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Letter from the Guest Editor

Forty Years of Shape and Shape Grammars, 1971 – 2011

Abstract. Guest Editor Lionel March introduces *Nexus Network Journal* volume 13 number 1 (Winter 2010) dedicated to Shape and Shape Grammar.

Introduction

To start with, a personal view and history. There were two of us at neighboring desks in the studio. Both of us had come up to Cambridge and were a year or so into reading the mathematical tripos when, independently, with the arrival of Leslie Martin as Professor of Architecture, we decided to move over to architecture. Christopher Alexander and I would occasionally walk along Kings Parade. Not surprisingly, having been born in Vienna, Chris would talk about Wittgenstein's *Blue and Brown Books* [1958]. I had read Wittgenstein's *Remarks on the Foundations of Mathematics* [1956] which had only recently been published from student notes taken from his lectures, including sharp interventions from Alan Turing. I had also come across Chomsky's early research work funded by the US Navy. This would have been in the late 1950s. Separately, in our subsequent careers, as an editor, I was privileged to promote rule-based 'shape grammars', and Alexander, with associates at the Center for Environmental Structure, University of California Berkeley, was to pursue and establish a 'pattern language'. Rules, grammar, and language are at the centre of Wittgenstein's philosophical questionings.

In the early 1970s, the Royal Institute of Architecture's Library Committee invited me to write a book relating the 'new' mathematics to architecture. In turn, I invited Philip Steadman to join me and *The Geometry of Environment* followed. As proofs were coming off the press, I came across a paper in the University's Computer Laboratory – George Stiny and James Gips' 'Shape Grammars and the Generative Specification of Painting and Sculpture' [1971] – just too late to reference. This paper started it all, and forty years later, the eight papers in this 2011 edition of the *Nexus Network Journal* provide a glimpse of progress today across the globe: from Stiny in the United States, to Li in Japan; from Özkar in Turkey, to Turkienicz in Brazil; and Duarte with colleagues in Portugal and the Netherlands.

Unfortunately, George Stiny was unable to present his paper in person at the 2011 conference because he was to deliver an address at the MIT memorial service for William J. Mitchell who had died just days before, on June 11. Readers of the *NNJ* will be aware of the considerable contribution to the promotion of 'mathematics and architecture' through computer-aided design that Bill Mitchell made during his career. Bill Mitchell had recommended George Stiny to me in a letter when I briefly held a Professorship in Systems Engineering at the University of Waterloo. On my subsequent appointment as Professor in Design at The Open University, I immediately offered George a lectureship. He accepted. It had been a blind appointment. We first met face to face on his arrival from California on a bitterly cold day in Woburn, England. His opening remark: "I agree with 95% of what you have written." Our conversation, since then, has never stopped.

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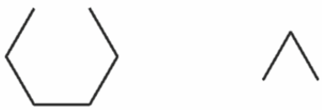
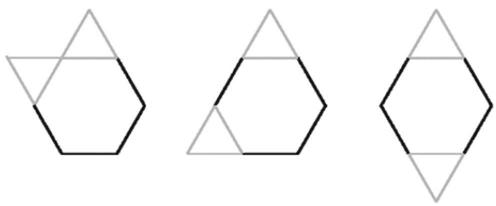
I used to quote Lord Kelvin: 'When you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind' (see [March 1976: 41]). But what numbers? Take a look at this example from Wittgenstein' *Remarks …* :

> An addition of shapes together, so that some of the edges fuse, plays a very small part in our life. – As when
>
>  and 
>
> yield the figure
>
> 
>
> But if this were an *important* operation, our ordinary concept of arithmetical addition would perhaps be different [1956: VII-61].

