
Oscillations and Waves in Sunspots

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Abstract

A magnetic field modifies the properties of waves in a complex way. Significant advances have been made recently in our understanding of the physics of sunspot waves with the help of high-resolution observations, analytical theories, as well as numerical simulations. We review the current ideas in the field, providing the most coherent picture of sunspot oscillations as by present understanding.

Keywords: Sunspots, Waves, Oscillations

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1 Introduction

Observations and theoretical modeling of sunspots have made significant progress in recent years. Nowadays, observational capabilities make it possible to study sunspot waves from the photosphere up to the corona. Traditionally, with ground-based telescopes operating in visible and near-infrared wavelengths, studies of wave properties from the low photosphere to the high chromosphere have been performed for decades. From them, it has been clearly established that umbral and penumbral waves have different properties. This result is undoubtedly related to their different magnetic configuration, with a magnetic field that is mainly vertical in the umbra and highly inclined in the penumbra. Waves above sunspots have also been shown to have a different behavior at the photosphere, chromosphere and above, where the amplitudes or periods of the disturbances have different values. The most prominent oscillations in sunspots have periods that are of the order of a few minutes. Global oscillations of sunspots as a whole also exist and have periods that range from hours to days.

In observations, wave properties can be determined using spectroscopic or imaging instruments. Spectrographs have the advantage that a number of spectral lines formed at different layers can be observed simultaneously to obtain the stratification with height of wave parameters like amplitude and phase variations and their distribution with frequency. The most evident parameter that can be derived from a spectral line is the line-of-sight velocity from the Doppler shift induced by waves. The variation of other parameters, like line width or line intensity at different levels from continuum to line core, can also be investigated. Line shape variations can be translated to temperature variations using inversion codes, at least for photospheric lines, where LTE conditions are usually assumed (Ruiz Cobo and del Toro Iniesta, 1992). For chromospheric lines, the use of inversion codes for wave propagation is not extended yet because of the complication derived from the NLTE conditions that govern their formation (Socas-Navarro *et al.*, 2015). Magnetic field strength and inclination and their temporal variations can also be studied from spectropolarimetric observations, at least, again, from photospheric spectral lines.

Complementarily, imaging instruments have the advantage that the spatial coherence of waves can be studied and their apparent propagation in the plane of the sky can be determined, an aspect that can not be studied with spectrographs. The co-alignment of images is also feasible, making it possible to use different instruments operating at various (ground- or space-based) telescopes to study different layers, which is a hard task for slit spectrographs. Narrow-band imaging instruments operating on the ground have often been used to study intensity variations in the chromosphere. The large amplitude of chromospheric Doppler shifts causes the spectral lines to move considerably to the blue and to the red and give rise to a significant variation of the observed brightness in the tuned wavelength, no matter whether it is line core or wing. In the transition region and corona, filters are usually broad and contain in their bandpass emission lines formed at high temperatures and their brightness variation can be interpreted in terms of density variations or variations of column depth (Cooper *et al.*, 2003). The comparison between the fluctuations observed with a number of filters that include lines of different temperature sensitivity facilitates the study of the propagation of disturbances with varying height.

From the ground, radio observations also make possible the observation of the corona above sunspots, but the real impulse for the study of waves in this region comes from space missions. In the upper layers, instruments like SMM (Gurman *et al.*, 1982), SUMER onboard SOHO (Wilhelm *et al.*, 1997) and the recently launched IRIS (De Pontieu *et al.*, 2014) have made possible the observation of emission spectra of ultraviolet lines formed in the high chromosphere, transition region and low corona. Sunspot waves can now be detected in these high atmospheric layers with adequate temporal and spatial resolutions. As with the layers below, line-of-sight velocities can be easily derived from Doppler shifts. These instruments are not provided with polarimetric capabilities and no direct information from the magnetic field can be obtained with the spectral

lines observed with them. The mechanisms of ultraviolet line formation are also different from those of typical photospheric and chromospheric absorption lines. Emission lines are formed by the recombination of an ion with an electron and this process depends on the squared electron density. This way, the relation between velocity and density variations can be studied.

With all this information, a global picture can be made of how the waves propagate through the three-dimensional sunspot atmosphere. Many observations point in the direction that wave phenomena at different layers of the sunspots are related with each other. To theoretically understand the observed wave properties, one has to take into account that different wave modes can exist in a magnetized medium (as, e.g., fast and slow magnetoacoustic modes and the Alfvén mode in a classical case of a homogeneous atmosphere). These waves have different dominant restoring forces in the wave equation, such as magnetic Lorentz force, gas pressure gradient or buoyancy. The photosphere and low chromospheres of sunspots are regions where restoring forces are of the same order of magnitude, making arbitrary the division into pure wave modes and complicating theoretical models. The plasma parameter β , defined as the ratio between the gas pressure and the magnetic pressure, conveniently separates regions of distinct physics. Below the photosphere, the plasma β is larger than one, while the opposite happens in the upper photosphere, chromosphere and corona. As a consequence, waves have different propagation speeds and physical properties in these regions. In addition, they may suffer transformation from one type to another around the $\beta = 1$ region or they may be subject to refraction and reflection. All this complex physics has to be taken into account for the correct interpretation of observations of sunspot waves. For an adequate understanding of all the observed phenomena, one has to have in mind the full 3D picture of wave propagation, and a simple explanation of them based on what is observed in a given layer should be disregarded. The interpretation of the observational material defines several groups of questions concerning the sunspot wave physics that can be summarized in the following way:

- What drives the waves observed in sunspots? Are they externally driven by the quiet Sun p -modes? Are there sources of oscillations inside the umbra due to weak convection?
- How can the oscillations observed at different sunspot layers be interpreted in terms of MHD waves? What are the relationships between photospheric and chromospheric oscillations? What causes the complex spatial pattern of oscillations such as chromospheric umbral flashes, penumbral waves, spatial coherency of waves over the umbra, etc.?
- Why is the wave power suppressed in the umbral photosphere compared to the quiet Sun? Why is it enhanced in the chromosphere?
- What mechanisms produce the change of the dominating frequency of waves in the umbra from 3 mHz in the photosphere to 5–6 mHz in the chromosphere? What are the reasons for the spatial power distribution at different frequency intervals observed in the umbra and penumbra?
- What are the propagation properties of sunspot waves in the upper layers, i.e., in the chromosphere, transition region and corona? What are the consequences of the non-linear shock wave oscillations in the umbra for the heating of the upper atmosphere?
- Is there observational evidence for mode transformation in sunspots? Is there observational evidence for a chromospheric resonant cavity?

The literature about sunspot waves is very extended. Mentioning all the works that have shed some light into this topic is impossible. We start by the description of the progress obtained on umbral and penumbral waves and oscillations from an observational point of view in Sections 2, 4, 3, 5, and 6. Section 7 provides a summary of theoretical models and introduces all important concepts necessary to interpret observations. Finally, in Section 8, we discuss theory and observations

together, following the list of questions defined above as a guideline. Previous reviews on the subject of sunspot waves together with theoretical issues are presented by [Bogdan \(2000\)](#), [Staudé \(2002\)](#), [Bogdan and Judge \(2006\)](#), and [Khomenko \(2009\)](#).

2 Waves and Oscillations in the Umbra

In the following sections, we describe observational properties of velocity and intensity fluctuations in the umbral photosphere, chromosphere, transition region and corona, as well as the propagation properties of the waves derived from simultaneous observations of various layers, where possible.

2.1 Velocity fluctuations

2.1.1 Basic properties of photospheric and chromospheric oscillations

Even at a spatial resolution of a few arcsec, some of the most relevant properties of sunspot oscillations were derived several decades ago. It was soon realized that sunspots show velocity oscillations similar to the surrounding photosphere, with periodic variations at time scales of few minutes. At photospheric layers, the amplitude of these fluctuations is clearly reduced in the umbra and penumbra of a sunspot ([Howard *et al.*, 1968](#)) when compared to its surroundings with peak-to-peak umbral velocities of the order of 0.5 km s^{-1} ([Bhatnagar *et al.*, 1972](#); [Soltau *et al.*, 1976](#)). [Figure 1](#) illustrates this early observational finding. The amount of power reduction of the wave velocity amplitude in the umbra compared to the quiet photosphere was found to be of the order to 40–60% ([Lites *et al.*, 1982](#); [Abdelatif *et al.*, 1984](#); [Braun, 1995](#)).

Power spectra of the temporal variations of the velocity measured in different spectral lines formed in sunspot umbrae at photospheric level show clear individual peaks and a dominant frequency that differs from one author to another ([Bhatnagar *et al.*, 1972](#); [Soltau *et al.*, 1976](#); [Balthasar and Wiehr, 1984](#); [Landgraf, 1997](#)). In general, velocity power spectra have their maximum power concentrated around five minutes, and traces of 3-minute oscillations appear with larger amplitude in those spectral lines that are formed higher in the photosphere (see, e.g., [Beckers and Schultz, 1972](#); [Schröter and Soltau, 1976](#); [Abdelatif *et al.*, 1984](#); [Lites and Thomas, 1985](#); [Landgraf, 1997](#); [Bellot Rubio *et al.*, 2000](#)). In most of the cases reported in the literature, measurements of velocity oscillations are done for sunspots located close to the solar disc center. Such measurements provide the component of oscillation velocity in a direction nearly parallel to the magnetic field of the umbra. Measurements of the transversal velocity component are very rare. One exception is the recent work of [Zharkov *et al.* \(2013\)](#) who studied oscillation power maps for a sunspot on its passage over the solar disc from the SOHO/MDI Dopplergrams. Such maps reveal an acoustic power enhancement in the sunspot umbra in the 3-min band in the velocity component perpendicular to the magnetic field, as can be seen in the maps taken at large heliocentric angles shown in [Figure 2](#).

Given the similarity between these oscillations and those typical of the photosphere outside sunspots, a number of strategies have been followed in the past to observe signatures that come directly from the umbra of sunspots. This is the case of the observation of atomic or molecular lines formed exclusively in the cool umbral atmosphere ([Abdelatif *et al.*, 1984](#); [Penn and LaBonte, 1993](#); [Aballe Villero *et al.*, 1990](#)) or the analysis of the V-zero crossing point ([Balthasar and Wiehr, 1984](#)), leading to the confirmation that the oscillations exist in the sunspot atmosphere and do not come from external stray light contamination.

[Penn and LaBonte \(1993\)](#); [Abdelatif *et al.* \(1986\)](#), and recently [Zhao and Chou \(2013\)](#) made an attempt to construct a $k - \omega$ diagram to represent the power of photospheric velocity oscillations as a function of their horizontal wave number and frequency. Such a diagram happens to be more noisy and have poorer resolution compared to the diagrams constructed from the quiet Sun data,

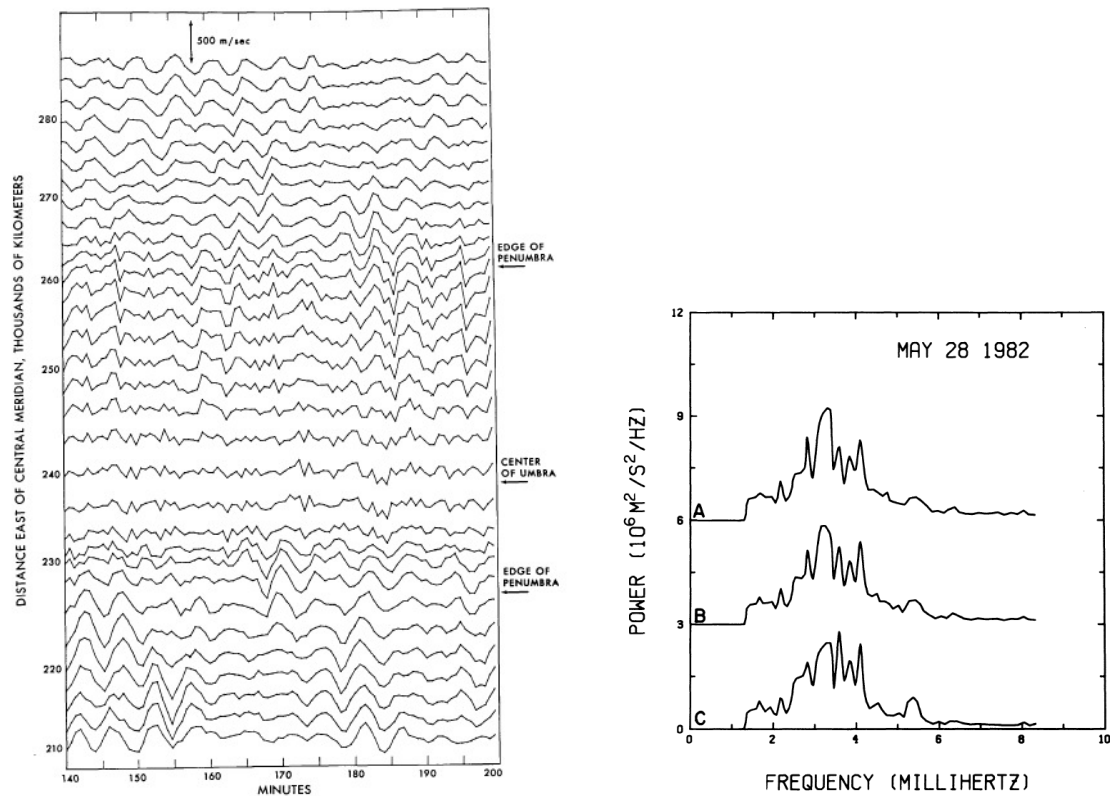


Figure 1: *Left:* Temporal variation of the velocity measured in the vicinity of a sunspot. The amplitude reduction in the umbra is apparent. *Right:* Average power spectra of the fluctuations of umbral photospheric velocity, spectral A, B and C are averaged over the different locations in the umbra. Image reproduced with permission from [left] Howard *et al.* (1968), copyright by D. Reidel, [right] Abdelatif *et al.* (1984), copyright by AURA.

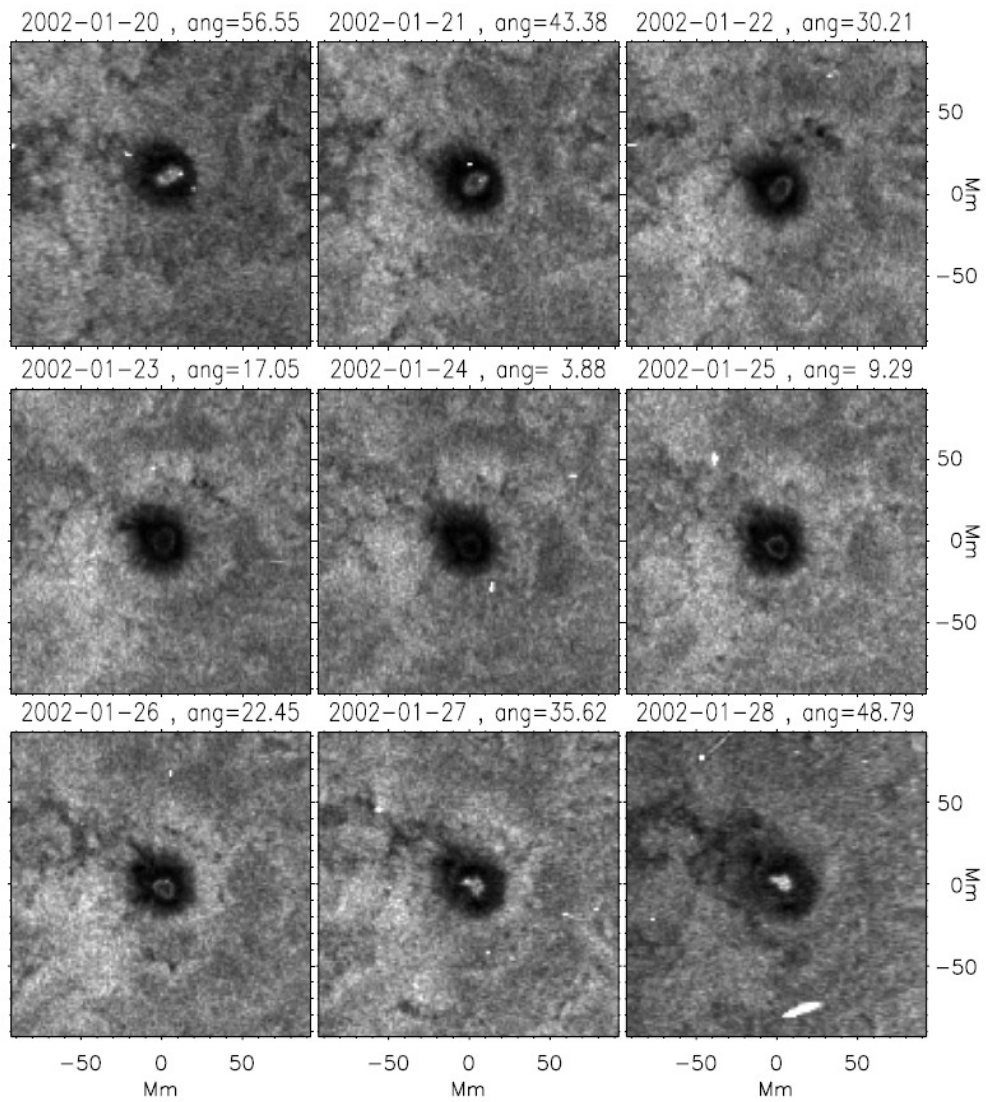


Figure 2: Center-to-limb variation of the acoustic power estimated from SOHO/MDI line-of-sight velocity data in the 3-min band for the NOAA AR9787 region on its passage over the solar disk from 56W to 61E on the solar surface. Image reproduced with permission from [Zharkov *et al.* \(2013\)](#), copyright by the author(s).

since the spatial and temporal resolution for observations of waves in the umbra is not good enough, for obvious reasons. Nevertheless, it was found that in deep photospheric layers, umbral oscillations display power in the same spatial and temporal frequency bands as the quiet Sun oscillations. A longer time series of SDO/HMI data allowed [Zhao and Chou \(2013\)](#) to convincingly show that the position of the ridges is similar to the quiet Sun, but slightly shifted according to the difference in the phase velocity of waves propagating below sunspots. Such observations are very useful to shed light into the driving mechanisms of waves in sunspots, see the discussion in [Section 8.1](#).

The 3-minute photospheric oscillations often appear as individual peaks in velocity power spectra, rather than a natural continuation of the broad 5-minute peak. This has led to the suggestion that they might be resonant modes of sunspots, with a cavity that might be located either in the sunspot umbra at subphotospheric layers ([Scheuer and Thomas, 1981](#); [Thomas and Scheuer, 1982](#); [Thomas, 1984](#)) or at chromospheric layers ([von Uexküll *et al.*, 1983](#); [Lites and Thomas, 1985](#); [Gurman, 1987](#)). An interesting relation between the power in a sunspot umbra in the bands of five and three minutes was obtained by [Schröter and Soltan \(1976\)](#), who found that the amplitude of the velocity oscillations in the two bands was anticorrelated, i.e., when one increased the other decreased. Similarly, [Lites \(1986a\)](#) found that the regions with high oscillatory power in the 3-min band were uncorrelated with those with high oscillatory power in the 5-min band. Derived from that fact, [Lites \(1986a\)](#) argued that the 5-min oscillations in the photosphere do not drive the 3-min chromospheric oscillations. Rather, the change in the frequencies and amplitudes of the oscillation modes observed in umbrae appear to arise from an interference of modes in the driving source in the photosphere, and not from changes in the chromospheric structure, i.e., chromospheric resonant cavity. Theoretical aspects of the various models of resonant cavities are discussed later in [Section 7.4](#).

While photospheric velocity oscillations have rather modest amplitudes, chromospheric signatures are very apparent and easily detectable. Spectral lines like Na I D₁ and D₂, Mg b₁, Ca II H and K, the Ca II IR triplet, H α or the He I 10830 triplet have traditionally been observed to sample the chromosphere at different heights. Except for the He I triplet, all spectral lines are rather deep and broad. This means that, even at their core, they have contribution from a wide range of layers in the solar atmosphere and it is difficult to assign a precise layer for their formation. In addition, they are affected by NLTE effects, which complicates their interpretation. Despite these difficulties, there is agreement that their cores are formed from the low to the high chromosphere (see, e.g., [Vernazza *et al.*, 1981](#); [Lites *et al.*, 1988](#); [Tsuneta *et al.*, 2008](#); [de la Cruz Rodríguez *et al.*, 2013](#)). The He I triplet represents a special case. These lines are almost absent in the quiet Sun, while in active regions they are enhanced presumably by UV coronal illumination ([Centeno *et al.*, 2008](#)) and thought to be formed in most cases in a narrow layer in the high chromosphere. The triplet is composed of two overlapped components and a third fainter line slightly shifted to the blue. For this reason, it is usual to talk in terms of the He I spectral line rather than triplet.

In contrast to the prevailing 5-min photospheric umbral oscillations, many works have confirmed that 3-min oscillations dominate the umbral chromosphere, transition region and corona ([Gurman *et al.*, 1982](#); [Gurman, 1987](#); [Thomas *et al.*, 1987](#); [Kentischer and Mattig, 1995](#); [De Moortel *et al.*, 2002](#); [Roupe van der Voort *et al.*, 2003](#); [Centeno *et al.*, 2006](#)). In the chromosphere, the peak to peak amplitudes of three-minute oscillations are of the order of 5–10 km s⁻¹ or larger. From time series taken at Ca II λ 8498– λ 8542, [Lites \(1984\)](#) found that 3-min chromospheric oscillations in sunspot umbrae are nonlinear and develop shock waves. Clear evidence for periodic nonlinear umbral oscillations was also revealed from observations in the He I line by the same author ([Lites, 1986b](#)). This line was observed to be in weak absorption during all phases of the oscillation, undergoing periodic shifts with amplitudes up to 11 km s⁻¹ and the waveforms of the Doppler shifts were indicative for shocks. It was suggested that these shocks might contribute to the heating of the upper chromosphere.

2.1.2 Propagation from the photosphere to the chromosphere

Many studies have been based on simultaneous observations of photospheric and chromospheric lines to derive the properties of sunspot oscillations at different heights. This way, the stratification with height of wave parameters like velocity amplitude, as well as their phase variations and their distribution with frequency can be determined. From these parameters, some authors have suggested that the three-minute chromospheric oscillations above sunspot umbrae are standing acoustic waves (Christopoulou *et al.*, 2000), while others found them to be propagating waves able to reach the upper layers of the solar atmosphere (Lites, 1992; Banerjee *et al.*, 2002; O’Shea *et al.*, 2002; Brynildsen *et al.*, 2003, 2004; Kobanov and Makarchik, 2004).

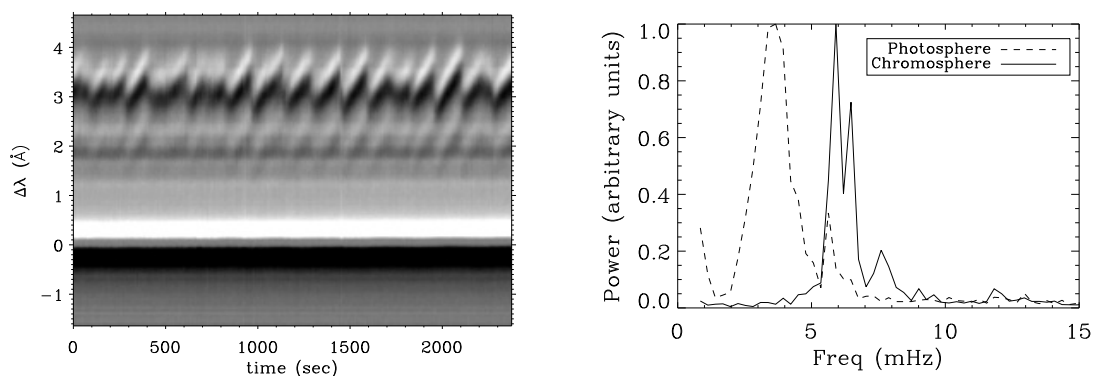


Figure 3: *Left:* Temporal evolution of Stokes V profiles observed in a sunspot umbra in a spectral region containing the photospheric Si I $\lambda 10827$ and the chromospheric He I 10830 spectral lines. The horizontal axis represents time, increasing to the right, and the vertical axis represents wavelength (with the origin set at the position of the silicon rest wavelength). The silicon Stokes V profile (lower part of the figure) shows no apparent change with time in this presentation, while helium profiles (upper part) show periodic Doppler shifts with a clear sawtooth shape. *Right:* Photospheric and chromospheric velocity power spectra. Image reproduced with permission from Centeno *et al.* (2006), copyright by AAS.

