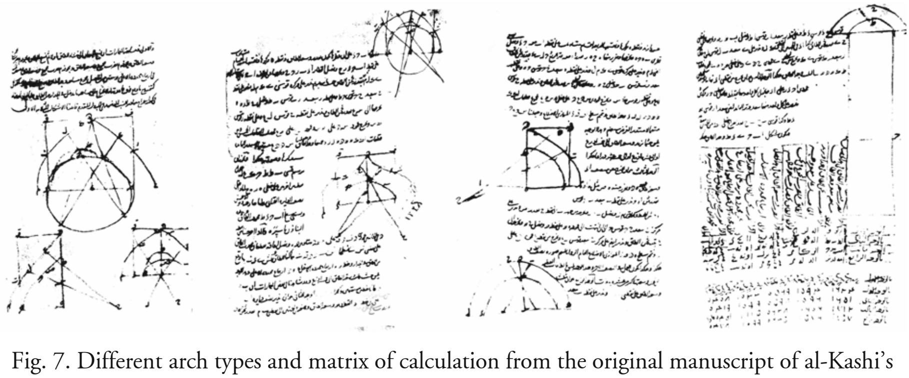


Fig. 7. Different arch types and matrix of calculation from the original manuscript of al-Kashi's *Risāleh Tāq va Ajaz* [Jazbi 1987]

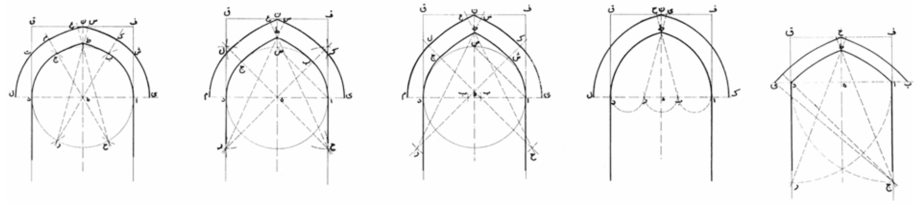


Fig. 8. Different arch types from a Farsi translation of al-Kashi's treatise [Jazbi 1987]. From left to right, arches # 1, 2, 3, 4, and 5

Ghiyath al-Din Jamshid Kashani (1380-1429), known as al-Kashi, Persian Islamic scientist and geometer of the fifteenth century, extensively wrote on measurement, calculation, and making arches, vaults, and domes in his seminal treatise, *Miftah al-hisab* (Key of Arithmetic).[13] Al-Kashi also introduced five major types of arches and methods for drawing them (figs. 7 and 8) in his work *Risāleh Tāq va Ajaz.* He suggested that Islamic arches were often pointed arches and that by spanning two pillars, thereby defining a chamber, they provided support for the vault or the dome [Jazbi 1987]. Depending on the size of the span, al-Kashi also introduced methods of calculation for making arches and vaults.

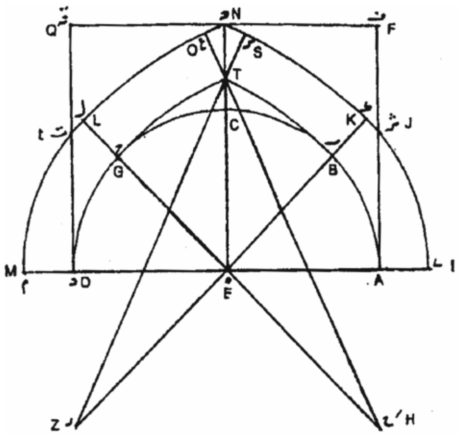


Fig. 9. Arch type 2, which according to al-Kashi was the most popular at his time [Jazbi 1987]

Al-Kashi indicated that in his day the second type of arch was more popular. In the fourth chapter of *Miftah al-hisab*, a treatise on arches and vaults, he explained the method by which the arch is found:[14]

> Draw a half circle on AD, span of the arc. Continue AD from both sides equal to the thickness of the arch until points I and M are found. Consider E as the center of the half circle. Divide the arch of the half circle to four equal segments, points A, B, C, G, and D. Extend BE and GE [respectively] to EH and EZ equal to AC, and the lines of BK and GL equal to DM, which is the thickness of the arch. On center E draw JM and KL arcs, on center H, draw arc GT, and on center Z draw arc BT. Connect HT and ZT and extend them equal to the thickness of the arch until points O and S are found. Draw arc LO from center H and arc KS from center Z. draw perpendicular lines of SN and ON from TS and TO.[15]

When this procedure is followed, the compound arc ABTGD comprises the arc of the entry and TN represents the thickness of the arch (fig. 9).

