

Effects of Connection Deformation Softening on Behavior of Steel Moment Frames Subjected to Earthquakes

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Abstract

In the years since the 1994 Northridge earthquake, the profession has paid significant attention to the potential effects of various forms of deterioration in connection strength and stiffness that steel moment-resisting frames can experience during severe seismic excitations. The brittle connection fractures that a number of welded steel moment-resisting frame structures experienced during recent earthquakes have been the most extensively studied to date. However, cyclic testing of post-Northridge beam-column connections demonstrates that ductile connections may suffer other forms of deterioration. Negative post-yield tangent stiffness or capping, hereafter referred to as deformation softening, is a behavior of particular interest because it may have significant adverse effects on frame system behavior. The effects of deformation softening on frames subjected to pulse excitations were examined as part of an integrated experimental and analytical investigation of the effect of various forms of hysteretic deterioration on the overall system behavior of moment resisting steel frames. Pulse excitations, and the near-field ground motions they represent, can be highly damaging to structures and are therefore the primary focus of the results presented in this paper. The experimental portion of this study consisted of a series of thirty-two shaking table tests, which were performed on a one-third scale, two-story, one bay, steel moment frame with idealized, mechanical connections. These tests and subsequent analytical studies show that, in general, significant loss of connection strength capacity, whether from deformation softening or other types of deterioration, leads to large residual drifts and, for large pulse excitations with durations longer than the fundamental period of the structure, to collapse. In particular, frames with connections exhibiting negative post-yield stiffness tend to have substantially increased peak and residual displacements when subjected to pulse excitations.

Keywords: connections, dynamic tests, earthquake engineering, nonlinear analysis, shake table tests, steel frames, structural behavior, structural dynamics

1. Introduction

In the aftermath of the 1994 Northridge, California earthquake, structural engineers discovered brittle fractures at the beam-column connections of a number of steel moment frames in the greater Los Angeles area (Bertero *et al.*, 1994). The profession had previously assumed that these connections were very ductile, and the discovery led to a great deal of research into the causes of the fractures and ways of preventing them (FEMA 2000a, b). The research led to the development of a number of new, post-Northridge connection designs that were not prone to brittle fracture. However, these new designs displayed

other forms of hysteretic deterioration during testing that involved repeated cycling into the inelastic range (FEMA 2000b). One of the most interesting forms of hysteretic deterioration observed in these tests, with significant potential impact on system behavior, was deformation softening.

Deformation softening is defined as the presence of negative post-yield tangent stiffness in the connection hysteresis that results from inelastic behavior of the connection's members. Deformation softening is also referred to as capping behavior by some researchers (e.g., Ibarra and Krawinkler 2005), and typically occurs when the beam begins to buckle and loses strength during an inelastic excursion. Figure 1 shows two examples of deformation softening observed in quasi-static cyclic tests of post-Northridge beam-column connections. The deformation softening, as indicated by the negative post-yield slope of the hysteretic loop, is most evident in the large, early cycles. As the test progresses, the connections shown also experience the other major types of ductile hysteretic degradation: cyclic strength degradation (isotropic

Note.-Discussion open until August 1, 2011. This manuscript for this paper was submitted for review and possible publication on March 13, 2010; approved on November 17, 2010.
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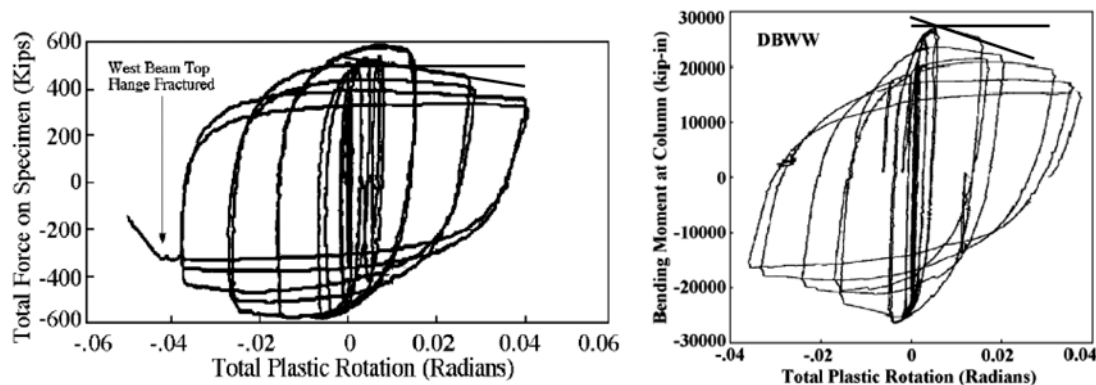