The typical sawtooth pattern of propagating shock wave fronts has been made patent in many observations using different chromospheric and transition region lines (see, e.g., Rouppe van der Voort *et al.*, 2003; Centeno *et al.*, 2006; Tian *et al.*, 2014). Examples of such profiles are given in Figures 3 and 5 as derived from the observations in He I and Ca II H lines in the above mentioned works. In all cases chromospheric velocity oscillations show a 3-min period. Simultaneous photospheric velocities vary basically with a 5-min period, although the power spectrum also shows a secondary peak in the 3-min band, as is evident from the example in Figure 3 extracted from the work of Centeno *et al.* (2006). Spectra of the phase difference between the photospheric and chromospheric velocity oscillations give evidence for the upward propagation of waves with periods shorter than the cut-off period in the umbra, that is approximately equal to ~ 4 min. The phase spectrum was shown to be reproduced with a simple model of linear vertical propagation of slow magnetoacoustic waves in a stratified magnetized atmosphere that accounts for radiative losses through Newton’s cooling law, as is illustrated in Figure 4. With this model the theoretical time delay between the photospheric and chromospheric signals can be computed, and happens to have a strong dependence on frequency. A very good agreement is found with the time delay obtained directly from the cross-correlation of photospheric and chromospheric velocity maps filtered around the 3-min band. Based on these reasonings, Centeno *et al.* (2006) argue that the 3-min power observed at chromospheric heights comes directly from the photosphere by means of linear wave propagation, rather than from nonlinear interaction of 5-min modes.

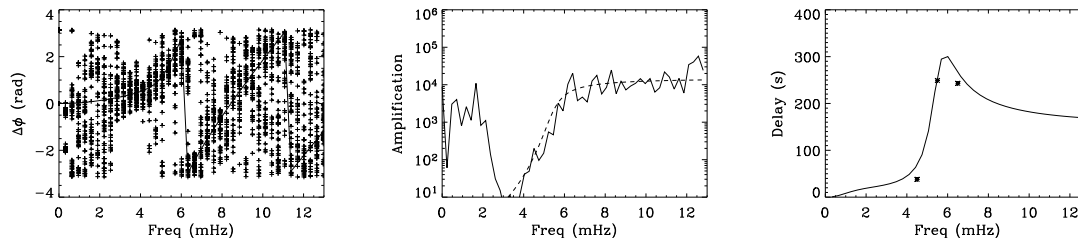


Figure 4: *Left:* Phase spectra for all observed points in the umbra of a sunspot. The y-axis indicates the phase difference between the Fourier transform of the chromospheric He I $\lambda 10830$ velocity oscillation and the photospheric Si I $\lambda 10827$ velocity oscillation (in units of radians). The solid line represents the best fit from a theoretical model of linear vertical propagation of slow magnetoacoustic waves in a stratified magnetized atmosphere that accounts for radiative losses through Newton's cooling law. *Center:* Ratio of chromospheric to photospheric power as a function of frequency. The dashed line represents the best fit from the same model. *Right:* Time delay between photospheric and chromospheric oscillations. The solid line represents the theoretical time that it would take for a quasimonochromatic photospheric perturbation to reach the chromosphere, as a function of frequency, obtained directly from the fits to the phase spectrum. Asterisks represent the measured values of the time delay within three narrow filtering bands. Image reproduced with permission from [Centeno *et al.* \(2006\)](#), copyright by AAS.

With a similar analysis, and using a larger set of photospheric and chromospheric spectral lines, [Kobanov *et al.* \(2013b\)](#) derived similar conclusions as [Centeno *et al.* \(2006\)](#) but with substantially smaller time delays. [Reznikova *et al.* \(2012\)](#) also traced the variations of the cutoff frequency across the umbra.

[Felipe *et al.* \(2010a\)](#) studied the amplitude and phase velocity variations in several spectral lines formed at different heights in the photosphere and chromosphere. Figure 6 shows the temporal variation of the velocity measured at a fixed location in a sunspot umbra with the spectral lines He I $\lambda 10830$, Ca II H $\lambda 3969$, Fe I $\lambda 3969.3$, Fe I $\lambda 3966.6$, Fe I $\lambda 3966.1$, Fe I $\lambda 3965.4$, and Si I $\lambda 10827$. The figure shows how the amplitude of the velocity fluctuations increases with height and the average period decreases. The authors explain this behavior as due to the propagation of the slow mode in the direction of the magnetic field in a stratified medium with radiative losses.

Interestingly, [Beck \(2010\)](#) reports observations of unusually strong photospheric and chromospheric velocity oscillations in a sunspot. The wave pattern found consists of a wave with about 3 Mm apparent wavelength, which propagates towards the sunspot. This wave seems to trigger oscillations inside the sunspot umbra, which originate from a location inside the penumbra on the side of the impinging wave. The wavelength decreases and the velocity amplitude increases by an order of magnitude in the chromospheric layers inside the sunspot. On the side of the sunspot opposite to the impinging plane wave, circular wave fronts centered on the umbra are seen propagating away from the sunspot outside its outer white-light boundary. They lead to a peculiar ring structure around the sunspot, which is visible in both velocity and intensity maps.

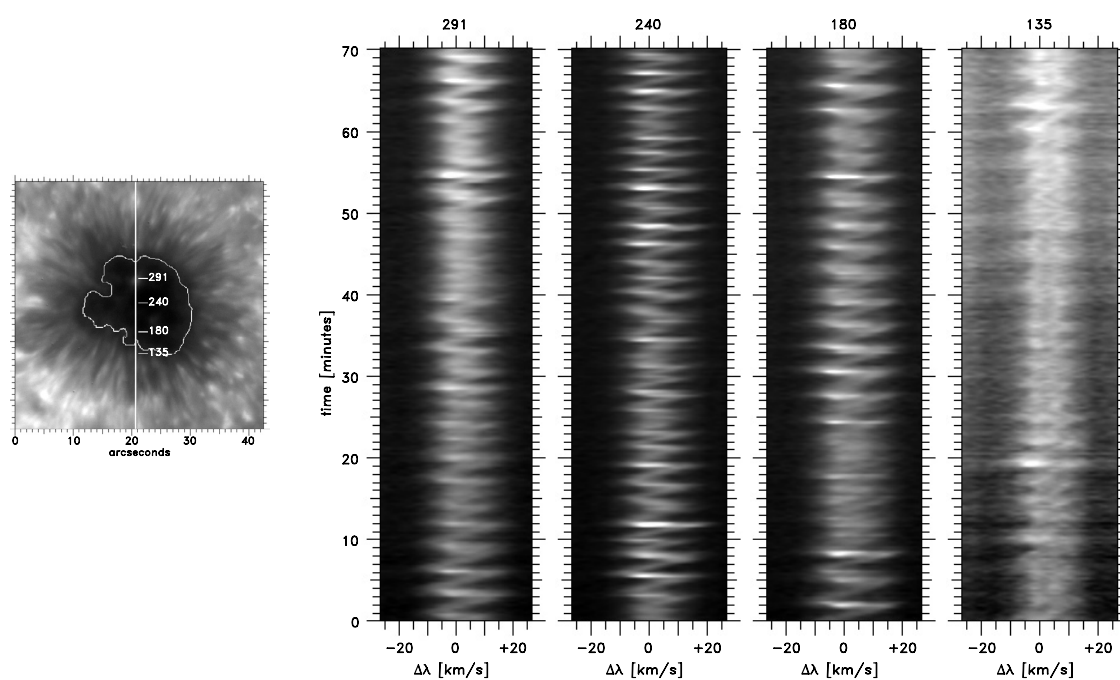


Figure 5: *Left:* Ca II H Dopplershift charts. The Ca II H image (left) specifies four slit sample locations, with pixel numbers. The four strip charts show the temporal evolution of a short spectral segment containing the central emission feature of the Ca II H line at these locations. The fourth chart samples the inner penumbra close to the umbral boundary. Image reproduced with permission from [Rouppé van der Voort *et al.* \(2003\)](#), copyright by ESO.

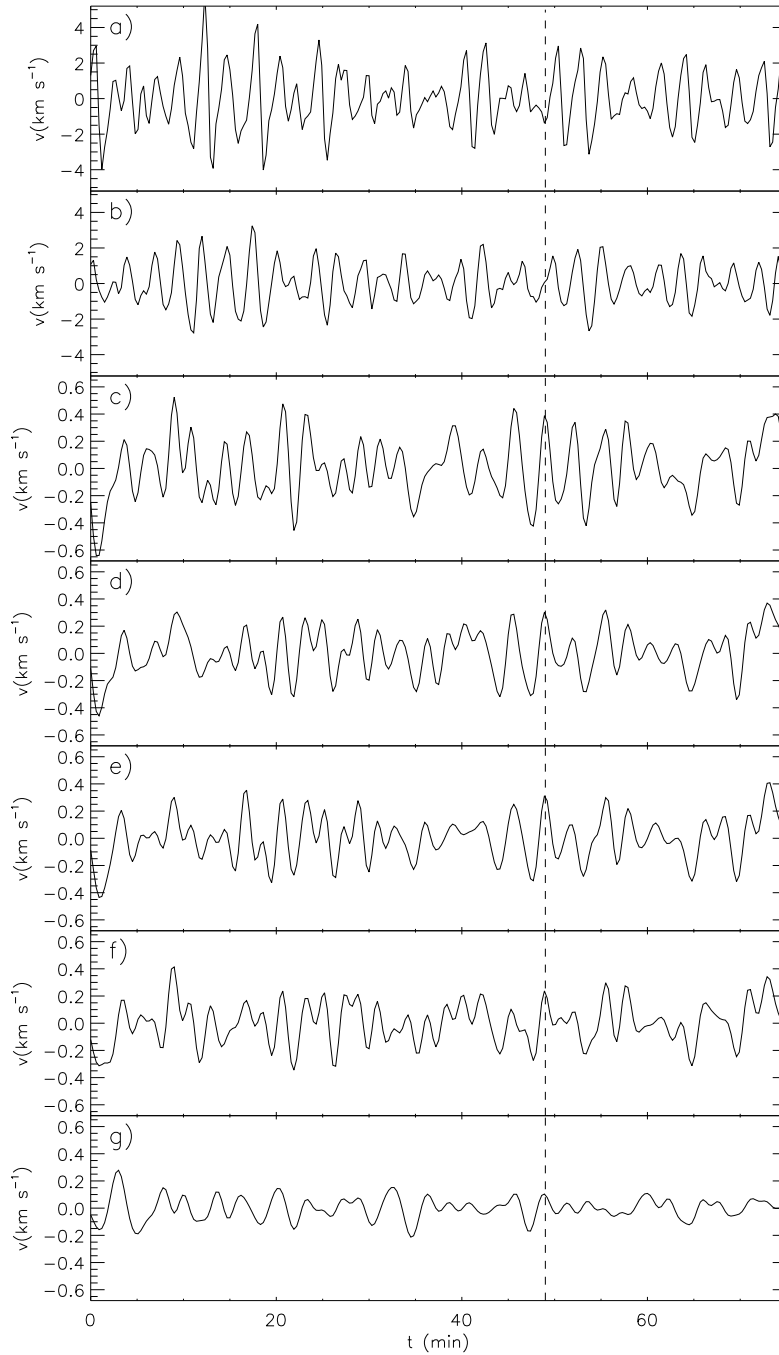


Figure 6: Temporal variation of the velocity measured in the umbra of a sunspot. The four bottom (top two) plots correspond to photospheric (chromospheric) spectral lines sorted by height of formation. From top to bottom: He I $\lambda 10830$ (a), Ca II H $\lambda 3968.5$ (b), Fe I $\lambda 3969.3$ (c), Fe I $\lambda 3966.6$ (d), Fe I $\lambda 3966.1$ (e), Fe I $\lambda 3965.4$ (f), and Si I $\lambda 10827.1$ (g). Image reproduced with permission from Felipe *et al.* (2010b), copyright by AAS.

2.2 Intensity fluctuations

Intensity fluctuations induced by sunspot waves have very different properties in the photosphere and the chromosphere. Usually, in the photosphere the amplitudes of intensity fluctuations are rather small. In the chromosphere, intensity fluctuations are dominated by so-called umbral flashes, one of the prominent features of sunspots dynamics, detected rather early in observations of chromospheric lines. Below we describe separately the properties of intensity fluctuations in sunspots in the photosphere and the chromosphere.

2.2.1 Photospheric oscillations

Photospheric line intensity fluctuations are difficult to detect. They are caused by the variations in temperature and density induced by the waves, which are very small in the photosphere. In addition, different spectral lines may have a different response to temperature variations (they may weaken or strengthen under given local temperature perturbations), which makes mandatory a deep study of the behavior of each spectral line used. It might be the case that, for instance, a density reduction caused by the waves leads to an opacity decrease which has the consequence that radiation comes from deeper and hotter layers and temperature and density effects may tend to compensate each other. From the ground very stable seeing conditions are required to have a coherent time series where photospheric intensity oscillations may be measured. For these reasons, it is not surprising that attempts to measure line intensity fluctuations in sunspots from the ground have often failed, (see, e.g., [Beckers and Schultz, 1972](#); [Bellot Rubio *et al.*, 2000](#)), despite velocity oscillations being detected. As a matter of example, one can compare the power of the velocity oscillations with those of temperature oscillations (related to the intensity oscillations) extracted from the infrared spectra of Fe I lines at $1.56 \mu\text{m}$ by [Bellot Rubio *et al.* \(2000\)](#) in Figure 14. These lines are formed in the deep photosphere. The power of velocity oscillations is significantly above the confidence level (horizontal line), while the power of temperature oscillations is essentially below that level.

Space missions give the required stability to measure intensity oscillations and important results have been obtained using data from a number of space-based instruments. Using Hinode G-band temporal series of almost five hours of duration and one minute cadence, [Nagashima *et al.* \(2007\)](#) calculated the photospheric power distribution of the intensity fluctuations in the G-band in a region containing a sunspot. Figure 7 shows the power distribution from 0.5 mHz to 5.5 mHz, at intervals of 1 mHz, revealing several important results. The G-band intensity power decreases almost monotonically with frequency and is suppressed inside the sunspot at all frequencies. Even at the largest frequency bin (4.5–5.5 mHz) the oscillation power outside the spot is non-negligible. The latter is an important result to understand whether three-minute oscillations in the sunspot are a consequence of the response of this structure to the external p -modes or if, on the contrary, they represent an own oscillation mode, see the discussion in Section 8.1. It is interesting to note the existence of a bright ring in the inner penumbra-umbra boundary that, as suggested by the authors, must be the result of the propagation of MHD waves in the complicated magnetic topology of the sunspot. A similar feature was also reported by [Hill *et al.* \(2001\)](#) in SOHO/MDI intensity power maps in their lowest frequency range (0–1 mHz).

2.2.2 Chromospheric umbral flashes

Periodic intensity disturbances in the umbra of sunspots are significantly more prominent in the chromosphere than in the photosphere and were detected rather early. The first manifestations of these disturbances were obtained by [Beckers and Tallant \(1969\)](#) and [Wittmann \(1969\)](#) who noticed sudden brightness increases in the core of the Ca II K_{2V} line, which they called “*umbral flashes*” (see Figure 5). The flashes occur with periods of about 2–3 min and are accompanied

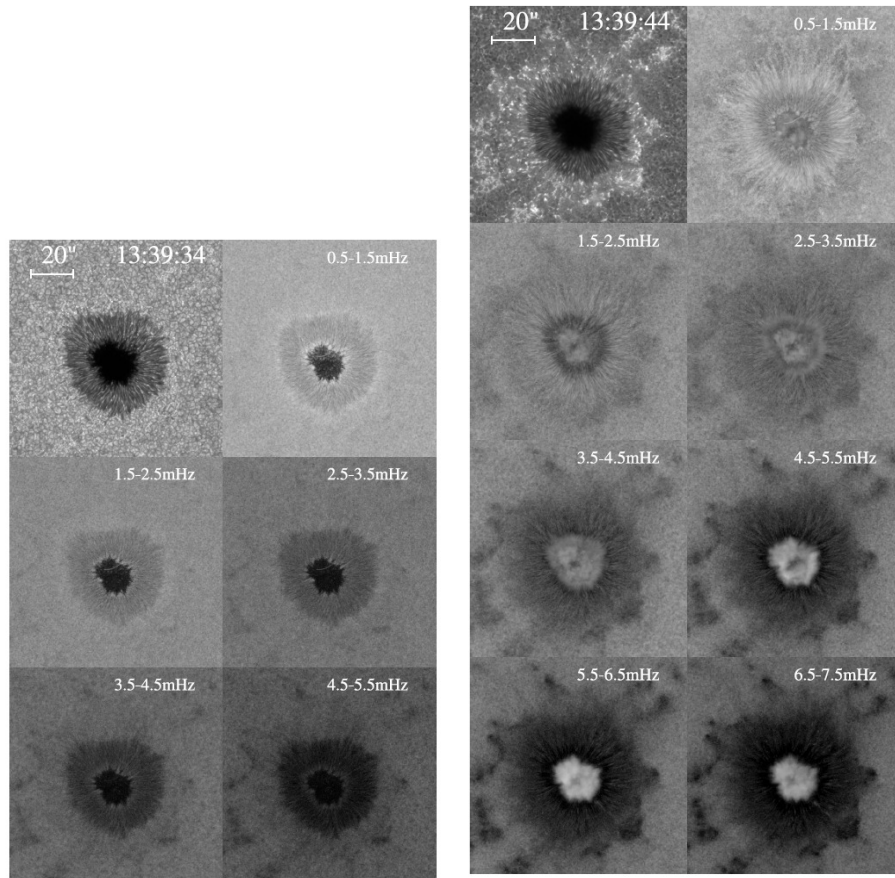


Figure 7: *Left:* Spatial distribution of the photospheric power in different frequency bands calculated from G-band intensity fluctuations of the sunspot region shown in the top-left image. The power distribution is shown in the indicated frequency ranges. *Right:* The same for the Ca II H line core intensity variations. The power is displayed in logarithmic grayscale. Image reproduced with permission from Nagashima *et al.* (2007), copyright by ASJ.

by up and down motions with the same period and amplitudes of $\pm 10 \text{ km s}^{-1}$ (Beckers and Schultz, 1972; Phillis, 1975), with an upward motion at the onset of each flash. Umbral flashes take place at any time everywhere in the umbra, with a clear correlation between the flashes and the intensity and velocity fluctuations in chromospheric lines (Kneer *et al.*, 1981). Giovanelli *et al.* (1978) found that umbral intensity oscillations were maximized in $H\alpha$ at $\pm 0.39 \text{ \AA}$ from line center and occurred simultaneously with downward velocity maxima measured at the same wavelength positions. Intensity maxima at line center lagged by 12 s, suggesting that these are waves propagating upwards. Using the time lags between the velocities measured in other spectral lines, they also concluded the outward wave propagation.

Most observations are compatible with a vertical wave propagation associated to the umbral flashes. An example of the power and phase spectra of simultaneously measured photospheric and chromospheric waves is provided in Figure 8, extracted from Lites (1984). The panels on the left show the power spectrum of the monochromatic intensity at a fixed wavelength, given by the average position of the core of Ca II $\lambda 8498$, and the power spectrum of the velocity fluctuations measured simultaneously with Fe I $\lambda 5434$. The latter spectral line is formed slightly above the

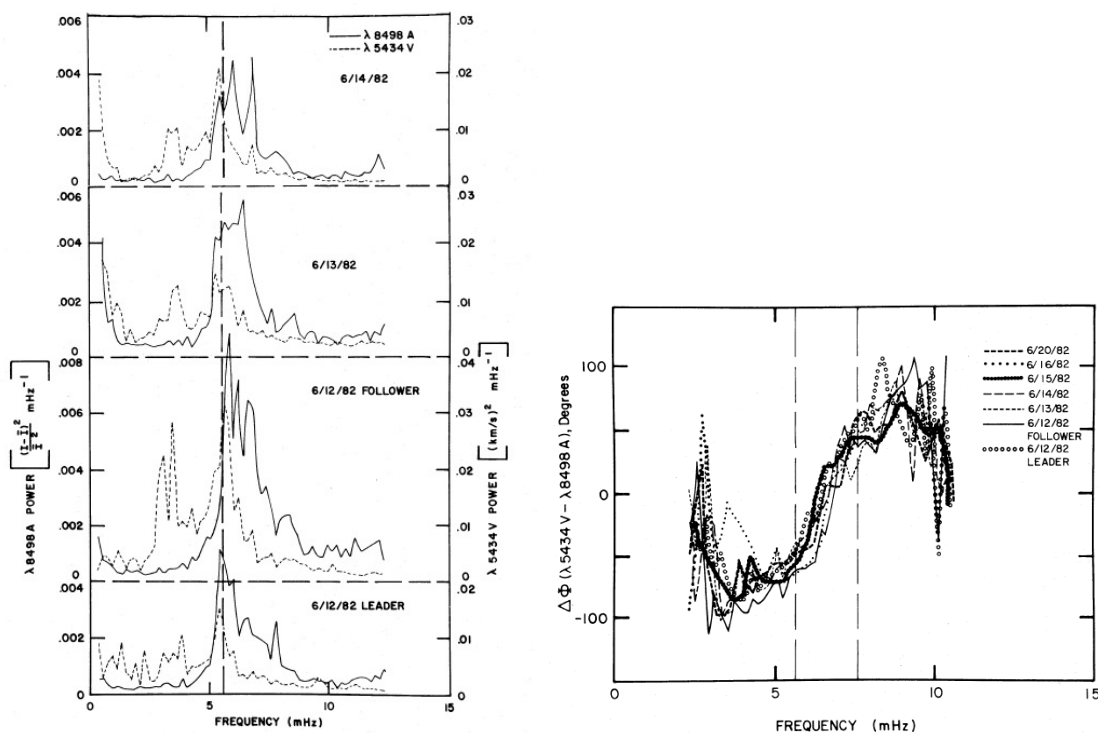


Figure 8: *Left:* Power spectrum of fluctuations of the monochromatic intensity at the fixed wavelength given by the average position of the core of Ca II $\lambda 8498$ (solid line) and power spectrum of the velocity fluctuations measured simultaneously with the Fe I $\lambda 5434$ (dashed line). The vertical line corresponds to a period of 180 s. *Right:* Phase difference spectrum for the same lines. Image reproduced with permission from Lites (1984), copyright by AAS.

temperature minimum and its power spectrum indicates the presence of periodic fluctuations with peaks in the five and three minutes bands, with similar amplitudes. Contrarily, the maximum power is located in the chromosphere at three minutes, with negligible power in the 5-min band. The spectrum of chromospheric intensity–photospheric velocity phase difference observed with these two spectral lines (right panel in Figure 8) has a linear trend in the 3-min band and almost null phase difference in the 5-min band. This behavior is compatible with the vertical wave propagation for the 3-min band and with standing waves in the 5-min band.

Unlike the results above, some authors claim that the velocity–intensity phase differences in their data are suggestive for the superposition of upward- and downward-propagating waves. This is the case of, for example, Gurman (1987) who used time series of the chromospheric Mg II K line at 2795 Å to study the oscillation properties in the umbra of five sunspots. Significant oscillations with periods in the range of 2–3 min were found in integrated line intensity and line centroid. This author argued that the shape of the power spectrum was consistent with a model of resonant transmission of acoustic waves (Section 7.4).