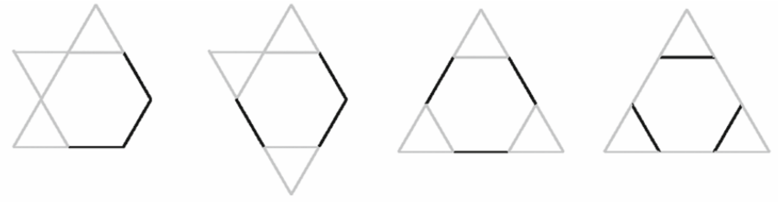
How different? Typically, Wittgenstein asks a question, but does not follow through. For the past forty years, Stiny has sought answers to what he deemed to be no small part and a very important operation in design: how to compute with shapes. In this arithmetical expression: 3 plus 6, the sum equals 9; however 9 take away 3 gives back 6, or 9 take away 6 gives back 3. Yet, what happens with shapes? Taking away the triangle from the combined figure does not give back the hexagon. Or, take away the hexagon and the triangle has gone.



Clearly, the traditional numbers game is up. Traditionally, if apples on a plate are counted, each apple can be taken one at a time and transferred to another plate. That is, counting means subtracting one at a time. Let's go on. There are three ways of adding two triangles to the hexagon under symmetry – a new notion, absent in arithmetical addition.

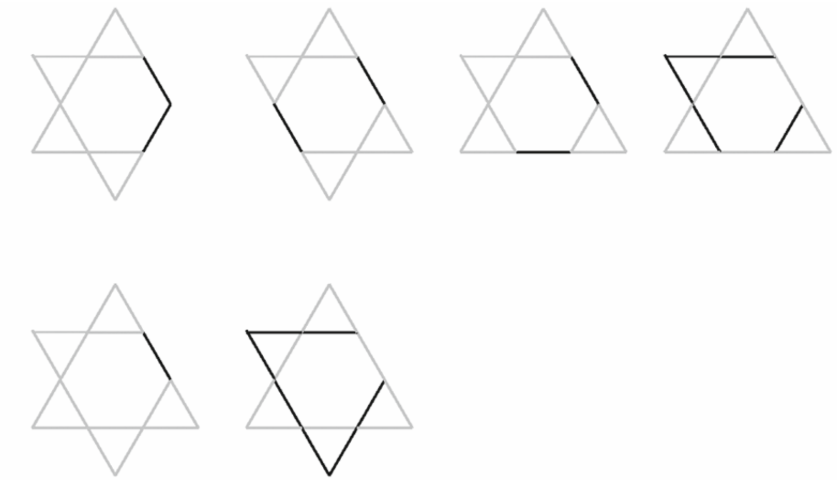


Subtracting the triangles (in grey) gives us back the two we added, but where is the hexagon? Three new and distinct shapes occur in place of the hexagon. If we had subtracted the hexagon instead, no triangles would have been counted. With three triangles



there are again three distinct figures, but with a difference. The second figure has a handed partner, or enantiomorph – yet another concept. The third figure has two counts of triangle. Three small triangles can be subtracted, or one large emergent, scaled-up triangle. Emergence, scaling – more concepts. Taken sequentially, only one of these answers can arise: the answer is ambivalent. Both three and one are correct in isolation. Four different shapes replace the hexagon. No triangles can be counted, if the hexagon is subtracted.

Four and five triangles added to the hexagon give similar ambivalent results. There is only one figure for five under symmetry, but two answers depending on whether small triangles are subtracted, or one large emergent triangle:



Things are more interesting with six triangles added. The combined figure has the symmetry of the hexagon. Six small triangles can be subtracted to leave no hexagon, or two large triangles can be subtracted from the hexagon which vanishes. Finally, the hexagon is subtracted leaving angles spanning the sides. How many triangles are there? 0, 2, 6, or – counting small and large triangles together – 8? The latter result can only be computed, if the deletion rule is applied in parallel.

