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Keywords: Leonardo da Vinci, reciprocal frames, deresonated tensegrity, geodesic domes, Buckminster Fuller, tensegrity, polyhedra

Research

The Wooden Roofs of Leonardo and New Structural Research

Abstract. The two types of spatial patterns reproduced in the *Codex Altanticus* fol. 899v can be deciphered in light of recent studies on reciprocal and tensegrity frames. For the construction of his wooden component roofs, Leonardo utilized two main modules: a grid of square modules and a grid of a tri/hexagonal module. Leonardo's drawings offer an opportunity to attempt a synthesis between the two structural systems, demonstrating the affinity that exists between the reciprocal frames used by Leonardo and the rigid tensegrities developed by Fuller. The continual observation, study and construction of models have permitted the verification of this hypothesis.

> The framework idea reappears in a most ingenious system of dismountable system of "geodesic" roofing for vast areas of land, which can be seen in a later sheet of the *Codex Atlanticus*, f. 328 v-a of circa 1508-10, anticipating the daring constructions of Buckminster Fuller!
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> Carlo Pedretti [1978: 151]

Reciprocal frames

Reciprocal frames are three-dimensional structures, the beginning module of which must contain at least three sticks (the triangle is the first manifestation of a minimal surface) arranged in such a way as to form a closed circuit of mutually supporting elements. These structures permit the realization of any form whatsoever, so obtaining final configurations that are surprisingly stable.

Reciprocal frames are capable of supporting considerable loads. The eventual breaking of only one element jeopardizes the whole system, as is generally the case with synergetic structures. They can be rapidly constructed with local materials so that the result is particularly appropriate in emergency situations.

Reciprocal frames can be observed in the iris of the eye, in the aperture of a photographic camera, in the nests of some birds, in the art of basket weaving (especially Chinese and Thai), and in other varied examples.

Medieval Chinese and Japanese architecture

The reciprocal frames in wood used since the twelfth century in Chinese and Japanese architecture were destroyed by fire or allowed to deteriorate naturally with time, so that there remains little or no trace of the ancient traditions of construction. In more recent times we find them in the architecture of Ishi (the Spinning House), Kan and Kijima (Kijima Stonemason Museum), used particularly as roofing.

The Museum of Seiwa Bunrakukan by Kazuhiro Ishi (1992) was the object of a 1999 study on the part of groups of students of the University of Hong Kong, who had already had experience with reciprocal studies under the guidance of Ed Allen. As Gotz Gutdeutsch reports in his book *Building in Wood* (1996), Ishi said that he was inspired by the game of

Nexus Network Journal 10 (2008) 27-38
1590-5896/08/010027-12 DOI 10.1007/ s00004-007-0054-x

waribashi in which the sticks (chopsticks, usually used for eating) were joined by rubber bands in order to form houses, bridges and other objects. Another source of inspiration was the temples of Buddhist Chogen monks (1121-1206).

The puppet theatre has been part of Japanese culture since the sixteenth century. Each puppet was manipulated by three puppeteers in connection with the musicians and narrators. Even if the new types of media have largely supplanted this antique art, at Seiwa, on the island of Kyushu, it has been fortunately maintained. For the restoration of the theatre, the architect relied heavily on wood, in support of the regional industry for this material. The entire operation was notably successful.

The complex is divided into three principle areas: the auditorium, with a square plan and pyramidal roof; the theatre, with a rectangular plan; the exhibit hall, similar to a pagoda, connected to the auditorium by a covered walkway. In this room, metal joints were used to connect the large and elegant reciprocal roof.

One other architect deserving mention with regard to reciprocal structures is Tadashi Kawamata [Gould 1993]. The use of bamboo in the construction traditions of China should also not be neglected.