Excellent datasets of umbral flashes have been presented by Rouppe van der Voort *et al.* (2003), see Figure 9. According to these authors, umbral flashes are the result of the upward shock propagation and a posterior spreading of the disturbance coherently over the entire sunspot in the horizontal direction. The latter marks a possible relation between the umbral flashes and running penumbral waves, discussed below in the next section. The flash brightening was attributed by Rouppe van der Voort *et al.* (2003) to the large redshift by post-shock material, but de la

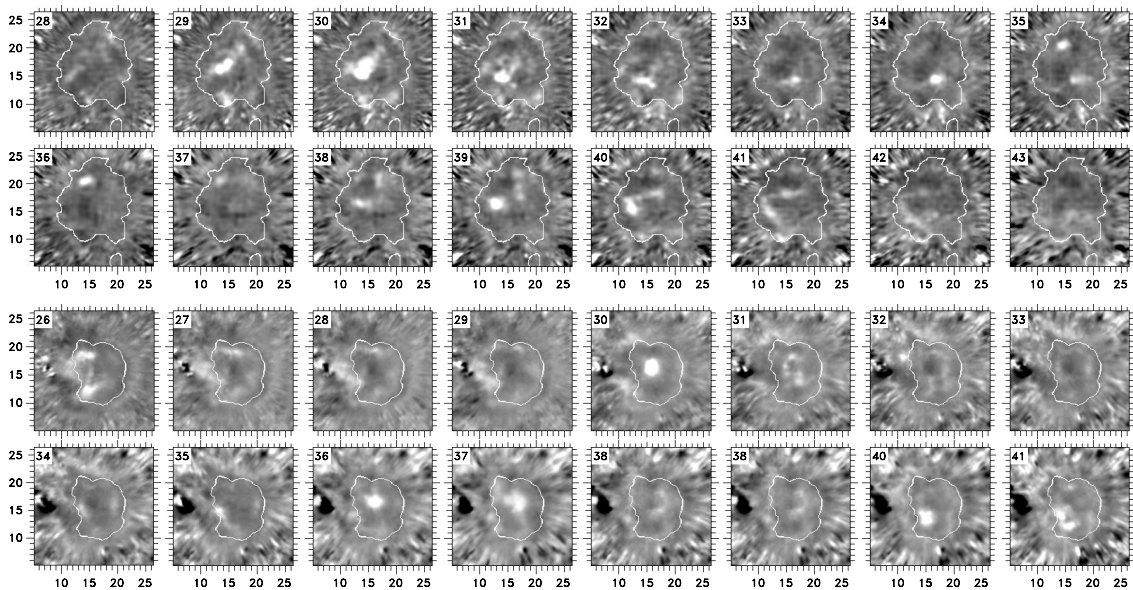


Figure 9: Two Ca II H cut-out sequences from DOT and SST data sets. A five-minute running mean is subtracted from each image. The cadence is 15.8 s between successive frames for the first sequence, irregular with a mean interval of 23.7 s for the second sequence. *Axes:* spatial scales in arcsec. The contours mark the umbral boundary. The grayscale is clipped so that the flashes are overexposed. Image reproduced with permission from Rouppe van der Voort *et al.* (2003), copyright by ESO.

Cruz Rodríguez *et al.* (2013) have measured that the shock front is roughly 1000 K hotter than the surrounding material. The brightening elements that appear in the umbrae are 3–5 arcsec wide, which is similar in size to the umbral dots. Nevertheless, no obvious relation between umbral flashes and umbral dots as potential driving sources of the disturbances was found neither by Rouppe van der Voort *et al.* (2003) nor in any other studies. The shock wave behavior of the Doppler velocity variations associated to the umbral flashes was also found by Tziotziou *et al.* (2006).

The group of the panels on the right of Figure 7 shows the chromospheric Ca II H intensity power maps measured by Nagashima *et al.* (2007) simultaneously to the photospheric G-band intensity maps, discussed above. This figure shows that the higher the frequency of chromospheric umbral oscillations, the higher is the relative power these oscillations have, when compared to the surrounding penumbra and even to the quiet Sun.

2.3 Oscillations in the transition region and corona

The first observation of sunspot waves in the transition region were reported by Gurman *et al.* (1982) and Henze *et al.* (1984). These authors analyzed time series of spectra of eight sunspots observed with the C IV $\lambda 1548$ Å line obtained with the Ultraviolet Spectrometer and Polarimeter onboard the Solar Maximum Mission. All sunspots showed significant oscillations in line of sight velocity with periods in the range 2–3 min. Intensity oscillations were also detected in four cases, with the maximum intensity in phase with the maximum blueshift. Fludra (1999, 2001) and Maltby *et al.* (2001) also found 3-min oscillations in transition region lines formed in the sunspot umbrae, and O’Shea *et al.* (2002) and Banerjee *et al.* (2002) found them at all umbral layers from the temperature minimum up to the upper corona. Such oscillations are often referred to as

oscillations of sunspot plumes, i.e., regions with enhanced EUV emission above sunspots. According to Fludra (1999, 2001), Maltby *et al.* (2001), Banerjee *et al.* (2002), umbral oscillations are present both inside and outside of sunspot plume locations which indicates that umbral oscillations can be present irrespective of the presence of these sunspot plumes. Oscillations also observed in magnetic coronal fan loop systems in AIA/SDO images (e.g., Uritsky *et al.*, 2013).

In a series of papers, Brynildsen *et al.* (1999a,b, 2000, 2001, 2002, 2003, 2004) analyzed data from the instrument SUMER onboard SOHO (Wilhelm *et al.*, 1997) and the filter channel at 171 Å of TRACE (Handy *et al.*, 1999) to get the velocity pattern at chromospheric and transition region heights as well as intensity variations in the corona. The results show a distinct oscillatory power in the 3-min band, with a clear connection between the chromospheric and the transition region velocities. The 3-min disturbances have progressively larger time lags for spectral lines with higher formation heights in the transition region and corona. This leads to a natural interpretation in terms of upward propagating waves (Brynildsen *et al.*, 1999a,b, 2000; Tian *et al.*, 2014). The first interpretation of the TRACE observations of propagating slow waves in the EUV band was provided by Nakariakov *et al.* (2000). While in the chromosphere and in the transition region the oscillations fill all the umbra, in the corona they occupy smaller areas, that may be identified as foot-points of coronal loops, reinforcing the idea that wave propagation along the magnetic field lines makes it possible their propagation toward the corona.

Figure 10, extracted from Brynildsen *et al.* (1999a), shows that the intensity-velocity time lags are very small, characteristic for the vertical wave propagation. With a better temporal resolution, Tian *et al.* (2014) find that the intensity enhancement of the Mg II $\lambda 2796$ Å line formed in the high chromosphere occurs slightly before the maximum blueshift is reached. A similar behavior was also found for the Ca II H and K lines by Ruppe van der Voort *et al.* (2003). The transition region lines C II $\lambda 1336$ Å and Si IV $\lambda 1394$ Å provide yet the opposite behavior and the maximum intensity oscillation occurs slightly later than the maximum blueshift. The different behaviors might be explained by the fact that the chromospheric Mg II is optically much thicker than the transition region lines. In that case, the intensity is determined by not only the density but also the temperature, and a careful radiative transfer analysis has to be done for an accurate explanation of the behavior of this line.

Brynildsen *et al.* (1999a) found that oscillations in line-of-sight velocities measured from the transition-region lines of O V and N V showed a characteristic nonlinear sawtooth shape, while those in the chromospheric Si II line did not. Using phase difference measurements they showed that these oscillations are due to upwardly propagating non-linear (shock) acoustic waves from the chromosphere to the transition region. Tian *et al.* (2014) present the first results of sunspot oscillations from observations by the Interface Region Imaging Spectrograph (IRIS), confirming the strong nonlinear oscillation in the spectra of several emission lines formed in the chromosphere and transition region. These authors find a positive correlation between the maximum velocity and deceleration, a result that is consistent with numerical simulations of upward propagating magnetoacoustic shock waves.

Oscillations in the umbra were also detected at coronal heights in radio observations. Gelfreikh *et al.* (1999), Shibasaki (2001), Nindos *et al.* (2002), and Sych and Nakariakov (2008) have observed oscillations in the microwave emission of a sunspot, with periods in the 2–4 min range. Gelfreikh *et al.* (2004) have also investigated quasi-periodic variations of microwave emission from solar active regions with periods smaller than 10 min detecting oscillations with periods of 3–5 min and 10–40 s, suggesting that the former might have an acoustic nature, while the latter might be associated with Alfvén disturbances.

Coronal loops in active regions are also found to oscillate with typical periods of a few minutes. Berghmans and Clette (1999), Nightingale *et al.* (1999), and Li *et al.* (2013) observed intensity disturbances propagating along active region loops in the EIT, TRACE 195 Å and AIA data, respectively. Similarly, De Moortel *et al.* (2002) detected intensity oscillations with TRACE 171 Å

data at the footpoints of coronal loops situated above sunspot regions. Coronal loop oscillations above sunspots are usually of 3-min period, however, detection of the 5-min oscillations also exist, see [Marsh *et al.* \(2003\)](#) and [Marsh and Walsh \(2006\)](#). The propagation speed of these disturbances is found to have temperature dependence, therefore, they were associated to be slow magnetoacoustic waves, see [Section 7](#).

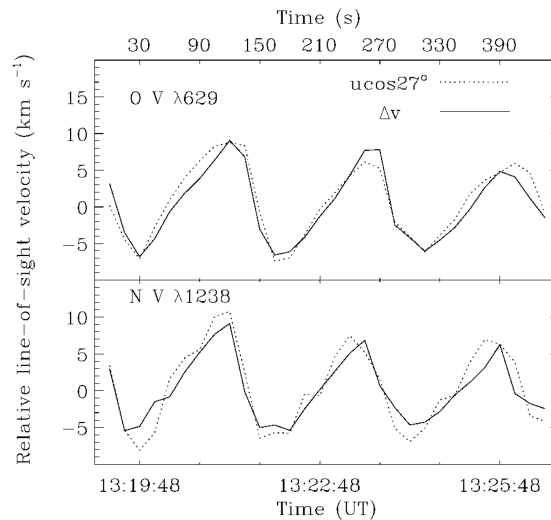


Figure 10: Temporal variation of the line-of-sight velocity measured in the umbra in the transition region lines O V $\lambda 629$ and N V $\lambda 1238$ (solid line) and the projected velocity derived from the intensity variations assuming an upward propagating acoustic wave (dashed line). Image reproduced with permission from [Brynildsen *et al.* \(1999a\)](#), copyright by AAS.

2.4 Fine structure of umbral waves

The umbral regions of sunspots are not homogeneous and show fine structure, such as umbral dots, light bridges, etc. The relation of the oscillations to the umbral structure was investigated by, e.g., [Aballe Villero *et al.* \(1990, 1993\)](#) who studied two dark cores inside the same umbra and concluded that the two cores had independent oscillatory patterns, but keeping a constant ratio of power between the five and three minutes bands. They also found a clear correlation between the power in 3 minutes and the dark cores brightness. This correlation was absent, though, for the five minute oscillations. [Soltau and Wiehr \(1984\)](#) also found a correlation between the umbral fine structure and the velocity pattern with a period of 4 minutes.

Another kind of fine structuring of oscillations was investigated by [Socas-Navarro *et al.* \(2001\)](#), who constructed a time-dependent semiempirical model of the chromospheric umbral oscillation in sunspot umbrae. The model consists of two optically thick unresolved atmospheric components: a “quiet” component with downward velocities covering most of the resolution element and an “active” component with upward velocities as high as 10 km s^{-1} with a smaller filling factor and a higher temperature at the same chromospheric optical depth. According to the authors, this semiempirical model accounts for all the observational signatures of the chromospheric oscillation when the filling factor of the active component oscillates between a few percent and 20% of the resolution element. [De la Cruz Rodríguez *et al.* \(2013\)](#) and [López Ariste *et al.* \(2001\)](#) also find that only a fraction of the resolution element appears to be emitting flashlike profiles, as if the waves were propagating only within localized magnetic field lines. It is worth mentioning that [Brynildsen *et al.* \(2001\)](#) detected the existence of two or more flows in the transition region above

sunspots within the same resolution element, where the component with the smallest velocity had an oscillatory character, whereas the high-velocity flow did not.

A recent study of the relation of the umbral fine structure and waves from AIA/SDO images by [Sych and Nakariakov \(2014\)](#) revealed the enhancements of the oscillation amplitude that have a peculiar structure consisting of an evolving two-armed spiral and a stationary circular patch at the spiral origin, situated near the umbra center. This structure is observed at all layers observed by AIA, from the temperature minimum to the corona. The spirals are more evident during the maximum phase of oscillations in the bandpasses with the highest oscillation power at 304 Å and 171 Å.

3 Waves and Oscillations in the Penumbra

Similar to the umbra, observational properties of oscillations differ from one layer to another. In the subsections below we describe properties of penumbral waves starting from the photosphere and moving to the higher layers.

3.1 Photospheric velocity and intensity oscillations

Photospheric velocity oscillations in penumbral regions have a similar power spectrum to the umbral oscillations with a maximum around five minutes. [Lites \(1988\)](#) found that 5-min velocity oscillations dominated in the outer penumbra at photospheric heights, with a ring of minimum power halfway between the inner and the outer penumbral boundary. This behavior was later confirmed by [Balthasar \(1990\)](#). [Marco *et al.* \(1996\)](#) also found indications of penumbral oscillations in deep photospheric layers using an Fe II spectral line, with significant variations between the inner and the outer parts of the penumbra. The maximum power was located at 5-min periods, but power in the 3-min band was also detected with a factor 3–4 less. [Sigwarth and Mattig \(1997\)](#) detected 5-min velocity oscillations from the inner to the outer penumbra up to the low chromosphere. According to [Figure 7](#), taken from [Nagashima *et al.* \(2007\)](#), intensity oscillations also exist at frequencies at least up to 5.5 mHz, all around the penumbra. The power of photospheric intensity and velocity oscillations in the penumbra is reduced compared to the surrounding quiet Sun.

From “time slice images” in the photospheric Fe I 5576 Å line, [Georgakilas *et al.* \(2000\)](#) revealed inward slow propagating waves in the photospheric penumbra and outward propagating waves in the area around the sunspot. The phase velocity of the waves was measured to be near 0.5 km s^{-1} in both cases and their horizontal wavelength was about 2500 km.

The penumbral fine structure, together with the presence of highly inclined magnetic fields, makes the detection and interpretation of the oscillations difficult for observations with moderate spatial resolution from the ground. [Balthasar and Schleicher \(2008\)](#) tried to detect oscillations observing an Fe I line formed in the lower to middle photosphere and did not find any evidence for penumbral waves. [Bellot Rubio *et al.* \(2000\)](#) found that the amplitude of the velocity oscillations increases toward the umbra/penumbra boundary, in agreement with the results by [Nagashima *et al.* \(2007\)](#) and [Lites \(1988\)](#).

Fine structure of photospheric penumbral oscillations is better evident at high spatial resolution. [Bharti *et al.* \(2012\)](#) analyzed high-resolution G-band intensity time series of penumbral filaments in a sunspot located near disk center and found that some filaments show dark striations moving to both sides of the filaments. Since the same phenomenon was detected in numerical simulations, they concluded that the motions of these striations are caused by transverse oscillations of the underlying bright filaments.

3.2 Chromospheric running penumbral waves

In the chromosphere, running penumbral waves were detected as brightness disturbances propagating radially outwards from the inner to the outer penumbral boundary ([Zirin and Stein, 1972](#); [Giovanelli, 1972](#)) and extend more than 15 arcsec beyond its observable boundary ([Kobanov, 2000b](#)). The disturbances are easily detectable in the core of the H α line and have a dominating period of five minutes in the inner penumbra. Running penumbral waves appear to be emitted from the umbra and expand concentrically with constant velocity around $10\text{--}15 \text{ km s}^{-1}$, although some authors have estimated speeds up to 50 km s^{-1} ([Nagashima *et al.*, 2007](#)). [Zirin and Stein \(1972\)](#) argued that umbral flashes happen with exactly half the period of running penumbral waves, which led them to suggest that running penumbral waves might be physically related to the flashes. Following this reasoning, some authors have suggested that three-minute oscillations

penetrate from the umbra into the penumbra and propagate farther as running penumbral waves (Zirin and Stein, 1972; Tziotziou *et al.*, 2002; Rouppe van der Voort *et al.*, 2003). However, other authors consider three-minute umbral oscillations and running penumbral waves to be independent phenomena (Giovannelli, 1972; Moore and Tang, 1975; Christopoulou *et al.*, 2001). Questions like the orientation of the velocity perturbations with respect to the magnetic field vector or the reason for the later detected decrease of the frequency of oscillations from the umbra to the outer penumbra were difficult to measure at that time.

A number of works followed to determine the properties of running penumbral waves as seen at different chromospheric heights and their relation to the umbral chromospheric oscillations (Lites, 1992; Tsiropoula *et al.*, 2000; Kobanov, 2000a,b). The comparison with the waves at photospheric heights was also investigated to find out whether this pattern was propagating horizontally from the umbra to the penumbra or vertically from the deep photosphere (see, e.g., Tziotziou *et al.*, 2002). Briskin and Zirin (1997) found that penumbral waves decelerate from 25 to 10 km s⁻¹ at the outer edge of the penumbra, with no appreciable decrease in the amplitude.

Lites (1988) measured simultaneously two spectral lines formed at different heights (Fe I λ 5434 and Ca II λ 8498), finding that the velocity perturbations were aligned with the magnetic field in the inner penumbra at photospheric heights and in the outer penumbra at chromospheric heights. Tsiropoula *et al.* (2000) showed several clear cases where waves that originate inside the umbra continue to propagate in the penumbra. However, the often abrupt termination of three-minute wave patterns at the umbra/penumbra boundary was noted by Kobanov and Makarchik (2004) and Kobanov *et al.* (2006). Not all three-minute wave fronts can be traced out from the umbra into the penumbra. This argument has been used to suggest that running penumbral waves are not associated with similar waves in the umbra and that very likely umbral and penumbral oscillations initially propagate along different magnetic field lines. Through careful consideration of the magnetic vector, Bloomfield *et al.* (2007a) provided evidence that velocity signatures of running penumbral waves observed in the He I λ 10830 multiplet are more compatible with upward-propagating waves than with trans-sunspot waves, see Section 8.

Reznikova *et al.* (2012), Reznikova and Shibasaki (2012), Jess *et al.* (2013), and Kobanov *et al.* (2013a) have investigated the role of the magnetic field topology in the propagation characteristics of umbral and running penumbral waves at chromospheric, transition region and coronal layers. They find an increase of the oscillatory period of brightness oscillations as a function of distance from the umbral center. The peculiar distribution of power of oscillations at different frequencies and heights is shown in Figures 13 and 11, extracted from Reznikova *et al.* (2012) and Jess *et al.* (2013). Figure 13 displays the same trend in the power distribution at all heights from the temperature minimum to the chromosphere, transition region and corona. At high frequencies (7 mHz), the maximum power is concentrated at the umbra, but at progressively lower frequencies the power extends in a ring-like structure over the penumbra. Therefore, the spatial distribution of dominant wave periods directly reflects the magnetic geometry of the underlying sunspot. As shown in Figure 12, the period of oscillations increases gradually outwards through the penumbra. This result is very important for the interpretation of the nature of running penumbral waves, see Section 8. Based on the intrinsic relationships they find between the underlying magnetic field geometries connecting the photosphere to the chromosphere, and the characteristics of running penumbral waves observed in the upper chromosphere, Reznikova *et al.* (2012) and Jess *et al.* (2013) conclude that running penumbral wave phenomena are the chromospheric signature of upwardly propagating magneto-acoustic waves generated in the photosphere. Kobanov *et al.* (2013a) obtained similar results for the wave power distribution at different frequencies and heights. In addition, the latter authors measured the time lag between oscillations at different levels. The deduced upward propagation velocities were of 28 km s⁻¹, 26 km s⁻¹, and 55 km s⁻¹ for the (Si I 10827 Å, He I 10830 Å), (1700 Å, He II 304 Å), and (He II 304 Å, Fe IX 171 Å) pairs of lines, respectively.

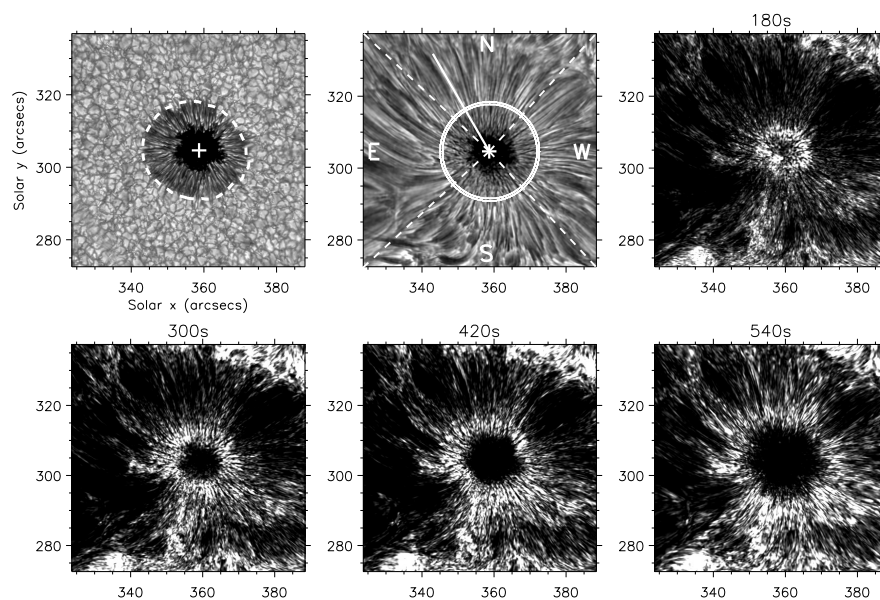


Figure 11: Simultaneous images of the blue continuum (photosphere; upper left) and $H\alpha$ core (chromosphere; upper middle) of a sunspot. The remaining panels display the chromospheric power maps extracted from the $H\alpha$ time series, indicating the locations of high oscillatory power (white) with periodicities equal to 180, 300, 420, and 540 s. Image reproduced with permission from [Jess *et al.* \(2013\)](#), copyright by AAS.

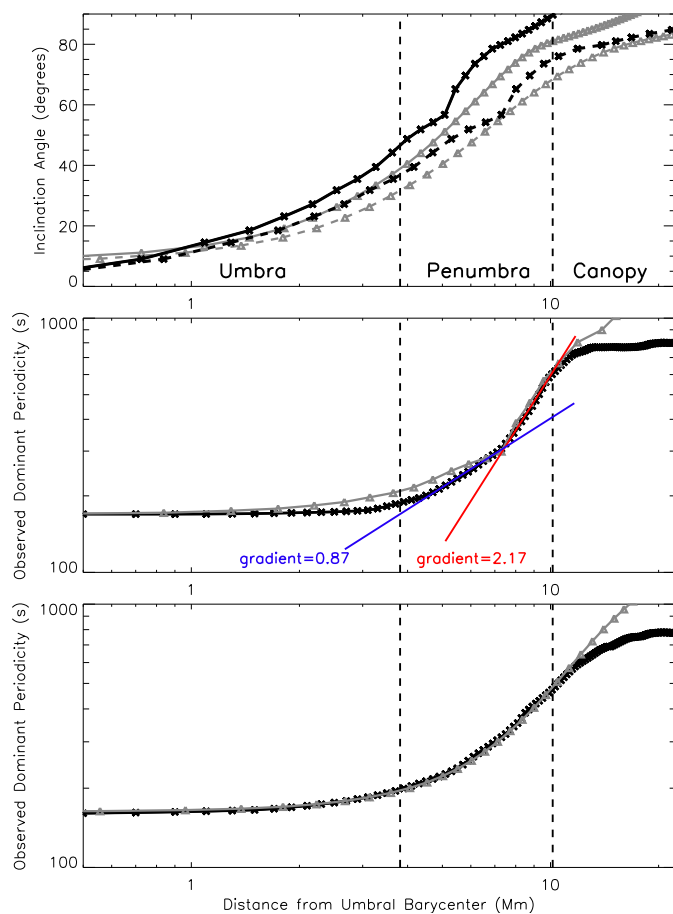


Figure 12: Average magnetic field inclination for the N (solid black line) and W+S+E (solid gray line) quadrants of Figure 11 as a function of photospheric distance from the umbral barycenter. The observed dominant periodicity for the N sunspot quadrant is displayed in the middle panel. The solid gray line displays the acoustic cutoff period determined from the magnetic field inclination angles. The lower panel displays the same information for the values averaged over the remaining three quadrants (W, S, and E). The vertical dashed lines indicate the inner and outer penumbral boundaries. Image reproduced with permission from Jess *et al.* (2013), copyright by AAS.