Figure 1. Deformation softening in post-Northridge beam-column connection tests, reprinted from FEMA-355D (FEMA 2000b). Note the negative post-yield slope, in the first several large inelastic cycles. Lines show the deviation of the most severe negative post-peak slope from zero slope.

softening), and stiffness degradation (reduction in reloading and/or unloading stiffness).

Deformation softening differs from these other types of ductile hysteretic degradation in several important ways. The first important difference is that significant deformation softening can occur in the first large inelastic excursion, as in the test shown on the right of Fig. 1. Other types of ductile deterioration require multiple large inelastic cycles before the connection loses a significant fraction of its strength and stiffness. Deformation softening therefore increases vulnerability to large near-field ground motions, during which the majority of the seismic demand can occur in one or two cycles of motion. The second important difference is that deformation softening in enough connections simultaneously can result in a negative slope in the global story shear-story drift hysteresis. The negative slope associated with connection hysteretic behavior can add to the global negative stiffness caused by P- Δ effects. Depending on the excitation, adding these two contributors to negative tangent stiffness together might be enough to cause collapse.

2. Background

A limited number of analytical studies have examined the effects of negative post-yield stiffness either directly (Mahin and Morishita 1998, Aschheim and Black 1999, Miranda and Akkar 2003, Ibarra and Krawinkler 2005), or as a way to model P- Δ effects in single degree of freedom (SDOF) systems (Bernal 1992; Rahnama and Krawinkler 1993). Until recently (Lignos and Krawinkler 2007), only one of these studies (Ibarra and Krawinkler 2005) considered multiple degree of freedom (MDOF) systems; the remaining studies were conducted on SDOF oscillators. The studies by Mahin and Morishita (1998) show that deformation softening can increase ductility demand, particularly in the short period range, and that it can lead to the accumulation of displacements in one direction, called “ratcheting”. Aschheim and Black (1999)

found that severe negative post-yield tangent stiffness (they used negative 10% of the initial elastic stiffness) makes SDOF oscillators prone to collapse, especially weak oscillators subjected to forward-directivity near-field motions. Miranda and Akkar (2003) reached similar conclusions for a range of negative post-yield stiffnesses; the more severely negative the stiffness, the more strength required to avoid collapse. The studies by Rahnama and Krawinkler (1993) show that deformation softening has significant adverse effects on system behavior, and that softening systems cannot achieve the same ductility as systems without softening unless the strength of the softening system is greatly increased. It should be noted here that Rahnama and Krawinkler’s study explicitly considered P- Δ effects rather than connection deformation softening, but they considered P- Δ by using a hysteretic loop with negative post-yield stiffness, making the study directly applicable to the problem at hand.

A more recent experimental study of ductile deterioration (Lignos and Krawinkler 2007) using a 1:8 scale model of a steel frame shows that the presence and severity of deformation softening are important contributing factors to collapse. Experimental tests of SDOF bridge pier specimens to collapse by Vian and Bruneau (2003) showed similar results to some of the analytical studies above; specimens with greater equivalent severity of negative post-yield stiffness due to P- Δ effects collapsed more readily.

Prior to the study reported herein, the effects of deformation softening on overall structural response of MDOF systems had not been comprehensively studied, and no experimental studies of MDOF systems had been conducted to validate the findings based on analytical simulations. Several studies discussed above (Ibarra and Krawinkler 2005; Lignos and Krawinkler 2007) have since examined various aspects of deformation softening. In the study presented here, deformation softening was included in an integrated experimental and analytical investigation into the effects of various forms of hysteretic