Rainbow Bridge. Another example of a reciprocal frame can be observed in the painting *Going up the river on the Rainbow* Bridge by Zhang Zeduan (twelfth century). In his book *Zhongguo Quiaoliang Shihua* (Taipei, 1987), Mao Yisheng reproduces a section of the Rainbow Bridge, demonstrating that the beams are placed so that they can be fixed in a reciprocal way. Realized for the first time in the province of Shandong, the Rainbow Bridge was one of the most important inventions during the time of the Chinese Song dynasty (960-1280 A.D.). It has the same place in Chinese culture as the Coliseum of Rome has in our own. According to Robin Yates of McGill University, the first foreigner to document that period of vigorous economic prosperity was Marco Polo, who arrived in China in 1275, at the end of the Song Dynasty. Before this dynasty, there were no printed books, gunpowder, compasses for navigation, paper money, restaurants, or bridges.

After many years of study and research, the engineer and historian Tang Huan Cheng recently reconstructed the Rainbow Bridge at full scale, collaborating with two experts from MIT, Marcus Brandt for the geometry, and Bashar Altabba for the structural engineering. The construction took place by starting on the two opposite sides of the river and connecting the whole structure with many timbers arranged in a reciprocal manner. Leonardo da Vinci has written that an arch is made of two weak halves that become strong when united. He also appreciated the way for spanning distances with short elements as illustrated in his three-dimensional grillage structures and temporary timber bridges. The principle used by Leonardo is identical to that used by the Chinese several hundred years before.

Other examples of reciprocal frames can be found in the wooden roofs designed by Sebastiano Serlio in the sixteenth century, and in the works of Villard de Honnecourt.

Hybrid systems

For Tony Robbin, "the use of hybrid systems constitutes a new constructive paradigm of resistance similar to that which exists in living organisms. The overloading of a structure is not only a waste of material but can be very dangerous." The Indian teepee is a hybrid system based on the interaction of various independent systems that come into play only when they are necessary. That is, an active system exists for normal loads while the other systems become active as they are needed. The sticks of a teepee are a distant relative of a reciprocal structure. As happens in all tents, the covering fabric provides additional strength to the structure. In the summer the lower folds are opened to favor the air exchange and circulation.

Further studies on reciprocal frames have been conducted for the past fifteen years by John Chilton, Olga Popovic, Wanda Lewis and others. The results of their research have been published periodically in the magazine *International Journal of Space Structures.* For example, IJSS no. 2-3 (2002) contains the results obtained by students in the structural engineering course held by John Chilton and Wanda Lewis at the University of Warwick, who examined some models of reciprocal frames with a Testometric machine with a 100 Kn capacity, loading the models with weights. Chilton, Saidani and Rizzuto give the following definition of a reciprocal frame: "A Reciprocal Frame system is a three-dimensional grillage structure constructed of a closed circuit of mutually supporting beams. A number of RFs connected to each other at the outer end of each radiating beam results in the formation of a Multi-Reciprocal Element space structure". According to Chilton, one of the first to experiment with these kinds of structures was Emilio Perez Pinero.

Lincoln Cathedral (James Essex, 1762). Clearly described by John Chilton and Thibault Devulder of the University of Nottingham, the Gothic cathedral of Lincoln is one of the largest English cathedrals with a decagonal plan, constructed between 1220 and 1235 by master craftsmen and able carpenters. It is composed of two principle bodies: a structural base in dense stone of a diameter of some 21 m, and a height of some 13 m; and a wooden roof organized in two separate structures where, in the lower part, the heavy reciprocal structure realized by architect James Essex in the 1762 restoration can be noted. Essex broke with the current structural tradition by utilizing pine beams rather than oak.

Rigid Tensegrity Structures

I introduced non-resonant tensegrity structures – called "deresonated tensegrities" by Amy Edmondson (1986), and "Rigid Tensegrities" by Hugh Kenner (1966) – in the article "Le strutture tensegrali" published in *L'architettura naturale* 10 (2001):

> "Deresonated tensegrity domes" were Fuller's major interest in the last years of his life. Increasing the frequency of subdivision of the polyhedron of departure decreases the distances between the struts and gives importance to the thickness of the struts themselves, which can be dimensioned in such as way so as to permit adjacent struts to touch. This thickness can be calculated by taking into consideration the value of the respective geodesic arch. In this way the structure is without resonance (the tensegrity is deresonated), since the struts are no longer hung but touch each other and can be bolted together at their tangency points.
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> The tension force otherwise visible in the preceding models becomes invisible in this type of structure. There is an evident reduction of the materials, that is, the number of struts of different lengths is reduced. In fact, there are only two different struts for a non-resonant 4v, in contrast to the eight struts necessary to construct the equivalent geodesic (p. 62).