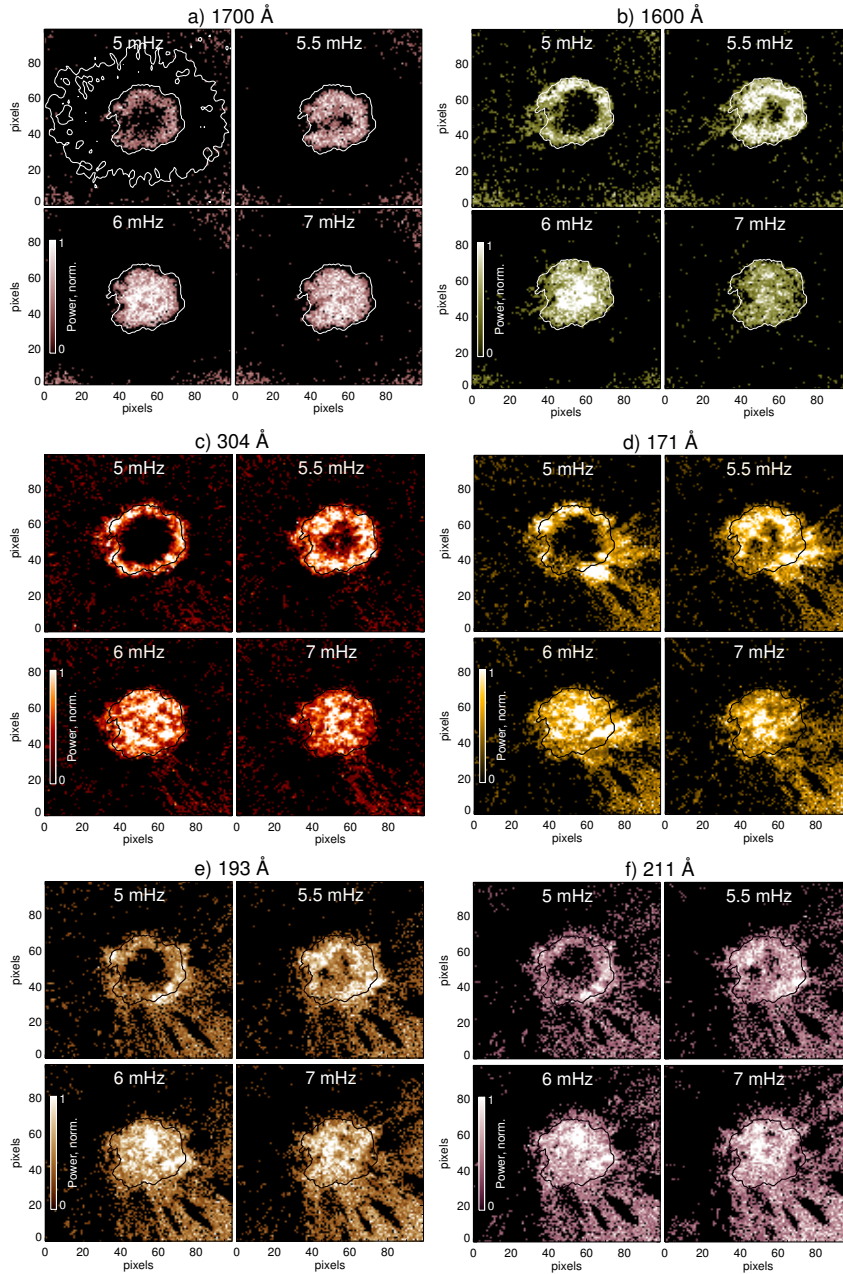


Figure 13: Spatial distribution of normalized Fourier power in four frequency bands with the central frequencies at 5, 5.5, 6, and 7 mHz (± 0.2 mHz in each band) at the wavelengths indicated in the images. Different wavelengths span progressively higher heights from the temperature minimum to the corona in the order given in the figure. Power grows with brightness. The umbra–penumbra boundary is shown by contour lines, white on panels (a and b) and black on panels (c to f). Image reproduced with permission from [Reznikova *et al.* \(2012\)](#), copyright by AAS.

4 Magnetic Field Fluctuations

The variations induced in the magnetic field by the oscillations are rather uncertain. Relatively little is known about the relations between the magnetic field, velocity and intensity oscillations. Several authors measured fluctuations of the magnetic field in sunspots from observations of different kind, with periods around 3–5 minutes and amplitudes ranging from a few gauss up to tens of gauss. All the measurements are done at photospheric heights so far, as a consequence of the absence of sufficiently good quality spectropolarimetric and magnetogram data, as well as intrinsic difficulties associated to the interpretation of measurements in the chromosphere and above.

Landgraf (1997) did not find significant oscillations of the magnetic field in sunspot umbra after analyzing polarization spectra of Fe I 5250 Å photospheric line with a large sensitivity to the magnetic field done with the Gregor telescope at the Observatorio del Teide in Tenerife. Conversely, Horn *et al.* (1997) reported significant magnetic field oscillations with periods of 3 and 5 minutes with another photospheric line, Fe I 6173.4 Å done with the FPI instrument at the VTT at the Observatorio del Teide. The oscillations were found to be specially apparent in those locations where the magnetic field lines were parallel to the line of sight. Lites *et al.* (1998), based on a full Stokes inversion of the Fe I 6301.5 Å and Fe I 6302.5 Å lines, reported an upper limit of about 4 G for the amplitude of the magnetic field oscillations, and considered them to be of instrumental rather than of solar origin. Rüedi *et al.* (1998) analyzed the velocity and magnetic field oscillations observed in sunspots using the MDI instrument onboard SOHO, and concluded that the data clearly showed highly localized oscillations of the magnetogram signal in different parts of the sunspots, with an rms value of 6.4 G. Kupke *et al.* (2000) detected an oscillatory behavior in the longitudinal field strength, with an rms amplitude of 22 G, in the 5-min band, localized at the umbral/penumbral boundary. Balthasar (1999b) obtained substantially larger amplitudes up to 50 G in individual patches of enhanced oscillations. Bellot Rubio *et al.* (2000) studied the magnetic field strength and velocity oscillations in a sunspot umbra based on the inversion of the full Stokes vector of the extremely magnetic sensitive Fe I lines at 15650 Å, formed in the deep photosphere, obtaining fluctuations with an amplitude of about 10 G and a period of 5 min, as is provided at the upper left panel of Figure 14, taken from this paper. No power was detected in temperature or magnetic field inclination. Different to that, Balthasar (2003) detected small periodic variations of the magnetic field strength or the magnetic inclination and azimuth restricted to very narrow areas in the penumbra. Kallunki and Riekhokainen (2012) analyzed magnetic field synoptic maps from SOHO/MDI to investigate the variation of the amplitude of the magnetic field strength, getting several oscillation periods in the sunspots above the 95% significance level, one in particular in the range 3–5 minutes. de la Cruz Rodríguez *et al.* (2013) did not observe significant fluctuations of the magnetic field in the umbra. In the penumbra, however, these authors measured that the passage of the running penumbral waves alter the magnetic field strength by some 200 G (peak-to-peak amplitude), without modifying the field orientation.

There is no established opinion on the spatial distribution of the magnetic field oscillations over sunspot regions. Norton *et al.* (1999) reported a decrease of the frequency of oscillations with decreasing magnetic flux. Balthasar (1999a), Zhugzhda *et al.* (2000), and Kupke *et al.* (2000) locate magnetic field oscillations at the umbra-penumbra boundary, while the opposite behavior was found in Bellot Rubio *et al.* (2000). At the same time, no difference in the power pattern for all frequency ranges was found by Balthasar (1999a).

As the amplitude of magnetic field oscillations is very small, the contradictory results obtained by different authors can easily be a consequence of differences in the observational techniques, in the sensitivity of spectral lines to the magnetic fields, and in the spatial coverage of the data used. The measurements may often suffer from a cross-talk with another oscillating magnitudes, as intensity and velocity, as is shown to be the case of MDI data by Rüedi *et al.* (1999). Additionally, the gradient of the magnetic field in sunspot umbra may introduce spurious oscillations because

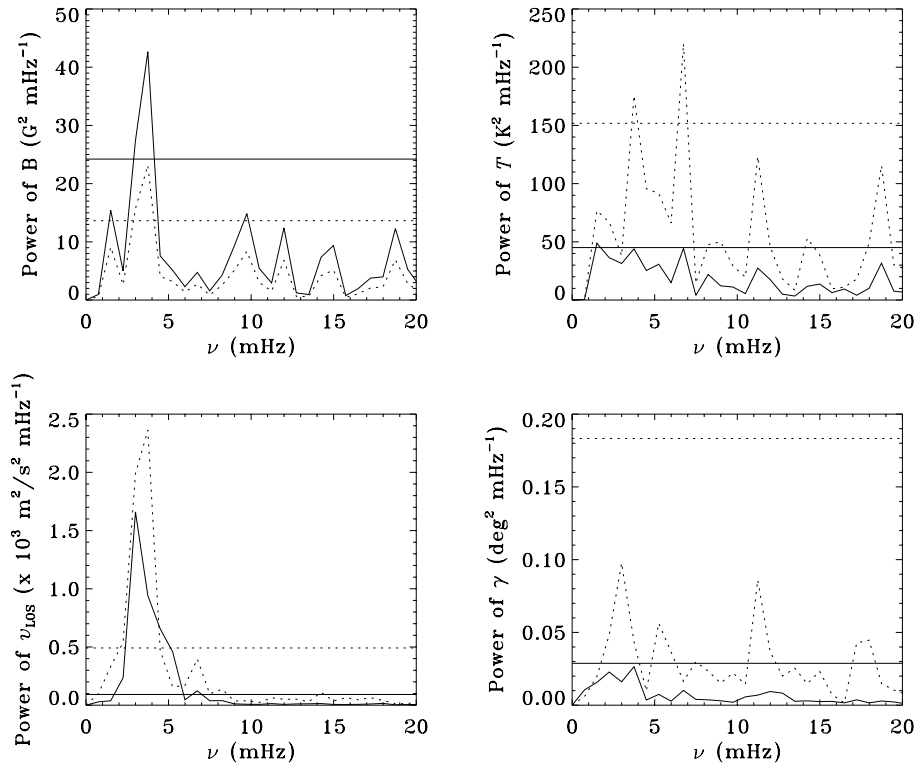


Figure 14: *Left:* Average power spectra of magnetic field (top) and velocity (bottom) fluctuations in a sunspot umbra at $\log \tau_5 = 0.0$ (solid lines) and $\log \tau_5 = -1.0$ (dotted lines). *Right:* Average power spectra of temperature (top) and magnetic field inclination (bottom) fluctuations at the same optical depths. Image reproduced with permission from Bellot Rubio *et al.* (2000), copyright by AAS.

the region of formation of spectral lines moves up and down due to opacity effects induced by oscillations in thermodynamic parameters, as was considered in Rüedi *et al.* (1999), Bellot Rubio *et al.* (2000), Rüedi and Cally (2003), and Khomenko *et al.* (2003).

Phase relations should be important for the diagnostic of the type of oscillatory phenomena observed. Most observations give values of about 90 degrees with upward velocity leading magnetic field (Rüedi *et al.*, 1998; Norton *et al.*, 1999; Balthasar, 1999b; Bellot Rubio *et al.*, 2000). Again, these measurements are very uncertain.

5 Sunspot Surroundings

An interesting aspect of the oscillations is how they are modified in the immediate surroundings of intense magnetic concentrations like sunspots. Acoustic power maps have been studied since [Braun *et al.* \(1988, 1990\)](#). As discussed in Sections 2 and 3, the measured oscillation power is reduced by some 40–60% in the photosphere of sunspots. This reduction appears as a dark area of suppressed acoustic power in the 5-min band that is spatially correlated with sunspots and active regions. [Penn and LaBonte \(1993\)](#) obtained further evidences that there is more power traveling toward the center of the umbrae than leaving their center, providing a direct measure of the absorption of p -modes by sunspot umbrae. This phenomenon has received the name of “absorption” in the literature on helioseismology ([Lites *et al.*, 1982](#); [Abdelatif *et al.*, 1986](#); [Brown *et al.*, 1992](#); [Hindman and Brown, 1998](#)). Such measurements also useful to clarify whether photospheric umbral oscillations are driven by the external p -mode oscillations or a result of an acoustic cavity that generates resonant frequencies, see Sections 8.1 and 7.4.

Subsequent works based on the analysis of temporal series of intensity and velocity maps reveal that there is a power enhancement of the oscillations in the 3-min band in the surroundings of active regions when compared to the quiet Sun. This phenomenon is observed at the photosphere ([Brown *et al.*, 1992](#)) and at the chromosphere ([Braun *et al.*, 1992](#); [Toner and LaBonte, 1993](#)) and is usually known as “*acoustic halos*”. The power enhancement is observed at high frequencies between 5.5 and 7.5 mHz with an increment of about 40–60% compared to the surrounding quiet Sun ([Hindman and Brown, 1998](#); [Braun and Lindsey, 1999](#); [Donea *et al.*, 2000](#); [Jain and Haber, 2002](#); [Nagashima *et al.*, 2007](#)). The halos are observed at intermediate longitudinal magnetic fluxes $\langle B \rangle = 50\text{--}300$ G in plage regions surrounding sunspots ([Hindman and Brown, 1998](#); [Thomas and Stanchfield II, 2000](#); [Jain and Haber, 2002](#)). [Schunker and Braun \(2011\)](#) pointed out that the largest excess of power in the halos occurs in regions with horizontal magnetic field, especially at locations between regions of opposite polarity, and that larger magnetic field strengths are accompanied by higher frequencies with maximum power.

The radius of the halo increases with height. In the photosphere, the halos are located at the edges of active regions, while in the chromosphere, they extend to a large portion of the nearby quiet Sun ([Brown *et al.*, 1992](#); [Braun *et al.*, 1992](#); [Thomas and Stanchfield II, 2000](#)). There are indications for the up- and downward propagating waves at the locations of halos ([Braun and Lindsey, 2000](#); [Rajaguru *et al.*, 2013](#)).

The power enhancement is most prominent in Doppler velocity and line intensity, but is absent in the continuum intensity. [Jain and Haber \(2002\)](#) analyzed data from MDI to derive power maps at different frequency bands of the fluctuations of the continuum and line core intensities, as well as of the velocity. A similar study was carried out by [Rajaguru *et al.* \(2013\)](#), using data from the Helioseismic and Magnetic Imager (HMI) ([Scherrer *et al.*, 2012](#)). The power maps of four active regions are shown in Figure 15, arranged in 2×2 boxes. From left to right the images correspond to continuum intensity, I_c (forming height at ~ 0 km), line core intensity, I_{co} (forming height at ~ 300 km), and velocity, v (forming height at ~ 140 km), while the top-to-bottom maps correspond to different frequency intervals. The power enhancement surrounding the sunspots is apparent in both the line core intensity and velocity in the 2–3-min bands. No such enhancement is detected in the 5-min band. There is a power deficit in the spots themselves in the three bands, as expected from previous investigations. The same happens with the continuum intensity, and the halos are not detected in the temperature variations in the deep layers where the continuum is formed.

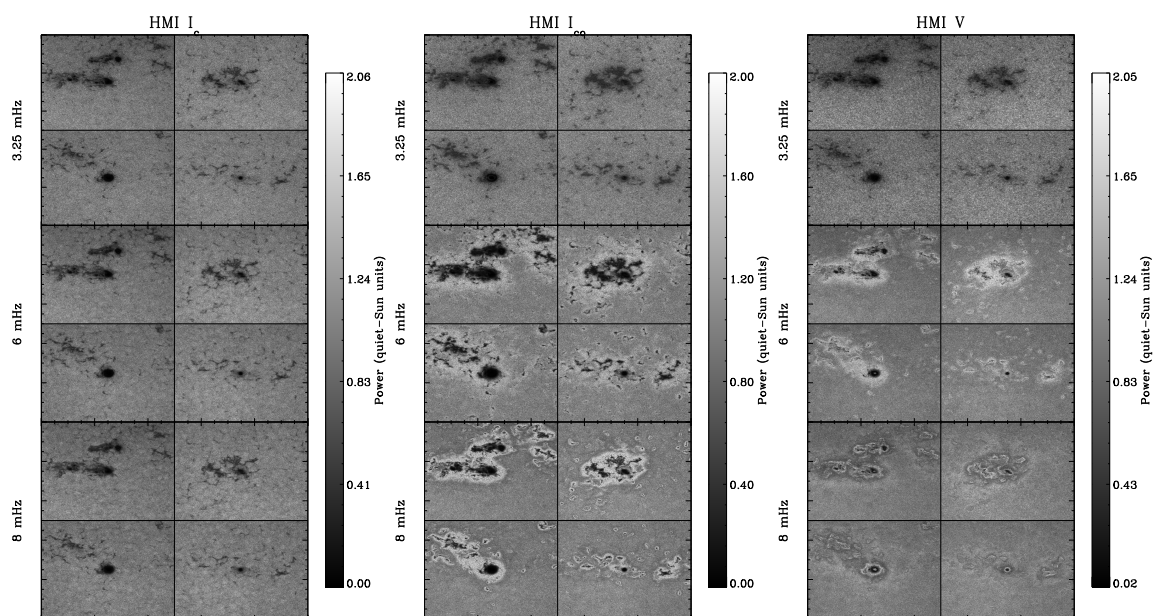


Figure 15: Power maps of four active regions, arranged in 2×2 boxes. *Left to right:* continuum intensity (I_c), line core intensity (I_{co}), and velocity (v). *Top to bottom:* maps correspond to the frequency intervals centered at 3.25, 6 and 8 mHz. Image reproduced with permission from [Rajaguru et al. \(2013\)](#), copyright by Springer.

6 Long-Period Oscillations

Apart from oscillations with periods of the order of a few minutes, discussed in the sections above, there exists another class of sunspot oscillations, with periods ranging from tens of minutes to several hours and days. Such long-period oscillations are difficult to detect. They require very stable observing conditions and instrumental performance (see, e.g., [Smirnova *et al.*, 2013a](#)). Therefore, not much observational material is available and the results are often not very reliable.

Long-period magnetic field oscillations were detected from SOHO/MDI and ground-based data at Pulkovo Observatory by [Nagovitsyna and Nagovitsyn \(2002\)](#), [Efremov *et al.* \(2007, 2009, 2012b, 2013\)](#), [Kallunki and Riehoainen \(2012\)](#), and with NoRH and SSRT instruments by [Abramov-Maximov *et al.* \(2013\)](#) and [Bakunina *et al.* \(2013\)](#), finding periods of 30–40, 70–100, and 150–200 and 800–1300 minutes and amplitudes of about 200 G. Some of the authors suggest that these oscillations can be related to sunspot global eigen-modes as a whole. [Efremov *et al.* \(2012a\)](#) have demonstrated that long-period ($T = 10\text{--}20$ hours) oscillations of the magnetic field in bipolar groups are excited synchronously in the main and tail spots of a group. At the same time, no correlation was found between long-period oscillations of the field of sunspots belonging to different active regions. The periods of these oscillations are not stable: they are different in different sunspots and in the same sunspot on different days. Theoretical models to explain these oscillations are discussed in Section [7.5](#).

In the microwave (gyroresonant) emission at 17 GHz above sunspots, oscillations of 50–150 min have been detected by [Chorley *et al.* \(2010, 2011\)](#) at the Nobeyama Radioheliograph. [Smirnova *et al.* \(2011\)](#) find two main ranges (10–60 and 80–130 min) of long quasi-periodic oscillations at 17, 37 and 93 GHz radio data. These long-period oscillations were found to be relatively stable and were suggested to be interpreted as a radial mode of sunspot oscillations. Even longer periods of 200–400 min were found in a later work ([Smirnova *et al.*, 2013b](#)).

The long period oscillations of the envelope signal of 3-min wave trains observed in the microwave emission show frequency drifts ([Sych *et al.*, 2012](#)). The speed of the drift is 4–5 mHz/h in the photosphere, 5–8 mHz/h in the chromosphere, and 11–13 mHz/h in the corona. The observed drifts can be positive or negative, but the latter are less frequent. These drifts can reflect the non-linear character of oscillations and the non-homogeneous structure of the umbra with different wave sources.

[Gopasyuk \(2005\)](#); [Gopasyuk and Gopasyuk \(2006\)](#); [Gopasyuk \(2010\)](#) detected torsional oscillations of sunspots from data in the Fe I 5253 Å line taken at the Crimean observatory. The periods of torsional oscillations were in the range of several days and larger periods were observed for sunspots located at larger latitudes, possibly related to the differential rotation of the Sun. These authors offered a mechanism based on the interplay between the life time of super-granular cells and the Coriolis force acting on sunspot flux tubes. Similar torsional oscillations, with period 3.8 days were confirmed from SOHO/MDI and TRACE data later by [Gopasyuk and Kosovichev \(2011\)](#).

All in all, given the observational difficulties, especially for observations from the ground, the detection of long-period oscillations remains an observational challenge and the nature of their driving mechanism remains uncertain. Several studies have sought to answer this question, see the discussion in Section [7.5](#).

7 Modeling Waves in the Magnetized Sunspot Atmospheres

Oscillatory motions observed in sunspots can be naturally attributed to different types of magneto-acoustic-gravity modes of a magnetized plasma. Depending on the approach, models of sunspot waves can be formally classified into local and global ones. In the local analysis the sunspot atmosphere is treated as horizontally homogeneous or weakly inhomogeneous. The large horizontal extent of sunspots allows to assume the horizontal direction to be infinite and the variations in the stratification essentially happening in the vertical direction, unlike for models of waves in small-scale magnetic flux tubes. Typical wavelengths of 3–5 min period acoustic waves in the photosphere are of the order of a few Mm, being smaller than the size of sunspot umbra and penumbra. Local models are used to explain umbral flashes and penumbral waves both in the photosphere and in the chromosphere. Global models consider the oscillations of a sunspot flux tube as a whole, and are used essentially to explain long-period oscillations. We review both classes of models separately below.