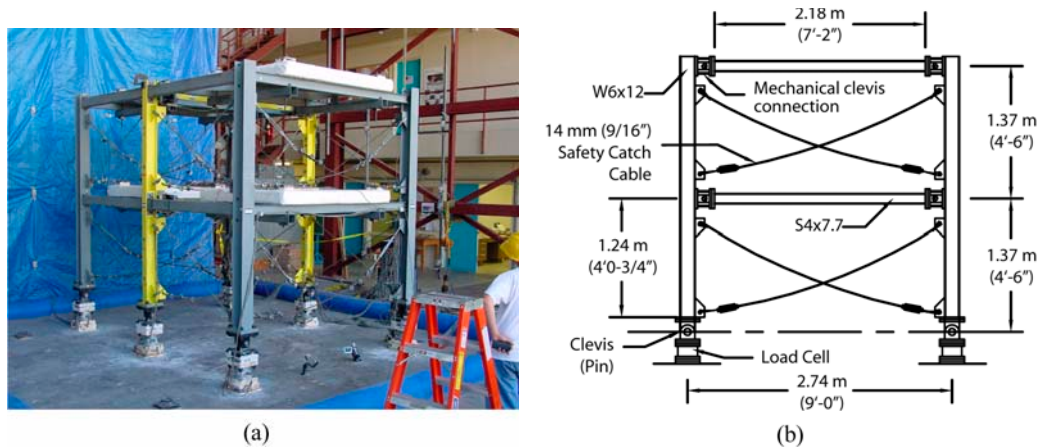


Figure 2. Experimental specimen on shaking table (a) and engineering drawing of main frame (b).

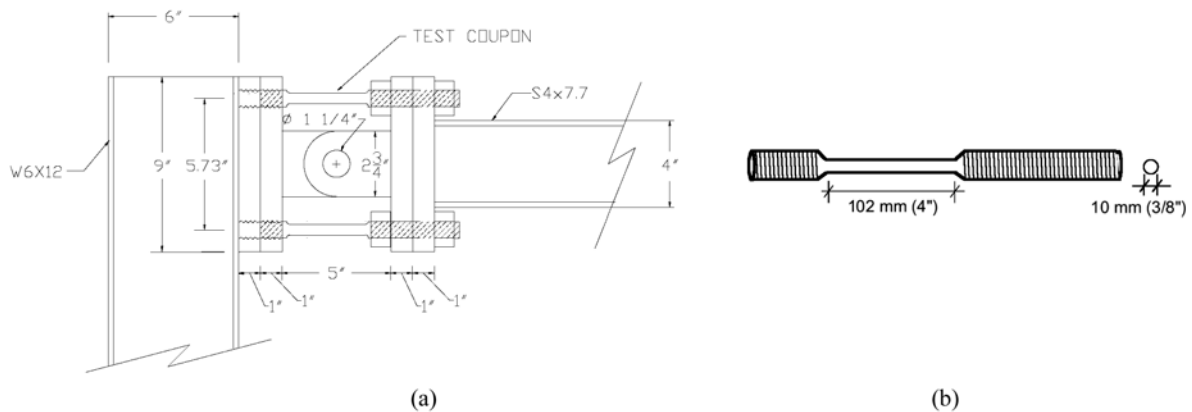


Figure 3. Idealized mechanical connection (a) and coupons used to generate deformation softening (b).

deterioration on the overall system behavior of moment resisting steel frames (Rodgers and Mahin 2004). This study was the first to examine the effects of deformation softening in steel moment frame connections on frame system behavior both in shaking table experiments and analytically. The overall investigation included the effects of fracture (Rodgers and Mahin 2006) and strength degradation as well. This paper presents the results of the investigation that pertain to the effects of deformation softening on structures subjected to pulse-type loading.

3. Experimental and Analytical Setup

3.1. Experimental specimen and plan

The experimental specimen, shown in Fig. 2, was a one-third scale, two-story, one-bay moment frame. Idealized mechanical connections (van Dam 2000; Rodgers *et al.*, 2006), described in greater detail below, represent the plastic hinge regions at the ends of each beam. Inelastic behavior was confined to replaceable components in the idealized connections so that the authors could conduct a large number of tests. The remainder of the frame was designed to remain elastic, and strain gage data show that it did so. The specimen had columns spaced at 2.74 m

(9.0 ft), floor heights of 1.37 m (4.5 ft), an elastic first-mode period of approximately 0.65 seconds, and a damping ratio of approximately 1.8%. To simplify testing and interpretation of results, pin ended (clevis) connections were used at the base of each column. Two simple pin-ended frames were placed parallel to the moment frame as shown Fig. 2, one on each side. These frames supported the inertial reactive mass of 4930 kg (10.9 kips) and provided out-of-plane bracing. Slack wire cables in the in-plane direction provided a mechanism to catch the test specimen when it collapsed during testing. Rodgers and Mahin (2004) provide more information on the experimental setup.