In deresonated tensegrity structures the dynamic quality that permits the structures to oscillate from their position of initial equilibrium is blocked (or rendered non-resonant). Increasing the frequency of subdivision, the central corners near the struts are modified and tend towards a form that is less acute and nearer to spherical. The struts try to touch each other and can then be fixed with nuts and bolts which take the place of the tension cables. The resulting structure will be more robust and subject to very few bending forces. In this way the tensegrity structures are changed from resonant to rigid, subsequently consolidating into geodesic structures.

Fig. 1. Model of a rigid tensegrity icosahedron in bamboo

Observing a deresonated tensegrity structure, we are led to the conclusion that the underlying rods support those above, and that the structure works in compression as happens in traditional structures in which the bolts serve to prevent lateral sliding, thus giving rise to a mistaken idea of the dynamics of the system. In reality, the structure is pushed towards the exterior by a hidden tension system that recalls the latent explosion of a soap bubble, with the difference that in these structures the superficial external membrane is supported by tension forces that derive from the membrane itself.

The tensegrity universe of R.B. Fuller: Brief chronology

1927, Greenwich Village Studio. The first experiments of Fuller in the search for an integrated architecture in tension (tensegrity).

1949. From the collaboration with sculptor Kenneth Snelson (at that time a student of Fuller's), are born the first models of tensegity columns.

1953, Minnesota University. Realization at large scale of a rigid tensegrity structure with 270 non-identical rods. The structure would be patented in 1962. The patent covered tensegrity structures with identical rods as well.

1959, Oregon University. The first model of a deresonated tensegrity with 270 identical rods. The 270-rod structure was described on p. 394 of *Synergetics 1* as "isotropic tensegrity geodesic sphere: single bonded turbo triangles, forming a complex six frequency triacontahedron tensegrity."

1960, Long Beach State College. The realization of a rigid tensegrity structure in bamboo with a diameter of 14m. The outer ends of each element were calibrated with regards to the central point of the adjacent elements.

1961, Bengal College of Engineering, Calcutta. Bamboo dome, hypothesis of low-cost shelters that could be realized with local materials and technology. The bamboo dome was reproduced in *Domebook 2* and corresponds to the alternate breakdown of a 4v icosahedron, sectioned to 5/8 of a sphere.

1961, Southern Illinois University. Basket weave tensegrity. An interwoven dome of a 22m diameter, ¾ of a sphere, approximately 15m high. The struts were made of wooden centine, segmented and interwoven, with permitted the cost to be reduced to 1/5 of the equivalent traditional construction. This geodesic also corresponds to the alternate breakdown of a 4v icosahedron, sectioned to 5/8 of a sphere.

John Warren and Norman Foster

John Warren was born in 1946 in Corona del Mar, California. He studied art and biology, but preferred sculpture to all other activities. Today he lives in the Kaui islands of Hawaii. He worked with Fuller from 1971 on, realizing the models of the *Fly Domes* as well as the rigid tensegrity structures (deresonated tensegrities) having the geometry of the alternate breakdown of a 6v icosahedron sectioned to 5/8 of a sphere. In 1982-3 he collaborated with Norman Foster & Associates, realizing the models of the Autonomous House. The Autonomous House was to be the residence of the Fullers in Los Angeles. Partly transparent and partly opaque, the dome was fitted with two rotating spherical caps that could be darkened and were able to follow the course of the sun during the day. The interstices between the two domes enclosed hot or cold air to heat or cool the interior. Tubes in carbon fiber were envisioned.