7.1 Behavior of pure modes

An extensive analytical theory exists for describing the propagation of waves (either adiabatic, or including radiative losses) in a gravitationally stratified atmosphere with an arbitrary inclined constant magnetic field (Ferraro and Plumpton, 1958; Osterbrock, 1961; Nye and Thomas, 1974, 1976; Zhugzhda and Dzhililov, 1982, 1984a,c,b; Leroy and Schwartz, 1982; Bogdan and Knölker, 1989; Babaev *et al.*, 1995a,b; Cally, 2001). Neglecting radiative losses, and the temperature gradient, the system of linear equations for adiabatic perturbations in an isothermal stratified atmosphere has the following form (e.g., Zhugzhda and Dzhililov, 1984a)¹

$$\frac{\partial \rho_1}{\partial t} + \vec{\nabla}(\rho_0 \vec{v}_1) = 0 \quad (1)$$

$$\rho_0 \frac{\partial \vec{v}_1}{\partial t} + \vec{\nabla} P_1 - \vec{g} \rho_1 + \frac{1}{\mu} [\vec{\nabla}(\vec{B}_0 \vec{B}_1) - (\vec{B}_0 \vec{\nabla}) \vec{B}_1] = 0 \quad (2)$$

$$\frac{\partial P_1}{\partial t} + \vec{v}_1 \vec{\nabla} P_1 = c_S^2 \left(\frac{\partial \rho_1}{\partial t} + \vec{v}_1 \vec{\nabla} \rho_1 \right) \quad (3)$$

$$\frac{\partial \vec{B}_1}{\partial t} - (\vec{B}_0 \vec{\nabla}) \vec{v}_1 + \vec{B}_0 (\vec{\nabla} \vec{v}_1) = 0 \quad (4)$$

These are the equations of conservation of mass, momentum and energy, and the ideal induction equation for the magnetic field, with a standard notation. This system can be reduced to a single equation for the plasma velocity:

$$\begin{aligned} \frac{\partial^2 \vec{v}_1}{\partial t^2} = & c_S^2 \vec{\nabla}(\vec{\nabla} \vec{v}_1) + \vec{\nabla}(\vec{g} \vec{v}_1) + (\gamma - 1) \vec{\nabla} \vec{g} \vec{v}_1 \\ & - \frac{1}{\mu \rho_0} \left[\vec{\nabla} \left(\vec{B}_0 \left((\vec{B}_0 \vec{\nabla}) \vec{v}_1 - \vec{B}_0 (\vec{\nabla} \vec{v}_1) \right) \right) - \vec{B}_0 \vec{\nabla} \left((\vec{B}_0 \vec{\nabla}) \vec{v}_1 - \vec{B}_0 (\vec{\nabla} \vec{v}_1) \right) \right] \end{aligned} \quad (5)$$

Further simplifying to the case of absence of gravity and stratification, this equation is reduced to a partial differential equation with constant coefficients and its solution is obtained in the form

¹ In the equations here and all over the paper we omit the dot in the scalar product of vectors, using a simpler $\vec{a}\vec{b}$ notation.

of Fourier harmonics, $\vec{v}_1 = \vec{v}_0 e^{i(\vec{k}\vec{r} - \omega t)}$, and can be found in text books on plasma physics, see e.g., [Priest \(1982\)](#). A dispersion relation is obtained:

$$\omega^2 \vec{v}_1 = c_s^2 \vec{k}(\vec{k}\vec{v}_1) + \left[\vec{k} \times \left(\vec{k} \times (\vec{v}_1 \times \vec{B}_0) \right) \right] \times \frac{\vec{B}_0}{\mu\rho_0} \quad (6)$$

This dispersion relation supports three modes: fast and slow magneto-acoustic mode and the Alfvén mode, see [Priest \(1982\)](#) for details.

Understanding the behavior of these pure modes is useful for interpreting sunspot oscillations, despite the fact that such pure modes cannot exist in the stratified and inhomogeneous atmosphere of sunspots. It is convenient to separate the cases with a weak and a strong magnetic-field regime. Seen from the point of view of wave propagation, the weak-field regime holds when the ratio of the squared sound, c_S , and Alfvén, v_A , speeds, is much larger than one, $c_S^2/v_A^2 \gg 1$. The opposite holds for the strong field regime.²

In the limit, when c_S^2/v_A^2 (or plasma β) is either much larger or much smaller than one, the dispersion relation for the fast and slow magneto-acoustic waves is simplified to give:

$$\begin{aligned} \omega &\approx c_S k = c_S \vec{k} \vec{e}_k; & (c_S \gg v_A); \\ \omega &\approx v_A k = v_A \vec{k} \vec{e}_k; & (c_S \ll v_A), \end{aligned} \quad (7)$$

for the fast mode and

$$\begin{aligned} \omega &\approx k v_A \cos \psi = v_A \frac{\vec{k} \vec{B}_0}{|\vec{B}_0|}; & (c_S \gg v_A); \\ \omega &\approx k c_S \cos \psi = c_S \frac{\vec{k} \vec{B}_0}{|\vec{B}_0|}; & (c_S \ll v_A), \end{aligned} \quad (8)$$

for the slow mode, with $\vec{e}_k = \vec{k}/k$, and ψ being the angle between \vec{k} and \vec{B}_0 .

The phase, $\vec{v}_{\text{ph}} = (\omega/k)\vec{e}_k$, and the group, $\vec{v}_g = \partial\omega/\partial\vec{k}$, velocities of the fast and slow modes can be trivially calculated from the above expressions. In the case of the fast mode, the group velocity is equal to either c_S or v_A and its direction is that of \vec{k} . The directions of the phase and group velocities coincide and the medium can be considered isotropic for the fast wave in this regime.

In the case of the slow mode, \vec{v}_g is directed along the vector \vec{B}_0 , while \vec{v}_{ph} is by definition directed along \vec{k} . Curiously, when c_S^2/v_A^2 approaches 1, the direction of propagation of the slow mode, given by \vec{v}_g , can depart by a maximum of 27 degrees from the field direction (see [Osterbrock, 1961](#); [Khomenko and Collados, 2006](#)). Because of their propagation speeds, the fast mode is essentially acoustic when $c_S \gg v_A$ and is essentially magnetic when $c_S \ll v_A$, and the opposite is true for the slow mode.

The dispersion relation for the pure Alfvén mode of the homogeneous atmosphere is independent of the ratio c_S^2/v_A^2

$$\omega = k v_A \cos \psi = v_A \frac{\vec{k} \vec{B}_0}{|\vec{B}_0|}, \quad (9)$$

therefore, while its phase velocity is always directed along \vec{k} by definition, its group velocity is parallel to \vec{B}_0 . Pure Alfvén waves in a homogeneous atmosphere are incompressible (\vec{k} is perpendicular to \vec{v}_1) and represent an equipartition between kinetic and magnetic energies.

² The ratio between the square of characteristic wave speeds is related to the plasma $\beta = P/P_{\text{mag}}$ as $\beta = (2/\gamma)c_S^2/v_A^2$. Since the coefficient $2/\gamma$ is close to 1, for many practical purposes, both, β and c_S^2/v_A^2 can be used indistinguishably.

7.2 Vertically stratified atmosphere: wave propagation and conversion

7.2.1 Eikonal approximation

Strictly speaking, the wave vector \vec{k} of a propagating mode is constant all over homogeneous atmosphere, limiting the application of the equations of a homogeneous medium for the description of sunspot oscillations. However, in practice, these equations can still be applied locally if one assumes that the wavelength of the perturbation is smaller than the characteristic scale of the variations of the sunspot atmosphere (i.e., pressure scale height or the typical scales of horizontal variations in the umbra and penumbra). This assumption allows to obtain a simple approximate solution of the wave equation (5) by an eikonal method (e.g., Gough, 2007; McLaughlin and Hood, 2006; McLaughlin *et al.*, 2008; Khomenko and Collados, 2006; Khomenko *et al.*, 2009b). In the zero-order eikonal approximation, one neglects the variation of the wave amplitude and considers only the variation of its phase, i.e., it is assumed that the perturbation velocity in Eq. (5) depends on x, y, z and time as $\vec{v}_1 = \vec{v}_0 e^{i\phi(x,y,z)} \cdot e^{-i\omega t}$ (where \vec{v}_0 is constant). In the works of Barnes and Cally (2001); Cally (2006) and Moradi and Cally (2008), the effects of the acoustic cut-off frequency were incorporated in the eikonal solution for waves in an isothermal atmosphere with constant inclined magnetic field. The following three-dimensional dispersion relation is obtained (Moradi and Cally, 2008):

$$F(\vec{r}, \vec{k}) = \omega^2 \omega_c^2 v_{Ay}^2 k_h^2 + (\omega^2 - v_A^2 k_{\parallel}^2) \times \left[\omega^4 - \omega^2 (c_S^2 + v_A^2) k^2 + c_S^2 v_A^2 k^2 k_{\parallel}^2 - \omega_c^2 (\omega^2 - v_{Az}^2 k^2) + c_S^2 N^2 k_h^2 \right] = 0, \quad (10)$$

where k_h and k_{\parallel} are horizontal and parallel to the magnetic field components of the wave vector ($k_x = \frac{\partial \phi}{\partial x}$, $k_y = \frac{\partial \phi}{\partial y}$, $k_z = \frac{\partial \phi}{\partial z}$), and $\vec{r} = (x, y, z)$ is the coordinate vector. The parameter $\omega_c = c_S/2H$ is the isothermal cut-off frequency, N is the Brunt–Väisälä frequency, $N^2 = g/H - g^2/c_S^2$ and H is pressure scale height. The parameters c_S , v_A , N and ω_c are allowed to vary smoothly with coordinate \vec{r} . Equation (10) can be solved by Charpit's method of characteristics by transforming it into the following system of ordinary differential equations:

$$\frac{d\vec{k}}{ds} = -\frac{\partial F}{\partial \vec{r}}; \quad \frac{d\vec{r}}{ds} = \frac{\partial F}{\partial \vec{k}}. \quad (11)$$

The variable s is the distance along the characteristic wave propagation path. The solution of Eqs. (11) gives $\vec{r}(s)$ and $\vec{k}(s)$ along the wave path s . The lines $z(x, y)$ give the trajectory of the group velocity of the wave. Calculations of the trajectories and phase velocities of sunspot waves by the eikonal method were done by Cally (2006); Moradi and Cally (2008); Khomenko and Collados (2006); Khomenko *et al.* (2009b) to investigate the refraction and reflection of fast-mode waves in sunspots, as well as their coupling to slow-mode waves. Figure 16 gives an example of the fast and slow magneto-acoustic wave paths, calculated for a two-dimensional case in the photospheric and chromospheric part of a sunspot model from Khomenko and Collados (2006). This figure shows how the slow mode (excited in the photosphere) propagates to the upper layers and gets gradually aligned with the magnetic field (dotted lines). The fast-mode wave propagates at an angle with respect to magnetic field lines. Reaching the chromosphere, its group speed becomes proportional to the Alfvén speed that possesses strong vertical and horizontal gradients (dashed lines mark the direction of the gradient of the Alfvén speed, $\vec{\nabla} v_A$ at different locations). Because of these gradients, the fast mode gets refracted and finally reflected at some height between the upper photosphere and the low chromosphere. Such behavior is typical for sunspot waves, and the height of reflection depends on the height of the layer where $c_S = v_A$ (solid line in Figure 16) and on the gradients of the Alfvén speed.

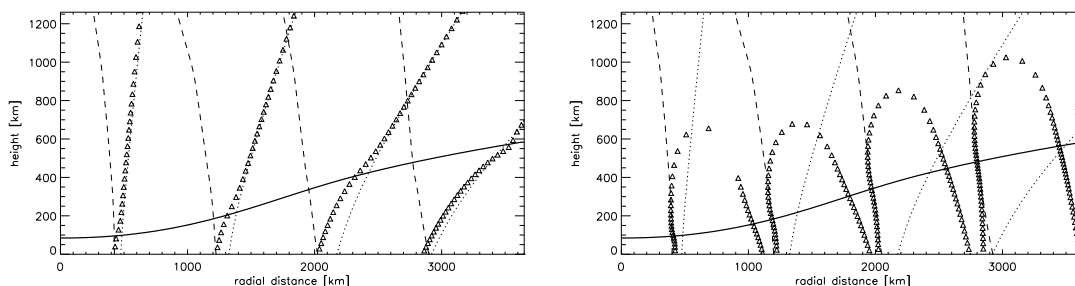


Figure 16: Symbols: paths of the slow (left) and fast (right) modes calculated in the eikonal approximation in the photospheric and chromospheric parts of a sunspot model. Dotted lines are the magnetic field lines; dashed lines indicate the direction of the gradient of the Alfvén speed, $\vec{\nabla}v_A$, at several locations; solid line is $v_A = c_S$ layer. The bottom level corresponds to $z = 0$ at the axis of the sunspot umbra photosphere (shifted 350 km below the quiet photospheric level due to Wilson depression). Image reproduced with permission from [Khomenko and Collados \(2006\)](#), copyright by AAS.

7.2.2 Fast-to-slow mode conversion in a vertical field

An analytical solution of Eq. (5) can still be obtained even if no restriction is set to the wavelength of the perturbation. Its Fourier-transformation in the direction perpendicular to the gravity ($x - y$ plane) leads to a system of coupled ordinary differential equations for the components of the plasma velocity with derivatives in the vertical direction z . An analytical solution of this system can be searched for in the form of a Frobenius series expansion, as suggested by [Ferraro and Plumpton \(1958\)](#) for the case of the magnetic field parallel to the gravity direction. This method was followed by [Zhugzhda and Dzhalilov \(1982, 1984a,c\)](#) in a series of papers, developing the theory of wave propagation and conversion in an isothermal atmosphere permeated by a constant arbitrary inclined magnetic field. A particular case of the propagation in a purely horizontal magnetic field varying with height was considered by [Nye and Thomas \(1974, 1976\)](#) who addressed the problem of running penumbral waves. [Zhugzhda and Dzhalilov \(1982, 1984a\)](#) obtained the solution in terms of Meijer G-functions, and the implementation of this solution in practice (for example, the application to the propagation of waves in layers of the solar atmosphere with plasma β close to 1) leads to computational difficulties. Later, [Cally \(2001\)](#) proposed to rewrite the solution of [Zhugzhda & Dzhalilov](#) for the vertical field case in terms of simpler hypergeometric ${}_2F_3$ functions, that are easier to evaluate numerically. The solutions allow to describe the whole spectrum of magneto-acoustic-gravity waves and their propagation and conversion properties in different frequency domains of the $k - \omega$ diagram. Asymptotic solutions exist in different regions of the diagram for high and low β regimes, and can be classified as more or less pure modes. Wave mode conversions are possible between them.

In sunspots, the plasma β is expected to be $\beta \gg 1$ below the photosphere, and $\beta \ll 1$ in the chromosphere and higher ([Mathew et al., 2004](#)). In between, there is an equipartition layer with $\beta \approx 1$ located at some height in the photosphere. The exact location of this layer depends on the magnetic field strength and the temperature stratification. It is important to note that waves, detected in observations with photospheric lines, both in the umbra and penumbra, propagate in layers where the plasma β is around one, where no pure modes (as in the sense described above) can be identified. Approaching the layer where $\beta \approx 1$, the propagation speeds of the different modes become similar and the energy can be transferred between the different branches of the dispersion relation. This process is referred to as “mode conversion”.³

³ The process of mode conversion can be defined from the physical or mathematical points of view, by labeling

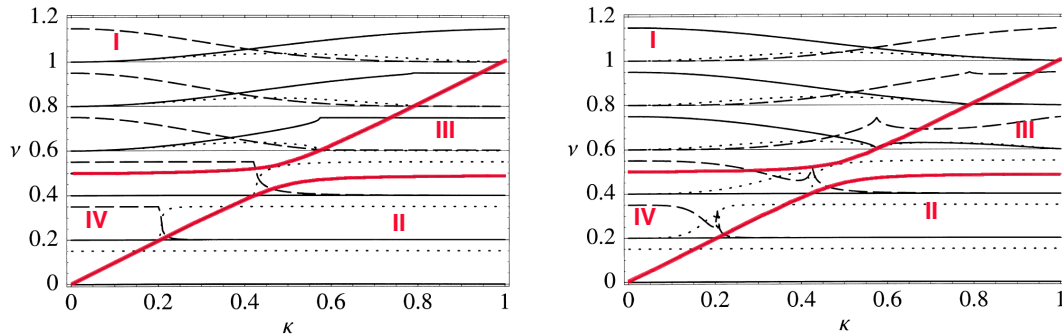


Figure 17: *Left panel:* transmission (solid curves), reflection (dashed curves), and conversion (dotted curves) coefficients for a pure slow-mode incident from high- β region, as a function of position in the diagnostic diagram. The diagram is calculated for an isothermal atmosphere permeated by a constant vertical magnetic field. The baseline of each graph is placed at its fixed frequency $\nu = 0, 0.2, 0.4, 0.6, 0.8,$ and $1.0,$ and wavenumber k is varied continuously. The dimensionless frequency on the vertical axis is defined as $\nu = \omega H/c_S$; the dimensionless horizontal wavenumber is $k = k_{\perp} H$, where H is the pressure scale height. $\nu = 1/2$ is the cut-off frequency. Vertical wave number $k_z^2 > 0$ in the domains (I) and (II) for acoustic and gravity waves; $k_z^2 < 0$ in the domains (III) and (IV) and $k_z = 0$ on the red curves. *Right panel:* same for the fast-mode incident from high- β region. Image reproduced with permission from Cally (2001), copyright by AAS.

Figure 17 shows the diagnostic diagram for transmission, reflection and conversion coefficients of magneto-acoustic-gravity waves in an isothermal atmosphere and vertical magnetic field, taken from Cally (2001). This figure nicely summarizes the properties of waves in the different frequency domains and mutual conversions from fast to slow magneto-acoustic waves (excluding Alfvén waves). The coefficients are calculated as the ratio between the vertical wave energy fluxes of the transmitted, reflected and converted modes and the flux of the incident high- β slow mode (left panel) and fast mode (right panel). According to the nomenclature used by Cally (2001), the “transmission” takes place when the wave nature (either acoustic or magnetic) and direction are preserved from $\beta \gg 1$ to $\beta \ll 1$ atmosphere; the “reflection” takes place when the direction of the energy flux is changed to the opposite; the “conversion” takes place when the nature of the wave is converted from acoustic to magnetic or vice versa.

The left panel of Figure 17 shows that, in regions (I) and (III) above the adimensional cut-off frequency ($\nu > 1/2$), the transmission coefficient of the incident slow mode increases from 0 to 1 (solid curve), while the reflection coefficient decreases from 1 to 0 (dashed curve), with increasing wave number k . Partial slow-to-fast mode conversion occurs for intermediate values of k (dotted curve). In region (III) the conversion coefficient from slow to fast mode is zero because the fast mode is evanescent. Below the cut-off frequency ($\nu < 1/2$) the transmission of the slow mode is zero because this mode is evanescent in the low- β plasma. Accordingly, the reflection is total in region (IV), and effective slow-to-fast conversion takes place over region (II).

As for the fast-mode incident from the high- β region (right panel of Figure 17) the transmission cannot occur below the acoustic cut-off frequency since the fast mode is evanescent in the low- β

the modes as “acoustic” and “magnetic” or as “fast” and “slow”. In the physical definition, the conversion or transformation happens when the mode nature is changed from magnetic to acoustic and vice versa (the propagation speed is maintained either fast or slow). In the mathematical definition, the conversion happens when the propagation speed changes from fast to slow or vice versa, but the nature of the wave (acoustic or magnetic) is maintained. In most of the literature the mathematical definition of the mode conversion is used. Although we consider the physical definition to be more appropriate, through the present review we will follow the mathematical definition for consistency with most of the wave studies.

atmosphere. The transmission decreases with increasing k in the region (I). There is also some transmission in the low-frequency part of the region (III) due to the mode tunneling. The fast-to-slow mode conversion takes place for intermediate k in region (I), and over all region (II). These results facilitate having an idea about the coupling of the wave modes as they propagate from sub-surface, high- β layers of sunspots, to the upper, low- β layers and help to interpret observational results for more complex magneto-atmospheres.

Fast-to-slow wave conversion within a modal description has been analyzed in great detail by Spruit and Bogdan (1992); Cally and Bogdan (1993); Cally *et al.* (1994); Cally and Bogdan (1997), with the aim to explain both the decrease of the amplitude (so-called “absorption”) and phase shift of p -modes observed to occur in sunspots (Braun, 1995; Cally *et al.*, 2003). Spruit and Bogdan (1992); Cally and Bogdan (1993); Cally *et al.* (1994) explore the possibility of the existence of trapped modes of oscillations in a sunspot atmosphere in the presence of a vertical magnetic field. These modes were named π -modes to distinguish them from quiet Sun waves, where the reason for the trapped modes is different. Fast magneto-acoustic-gravity waves experience reflection due to the rapid increase of the Alfvén speed with height, and also due to the increase of the acoustic speed with depth. On each passage through the equipartition $\beta = 1$ layer, slow longitudinal modes are produced by mode conversion and extract energy from the fast mode. Therefore, a part of the fast-mode energy remains trapped, and a part of it leaks away due to mode conversion, resulting in either spatial or temporal damping of oscillations. The problem was approached by prescribing a real wave number k and computing the complex frequencies (Cally and Bogdan, 1993), or by prescribing a real oscillation frequency ω forced by external p -modes and computing the complex wave numbers k (Cally *et al.*, 1994). These complex wave numbers produce a spatial decay of the wave from the borders of the umbra inwards, resulting in a visible “absorption” of the wave power.

Spruit and Bogdan (1992) considered absorption of waves by sunspots and the surrounding plage by means of the above model, and found that, for the f -mode, the absorption coefficient increases monotonically from small to large horizontal wavenumbers. The same phenomenon occurs along the p -mode ridges. Numerical simulations of the p and f -mode interaction with a vertical magnetic field concentration have demonstrated that, indeed, a significant power of the incident modes is converted into slow high- β magneto-acoustic modes propagating downwards along the magnetic field lines and leading to a visible power reduction at the surface (Cally *et al.*, 1994; Cally and Bogdan, 1997; Rosenthal and Julien, 2000). In addition, these simulations have shown that the fraction of the f -mode power transformed to slow modes is sufficient to explain its power reduction in observations of the f -mode absorption. On the contrary, p -modes were not transformed sufficiently in a vertical magnetic field in order to be explained by this mechanism.

7.2.3 Fast-to-slow mode conversion in an inclined magnetic field

Mode conversion causes important effects on the observed wave propagation in sunspots and active regions, therefore we discuss it here in more detail. As mentioned above, the bases of the MHD wave conversion theory in a stratified solar atmosphere were developed by Zhugzhda and Dzhililov (1982, 1984a,c) and Cally (2001). Later on this theory was improved and extended to the case of inclined magnetic fields, by means of the ray theory and eikonal approximation (e.g., Cally, 2005, 2006; Schunker and Cally, 2006; Cally and Goossens, 2008).