The idealized connections, shown in Fig. 3, were designed with interchangeable parts that could, through differences in cross-sectional properties and end conditions, reproduce a wide variety of hysteretic characteristics repeatedly and dependably. The intent was to reproduce a single primary hysteretic characteristic of interest with each different configuration of interchangeable parts, so that the effects of each primary characteristic could be studied individually. The primary hysteretic characteristics of interest in this paper are shown in Fig. 4: ideally ductile behavior with a flat to positive post-yield slope (a)

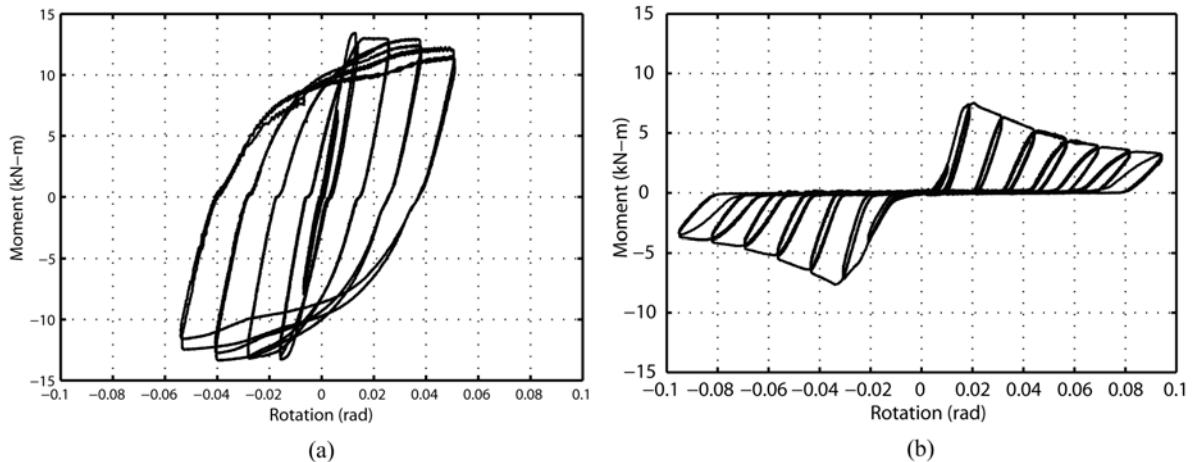


Figure 4. Quasi-static test results showing ductile baseline (a) and deformation softening (b) hysteretic behavior of idealized mechanical connections.

and deformation softening (negative post-yield stiffness, b). The larger experimental study considered fracture and strength degradation behavior as well, and design considerations to allow the idealized connections to reproduce fractures governed many connection design features.

The idealized connection resists moment with a force couple of replaceable tension and compression coupons, similar to material test coupons, placed on either side of a clevis pin. Coupon cross-section properties and end conditions can be varied to produce different primary hysteretic characteristics. Deformation softening behavior was achieved by buckling of the compression coupons. To prevent the coupons from resisting tension and undergoing strain hardening, which earlier prototype tests by van Dam (2000) show causes a positive post-yield slope rather than the negative post-yield slope needed for the experiment, nuts were not installed on the outside of the coupons. This design necessity caused the extremely pinched hysteretic behavior under cyclic loading shown in Fig. 4. Due to the excitations used in this study, the negative post-yield stiffness of the connections controlled the behavior of the specimen during the initial cycle of pulse loading (the loading period of greatest interest), rather than pinching. This paper considers experimental results only from the first part of the loading cycle, when negative post-yield stiffness governs. During the first major inelastic excursion, the behavior of the idealized connection in Fig. 4 (b) is very similar to the behavior of the real connections shown in Fig. 1.