1986. Windstar Biodome, Snowmass, Colorado. I described this dome on p. 15 of the magazine *Bioarchitettura* 18 (June 2000) and is related to the pioneering work of the New Alchemy Institute (1977), as well as to the concept of "permaculture" developed by Bill Mollison in 1978. The Biodome was realized on the model of the interwoven dome (basket weave), with the result that it is a rigid tensegrity structure. It is covered with inflatable three-ply plastic (EFTE) cushions that contribute to the greenhouse effect, optimizing the values of insulation, light transmission and durability (lasting about 12 years). The same panels were used in the Eden Project by Grimshaw and Partners in St. Austell in Cornwall (1988). The whole structure was assembled by hand. The reflective surface of the cushions reduced the nighttime heat loss to a minimum. Acquaculture was carried on in the interior of the Biodome, the water remaining tepid thanks to the constant production of passive solar heat.

The alternate breakdown of the iscosahedron

In alternate breakdown, faces of the iscosahedron are divided into multiple frequencies of 2. The frequency of subdivision was indicated by Fuller with "v," due to the similarity between the letter v and the triangle. After the great circles (GC, the equator that divides the sphere into two equal parts), next to be taken into consideration are the small circles (SC, all the circles of the sphere whose centers do not coincide with the center of the sphere). This distinction is necessary because the sphere of frequencies 4 and 6 present a degree of symmetry that is distorted and modified with respect to the 2v geodesic. The introduction of the SC allows the sectioned geodesic to rest on a single plane.

The icosahedron Alt 2v (fig. 2) has 6 decagonal GCs arranged so as to form a spherical icosadodecahedron with 12 pentagons and 20 triangles. A vertex of the pentagon meets the vertex of another pentagon. The circuit model has 30 struts that form 6 interwoven pentagonal circuits. The 60 tendons define the corners of the icosadodecahedron (figs. 3-6).