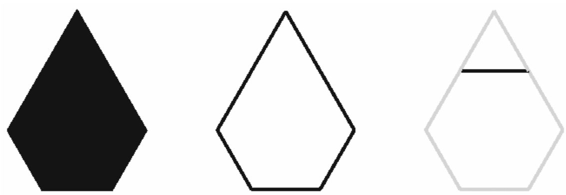
This figure was looked at by Hermann Lotze: "We see it sometimes as two large triangles superimposed, sometimes as a hexagon with angles spanning its sides, sometimes as six small triangles stuck together at their corners" (cited in a footnote by William James [1950: 442-443]; also in [Stiny 2006: 152]). In brief, the arithmetic of shape is a matter of looking. There is no one way of seeing. There is ambivalence, ambiguity: the lifeblood of creativity. In related observations, I have used the relationship of two equilateral triangles to illustrate a point about Charles Babbage's (1791–1871) computing 'miracles' [March 1996].

Addition and subtraction of shapes are Boolean operations. Moving the original triangle to add to the hexagon exhibits Eulidean symmetry operations – translation, rotation, reflection, and scaling. Elementary topology is introduced with changes in dimension through a boundary operator. A 1-dimensional line is not made up of points, but it is bounded by its two 0-dimensional endpoints. A 2-dimensional face is bounded by 1-dimensional lines, a 3-dimensional solid by 2-dimensional faces. Take the boundary of the original figure

which comprises seven points. Take away the triangle – now defined by three vertices – and four points replace the hexagon. Or, take away the six vertices of the hexagon, and one point remains of the original triangle:

Note that these 0-dimensional shapes conform to normal arithmetic. Put another away, computation in ordinary arithmetic is inevitably 0-dimensional, even when graphic coordinates – (x, y) or (x, y, z) – boast 2- or 3-dimensions.

The inverse of the boundary operator, the co-boundary operator, fills in the 2-dimensional face of the figure



from which arbitrary numbers of equilateral triangles may be subtracted under scaling. The boundary of this face is the Boolean 'symmetric difference' of the triangle and hexagon, while the missing edge is their Boolean 'product'.

I run ahead of myself in what the English novelist Muriel Sparks calls 'narrative looping', or telling events in advance of their occurrence. To go back to the birth of shape grammars. James Gips' *Shape Grammars and their Uses: Artificial Perception, Shape Generation and Computer Aesthetics* was published by Birkhäuser in 1975; this was followed three volumes later in the same year by George Stiny's *Pictorial and Formal Aspects of Shape and Shape Grammars: On Computer Generation of Aesthetic Objects.* The former book derived from James Gips's doctoral thesis in Artificial Intelligence, Stanford University; the latter from George Stiny's doctoral dissertation submitted to UCLA's System Science Department. Both authors had been undergraduates at MIT where they attended classes given by Marvin Minsky and Naom Chomsky. It was in tackling an assignment designing handtools set by Minsky that the seeds of shape grammars were first sown. My own essay 'A boolean description of a class of built forms' was finally published in *The Architecture of Form* [March 1976]. The argument followed a personal interpretation of Max Newman's *Elements of the Topology of Plane Sets of Points* [1939]. I designed the cover of the first paperback edition of Newman's book for Cambridge University Press in 1964. Here is a fundamental difference. In my early work a line was understood to be a set of points, while for George Stiny, "all lines in a shape have finite, non-zero length" [1975: 4]. This harks back to Anaxogoras's (c. 500 BCE–428 BCE) principle of homeomerity and to its medieval reprise in the philosophical dispute between the atomism of Richard Grosseteste (1175–1253) and Thomas Bradwardine's (1290–1349) later view that a continuum is decomposable only into self-similar continua; that is, a line divides into lines, not points [March 2008].