Consider the conversion from a fast high- β mode (essentially acoustic and isotropic) to a slow low- β mode (also acoustic, but propagating along magnetic field lines) magneto-acoustic wave in a two-dimensional situation. This situation is of practical interest for the theory of sunspot waves. Solar p -modes excited in sub-photospheric layers in the quiet Sun by convective motions are essentially acoustic. It is not clear whether sub-photospheric motions in sunspots excite their proper wave spectrum (see Section 8.1). Therefore, it is understood that quiet Sun p -modes propagate into the magnetized atmosphere of sunspots. This is a common setup of the problem

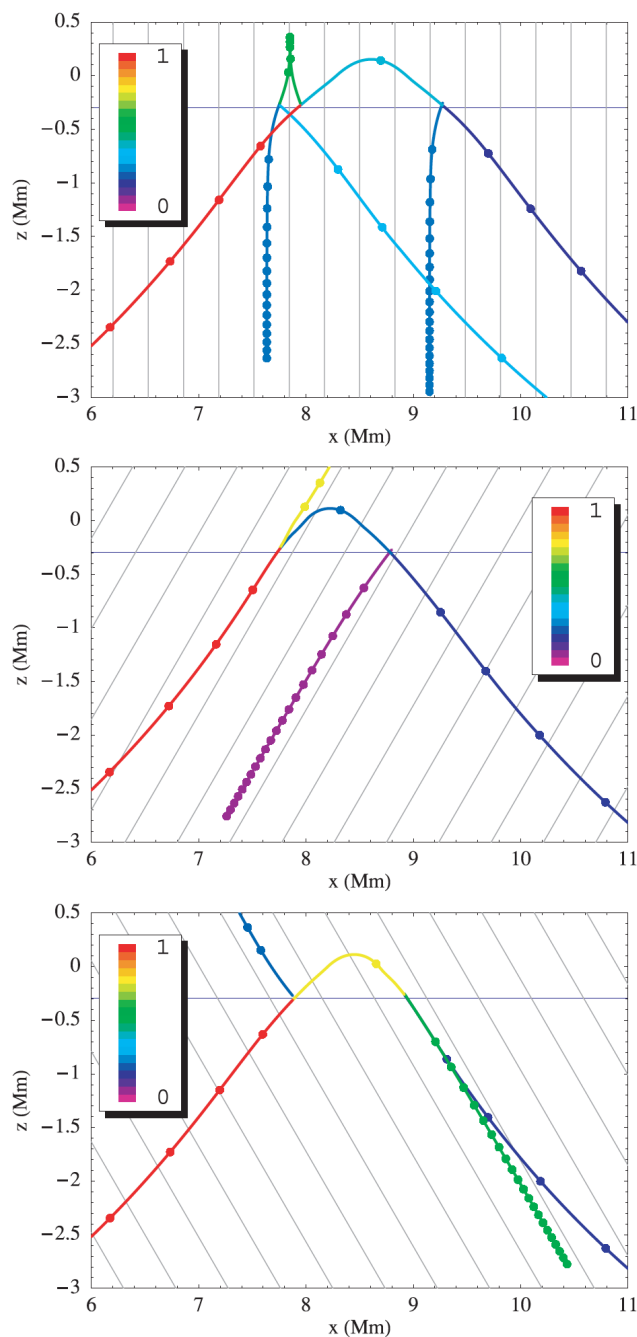


Figure 18: Ray path diagrams in $x - z$ space for 5-mHz acoustic (fast-mode) rays launched horizontally from $x = 0$, $z = -5$ Mm in the presence of a 2-kG magnetic field which is vertical (top panel), inclined at 30° (middle panel) and at -30° (bottom panel). The horizontal line at $z \approx -0.3$ Mm indicates the equipartition level. The dots on the ray paths represent 1-min time intervals, making it easy to distinguish fast from slow branches. The background straight gray lines represent the magnetic field. Image reproduced with permission from [Schunker and Cally \(2006\)](#), copyright by the authors.

solved in local helioseismology of sunspots. As we discuss below, mode conversion is believed to be responsible for the power redistribution observed in sunspots and surrounding plage regions. By means of the mode conversion theory it is possible to calculate the efficiency of the energy conversion of the fast acoustic mode, propagating and refracting from its lower turning point upwards to the surface, into slow longitudinal acoustic mode.

The ray theory shows that the direction and the efficiency of the mode conversion depends, among other parameters, on the wave frequency and the attacking angle between the wave vector \vec{k} and the magnetic field (e.g., Cally, 2005, 2006; Schunker and Cally, 2006; Cally and Goossens, 2008). In the two-dimensional case, an approximate formula for the fast-to-slow mode (acoustic to acoustic) transmission coefficient was derived by Cally (2005) for high-frequency waves (above the acoustic cut-off):

$$T = \exp\left(-\frac{\pi k_{\perp}^2}{k_z \frac{d}{dz} \left|\frac{c_s^2}{v_A^2}\right|}\right), \quad (12)$$

where $k_{\perp} = k_x \cos \theta - k_z \sin \theta$ is the component of the wave vector perpendicular to the magnetic field, and the attack angle is given by $\alpha = \arctan k_{\perp}/k$ with $k = \sqrt{(k_x^2 + k_z^2)}$. According to this equation, the fast-to-slow mode conversion is complete ($T = 1$) for waves with \vec{k} directed along \vec{B} . For larger angles α the efficiency of the fast-to-slow mode conversion rapidly decreases. The higher the frequency of waves (implying generally larger k), the smaller is the cone of ψ 's where the fast-to-slow mode conversion is effective. The above equation for the transmission coefficient is only approximate, in the full version T does not reach one at zero attack angle, see Hansen and Cally (193–202).

Figure 18 shows an example, taken from Schunker and Cally (2006), of the trajectory of the fast-mode ray incident from its sub-photospheric lower turning point to the surface through the $\beta = 1$ layer where a 2 kG magnetic field is either vertical (top), inclined by 30° (middle) or by -30° (bottom). The coloring of each ray indicates the amount of energy it contains after each transformation. By comparing the three panels, it can be seen that the most efficient conversion between fast and slow modes happens when the field inclination is 30° because the direction of propagation of the fast-mode ray is aligned with the magnetic field (i.e., the attack angle α is zero) (Crouch and Cally, 2003; Cally, 2006; Schunker and Cally, 2006). The most efficient secondary conversion from fast magnetic mode to the slow magnetic mode happens on the second passage of the fast mode through the $\beta = 1$ layer on its return produced after the refraction and reflection (see Section 7.1), for the -30° inclined field, again due to the almost perfect alignment between the ray and the field direction. That the p -mode conversion of high frequency waves is significantly enhanced by moderate magnetic field inclinations was also confirmed in the 3D analysis by Cally and Goossens (2008). All in all, the mode transformation extracts energy from the incident fast acoustic mode ray as it returns to the sub-surface layers.

The critical role of the magnetic field inclination for mode conversion is confirmed by numerical simulations of MHD wave propagation in the chromosphere by Carlsson and Bogdan (2006), where the authors considered oscillations with wavelengths comparable to magnetic field scales. At angles smaller than 30 degrees, much of the high- β acoustic-like fast-mode power is transformed into low- β slow modes propagating along the magnetic field lines. At larger inclination angles the low- β fast modes are refracted and reflected and return back to the photosphere. Note that larger inclinations correspond to regions where the field strength is smaller and the conversion layer is located higher in the atmosphere. When observed at a particular height, the interference pattern produced by the upward and downward propagating waves in these regions creates a ring of enhanced power around the magnetic field concentrations (Carlsson and Bogdan, 2006) which can be an explanation of the acoustic halos observed at the edges of active regions (see the discussion in Section 8).

7.2.4 Conversion to Alfvén waves

In a fully three-dimensional situation, mode conversion into the Alfvén mode becomes possible. For that, it is essential to have the magnetic field vector out of the wave propagation plane. Cally and Goossens (2008) pioneered the study of 3D mode conversion from fast to Alfvén waves for homogeneous inclined fields. They found that, for an acoustic (fast-mode) ray propagating from its sub-photospheric lower turning point, the conversion is most efficient for preferred magnetic field inclinations between 30 and 40 degrees, and azimuth angles between 60 and 80 degrees. This orientation allowed the best alignment between the wave vector of the mode, \vec{k} , with the magnetic field, \vec{B} . The theoretical study by Cally and Goossens (2008) has shown that Alfvénic fluxes transmitted to the upper atmosphere were found to be similar, or larger, than acoustic fluxes at some orientations. Newington and Cally (2010) showed that low-frequency gravity waves can also be converted efficiently into Alfvén waves at large magnetic field inclinations. The inclusion of radiative losses into the model does not change significantly the wave paths or the conversion picture (Newington and Cally, 2011).

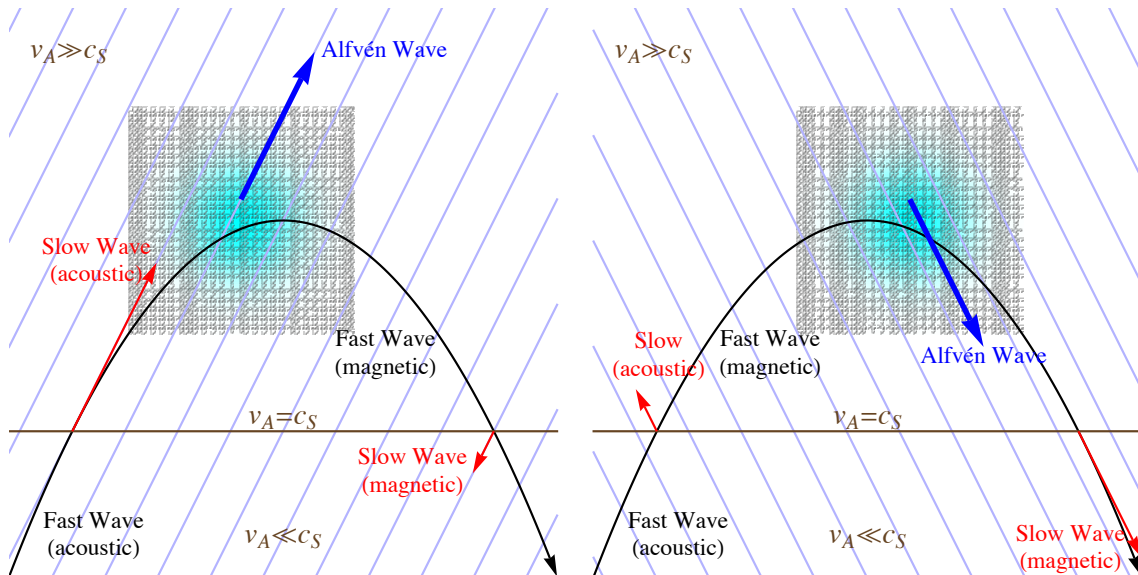


Figure 19: Schematic diagram illustrating the various mode conversions and reflections as the acoustic (fast-mode) ray enters the solar atmosphere in a region of strong inclined magnetic field. Field lines (pale blue) are oriented out of the plane and are shown here in projection, with the azimuthal angle $\phi < 90^\circ$ in the left panel and $\phi > 90^\circ$ in the right panel. The fast wave experiences reflection above $\beta = 1$ layer due to the rapidly increasing Alfvén speed with height. In a nebulous region around and above the fast wave reflection point, fast-to-Alfvén conversion occurs, in the case on the left predominantly to an upgoing Alfvén wave. For the $\phi > 90^\circ$ (right), the downgoing Alfvén wave is favored. The fast-to-Alfvén conversion may only occur where the wave vector and the magnetic field lines are not in the same vertical plane. Image reproduced with permission from Khomenko and Cally (2012), copyright by AAS.

Figure 19 summarizes the properties of this conversion for the case of two different orientations of the magnetic field, taken from Khomenko and Cally (2012). While the fast-to-slow mode conversion occurs at the layer where plasma $\beta = 1$, the fast-to-Alfvén conversion takes place around and above the fast wave reflection height z_{ref} in 3D (see Figure 19), localized close to z_{ref} as the frequency increases (Cally and Hansen, 2011). For 3–5-min waves, the conversion process is

typically spread over much of the chromosphere. Both up-going or down-going Alfvén waves can be produced depending on the relative orientation of the wave vector and magnetic field at heights where the fast mode undergoes the refraction, going up and down.

Numerical simulations of conversion to Alfvén waves require 3D or, at least, 2.5D geometries. Such simulations were recently performed by Felipe *et al.* (2010b); Felipe (2012); Khomenko and Cally (2011, 2012). The 2.5D simulations by Khomenko and Cally (2012) show that the global picture of conversion to Alfvén waves remains valid when considering more complex field geometries, appropriate for large-scale sunspot magnetic structure. The conversion to Alfvén waves is particularly important for strongly inclined fields like those existing in sunspot penumbrae. In the 3D numerical simulations by Felipe *et al.* (2010b) some Alfvénic power is produced by a point wave source located in a sunspot model far from the axis. In this example, the Alfvén wave generation is not efficient, due to the particular geometry of the background model at the source location (small azimuth angles).

Felipe (2012) performs simulations similar to Khomenko and Cally (2012), but fully in 3D, covering a large portion of a sunspot model with a wide range of inclinations. This simulation demonstrates that a significant energy flux due to Alfvén waves can indeed be transmitted to the chromosphere at large field inclinations. The upward-propagating flux due to Alfvén waves at the peripheric parts of the sunspot model is found to be as large as the acoustic flux due to slow longitudinal waves propagating close to the axis.

7.3 Resonant absorption

Resonant absorption is yet another mechanism of interaction of quiet Sun waves with a sunspot flux tube. As is mentioned in Section 5, sunspots are often discussed in a local helioseismological context as “absorbers” of the external p -mode power (Braun *et al.*, 1987, 1988). Comparing the amplitudes of the waves traveling in and outside of the sunspot, the latter authors determined that up to 50% of the p -mode power can be lost after these waves travel through the sunspot atmosphere. Resonant absorption of sunspot waves was proposed for the first time by Hollweg (1988) by analogy to waves in thin flux tubes. This mechanism acts when the flux tube boundary has non-zero thickness, and mode conversion happens at the boundary between the tube and the external quiet Sun atmosphere, taking energy out from the incoming p -modes. The mechanism of resonant absorption is frequently used in coronal studies to explain the damping of coronal loop oscillations (Nakariakov and Verwichte, 2005).

The typical geometry of sunspot-wave interaction for the model of resonant absorption is given in Figure 20 (for a plane cartesian geometry). The physics of the resonant layer is the following. One may think of external pressure fluctuations associated to p -modes as a harmonic driver affecting individual magnetic field lines in the resonant layer. If the oscillations induced into each individual field line are in phase, the resonance occurs and the energy is taken out from the driver, and the external wave is “absorbed” or “damped”. This process should not be confused with a damping due to dissipative processes, since the energy of the initial wave is converted into the energy of another wave mode (Alfvén mode or slow mode) in the resonant layer, and not into heat. Unlike the mode conversion mechanism discussed in detailed above, the resonant absorption happens at the tube walls (Figure 20) and not at the horizontal layer where plasma β approaches unity (Figures 18 and 19). The alikeness between the mode conversion and resonant absorption was recently discussed by Cally and Andries (2010).

Hollweg (1988) finds that the resonant absorption in sunspots can be important under certain circumstances, i.e., when the waves approach the magnetic field at an angle not exceeding 40 degrees, and some restrictions on the sunspot internal density and specific heat ratio. He concludes that this mechanism is unlikely to be the primarily one to explain the observed power deficit in sunspots.

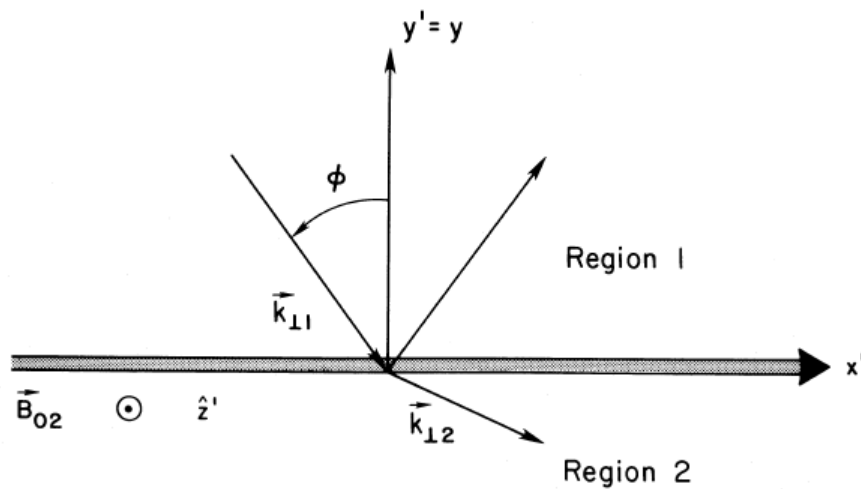


Figure 20: Geometry of sunspot-wave interaction for the model of resonant absorption. Region 1 is a field-free region, Region 2 is a sunspot with vertical magnetic field directed outside of the plane. The gray region around x -axis is the resonant payer. The perpendicular component of the incident and transmitted wave vectors are also indicated. Image reproduced with permission from [Hollweg \(1988\)](#), copyright by AAS.

Since the gradients are large in the transition layer, it is reasonable to assume that the dissipation mechanisms (viscosity, resistivity) will be enhanced, and therefore waves can be damped and converted into heat. The large gradients may also drive instabilities and turbulence. Then, the actual physical damping of the converted waves will be produced in the resonant layer. In subsequent works, the idealized model by Hollweg was improved by [Lou \(1990\)](#) by adding the effects of viscosity acting on Alfvén waves in the resonant layer. The new estimations of the absorption from this model give about 40–50%, which is a value similar to observations. [Goossens and Poedts \(1992\)](#) confirm this result in resistive MHD, showing that the amount of absorption is independent on the damping mechanism (viscosity or resistivity).

Twisting of the magnetic field lines also facilitates resonant absorption since it allows for the absorption of the axisymmetric ($m = 0$) modes at the Alfvén resonance ([Chitre and Davila, 1991](#); [Goossens and Poedts, 1992](#)). The latter authors have shown that resonant absorption is more efficient for larger sunspots with twisted magnetic field. Further improvements by [Stenuit *et al.* \(1993\)](#) reveal that by increasing the strength of the azimuthal magnetic field in the flux tube equilibrium, total absorption can be achieved over a relatively wide range of spot radii.

Resonant absorption gets even more efficient if, instead of a single monolithic structure, one assumes a sunspot made of fibrils ([Rosenthal, 1990, 1992](#); [LaBonte and Ryutova, 1993](#)). The analytical calculations by [Rosenthal \(1992\)](#) reveal that efficient absorption is possible even when the mean magnetic flux density is not so large in the case this flux is concentrated into slender fibrils. Again, twist is found to increase the absorption in this fibril model. [Rosenthal \(1992\)](#) concludes that resonant absorption can be an efficient mechanism acting in monolithic sunspots, fibril sunspots, as well as plage fields. All these models do not take gravity into account though. Wave absorption by fibril field is substantially modified in a gravitationally stratified medium ([Felipe *et al.*, 2013](#); [Hanson and Cally, 2014a,b](#)).

The ability of helioseismology to distinguish between the monolithic and the spaghetti models of sunspot based on phase shifts and absorption of waves was recently evaluated by [Felipe *et al.* \(2014\)](#). The authors find that the difference in absorption coefficient between both models can

be detected above the typical observational noise level, and that the phase shifts are also rather different. Therefore, different magnetic structures leave different fingerprints in the wave velocity field and can be used to reveal the internal structure of sunspots in future observations.

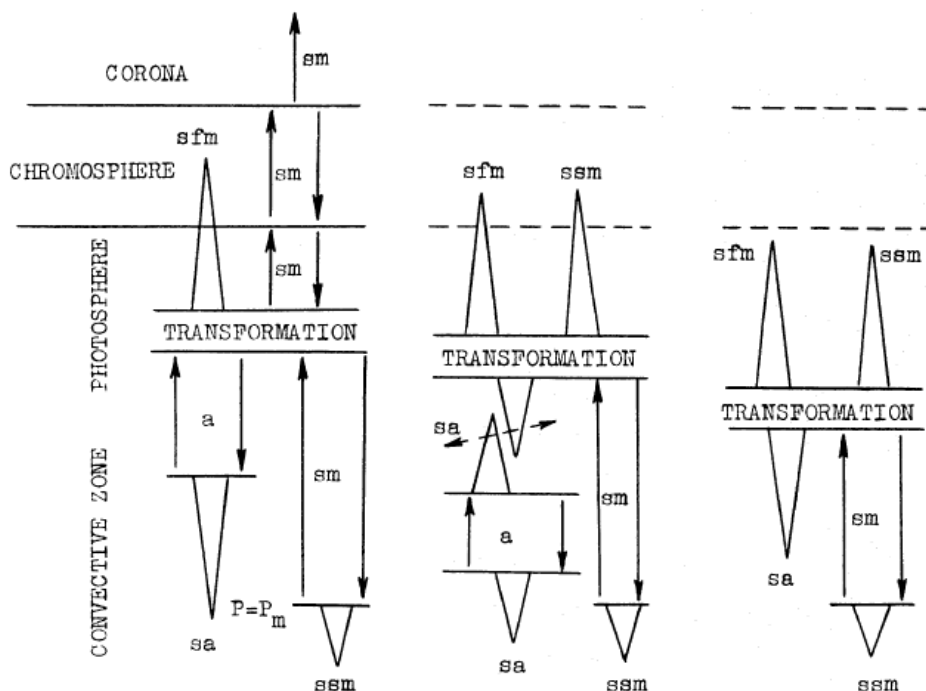


Figure 21: Schematic picture of resonance zone in the sunspot atmosphere for waves with 3-min (a), 5-min (b) and 30-min (c) period. The abbreviated mode classification is: *sm* (slow mode), *sfm* (surface fast mode), *ssm* (surface slow mode), *a* (atmospheric mode), *sa* (surface atmospheric mode). Image reproduced with permission from Zhugzhda (1984), copyright by RAS.

7.4 Resonance cavity model for chromospheric waves

Since real atmospheres are vertically stratified, they can produce reflections and the picture of trapped modes in the umbra has attracted a lot of attention in the past. The characteristic 3- and 5-minute periodicities of sunspot waves were suggested to be the evidence of the presence of resonance modes. This produced a discussion in the literature in the 1980s–1990s about the nature and particular properties of different resonances, in the context of their ability to describe the particular observed frequency distribution in the spectrum of photospheric and chromospheric oscillations.

Zhugzhda and Dzhililov (1982) and Zhugzhda (1984) define resonance layers for different types of waves and different frequencies, summarized in Figure 21. The nomenclature used by Zhugzhda, as given in the figure, is somewhat different from what we have used so far. That is, *sm* (slow mode) is used both in the low and upper atmosphere in the high- and low- β plasma, we note that it is essentially magnetic in the low atmosphere and essentially acoustic in the upper atmosphere. The atmospheric mode denoted by *a* is actually fast magneto-acoustic mode, that can be subdivided into acoustic, gravity and evanescent (or surface) acoustic modes (the latter are referred to as *sa*). The appendix “surface” is used to indicate that the mode is not propagating in the vertical direction. In the case of the *sfm* (surface fast mode), it refracts and reflects due to the

Alfvén speed gradients as discussed above. In the case of *ssm* (surface slow mode), the mode is affected by the cut-off frequency and therefore becomes evanescent. The figure defines three types of resonances, depending on the wave frequency.

Photospheric fast-mode resonance. This resonance layer is similar to that of acoustic-gravity waves defined by [Leibacher and Stein \(1981\)](#) if the field is sufficiently weak, i.e., the reflection at the lower boundary occurs due to acoustic speed gradients and the reflection at the upper boundary is due to cut-off frequency effects. If the field is strong and vertical, the conditions of the resonance change due to the presence of the transformation layer in between the reflection layers (middle panel of [Figure 21](#)). There some energy can escape into the upward and downward propagating slow modes. Depending on the frequency range of the considered waves, and on the strength of the field, the relative location of the boundaries of the resonance layer and the transformation layer changes, and can affect the resonance frequencies to a smaller or larger degree. According to [Zhugzhda and Dzhililov \(1982\)](#); [Zhugzhda \(1984\)](#), the fast-mode resonance is essential in producing waves in the 5-min band. Conversely, [Thomas and Scheuer \(1982\)](#); [Scheuer and Thomas \(1981\)](#) calculated numerically the fast-wave resonance, under the assumption that the reflection at the lower boundary is again due to acoustic speed gradients, and the reflection of the upper boundary is due to Alfvén speed gradients, and concluded that this resonance produces a lowest mode with a period of 153 s, i.e., in the 3-min band.

Chromospheric slow-mode resonance. The chromospheric resonance, as proposed by [Zhugzhda and Locans \(1981\)](#) and [Zhugzhda et al. \(1983\)](#), is due to reflections of high-frequency (periods around 3 min and lower) slow magneto-acoustic waves due to the temperature gradients around the photospheric temperature minimum, and at the transition region (see left panel of [Figure 21](#)). This resonator is completely located in the strong-field zone. The resonator is not perfect since the reflections are not total and the waves are propagating both below and above the resonator. Partial reflections define both the frequency of the trapped waves and the quality of the resonator. The chromospheric slow-mode resonator affects the fast-mode resonator through the transformation layer where the modes are coupled. [Locans et al. \(1988\)](#) calculated the spectrum of oscillations trapped by the chromospheric resonance cavity, and compared with observed oscillation frequencies by different authors, concluding that a general agreement exists. According to [Staude et al. \(1985\)](#), different assumptions about the structure of the transition region do not modify the resonant frequencies, but rather the amplitudes and the phases of oscillations, which allowed these authors to check different models of the structure of the chromosphere and transition region by comparing observations with the theory of chromospheric slow-mode resonator. The theory of the chromospheric resonator was followed by [Gurman and Leibacher \(1984\)](#) and [Gurman \(1987\)](#). The latter work reports evidence for upward and downward propagating waves from observations in Mg II K line, as a further observational evidence for the resonance transmission model (in addition to the measured wave frequencies in the 3-min band).