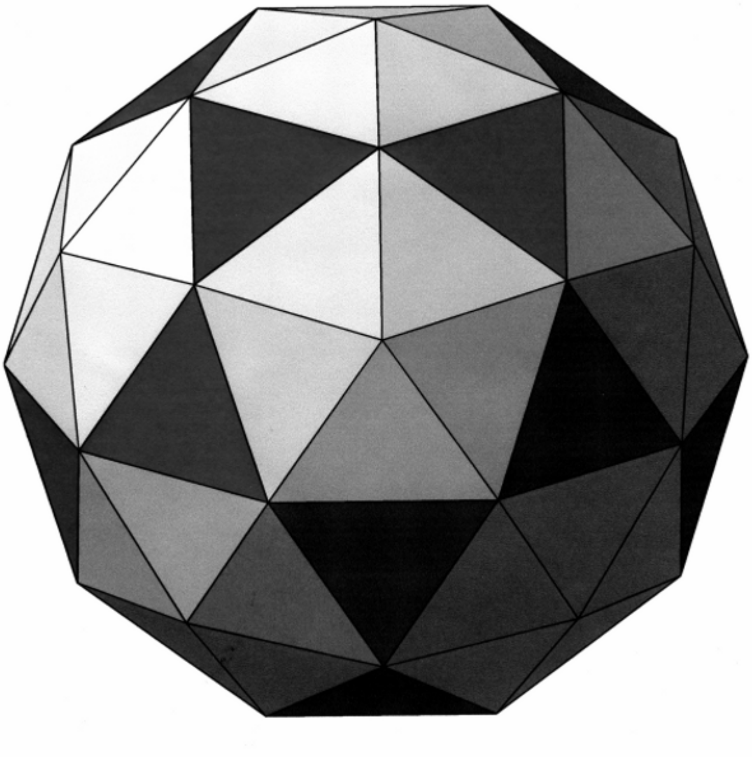

Fig. 2. Icosahedron Alt 2v geodesic sphere

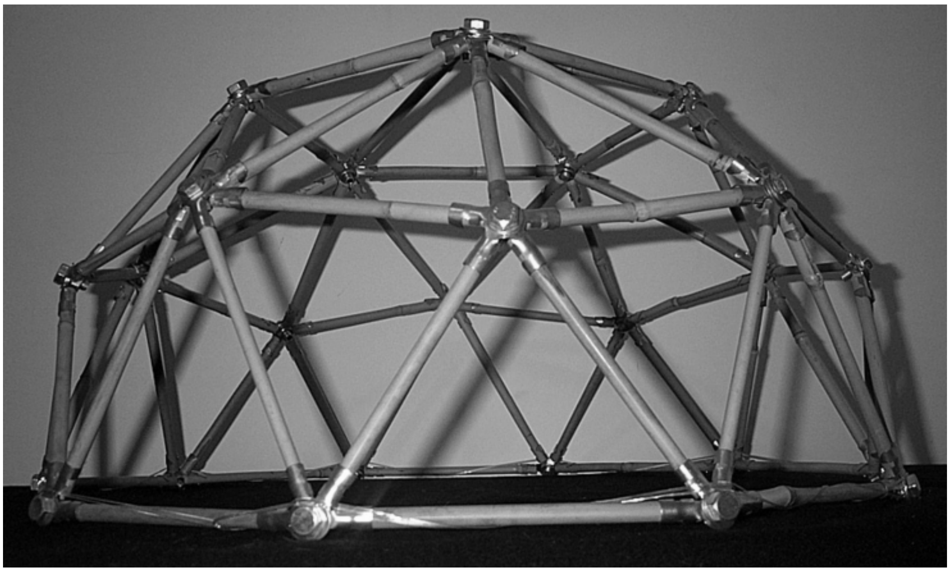

Fig. 3. Model of a geodesic dome icosahedron Alt 2v, in bamboo

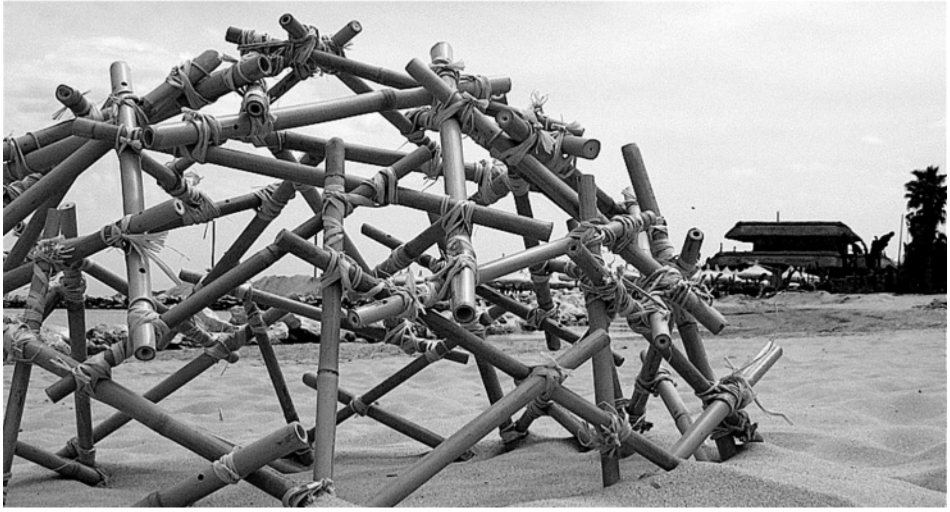

Fig. 4. Reciprocal model of the icosahedron Alt 2v in bamboo, built on sand

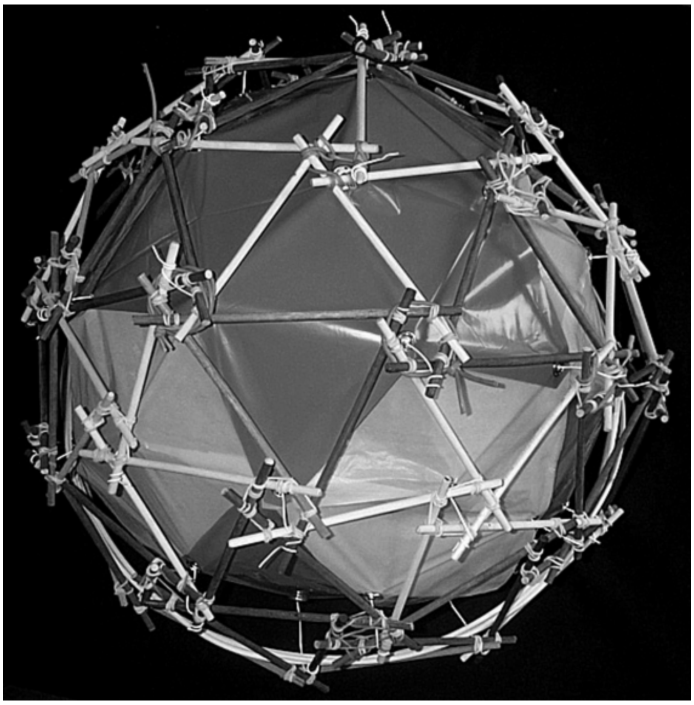

Fig. 5. Model of a reciprocal frame icosahedron Alt 2v, 5/8 of a sphere. The covering is made from a cloth hung on the interior