The Cambridge philosopher of science, Mary Hesse [1963] had set out a position arguing that many theories in a particular field started as analogues of pre-existing theories in another field [Echenique 1972]. James Gips and George Stiny modeled their initial proposals on linguistic analogies. Explicitly: "Where a phrase structure grammar is defined over an alphabet of symbols and generates languages of strings of symbols, shape grammars are defined over an alphabet of shapes and generate languages of shapes" [Stiny 1975: 28]. Both authors define a shape grammar as a 4-tuple $\langle V_T, V_M, R, I \rangle$, where V_T is a finite set of terminal shapes, effectively a vocabulary; V_M is a finite set of markers, shapes distinguishable from those in V_T; R is a finite set of shape rules of the form $u \rightarrow v$ with appropriate provisos; I is called the initial shape. This formal apparatus is nowhere to be found in George Stiny's *Shape* [2006]. There is no requirement for a vocabulary, the marker set is reduced to a point, rules are classified into a variety of schema, and everything can start with the empty shape, a blank sheet of paper. The linguistic crutch has gone. Shapes are not symbols. Symbols operate under the identity relation, but shapes operate under the embedding – partial order – relation. Shape computations extend Turing's classical machine: 'recursion and identity' for symbols extend to 'recursion and embedding' for shapes. Symbols, like points, are 0-dimensional, while shapes comprising lines, faces and solids are 1-, 2-, and 3-dimensional respectively. A point is identified as its sole and only point, but a line is an uncountable myriad of embedded lines; faces and solids likewise. For a formal comparison of point set approaches and shapes, see Christopher Earl [1997].

In 1978, George Stiny and James Gips published *Algorithmic Aesthetics: Computer Models for Criticism and Design in the Arts.* A theme issue on design and language of *Environment and Planning B: Planning and Design* was published in 1981. This issue included Stiny and March's 'Design machines' [1981], Downing and Flemming's 'The bungalows of Buffalo' [1981], Koning and Eizenberg's 'The language of the prairie: Frank Lloyd Wright's prairie houses' [1981]. In 1994, Terry Knight published *Transformations in Design: A Formal Approach to Stylistic Change and Innovation in the Visual Arts.* This book was the first to present shape grammars in a form suitable for pedagogical purposes as well as presenting the author's original research into stylistic change, for example, in the meander patterns of Greek pottery; the paintings of George Vantongerloo and of Fritz Glarner; and the transformation of the prairie house to the architecture of the Usonian homes by Frank Lloyd Wright. As an introductory text, Terry Knight's remains unsurpassed. In 1999, a special design and computation edition of *Environment and Planning B: Planning and Design* was published, guest edited by myself and Terry Knight. Apart from papers by myself, Knight and Stiny, the issue contained Djordje Krstic's setting of shape grammars into the frame of multi-sorted algebras; Scott Chase's demonstration of the use of shape grammars to permit 'emergence' in geographical information systems; Mark Tapia's computer implementation of a simple shape grammar; and Athanassios Economou's examination of the symmetries of Froebel kindergarten blocks employing Polya's counting theorem (see 'Kindergarten grammars: designing with Froebel's building gifts' [Stiny 1980]).

At the turn of the century, shape grammars were prominently featured in Erik K. Antonsson and Jonathon Cagan's *Formal Engineering Design Synthesis* [2001]. George Stiny's *Shape: Talking about Seeing and Doing* [2006] revisited familiar ground and radically revised the very foundations of computing with shapes. Two years later, guest edited by me [March 2008], *Planning and Design* reprinted seven selected shape grammar papers starting with George Stiny's 'Two exercises in formal composition' [1976] – the very first shape grammar paper that I published, as founding editor, in the third volume of *Environment and Planning B*. Of possible and particular interest to *NNJ* readers, will be Stiny and Mitchell's 'The Palladian grammar' [1978]. Papers by Flemming [1987], Knight [1989], and Earl [1997] are also included in the selection; together with a letter from Stiny [1982] in which he makes the important distinction between set grammars and shape grammars. Given the location of the conference recorded here – Porto, Portugal – it seems appropriate that this special issue of *Planning and Design* concluded with José Duarte's 'Towards the mass customization of housing: the grammar of Siza's housing at Malagueira' [2005], based on his MIT doctoral dissertation. This volume also included a perceptive review of *Shape* by Earl, reprinted the following year [Earl 2009]. For reasons of space, the volume regrettably did not include a reprint by Krishnamurti, who had contributed over fifteen noteworthy papers to *Planning and Design* since 1978, many related to the computer implementation of shape grammars.