[Scheuer and Thomas \(1981\)](#) and [Thomas and Scheuer \(1982\)](#) raised the controversy if 3-min umbral oscillations are due to a photospheric or a chromospheric resonance, suggesting that 3-min oscillations modes can be the result of the former as well. In their picture, the photospheric resonance upper reflection layer is due to the Alfvén speed gradients acting on the fast magnetic mode, leading to the lowest mode period of 153 seconds. The reflections from the transition region are unimportant for such a resonance and the chromospheric 3-min oscillations would be due to the slow-mode leakage along near vertical magnetic field lines. [Thomas \(1984\)](#) claims that observations of upward phase propagation of umbral oscillations in the chromosphere are in agreement with the photospheric resonance theory of [Thomas and Scheuer \(1982\)](#) and in contradiction to the chromospheric resonance theory of [Zhugzhda et al. \(1983\)](#).

Hasan (1991) tried to resolve to some extent the above discrepancy by performing calculation of the eigen modes of an umbra with a vertical temperature stratification described by Maltby's semi-empirical model (Maltby *et al.*, 1986). In the photosphere, he finds the resonant modes to be essentially fast, producing oscillations both with 5 and 3-min periods, while in the chromosphere the modes are slow acoustic ones with periods in the 3-min range. In a same vein, Lee and Yun (1987) proposed a model of two resonant cavities of the umbral 3-min oscillations, consisting of a weak-field cavity in the photosphere for fast modes and strong-field cavity in the chromosphere for slow modes. Further developments of the resonance cavity models were presented by, e.g., Wood (1990, 1997); Settele *et al.* (1999, 2001); Christopoulou *et al.* (2003); Yelles Chaouche and Abdelatif (2005); Botha *et al.* (2011).

Sub-photospheric slow-mode resonance. Yet another slow-mode resonance cavity may exist for slow magneto-acoustic waves propagating in the high- β region with essentially Alfvén speed, which is of special interest for low-frequency waves. Zhugzhda (1984) suggests that for typical conditions below sunspot umbrae the resonant period will be around 30 min and might be related to the life time of umbral dots (see right panel of Figure 21). Due to the rapid decrease of the wavelength of the slow mode with depth as the density increases, these waves might be able to penetrate sufficiently deep in the atmosphere if not destroyed by dissipative processes and not converted into turbulence.

Chromospheric Alfvén mode resonance. A resonance model for Alfvén waves trapped in the overstable region between the sub-photosphere and the transition region is developed by Uchida and Sakurai (1975) considering those waves to be generated by convection motions in the umbra. It was shown that the fundamental frequency of such resonator happens at 140–180 s and does not depend much on the parameters of the sunspot.

All in all, presently, it is considered that there is no strong observational support for an umbral resonance cavity model of any kind. As mentioned by Bogdan (2000), earlier observations tended to relate the multiple power peaks in the oscillation spectra as an evidence of discrete modes, as in the quiet Sun. However, the precise positions of the peaks change significantly from one observation to another, and even in time within the same observational sequence (see Section 2). In addition, an essentially upward propagation is observed for waves in the chromosphere and above. It is more probable that umbral oscillations are just a response due to forcing from outside p -modes and weak convection in the umbra rather than eigen modes of the umbra itself. A high-resolution $k-\omega$ diagram is a tool to resolve this issue. A successful attempt to construct such a diagram from sunspot data taken by SDO/HMI, reported recently by Zhao and Chou (2013), convincingly shows that the positions of the ridges are quite similar to the quiet Sun, except for the displacement produced by a different phase speed of waves beneath sunspots. Therefore, the latest data not seem to confirm the presence of a particular resonant cavity in the sunspot atmosphere.

7.5 Oscillations of sunspot entire flux tube

The nature of long-period oscillations with periods of the order of 100 of minutes (see Section 6) is far from being clarified. There are very few theoretical developments in this direction and further work of this puzzling feature is definitely needed.

It has been proposed that the long-period oscillations, of 60–80 min and larger, are representative of global vertical-radial oscillations of a magnetic element (spot, pore) as a whole around its position of stable equilibrium Solov'ev and Kirichuk (2008, 2009, 2014). Such oscillations are reminiscent of sunspot flux tube emergence and formation process, and represent deviations from the equilibrium position of the mass of gas from the region of the sunspot Wilson's depression. The

model by Soloviev & Kirichek is based on the following assumptions: (1) the forming sunspot undergoes radiative cooling in the photospheric layers which results in compression and a formation of a converging flow below the surface; (2) the penumbra is a surface feature and does not contribute significantly to the variations of sunspot energy; (3) a Wilson depression of a few hundreds of km is present; (4) the sunspot flux tube widens both at its upper end in the photosphere, but also below the photosphere where a presumably a hot zone exist (Kosovichev *et al.*, 2000). As the density is taken constant in the model, the gas pressure near the upper edge of a hot region necessarily exceeds the external gas pressure resulting in a tube widening sharply in sidewise directions at the boundary of the hot zone, extending inside the convective zone in an extremely weakened and scattered form. The schematic representation of this sunspot model is given in Figure 22.

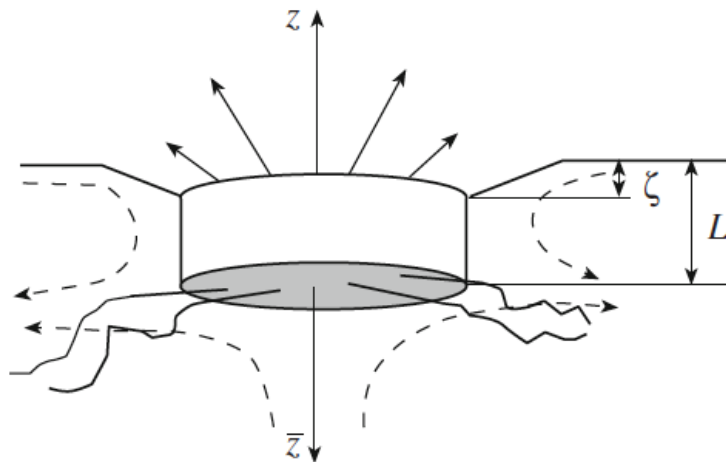


Figure 22: Schematic representation of the shallow-sunspot model. ζ is the Wilson depression and L a typical length. At depths $\zeta < h < L$, the spot, as a region of cooler plasma and strong magnetic field, is approximated as a straight cylinder of radius a . Below the level L , the magnetic lines of force become irregular. The dashed curves show plasma flows converging to the spot at depths above L and ascending, diverging flows in deeper layers under the spot. Image reproduced with permission from Solov'ev and Kirichek (2009), copyright by Pleiades.

Solov'ev and Kirichek (2008, 2009, 2014) suggest that the stability of the sunspot is provided by the cooling of the sunspot plasma and decreasing of its gravitational energy due to the vertical redistribution of the gas density when the geometric Wilson depression of the sunspot is formed. As the sunspot compresses and submerges to a larger depth the magnetic energy of the system increases, but the potential gravitational energy decreases. The balance between the two determines the equilibrium against the vertical displacement⁴.

The eigenfrequency of oscillations of sunspot as a whole is determined as $\omega \approx \sqrt{\kappa/M}$ where M is the mass of gas in the sunspot, $M = \pi a^2 L \rho$ (L is a typical vertical scale length, a is sunspot umbra radius, ρ is the average mass density), and κ is the so-called elasticity equal to the full magnetic tension applied to the system divided by the scale length, $\kappa = \pi a^2 B^2 / (4\pi L)$, see Figure 22. The

⁴ Apart from the oscillation analysis, the model by Solov'ev and Kirichek (2014) derives conditions of sunspot stability as a function of their radius and magnetic field strength. It shows that the magnetic field strength in sunspot varies monotonically with radius from about 700 G up to an asymptotic limit of about 4000 G. The depth of the Wilson depression grows linearly with B . The range of stable equilibria is limited in a way that larger sunspots (radius larger than about 12–18 Mm) are unstable, which may explain the absence of very large sunspot on the Sun, as well as the appearance of the light bridges in large sunspots dividing them in several parts. Sunspots with B in the range of 2.6–2.7 kG and an umbral radius of about 2 Mm are the most stable.

typical period of oscillations given by this formula is of 1.5 hours, i.e., falls in the range of periods derived in observations of long-period oscillations described in Section 6.

The assumptions underlining Soloviev & Kirichek model about the existence of a convergent flow below the sunspot and the existence of hot zone are derived from local helioseismology studies. The former is a controversial result since observations of the visible sunspot layers show the Evershed and moat outflows rather than a converging inflow. Likewise, the existence of the hot zone had been questioned. The interpretation of the inversion results of time-distance helioseismology encountered major critics when applied to magnetic active regions of the Sun. Since magnetic field modifies the wave propagation speeds in a similar way as the temperature perturbations do, the magnetic and temperature effects on the wave travel times are difficult to separate (see, e.g., [Moradi and Cally, 2008](#)). It is not clear, however, how the break of the above assumptions affect the model by [Solov'ev and Kirichek \(2008, 2014\)](#) since no studies exist so far, and no other models have been proposed, up to our knowledge.

8 Discussion of Observations and Theory of Sunspot Waves

In view of the current theoretical models discussed above, some of the questions raised in the introduction can be answered.

8.1 What drives the waves observed in sunspots?

No clear answer to this question exists so far, but it is plausible that both, magneto-convection in the umbra and external p -modes contribute to the observed wave spectra in sunspots. The quiet Sun waves undergo modifications in the magnetized sunspot atmosphere.

8.1.1 Magneto-acoustic waves

It is generally accepted that quiet-Sun oscillations are stochastically excited by solar convection at the top part of the convection zone (Goldreich and Kumar, 1990; Nordlund and Stein, 2001). The frequency spectrum and the temporal behavior of waves in the umbra is very similar to the quiet Sun, except for their reduced power (see Sections 2 and 3). It is systematically found in observations that more wave power travels toward the umbra than leaves the umbra in horizontal direction (Section 5). One of the most conclusive examples of the $k - \omega$ diagram in sunspots recently constructed by Zhao and Chou (2013) from SDO/HMI data shows a very similar system of ridges with reduced power at the location of p -modes, and, especially of the f -mode. The $k - \omega$ diagram also shows that convection in sunspots is significantly suppressed and its spectrum is shifted to large spatial scales. Therefore, it is natural to assume that sunspot waves may be driven by external p -modes, modified inside the magnetized sunspot atmosphere.

As discussed above, in Sections 7.2.2 and 7.2.3, many theoretical investigations of sunspot waves have been performed under the assumption of incoming p -mode waves incident on a sunspot-like magnetic field concentration (e.g., Cally *et al.*, 1994; Cally and Bogdan, 1997; Rosenthal and Julien, 2000). This has been a standard setup in wave mode conversion studies and in local helioseismology when studying the “absorption” and scattering of waves by sunspots (Gizon and Birch, 2005).

Despite the presence of strong magnetic fields in sunspot’s umbrae and penumbrae, magnetoconvection models indicate that these environments can be convective (Savage, 1969; Moore, 1973; Lee, 1993; Weiss *et al.*, 1990; Cattaneo *et al.*, 2003; Schüssler and Vögler, 2006; Spruit and Scharmer, 2006), see the review by Rempel and Schlichenmaier (2011). Narrow nearly field-free upflowing umbral dots adjacent to downflows can be the result of convection in a strong magnetic field (Weiss *et al.*, 1990). Recent observations seem to confirm this idea (see, e.g., Bharti *et al.*, 2007; Watanabe *et al.*, 2009, 2012). In particular, there are measurements suggesting that the upflowing mass flux of umbral dots and the downflowing flux in their surroundings can be balanced, which is characteristic for magnetoconvection (Riethmüller *et al.*, 2013). Thus, some waves can be generated inside the sunspot umbra as well. Jacoutot *et al.* (2008) performed numerical simulations of magnetoconvection and studied the spectra of the generated waves as a function of the magnetic flux in the model. Apart from the power suppression in regions with enhanced magnetic field, these simulations suggest an increase of high-frequency power (above 5 mHz) for intermediate magnetic field strengths (of the order of 300–600 G) caused by changes of the spatial-temporal spectrum of turbulent convection in a magnetic field, see Figure 23.

Thus, it can be concluded that both external driving and in-situ excitation by convection are able to explain the observed properties (e.g., power reduction) of waves in sunspots. A qualitative estimation of the effect of the reduced wave excitation in sunspots and the direct comparison to observations was reported by Parchevsky and Kosovichev (2007). The authors performed hydrodynamical simulations of waves generated by random sources, with the strength of the sources reduced to zero in the sunspot umbra. They obtained that, even though no waves were excited in the umbra, the velocity power measured there was about two times larger than actually observed.

Thus, the waves detected in sunspots may be a mixture of external p -modes (and other wave types produced by the mode conversion) and MHD waves generated directly inside the sunspot by weak convection. The relative contribution of the different effects is to be determined.

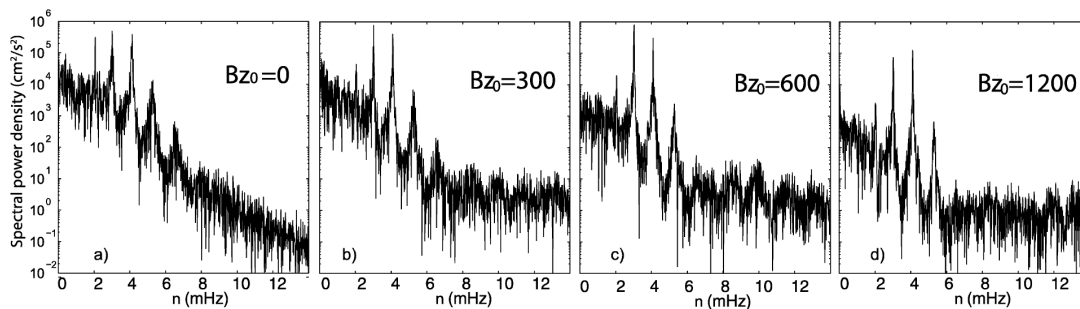


Figure 23: Power spectra of the horizontally-averaged vertical velocity at the visible surface in the simulations of magnetoconvection by [Jacoutot et al. \(2008\)](#). Different panels are for simulations with different initial vertical magnetic fields (indicated). The peaks on top of the smooth background spectrum of turbulent convection represent oscillation modes: the sharp asymmetric peaks below 6 mHz are resonant normal modes, while the broader peaks above 6 mHz, which become stronger in magnetic regions, correspond to pseudomodes. Image reproduced with permission from [Jacoutot et al. \(2008\)](#), copyright by AAS.

8.1.2 Excitation by solar flares

Solar flares have been seen to excite waves in the quiet Sun from the interior up to coronal heights ([Kosovichev and Zharkova, 1998](#)). There are also occasional observations of excitation of waves by flares in the chromosphere of sunspot umbra and penumbra ([Kosovichev and Sekii, 2007](#)). These events are relatively rare and difficult to detect. In the example shown by [Kosovichev and Sekii \(2007\)](#), a two ribbon flare X-class flare excited oscillations in the nearby sunspot with amplitudes 2–4 times larger than before the flare, and with higher frequencies as well. Due to the low cadence of observations, it was impossible to identify the type of the excited modes. However, using data recorded by the Nobeyama Radioheliograph at 17 GHz, [Sych et al. \(2009\)](#) measured a gradual increase in the power of the 3-min oscillations in a sunspot just before bursty flares with a period of 3 minutes took place. Thus, these authors suggested that the oscillations follow from sunspots towards the flaring site, rather than the flare triggering the oscillations. More effort is needed in the future to study these particular events and to make practical use of these measurements to infer sunspot properties.

8.1.3 Alfvén waves

In the quiet Sun, a standard picture suggests that Alfvén waves are generated by direct granular buffeting in the photosphere in large amounts ([Narain and Ulmschneider, 1996](#); [Cranmer and van Ballegooijen, 2005](#)). These Alfvén waves propagate upwards along the flux tubes, partly reflect at the transition region, and partially penetrate into the solar corona, and drive the solar wind. A less standard picture questions this paradigm since the low ionization fractions in the photosphere (one ionized particle over 10^4 neutrals) leads to the weakening of the coupling between the magnetic field and the gas and may decrease the rate of generation of Alfvén waves ([Vranjes et al., 2008](#)), but see [Tsap et al. \(2009, 2011\)](#); [Zaqarashvili et al. \(2011, 2013\)](#).

The generation of Alfvén waves is apparently even less efficient in the magnetized atmospheres of sunspots. Back in the 1960s, it was proposed that some form of mechanical energy transport

must act in sunspots derived from the impossibility to construct sunspot modes in static equilibrium with only radiative transport of energy (Schlüter and Temesváry, 1958) and also because of the observational detection of motions in the umbra. It was suggested that sunspots might be cooled by downward propagating Alfvén waves (or other magneto-acoustic waves) generated by overstable convection in the umbral sub photosphere (Parker, 1974a,b, 1975a,b). This suggestion was based on earlier works by Musman (1967); Savage (1969), and then followed in the literature for some years (see, e.g., Thomas, 1978; Wilson, 1975; Beckers, 1976) until finally rejected due to the lack of observational confirmation (Beckers and Schneeberger, 1977; Nye and Hollweg, 1980; Bel and Leroy, 1981). In fact, in his earlier paper, Musman (1967) suggests that the generation of Alfvén waves is efficient only under very specific conditions, making this mechanism of sunspot cooling unlikely. The flux density of such waves in observations was found to be essentially directed upwards, and not downwards, and was too small to produce efficient cooling (Nye and Hollweg, 1980; Bel and Leroy, 1981).

As of today, no clear observational detection of Alfvén waves in the sunspot lower layers has been made, and the mechanisms of their generation by magnetoconvection in the umbral sub-photosphere do not seem to be efficient. Yet, it is still challenging to determine if a significant flux of these waves can be transported into the solar corona in sunspots and play a role in the heating of these regions.

Another mechanism of Alfvén waves generation via conversion from fast acoustic waves, proposed recently by Cally and Goossens (2008), is discussed above in Section 7.2.4.

8.2 What types of MHD waves do we observe in sunspots?

Detection of one or another wave type depends on the particular conditions of the sunspot atmosphere and the height where the waves are observed. In this respect, it is crucial to know the location of the $\beta = 1$ surface relative to the height sampled by the observed spectral line (or spectral window in the case of filter observations), as well as the configuration of the magnetic field lines.

Measurements of the plasma β in sunspots are not frequently reported in the literature, because of the difficulties associated with spectropolarimetric inversions, and with the establishment of the zero height reference at the different sunspot parts, depending on the amount of the Wilson depression. One of the successful measurements was reported by Mathew *et al.* (2004), see Figure 24. By means of spectropolarimetric inversions of the pair of the infrared Fe I lines at $1.56 \mu\text{m}$ these authors established that the plasma β at heights of the continuum formation of radiation at $1.56 \mu\text{m}$ is below 1 in most of the umbra and inner penumbra reaching values above 1.5 in the outer penumbra. Jess *et al.* (2013) also estimated that the contours of plasma $\beta = 1$ in the photosphere comprise the whole visible continuum sunspot region, including its umbra and penumbra (see white dashed line in the continuum image in Figure 11 in Section 3). Most photospheric lines used typically in observations (like Ni I 6768 Å used by SOHO/MDI) are formed higher in the atmosphere, indicating that oscillations observed in sunspot umbrae should be due to the low- β MHD waves. The situation is more complex in the penumbra where the plasma β should be around unity at the typical heights of formation of photospheric spectral lines. Plasma β is definitely below 1 at chromospheric heights where running penumbral waves are detected. In the surrounding plage region, it is reasonable to expect that $\beta > 1$ in the photosphere and low chromosphere. It is therefore possible that, in the photosphere where the plasma β is close to or below 1, different types of observations favor the detection of different wave modes.

The analysis of magnetic field oscillations in a sunspot obtained from spectropolarimetric observations of Fe I $1.56 \mu\text{m}$ lines by Khomenko *et al.* (2003), led them to conclude that the waves observed in the deep photosphere are a mixture of fast and slow low- β waves. The contribution of slow low- β magneto-acoustic waves is larger in the sunspot umbra while the contribution of the

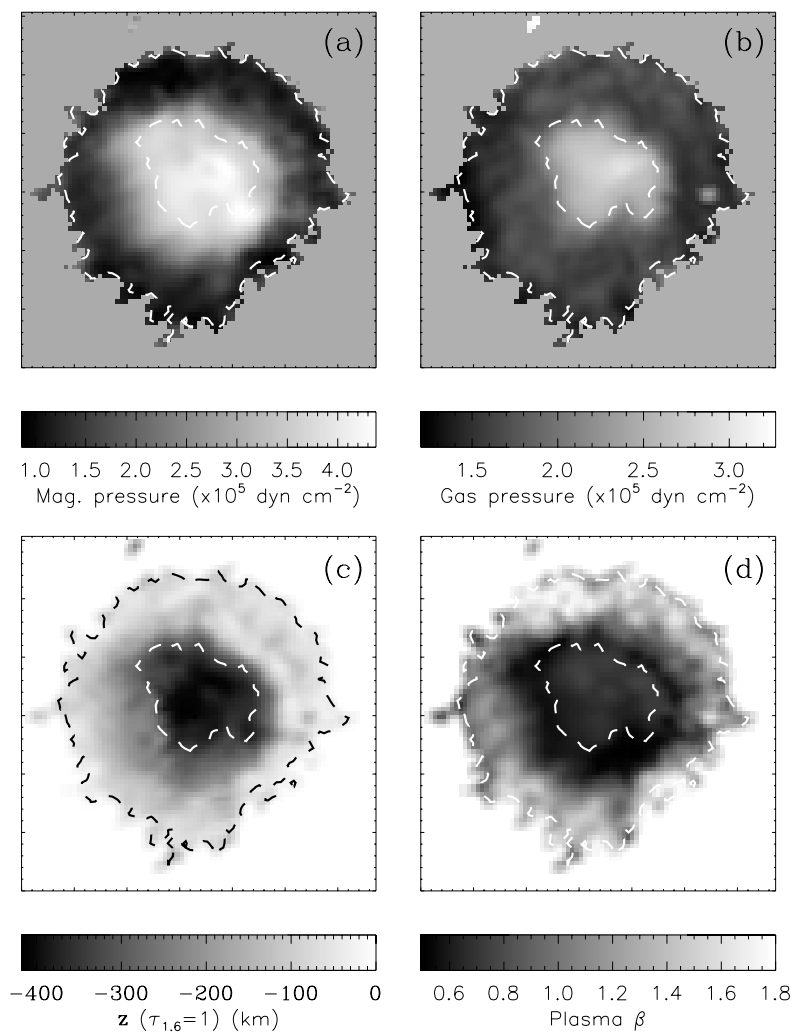


Figure 24: Magnetic pressure (a); gas pressure (b), Wilson depression (c) and plasma β (d) for a sunspot in FeI $1.56 \mu\text{m}$ and calculated via spectropolarimetric inversion. Image reproduced with permission from Mathew *et al.* (2004), copyright by ESO.

fast magneto-acoustic mode increases at the border of the umbra and penumbra.

Simultaneous observations of the umbra in the photosphere and chromosphere point toward the detection of slow magneto-acoustic waves propagating longitudinally along the magnetic field lines (Centeno *et al.*, 2006; Felipe *et al.*, 2010a; Bloomfield *et al.*, 2007b,a; Jess *et al.*, 2013), see Sections 2 and 3. In the umbra, spectropolarimetric observations by Centeno *et al.* (2006) and Felipe *et al.* (2010a) unequivocally show the gradual propagation of slow waves (with the phase speed that is expected for acoustic waves) vertically from the photosphere to the chromosphere at frequencies above the cut-off frequency of the umbra, 4 mHz.