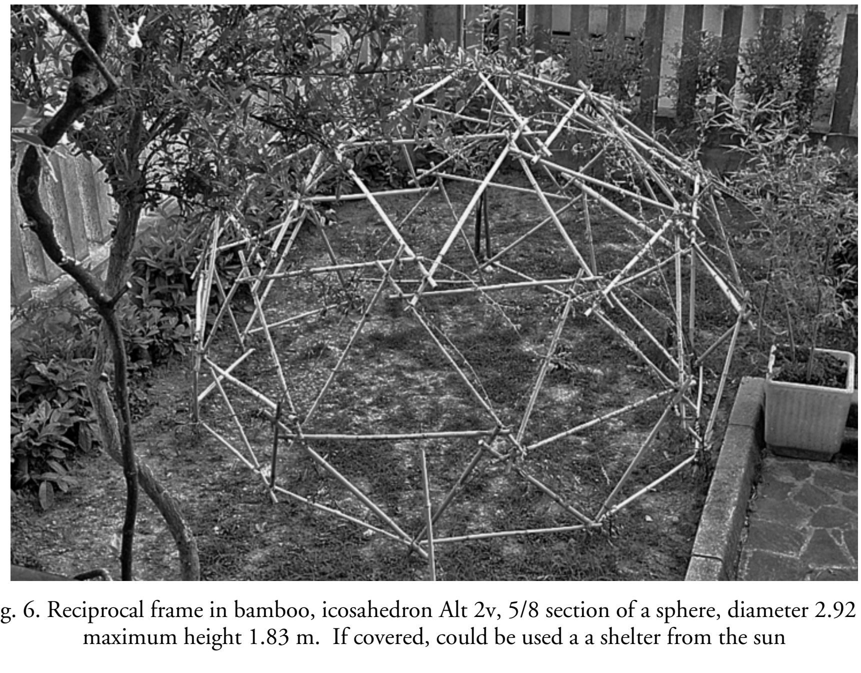

Fig. 6. Reciprocal frame in bamboo, icosahedron Alt 2v, 5/8 section of a sphere, diameter 2.92 m, maximum height 1.83 m. If covered, could be used a a shelter from the sun

The icosahedron Alt 4v, has 12 20-sided SCs. The SCs are arranged in parallel couples. Doubling the 6 GC of the preceding model gives rise to six parallel couples of circles, no longer great but small, because the GC is reduced in diameter as it gets further from the equator. The resulting polyhedron will have 12 regular pentagons, 30 hexagons and 80 irregular triangles. The geometric arrangement from vertex of pentagon to vertex of pentagon is penta/hexa/penta. The circuit model is made of 120 struts and 240 tendons arranged in 12 decagonal circuits (figs. 7-8).

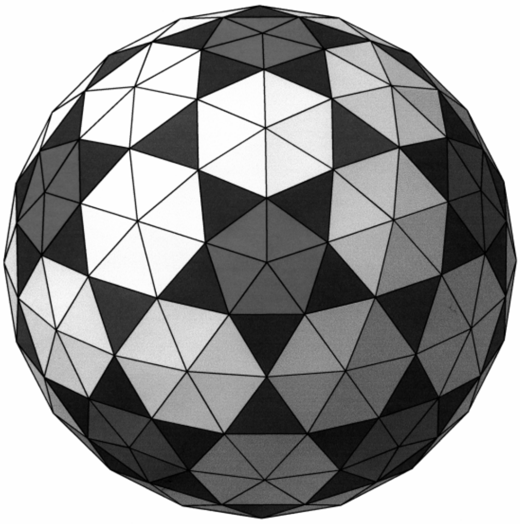

Fig. 7. Geodesic sphere icosahedron Alt 4v

Fig. 8. "Circuit" model, tensegrity structure icosahedron Alt 4v

The icosahedron Alt 6v has 270 struts, 540 tie rods and 18 SCs of 30 sides each with two different types of circuits. The geometric arrangement from vertex of pentagon to vertex of pentagon is penta/hexa/hexa/penta. This polyhedron is obtained by adding 6 SCs to the preceding model (fig. 9).