To conclude: this present issue of the *NNJ* contains eight papers that resulted from the 2010 Nexus conference section on Shape and Shape Grammars. George Stiny's paper provides the keynote for the section. Among the twelve contributors, José Duarte, Mine Özkar, and Andrew Li are former doctoral students of Stiny's at MIT; while Rudi Stouffs was a doctoral student under Ramesh Krishnamurti's supervision who, himself, had been Stiny's postdoc at The Open University. José Duarte is involved in four of the papers and his achievement and enthusiasm in bringing shape grammars to Portugal deserves recognition.

In selecting the papers, it seemed appropriate to spread themes as widely and representatively as possible. George Stiny provides further theoretical insights developing the concept of *schemas* first mooted in 'How to calculate with shapes' [2001] and advanced in *Shape* [2006], with some deft hints at Alberti's prescience in *Elementi di pittura* [Williams, March, Wassell 2010]; Mine Özkar, alert to visual schemas, having produced more than a thousand illustrations for Stiny's *Shape,* demonstrates examples of pedagogical applications in the architectural studio – a matter Stiny stresses on closing his keynote; Andrew Li takes up the theme of shape language and style in classical Chinese architecture. José Duarte collaborates with Filipe Coutinho and Mário Krüger – editor and commentator on the current Portuguese translation of Alberti's *De re aedificatoria* – in using shape grammars and descriptions to 'decode Alberti 'and to employ the results in arguments concerning influences on architecture during the Portuguese Renaissance. The paper by Alexandra Paio and Benamy Turkienicz uses a shape grammar to visualize the written rules of composition of colonial Portuguese cities and their subsequent cartographic presentation.

José Duarte, in the first of his three remaining collaborative pieces, joins with Sara Eloy in using shape grammar formalism as an aid to the rehabilitation of housing stock among *rabo-de-bacalhau* in Lisbon. In part, they incorporate graph techniques from 'space syntax'. The paper contributes to the concern for conservation and rehabilitation first broached in shape grammar literature by Flemming [1987]. At the contemporary urban scale, Duarte collaborates with José Beirão and Rudi Stouffs from TU Delft in creating a design system involving a parametric shape grammar with descriptions to generate urban block layouts within a defined spatial region. In their discussion, they evoke Alexander's pattern language as manifest in software design. José Duarte, finally, works with Maria da Piedade Ferriera and Duarte Cabral de Mello in presenting a novel example of a kinetic shape grammar simulating human body movements. The paper presents rules within a confined set of yoga movements for demonstration purposes; but it also presents a fascinating historical survey of the study of human movement.

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Professorships in Engineering (Waterloo, Ontario), Technology (The Open University), Architecture and Urban Design, and Design and Computation (University of California, Los Angeles). He was Rector and Vice-Provost (Royal College of Art, London). He was co-editor with Sir Leslie Martin of the twelve volume Cambridge Architectural and Urban Studies, Cambridge University Press. He was founding editor in 1967 of the bi-monthly refereed journal *Environment and Planning B*, now *Planning and Design*.

Relevant recent publications include *Architectonics of Humanism: Essay on Number in Architecture* (1999); 'Renaissance mathematics and architectural proportion in Alberti's *De Re Aedificatoria'*, *Architectural Research Quarterly* (*ARQ*), 2/1, 1996, 54-65; 'Proportional design in L. B. Alberti's Tempio Malatestiano, Rimini', *ARQ*, 3/3, 1999, 259-269; 'Palladio, Pythagoreanism and Renaissance Mathematics', *Nexus Network Journal*, 10/2, 2008, 227-243. Lionel March is currently Visiting Scholar, Martin Centre for Architectural and Urban Studies, Department of Architecture, University of Cambridge, and emeritus member, Center for Medieval and Renaissance Studies, University of California, Los Angeles.