Experiments were conducted on the frame with the ductile baseline and deformation softening connections arranged in the configurations of interest shown in Fig. 5. These configurations included a baseline case with all ductile connections, and cases where deformation softening occurred in the first story (B pattern) or first and second stories (C pattern). In Fig. 5, gray circles denote connections with deformation softening behavior; all other connections

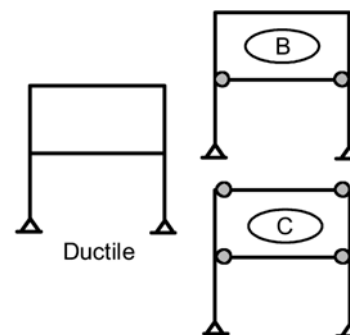


Figure 5. Connection configuration patterns of interest; gray circles denote softening connections.

are ductile baseline connections, if applicable. The two simplified cosine acceleration pulse earthquake excitations representing idealized fault-normal near-source ground motions used are shown in Fig. 6. The first cosine pulse was 0.6 seconds in duration, about the same as the fundamental period of the test specimen (0.65 sec.). The second cosine pulse was 1.2 seconds in duration, about twice the fundamental period. The B and C patterns were subjected to both 0.6 second and 1.2 second cosine acceleration pulses.

3.2. Analytical model

A two-dimensional analytical model of the experimental specimen was developed using OpenSees, an open-source computational framework for nonlinear static and dynamic structural analysis. The idealized mechanical connections were modeled as nonlinear zero-length springs, with general hysteretic and steel material models (see Mazzoni *et al.*, 2003) used to model the moment-rotation relations for deformation softening and ductile baseline behavior, respectively. The remainder of the frame was modeled with elastic beam-column elements because the test data confirmed that nonlinearity was

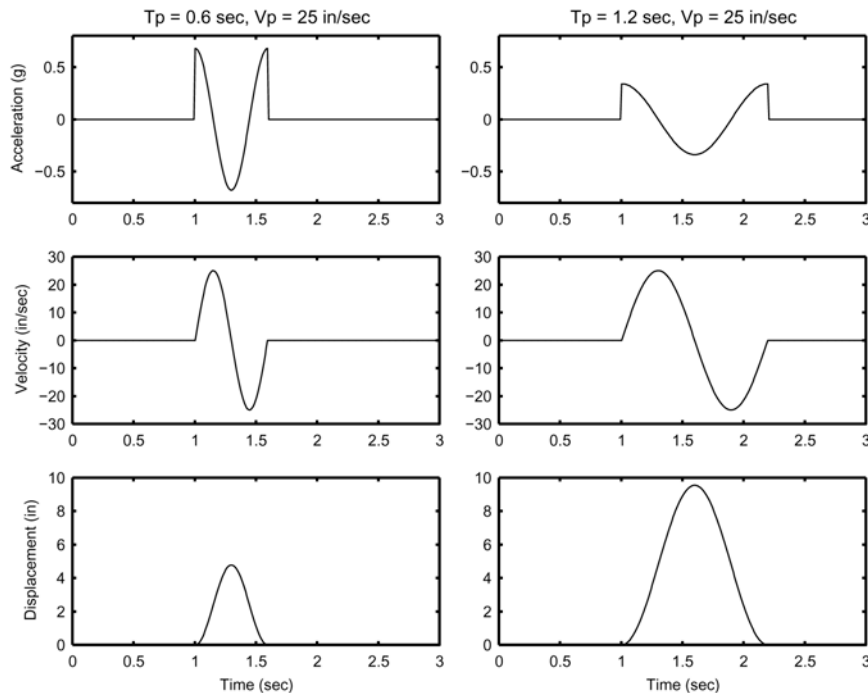


Figure 6. Cosine acceleration pulse excitations.

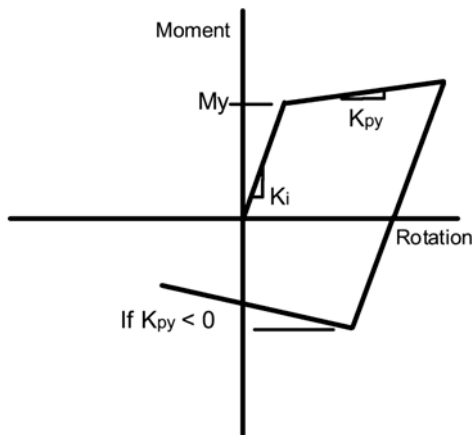


Figure 7. Definition of hysteretic characteristics examined by analysis; K_i denotes initial elastic slope, K_{py} denotes post-yield tangent slope, and M_y denotes yield moment.