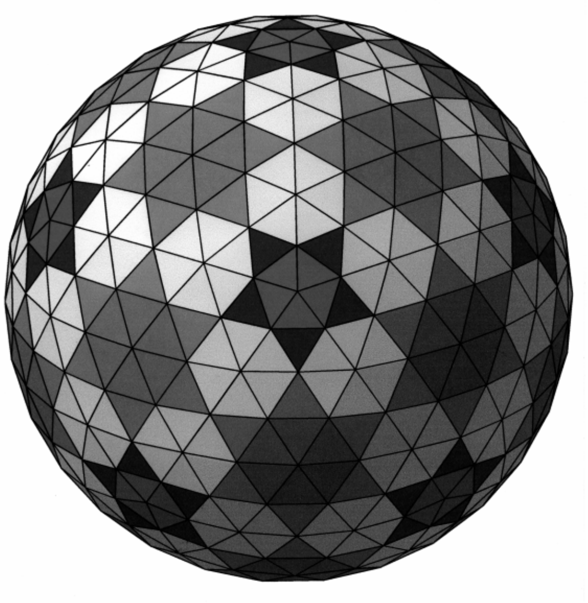

Fig. 9. Geodesic sphere icosahedron Alt 6v, often used as the reference geometry for Japanese thread balls known as *Temari*

Tensegrity structure Icosahedron Alt 4v, "circuit" model

The principle module is constituted of one strut (sliced strut) surrounded by a single or even a double tendon. The construction of the model begins with the upper pentagon, then five struts are added, followed by five more couples of struts and so on. It is practically impossible to visualize the figure without referring to drawings or to models.

For the construction of spherical models there are two principle references:

A) A first version with struts all equal and with the tendons about half as long as the struts results in a form that is less spherical and more angular. The struts wave in a way that is slightly disorderly.

B) A second, more spherical and more elegant, version utilizes struts with two different dimensions and three different types of tendons (which permits the visualization of the reference polyhedron). The dimensions of the tendons are the same utilized as geodesics in the struts of the bamboo dome. In this version [Pugh 1976: 62], 60 22.5 cm long (L) struts are used, and 60 20.31 cm short (S) struts are used, which alternated around the structure. The pentagon of departure is made of 5 struts and is visualized from the short tendons. The dimensions of the tendons are L= 11.87 cm; M (medium) = 11.25 cm; S = 9.67 cm. The tendons are arranged in a LMSML sequence.

Takraw Ball

The takraw ball is used in Malaysia and Thailand in a game called *Sepak Takraw*. The ball is created by interwearving six large circles, each of which is formed of from six to eleven strands of rattan. The modern version is made of plastic. The Thai ball (fig. 10) is based on frequencies equal to that of the iscosahedron. Observing this spherical geodesic it can be noted that the sides of the regular pentagon are smaller with respect to the sides of the irregular hexagon, in a ratio of about 2 to 3. It isn't easy to come across the data for the construction, because in Thailand this art is passed down orally, conserved in the memories of the basket masters. In the cardboard model of Iscosahedron Alt 4v that I made, to the side of the pentagon equal to 4 cm corresponds a height of the strip that equals 9 cm (by "strip" we mean one of the 6 GCs that make up the wicker ball).