In the penumbra, Bloomfield *et al.* (2007b) obtained that the propagation speeds of waves in the deep penumbral photosphere most closely resemble the fast-mode speeds of the modified p -modes, i.e., high- β acoustic-like waves propagating toward the sunspot at an angle ~ 50 degrees to the vertical. In a subsequent paper, Bloomfield *et al.* (2007a) concluded that running penumbral waves are the visible pattern of low- β slow mode waves propagating and expanding their wavefront along the inclined magnetic field lines in the penumbra. A similar interpretation of the running penumbral waves was also suggested by Bogdan and Judge (2006). Recently, Reznikova *et al.* (2012) and Jess *et al.* (2013) have shown a very good agreement between the cut-off frequency of acoustic waves modified according to the inclination of the penumbral magnetic field lines by a factor of $\cos \theta$, and the actual observed frequency of penumbral waves, providing therefore a strong argument that running penumbral waves are slow magneto-acoustic modes propagating along the inclined magnetic field of the penumbra.

When waves are observed at a single photospheric level, like the typical measurements done in local helioseismology, the correlation between the wave fronts is searched in horizontal direction (Gizon and Birch, 2005). Such measurements favor the detection of fast high- β magneto-acoustic modes (modified p -modes) (Khomenko *et al.*, 2009a). In the quiet Sun, these modes are usual acoustic-gravity waves. These external p -modes enter the sunspot atmosphere, suffer modifications and mode transformations due to the presence of the magnetic field, but it comes as no surprise that a part of the wave front corresponding to the initial fast-mode waves remains coherent with the exterior wave front. Immediately below the sunspot and in the deep photosphere the plasma β is around 1, so it is understood that several wave types exist simultaneously at these heights. It is plausible to conclude that fast high- β waves propagating beneath sunspots, with the speed and reflection conditions modified by the magnetic field (see Section 7), are selected by the types of measurements done in local helioseismology.

A new method of diagnostics of sunspot wave modes by means of wavefront dislocations was suggested recently by López Ariste *et al.* (2013), who detected the presence of dislocations in the chromospheric 3-min velocity pattern observed by Centeno *et al.* (2006). In them, the amplitude of the wave vanishes and the phase is undefined. An interesting property of dislocations is that they cannot be created or destroyed without external forces. Different types of dislocations exist and can be carried by MHD waves and their diagnostic potential to understand the nature of sunspot waves is still to be understood.

8.3 What produces a shift from 5 to 3 min with height in the umbra?

As was discussed in Section 7.4, back in the 1980s, a model of a resonant cavity was proposed by Zhugzhda and Dzhililov (1982); Zhugzhda (1984) and others to explain the spectrum of umbral waves. In particular, a chromospheric resonance cavity was suggested where slow magneto-acoustic waves are trapped due to reflections at the steep temperature gradient in the photosphere and at the chromosphere-corona transition region, producing an observed spectrum of 3-min chromospheric oscillations.

Nevertheless, a similar change of the dominant period of oscillations is also typical in the quiet non-magnetic internetwork solar regions. Assuming the waves observed in the umbral photosphere

and chromosphere are slow low- β acoustic-like waves propagating along nearly vertical magnetic field lines, the mechanism of the frequency variation with height of these waves should be similar to that of the ordinary acoustic-gravity waves in the non-magnetic Sun. [Fleck and Schmitz \(1991\)](#) have proposed that the change of frequency with height from 3 to 5–6 mHz is a basic physical phenomenon due to a resonant excitation at the atmospheric cut-off frequency, occurring even for linear waves in an isothermal atmosphere (see also [Kalkofen *et al.*, 1994](#)). In the solar case, the low temperatures in the upper photosphere give rise to the cut-off frequency around 5 mHz defining the spectrum of chromospheric waves. Non-linearities and non-adiabaticity of oscillations are not relevant for the action of this mechanism. Moreover, as it works even in an isothermal atmosphere, the assumption of chromospheric resonance cavity for 5 mHz waves seems not to be necessary.

The presence of non-linearities in chromospheric waves may also influence the power spectrum, producing high-frequency harmonics. [Fleck and Schmitz \(1993\)](#) showed that the response of the solar atmosphere to non-linear adiabatic shock wave propagation also leads to an appearance of the 5 mHz frequency peak in the power spectra, under the condition that the underlying photosphere has a low frequency 3 mHz component.

Yet another explanation was suggested by [Carlsson and Stein \(1997\)](#) based on the fact that at lower frequencies the wave energy falls off exponentially with height, stronger than for high-frequency waves, not affected by the cut-off. A similar effect was also discussed by [Bogdan and Judge \(2006\)](#). This simple explanation only implies that waves with 3-min periodicity are already present in the photospheric power spectrum, as indeed seems to be the case, shown by many observations discussed in Section 2. The amplitude of the evanescent 5-min oscillations grows with height as

$$v \sim \exp\left(z \left[1/2H - \sqrt{(\omega^2 - \omega_c^2)/c_S}\right]\right), \quad (13)$$

while that of 3-min oscillations increases as

$$v \sim \exp(z/2H). \quad (14)$$

Therefore, at some height the 3-min oscillations start dominating the power spectrum. A simple calculation can be made of how the photospheric power spectrum of slow magneto-acoustic waves is propagated from the photosphere to the chromosphere in a solar model atmosphere taking into account the effects of the cut-off frequency on the wave amplitude. The resulting wave spectrum at different heights (using the VAL-C model atmosphere) is given in Figure 25. The initial photospheric power distribution is modeled by a Fourier transform of Ricker wavelet with a peak frequency of at 3.3 mHz. The power spectra are calculated every 100 km from 0 to 1000 km. One can observe that already at 400 km in the photosphere the 3-min oscillations start to dominate. Also in agreement with observations we note the absence of a smooth transition of the dominant frequency from 3 to 5 mHz (see [Felipe *et al.*, 2010a](#)). The photospheric spectra peak at around 3 mHz, and the chromospheric ones peak at 5–5.5 mHz. The absence of the peak at 4 mHz at any height is due to the fact that oscillations at this frequency do not have sufficiently high amplitudes in the photospheric power spectrum, but their frequency is already sufficiently close to the cut-off frequency and their amplitude increase with height is also not significant. Therefore, this simple explanation might explain the observed periodicities in the umbra.

8.4 How do waves propagate through the transition region and corona?

Essentially vertical wave propagation from the photosphere to the chromosphere is detected in many observations. Not so many data exist, however for propagation through the transition region to the corona. Only recently do such data start becoming available from space missions such as SDO/AIA and IRIS. Non-linear waves are firmly detected at chromospheric and transition region heights. In the corona, waves observed in coronal loops do not show a shock wave behavior,

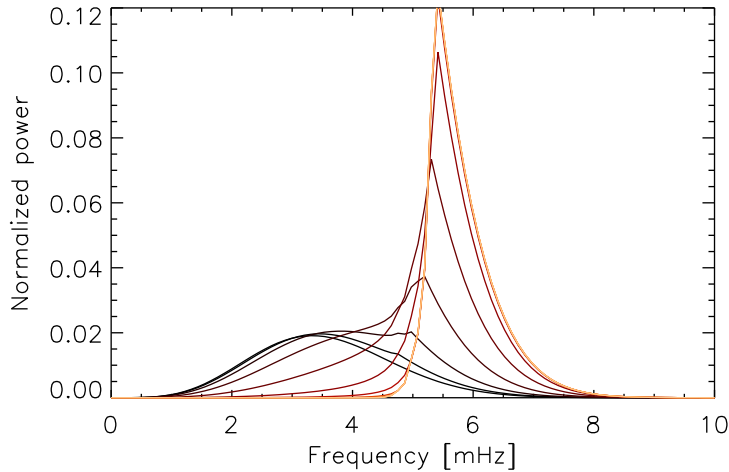


Figure 25: Power spectrum of umbral waves, theoretically propagated from the photosphere (black curve with a peak at 3.3 mHz) up to 1000 km in the chromosphere with a step of 100 km (progressively more red to orange curves), assuming purely vertical propagation of slow magneto-acoustic mode and taking as a reference the cut-off frequency calculated in the VAL-C model atmosphere.

implying a loss of energy at some height. It is not clear, as of today, whether a significant part of the wave energy is reflected from the transition region back to the photosphere, or if it manages to overcome it in one way or another. Possibly, magneto-acoustic waves are converted into Alfvén waves (see Section 7.2.4), that allows them to penetrate more easily into the corona. However, there is no observational confirmations yet.

Chromospheric umbral flashes are determined to be a consequence of longitudinal acoustic wave propagation and shock formation in the umbra. Numerical modeling of umbral flashes in the Ca II H line was performed by [Bard and Carlsson \(2010\)](#). These authors accomplished very detailed NLTE radiation hydrodynamic simulations of the propagation of acoustic waves in sunspot umbrae and concluded that umbral flashes result from an emission caused by the passage of the shock waves with frequencies in the range 4.5–7.0 mHz at chromospheric heights, while waves with frequencies below 4.5 mHz do not play a role despite their dominance in the photosphere.

Nevertheless, it is not clear if the energy contained in these shocks is sufficient to heat the umbral chromosphere. Theoretical calculations by, e.g., [Lee and Yun \(1985\)](#), by making use of the shock theory developed by [Ulmschneider \(193, 1971\)](#) and [Ulmschneider and Kalkofen \(1978\)](#) suggest that the chromosphere can be heated by the dissipation of shocks with a period of about 20 seconds, if their mechanical energy flux amounts to 2.6×10^6 erg/cm² s at a height of 300–400 km above the temperature minimum region. Waves with such small periods are difficult to observe, and no conclusive measurements were done up to date. For waves with a typical 3–5 min periods in the umbra [Felipe *et al.* \(2011\)](#) concluded that the mechanical energy contained in these waves is far too small already at photospheric heights, compared to the radiative energy losses of the chromosphere, see Figure 26. Further investigations of the energy budget of sunspot waves are required, with emphasis on the theory and observations of waves at the upper chromosphere, transition region and corona.

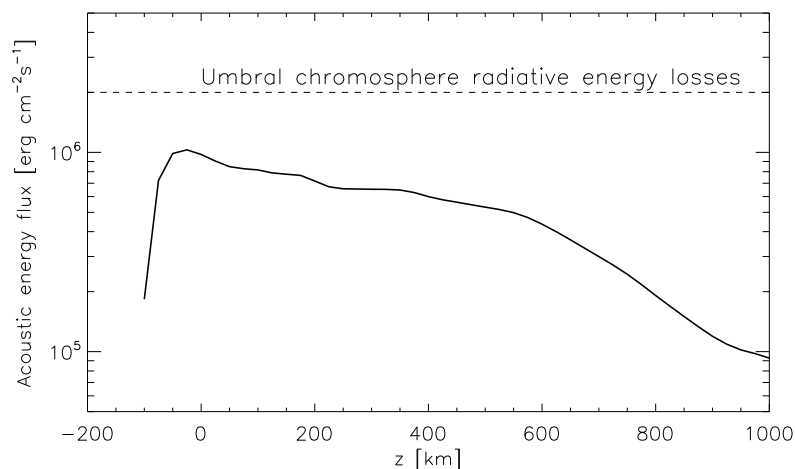


Figure 26: Acoustic flux calculated as $F = \langle \delta p \delta v \rangle$ in the simulations of sunspot umbral waves excited by the observed photospheric velocity field. Image reproduced with permission from Felipe *et al.* (2011), copyright by AAS.

8.5 What causes the wave power redistribution around sunspots?

As was discussed in Sections 2, 3 and 5, the power of velocity and intensity oscillations is distributed in a complex way across the sunspot umbra, penumbra and the surrounding plage regions (Jain and Haber, 2002; Tziotziou *et al.*, 2007; Nagashima *et al.*, 2007; Mathew, 2008; Reznikova and Shibasaki, 2012; Reznikova *et al.*, 2012; Jess *et al.*, 2013).

In the photosphere, the power of oscillations is generally suppressed at all frequencies, both in the umbra and in the penumbra. There are few peculiar observations of the enhanced power ring at the umbra-penumbra boundary by Nagashima *et al.* (2007), or contrarily, enhanced absorption was detected at the umbra-penumbra boundary by Mathew (2008). Acoustic halos are found at the edges of active regions depending on the magnetic field strength (Hindman and Brown, 1998; Braun and Lindsey, 1999; Donea *et al.*, 2000; Thomas and Stanchfield II, 2000; Jain and Haber, 2002; Gizon *et al.*, 2009; Schunker and Braun, 2011; Rajaguru *et al.*, 2013), being most prominent at high frequencies above 5 mHz.

In the chromosphere, the power is suppressed at low frequencies (below 2.5 mHz) in the umbra but then is enhanced at high frequencies, above 3.5 mHz, (e.g., Nagashima *et al.*, 2007; Reznikova *et al.*, 2012). The most prominent are 5–6 mHz oscillations observed in the umbral chromosphere and above. Running penumbral waves in the chromosphere have lower frequency around 3 mHz (Tziotziou *et al.*, 2007; Jess *et al.*, 2013). For progressively lower frequencies, the ring of the enhanced chromospheric wave power extends further away into the sunspot penumbra and even into the moat region (Reznikova *et al.*, 2012; Jess *et al.*, 2013). Wave power variations reveal fine structure following the penumbral filaments.

As for the power distribution at different frequencies of the penumbral waves observed in the chromosphere, several independent investigations convincingly point that these waves are the visible pattern of the longitudinal propagation of low- β slow magneto-acoustic waves (Bloomfield *et al.*, 2007a; Bogdan and Judge, 2006; Jess *et al.*, 2013). The calculations of the reduced cut-off frequency due to the inclination of the magnetic field lines in the penumbra are in good agreement with the actual observed frequency of these waves across the penumbra, explaining the observed wave power distribution.

The power suppression in the umbral photosphere is thought to be due to the mode conversion

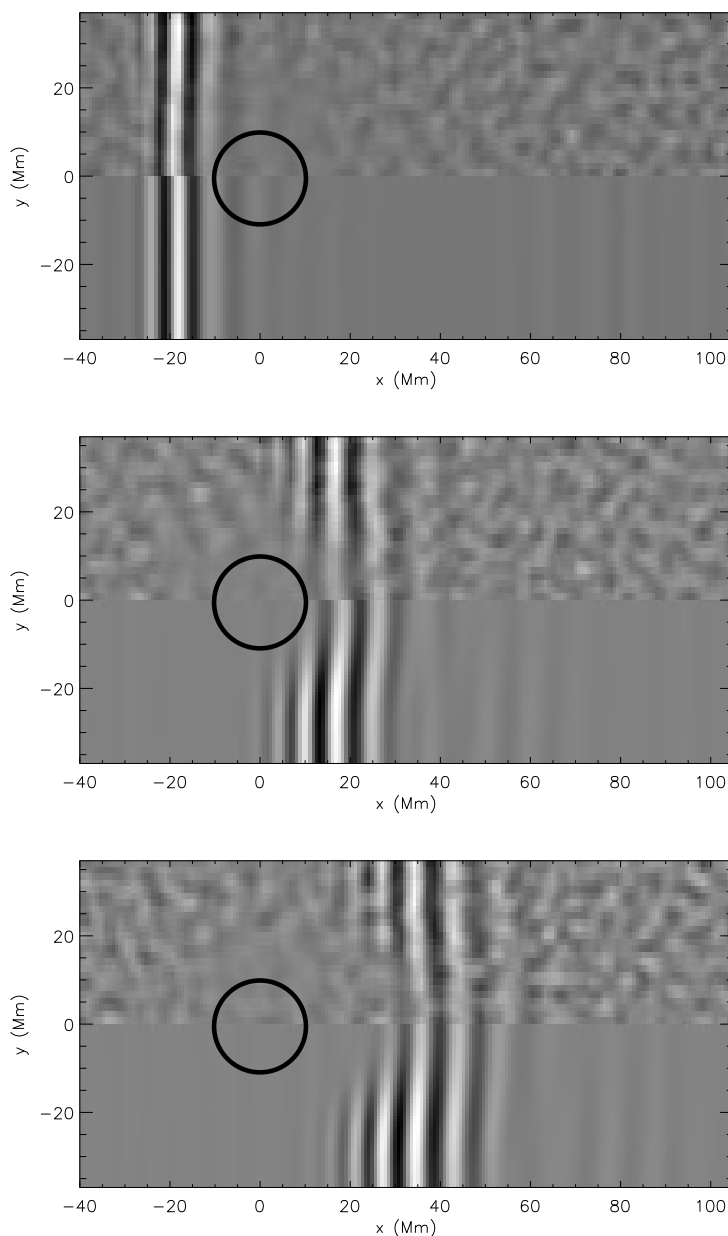


Figure 27: Comparison of the simulated vertical velocity (bottom part of each frame) and the wave cross-covariance calculated from the observational data (top part of each frame) for the simulations of the propagation of the f -mode wave packet through the sunspot region. The black circle indicates the location of the sunspot. Different panels are different instants of time (40, 100 and 130 min). Image reproduced with permission from [Cameron *et al.* \(2008\)](#), copyright by the author(s).

extracting the energy from the incident p -mode waves, see Sections 7.2.2 and 7.2.3. Mode conversion in a vertical field seems to be able to explain well the amount of the power reduction of the f -mode, but not the p -modes. The numerical simulations of the f -mode propagation through a model sunspot by Cameron *et al.* (2008) show a very good agreement with observational data regarding the wave power reduction and phase shifts, see Figure 27. Moreover, their simulations provide a way to constrain the magnetic field of the observed sunspots since the amount of the power reduction is found to be magnetic field dependent. The simulations by Cameron *et al.* (2008) show the best agreement with a model whose peak field strength is 3 kG at the photosphere. As for the p -modes, it is now understood that the inclination of the magnetic field has to be taken into account to produce the observed amount of power reduction due to the fast to slow mode conversion (Crouch and Cally, 2003; Cally *et al.*, 2003; Crouch and Cally, 2005). The amount of p -mode “absorption” and the phase shifts in the proposed models are in a good agreement with observations. Alternatively, the resonant absorption of the incoming p -modes is also found to be a rather effective mechanism that is able to reproduce the observed amount of power reduction in sunspots (Braun *et al.*, 1988). In general, it is agreed that the wave power is decreased in sunspots compared to the quiet Sun due to the leakage of the acoustic-gravity wave energy into one or other wave type. However the details of the particular mechanisms and the reasons of these conversions differ.

As for the high-frequency acoustic halo surrounding active regions, several plausible explanations exist as of today (Brown *et al.*, 1992; Braun *et al.*, 1992; Hindman and Brown, 1998; Jain and Haber, 2002; Kuridze *et al.*, 2008; Jacoutot *et al.*, 2008; Hanasoge, 2008; Khomenko and Collados, 2009). Hanasoge (2008) calculated the rms wave power after randomly distributing acoustic sources over a region containing a sunspot-like flux tube. Both power reduction in the “umbra” and halo effect in the surrounding appear naturally in these simulations, confirming that they are definitely an MHD effect. Hanasoge (2008) suggested that the power enhancement in halos is produced by mode mixing induced by the magnetic field, resulting in preferential scattering from low to high wave numbers. Alternatively, an explanation has been offered based on the enhanced high-frequency wave excitation at regions of intermediate magnetic field strength in halos (Jacoutot *et al.*, 2008). Bogdan and Cally (1995) discuss jacket modes produced after the scattering of the p -modes on a sunspot and resulting in a power excess surrounding sunspot and plages, though this can not explain halos.

Single-source numerical simulations of magneto-acoustic wave propagation in a magneto-static sunspot model demonstrate that the halo effect can happen in a natural way due to the additional energy input from the high-frequency fast mode waves, after their refraction above the $\beta = 1$ layer (Khomenko and Collados, 2009). The frequency and location of the halo is extremely sensitive to the relative location of the $\beta = 1$ layer, the cut-off layer and the formation height of a given observed spectral line. The halo is produced in those photospheric regions where the field is intermediate, implying that the Alfvén speed is lower than the sound speed. The halo is not observed at low frequencies because these waves are already reflected by cut-off below the transformation layer, and is not observed in the umbra of the sunspot because the refraction happens below the layer visible to spectral line observations. This model, as well as the model of mode mixing (Hanasoge, 2009) suggest that the halo power enhancements might be larger in the horizontal velocity component.

Yet another mechanism to explain power enhancements in halos was proposed based on acoustic waves trapped in field-free atmospheres lying below small-scale magnetic canopies of network cores and active regions (Kuridze *et al.*, 2008). A similar effect might also be responsible for the appearance of “magnetic shadows” around the cores of network magnetic elements (Nutto *et al.*, 2012). Alternative pictures of wave transformation and refraction (Kuridze *et al.*, 2008; Khomenko and Collados, 2009; Nutto *et al.*, 2012) are, in fact, similar, suggestive that both, halos and shadows, are different manifestations of the same physical mechanism. Recent observational results seem to favor the model based on wave transformation and refraction (Rajaguru *et al.*, 2013).

One can not exclude, though, that other effects may also play a role (Hanasoge, 2009; Jacoutot *et al.*, 2008). For example, acoustic halos are found to be co-spatial with acoustic glories, locations with enhanced seismic emission surrounding active regions (Donea and Newington, 2011). The nature of the glories is different to halos, since they tell us about an increase of seismic emission, i.e., power of waves emanating directly from the wave sources, whereas the acoustic halos are just measures of increased wave amplitudes, whatever their origin is. Thus, the last word on the nature of halos is not yet said.

8.6 Open questions

A global three-dimensional picture of oscillations and waves in sunspots is gradually emerging. The influence of the $\beta = 1$ level and the magnetic field inclination on the observed wave properties is being progressively clarified, allowing the interpretation of the observed oscillations in term of MHD wave modes. The observed wave pattern in the upper photosphere and the chromosphere is found to be compatible with the low- β acoustic-like slow mode propagation along inclined magnetic field lines. Several physical mechanisms have been proposed to be responsible for the frequency change with height in the umbra from 3 mHz in the photosphere to 5–6 mHz in the chromosphere. Mode transformation at the $\beta = 1$ level is proposed to be responsible (at least in part) for the wave power distribution over active regions, including wave suppression in the umbra and, possibly, enhanced power of acoustic halos. Still, the final issue in this field is far from clear and major developments are expected in the future. Below we provide an incomplete list of open questions to be addressed and clarified in future studies of sunspot waves.

- What is the dominant excitation mechanism of different wave modes in sunspots? Is it mainly external driving or weak convection?
- Is there observational evidence for Alfvén waves? At what height are they generated? How large is the energy carried out by Alfvén waves up to the transition region and corona? Are they able to penetrate into the corona without significant reflections?
- What is the observational evidence for fast mode waves in sunspots? How can the efficiency of mode transformation be measured from observational data?
- Are magnetic modes better detected in observations of sunspots at the limb? What is the power spectrum of the velocity component transverse to magnetic field?
- How large is the energy carried out by chromospheric shock waves to the transition region and corona in comparison to radiative losses? Can it be responsible for heating? How do these shocks interact with the discontinuity in wave speed represented by the transition region?
- Is there observational evidence for simultaneous up- and downward wave propagation in the umbra and penumbra, suggestive for reflections from the transition region? How do the reflections depend on the wave frequency?
- Are we able to distinguish between the monolithic and spaghetti sunspot models, from the observed behavior of sunspot waves?
- The photospheric penumbra is very inhomogeneous, how does it affect wave propagation?
- What is the relation between the fine structure of the umbra and waves?
- Why do running penumbral waves apparently disappear at the boundary of the visible penumbra? Where do they go?

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- What are the observational properties and physics behind long-period sunspot oscillations, as well as torsional oscillations of sunspot magnetic field, detected from magnetogram data? Longer time series of data from space missions should help to answer this question.
 - Are there observational detections of g -modes at low frequencies in sunspots? What are their properties?
 - What are the excitation mechanisms of waves at the foot points of coronal loops above sunspots?
 - What are the effects of the very low degree of atomic ionization of the plasma at the photosphere and chromosphere of sunspots on wave propagation, excitation and damping?

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