confined to the connections. Details of the analytical modeling assumptions and procedures used are available elsewhere (Rodgers and Mahin, 2004). The analytical model was used both to make direct comparisons to the specific experiments carried out on the shaking table, and to conduct a parametric investigation that examined the effects of several hysteretic and pulse excitation characteristics. Figure 7 defines hysteretic characteristics that were examined by analysis. For the parametric investigation presented here, the pinching that was characteristic of post-peak deformation softening connection hysteretic behavior in the experimental specimen was removed from the analytical model. Figure 8 compares

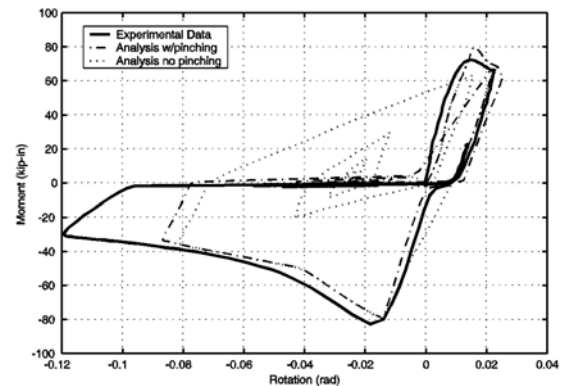


Figure 8. Comparison of deformation softening connection hysteretic behavior from experiment (pinching) and analysis (both with and without pinching) for 1.2 second cosine pulse excitation, deformation softening B pattern.

the hysteretic behavior obtained for deformation softening during experiments (with post-peak pinching) with that obtained using analytical models that include and remove pinching, respectively. As previously explained, the deformation softening connections in all experiments displayed pinching behavior out of necessity.

4. Experimental and Analytical Results

This section presents an integrated discussion of the results of the experimental and analytical portions of the study to provide a better understanding of the effects of deformation softening on steel moment frames subjected to pulse loading, as well as the interaction between

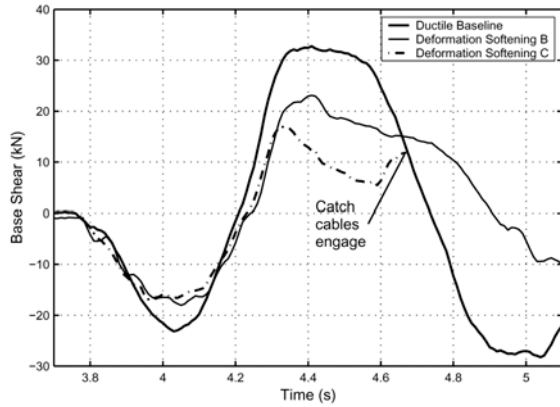


Figure 9. Base shear response, 1.2 second cosine pulse.

deformation softening and pulse excitation and connection hysteretic parameters. Parameters discussed include the number of degrading connections, pulse excitation amplitude and frequency characteristics, and hysteretic characteristics such as severity of post-yield tangent stiffness.

4.1. Effects of deformation softening on system behavior

When buckling of the compression coupons initiates, the moment capacity at the degrading connection begins to decrease substantially. Because of equilibrium, the reduction in connection moment capacity causes a subsequent decrease in the measured base shear response of the structure, as Fig. 9 shows. As the coupons buckle during the pulse, the base shear response shows one of two trends; the response either (a) initially increases, then plateaus, then decreases or (b) plateaus immediately before decreasing, depending on the number of deformation softening versus ductile connections. Increase-plateau-decrease behavior occurs in the B pattern case, which has the two ductile connections in the second story that are still in the elastic range when softening initiates in the first story. Because the second story connections have not yielded, they are able to resist additional moment as demands increase. Plateau-decrease behavior occurs in the C pattern case, when all connections begin to soften simultaneously, and there is no reserve capacity to resist additional demand. After the plateau, the base shear for the C pattern begins to decrease rapidly as the compression coupons continue to buckle and the connections suffer a severe within-cycle loss of moment capacity. The rate of base shear decrease depends on the post-yield tangent stiffness of the connections as well as the number of softening connections.