Al-Kashi's arches required the use of a compass and a straightedge on paper, followed by the use of rope, string, and plumb lines in the ensuing construction process. In the translation from paper to the actual space, the architect had to identify in space the center points of circles and intersecting points using accurate measurements, while having at the same time to imagine "invisible" lines that preceded the construction process. Pirniya [2007] believed that for smaller cupolas, the mason would not have used formwork for arches; instead, the mason would have relied on experience, memory and, above all, his imaginative capability to visualize the space prior to the actual construction.

The drawings in fig. 10 show the vault rib structures of chamber no. 60 (according to plans in [Galdieri 1984]) of the Friday Mosque.[16] The domical space results from the cross arches forming rib structure, which also produces an ornamental pattern. Geometry here functions both in two dimensions (the plan of the ribs represent a star in the square) and in three dimensions (the rib structure), which then simultaneously transforms the lower cubic space into the upper domical space. As such, the ribs form a series of interlocked arches providing structural support for the filling bricks. As the perspective demonstrates, the spaces between the arches are filled in as the ornamental architectural structures of muqarnas. Once the interior form is completed, then a second and third layer of bricks cover the vault, creating a smooth sphere-like volume in the space below.

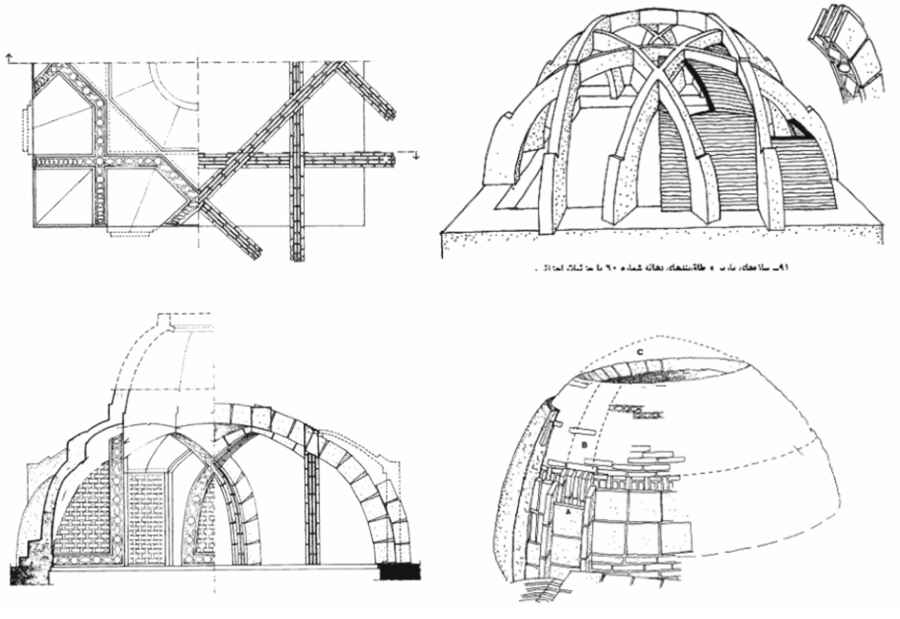


Fig. 10. Arch no. 60 of Friday Mosque demonstrating aesthetic role (upper left) and structural role (upper right, lower left) of the rib structure [Jabal Ameli 1995]

Masons and architects who were involved in conceiving and building the Friday Mosque were most likely well-versed in techniques of erecting arches and vaults, considering the significance of the city in which the mosque was to be located. According to Grabar,

> [these techniques] are wonderful examples of the operation of an ahistorical vernacular practice, which over the centuries cleaned and repaired the lonely covered spaces of the mosque. There is little point in trying to establish a chronology of the domes or of the transformations affecting supports. What does appear here is that, from some moment yet to be determined, the culture and technical competence of Isfahan built up and maintained the large space of its mosque in the consistent language of brick derived forms [Grabar 1990: 37].

Indeed, such ahistorical vernacular methods for building the three-dimensional vault space were not restricted to the Friday Mosque, but were widely executed across geographical and temporal horizons in pre-modern Iran.[17]

The transition to three dimensions

Through the use of spatial geometry, the four corners of the square morph into a circular shape, by means of a variety of patterns of squinches (fig. 11). The squinch evolved from somewhat simple forms during the pre-Islamic era (for example, Sarvistān Palace) to tectonic perfection during the Seljuk period. As Eric Schroeder indicated,

> By courageous experiment and the intelligent observation of failure, the Seljuks built in the twelfth century what is practically the ideal dome, made possible by the advance of mathematical science in the eighteenth [century] [quoted in Blunt 1966: 34].