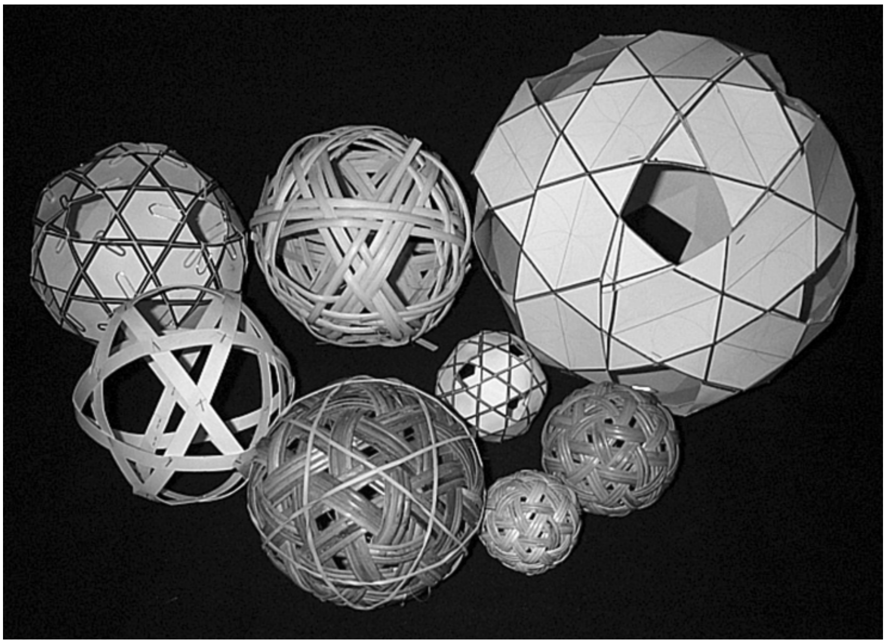

Fig. 10. Studies of the geodesic geometry of the Takraw ball.

Oriental culture is rich in other examples. In the Forbidden City of Beijing, in front of the bridge of Purity of Paradise, is the sculpture of the Lion holding in a geodesic ball in his front balls that dates from the Quian Long Dynasty (1736-1796), the geometry of which could refer to the alternate breakdown of the icosahedron at a frequency greater than 6. Some modules, however, turn out to be irregular. (Both Joseph Clinton and Russel Chu agree about the reference to the geodesic geometry.) It is supposed that the sphere derives from the art of the Temari, which consists of a ball made with strips of silk, used as a toy or as decoration in ancient China and in Japan.

The photograph on p. 8 of [Hargittai 1995] reveals the geometry of the truncated icosahedron (the fullerene) combined with the geometry of the icosahedron alt 6v. The alternate breakdown of the icosahedron 6v can be traced also in the geometrical arrangement of the pentagonal and hexagonal stars.

Conclusions

The reciprocal structures of Leonardo can be considered as forerunners to rigid tensegrity structures, which can themselves be considered the forerunners to geodesic structures. The rigid tensegrity systems turn out to be apparently more complex than reciprocal structures but in reality the only difference regards the systems of joints: in reciprocal structures the terminal point of a rod corresponds to the terminal point of another rod, and the final joint assumes the aspect of a "turbinated" star and is easily modified. In rigid tensegrity structures as well the final joint assume a "turbinated" aspect (as Fuller stated in his patent of tensegrity structures in 1962). Once again, today as in the past, the genius of Leonardo indicates new yet ancient solutions and points to the simplest possible way forward in research that is often apparently complex and confused.

Translated from Italian by Kim Williams

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About the author

Biagio Di Carlo is an architect as well as a graphic designer and a musician. He received his degree in architectural studies with honors in 1976 from the University of Architecture of Pescara; his thesis was published. His thesis advisors were Eduardo Vittoria, together with Giovanni Guazzo and Augusto Vitale. He taught at the Art Institute of Pescara, and often collaborates with the architectural faculties of Pescara and Ascoli, by giving lectures, lessons and seminars in synergetic geometry, geodesic domes, tensegrities, quasi crystals and four dimensional polytopes. He is the author of three self-published books: *Cupole geodetiche*, *Poliedri* and *Strutture Tensegrali*, and has published articles on synergetic geometry and geodesic domes in journals such as *Bioarchitettura* and *L'Architettura naturale*. Among his interests are architecture, molecular geometry, graphic arts and cartoon illustrations, Latin jazz, song writing and Brazilian music. More information about his activities is available on his website: http://www.biagiodicarlo.com.