Treatises of the time, such as those of al-Būzjānī and al-Kashi, documented the construction of domes and the calculation methods used to build them. Later, the squinch evolved to the augmented ornamentations of the Safavid period in the fifteenth to seventeenth, and eventually to non-structural and decorative geometric pattern finishes (*rasmi-bandi*).[18]

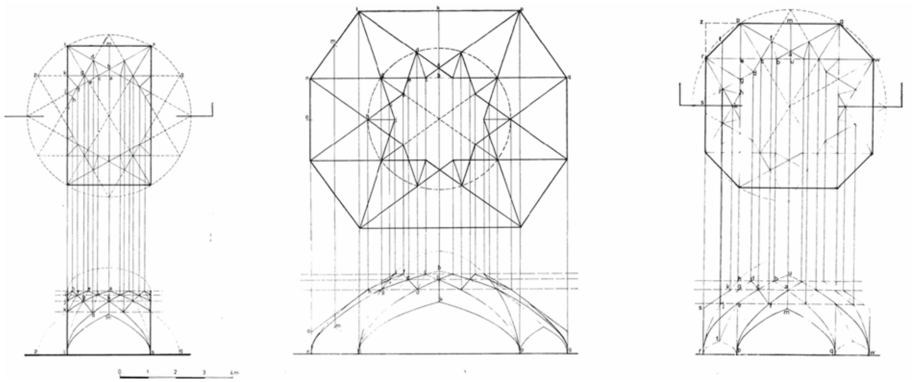


Fig. 11. Transition from square to circle through use of triangles and squinches. After the seventeenth and eighteenth centuries, non-structural transitional spaces emerged; see [Buzurgmihri 1992]

This progression from merely structural purposes (pre-Islamic, early Islamic) to more visual decorations (post-Safavid) reveals the immense significance of transforming squares into circles in Persian architecture as both structural and decorative solutions. This visual process reached its zenith during the post-Safavid period, when geometric patterns appear in the interior surface of a vault with no structural function. This progression from structural function to ornamentation, a constant desire to elaborate this transition through time, strongly suggests that the process of squaring the circle was seen as more than a structural solution. It can be considered a meditative practice associating essential cultural and symbolic conceptions to the artefacts built.

The squinch, from sensible reality to intellectual ideality

Geometric principles enabled architects and masons to push tectonic limits and create higher spaces with larger structural spans. In studying the transitional space as a material construct, major typologies of these three-dimensional structures and geometries were discovered. Geometric patterns in the Friday Mosque demonstrate an almost limitless variety of forms that were derived from brick. As Grabar indicated:

> The 476 cupolas in one building, almost all of them different from each other, create a festival of vaulting which does not fail to arouse all but the most jaded designers or historians. The cupolas can be seen from the outside as a sea of bubbles from which occasionally emerges an exhilarating island of arches and segments of vaults… . The ceilings consist of small cupolas among which the same anarchical voluptuousness rules as among supports. …[They] show a range in quality and success that is just as great as the time span separating them [Grabar 1990: 36-37].

While the three-dimensional geometric space is essential to the construction of the dome, it can also contain highly symbolic expressions in two-dimensional view. These two concurrent representations of geometry – one in the material construction of the actual domical space, and the other in the mental construing of the symbolism and intellectual content within that space – link geometry to our cognitive faculties. The interplay between "depth and surface" achieved by an imaginative participation constantly reminds us of the importance of the simultaneity of reality and ideality in an Irano-Islamic worldview.

The collection of geometric patterns seen in the vaults of Isfahan's Friday Mosque gave birth to masterful types of transitional elements we know as squinches, which have both symbolic and tectonic relevance. Constantly repeated through time and in elaborated artful forms, the spaces created by squinches can be perceived as semi-universal structural-conceptual truths. These patterns are "incorporated into a building by timeless piety and are not to be judged or evaluated in chronological terms" [Grabar 1990: 10-11]. Ardalan and Bakhtiar explained the symbolism of the chamber and dome as following:

> The square, the most externalized form of creation, represents, as earth, the polar condition of quantity, whereas the circle, as heaven, represents quality; the integration of the two is through the triangle, which embodies both aspects. The square of earth is the base upon which the Intellect acts in order to reintegrate the earthly into the circle of heaven. Reversing the analogy, the square, as the symbol of the manifestation of the last of the created worlds, reverts to the first; thus the heavenly Jerusalem is seen as a square in its qualities of permanence and immutability, and the circle is seen as earthly Paradise. The end of the world is seen symbolically as the "squaring of the circle" – the time when heaven manifests itself as a square, and the cosmic rhythm, integrating itself into this square, ceases to move [Ardalan and Bakhtiar 1973: 29].

The geometry used in erecting vaults offers meditative attributes capable of contributing to the creation of mental conceptions of the world. From archetypal shapes to exquisitely elaborated geometries, there is a consistent message – an expectation of geometry to express meanings belonging to a higher reality. The triangle, envisioned as a mediating influence between the circle and the square, unifies the structure and decoration. While the former is bound to the realm of reality, the latter seeks to achieve an idealized world. Residing between these two worlds, the triangle holds qualitative attributes and becomes an imaginal being, inviting the viewer to creatively participate in exploring the playful space of "depth and surface," to contemplate "matter and meaning," and to be reminded of the "temporal and eternal" nature of human existence.

The squinches of the Friday Mosque go far beyond demonstrating the technical skills of generations of masons in forming innovative asterisk-like structures seen in reflective plans. They, indeed, represent contemplative geometrical figures, like stars in the sky, guiding man to a higher space of reality, one based on imaginative perception. The space formed by the squinch is the space of qualified geometry echoing theoretical conceptions of individuals who found themselves between the earthy and heavenly worlds. The squinch is an intermediary that transforms these worlds into one another. As the Persian master architect would look "up" and see completeness of the heavenly being in the perfectness of the circle, he desired to bring that heaven to the earth, an act of squaring the circle. This spiritual will, however, started with practicing from "down," the earth. "Squaring the circle" is the intellectual drive that generates the practical solution of circling the square. The squinch, the mediator, manifests the mirror-like relationship of this transformation.

Notes

1. Greek and Hellenistic thought influenced Islamic Persian sciences, mathematics, and architecture as early as the eighth century, but most significantly since the tenth century of Abbasid dynasty in Baghdad. With the inception of the *Dar al-Hikma* (lit. House of Wisdom), a scholarly institution and movement for translating Greek texts into Arabic language, the early Islamic courts became acquainted with the Pythagorean's transcendental approach to the geometry and semantic dimensions of Platonic solids (for example, the association of the cube to the terrestrial world, and the sphere to heaven). It was in such an atmosphere that such early Persian and Muslim scientist and mathematicians as al-Būzjānī, Ibn al-Haytham, and Al-Karaji emerged. The first wrote a treatise on practical geometry to be used by architects, and the other two wrote treatises on building and construction.
2. Many masons and architects believed that architecture started with the arch and reached its zenith with the dome. This quote is attributed to *Ghiyath al-din Jamshid Kashani*, a Persian mathematician and astronomer of the fourteenth century [Ashrafi and Ahmadi 2005].
3. Al-Farabi was among the first to provide a classification of sciences for the Islamic world in the tenth century. In his classification, architecture and mechanical devices, namely *ilm al-hiyal* (lit. science of deception), were classified as a subcategory of the practical geometry. Ikhwan al-Safa also following al-Farabi, considered architecture as a subcategory of practical geometry.
4. *Iwan* is a half-vault entry structure that connects the courtyard to the interior spaces. This threshold space, often high enough to elaborate the façade and the building, is located on the central axis of the courtyard. The Friday Mosque in Isfahan, according to [Pirniya 2007], is considered to be the first four-*iwan* typology of mosques introduced from Persia to other Islamic lands.
5. *Shabistan* is an arcade-vaulted space usually surrounding the courtyard of the mosque. The structure of this space is based on squared grid plan that defines the location of the columns at the lower level and vaults sitting on the columns. The etymology of the word is also revealing in regard to the star-like patterns appearing under the vaulted structures. *Shabistan* is comprised of two words; *shab* (lit. night) and *stan* (lit. house), meaning "night house". Historic sources, such as Nazim al-Atibba, have mentioned *shabistan* as a place for Dervish and others to pray and sleep there in cold nights. Therefore, with sharp contrast to the courtyard in terms

of access to the light on the one hand, and a place to be used during the night, on the other hand, it would make sense that star-like patterns became semantically relevant to that space.

6. Safavid architecture contributed in taking squinch design to another level of ornamentation, introducing the use of glazed tiles in squinches, which became a foundation for *rasmi-badi,* a practice in which the transformation was reiterated at a decoration level and not necessary structural.
7. Haji Ghasemi suggested that "late in the [tenth century], this building contained 262 vaults and 55 circular bases, which formed 18 aisles along the length of the courtyard and 15 aisles along its breadth, the central aisle on the [Qibla] axis being broader than the others. At this time, the *shabistans* of the mosque had flat timber roofs. A 420-meter-long raw brick wall encircled the mosque" [2001: 121].
8. There is no direct evidence that al-Būzjānī's treatise was used in Isfahan at the time. However, the role of treatises on practical geometry in communicating theoretical geometric truths to the architects and artisans on the one hand, and the popularity of such treatises in that era on the other hand, indicate that such geometrical investigations were ongoing at the time of construction of the Friday mosque.
9. Translation from Farsi by Hooman Koliji from a Farsi edition by A. Jazbi [2005].
10. This is the northern dome of the Friday Mosque, built in 1098, one year after completion of the south dome. Grabar [1990] explains that that patron of the North dome wished to accomplish something more significant than that previous individual, therefore, the dome was built as a competitive act to the earlier one.
11. *Muqarnas* is a stalactite-like ornamental structure often appeared underneath vaults or half-vaults granting visual sophistication to the transition from the lower rectangular space to the upper area. Muqarnas is non-structural and is a second layer with bricks, glazed tiles on plaster, or paint on plaster. Muqarnas is hung beneath the main structure. Its sophisticated three-dimensional sub-volumes calls for mastery in the practical geometry and imagination, as the muqarnas drawing tradition had neither section or elevation drawings as far as we know. Therefore, such intricate structures required the master builder's presence to be completed while construction.
12. Translation from Farsi to English by Hooman Koliji from [Jazbi 2005]. Persian Jazbi provided a Farsi translation of al-Būzjānī's treatise based on historical manuscripts found in Iran's libraries. Other versions include an Arabic edition [Al-Būzjānī 1971].
13. *Miftah al-hisab*, originally written in the fifteenth century, was translated into Russian in 1954 and several Arabic editions of the book exist. Gülru Necipoğlu [1995] notes that al-Kashi's treatise was addressed to members of the state who where involved in supervising construction for taxing purposes, and not necessarily used by architects. However, such a level of precision in arch and vault computation must have come from the hitherto well-established existing knowledge, either in form of written/drawn documents or from the actual practice by architects of the time. This infers that such treatises must have existed during the time of the Friday Mosque construction. Thabit ibn al-Qurra's treatise, *On Mensuration of Parabolic Bodies*, from the ninth century, is an obvious indication of the use of such treatises. Additionally lost architectural treatises of Ibn al-Haytham and Al-Karaji might have contained relevant information similar to that of al-Kashi. It is also, nevertheless, quite reasonable to imagine that such treatises as al-Kashi's were used by masons and architects. *Miftah al-hisab* is comprised of five main sections, each with several chapters. The fourth section, "On Area," includes nine chapters, the last of which, "On Volume of Buildings and Edifices," discusses vaults and cradle vaults, dome, and muqarnas. Here I used the Farsi translations of the book given in [Jazbi 1987].
14. Translation from Farsi by the author, from [Ashrafi and Ahmadi 2005]. The authors of this reference consulted Yvonne Dold-Samplonius's writings, a mathematical historian of Heidelberg University, Germany, who has done extensive research on al-Kashi; see, for example, [Dold-Samplonius 2002].
15. It is not clear why al-Kashi suggests that from points T and S one should draw perpendicular lines (SN and ON) to find the external point of the arch, N. Considering the minimal length of SN and ON, they could be the continuation of the arcs KS and JO accordingly. The

difference in the height of point N would be minimal and negligible. However, his insistence on drawing perpendicular lines in all types of arches must have been related to construction practices at his time.

16. Farsi translation of [Galdieri 1984] by Jabal Ameli [1995].
17. What Grabar observes as "ahistorical vernacular technique" could also suggest the use of the practical geometry treatises (for example, al-Būzjānī, tenth century) and dome computations (Al-Kashi, fifteenth century) across time and not specific to their own period. Extant pages of treatise on parabolic dome calculations attributed to Thabit ibn al_Qurra (ninth century) are similar to al-Kashi's attempts to determine a method for computing the surface areas of domes.
18. Another term that is used for *rasmi-bandi* is *kār-bandi.* While these two terms have been used interchangeably, *rasmi-bandi* suggests a structural role for the ribs, while *kār-bandi* suggests a decorative role.

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