In contrast, no sudden changes were observed in the displacement response at the onset of buckling, as shown in Fig. 10 for the 1.2 second pulse case. However, deformation softening dramatically affected the amplitude of the displacement response for this specimen, which

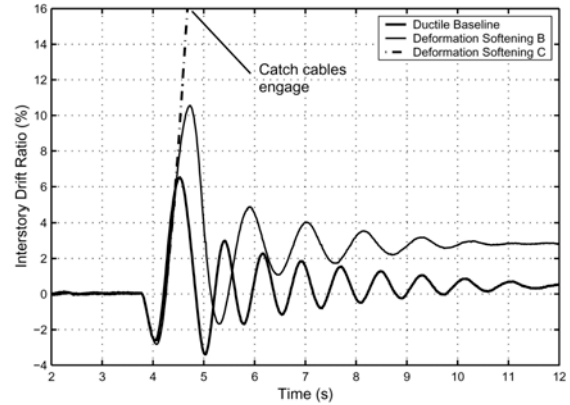


Figure 10. Story drift ratio, 1.2 second cosine pulse.

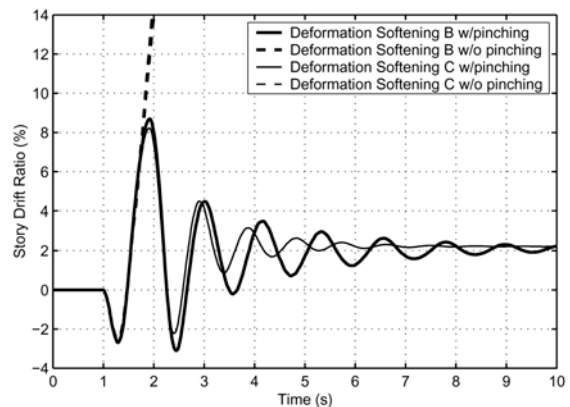


Figure 11. Analytical results for the 1.2 second cosine pulse showing the effects of pinching in the connection hysteresis on story drift. Both C cases collapse during the first positive excursion, and the curves coincide.

had an initial fundamental period of 0.65 seconds. The maximum story drift ratios are much greater for the cases with deformation softening than for cases with ductile connection behavior. Because the structure first undergoes substantial yielding during the half-cycle in which the maximum displacement occurs (though a small amount of yielding occurs in the previous half-cycle), the peak values are sensitive to post-yield stiffness and are relatively unaffected by pinching, as Fig. 11 shows. It can be noted that residual displacements increase in proportion to peak displacements for such systems. As expected, in Fig. 10 the deformation softening cases show significant period elongation when compared to the ductile cases, both within the first cycle (governed by deformation softening) and subsequent cycles (governed by pinching).

4.2. Effects of excitation amplitude

The effects of excitation amplitude were examined analytically using incremental dynamic analyses (IDAs) (Vamvatsikos and Cornell 2002). In these analyses, the computer model of the specimen was repeatedly subjected to the same excitation with increasing amplitude. Figure

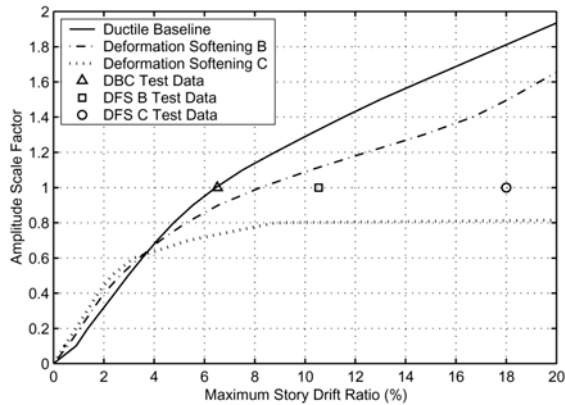


Figure 12. Incremental dynamic analyses showing effects of amplitude for the 1.2 second cosine pulse excitation.

12 shows the results for the 1.2 second cosine pulse excitation, where an amplitude scale factor of 1 corresponds to the amplitude of the excitation during the shaking table experiments. The IDAs show that maximum story drift response generally increases with increasing excitation amplitude.

4.3. Effects of number of deteriorating connections

The number of connections that display deformation softening behavior has significant effects on the response of the specimen. Figure 9 shows that the case with all connections deteriorating (the C pattern) has a much larger decrease in strength, and much larger maximum and residual displacements (it suffers collapse) than the B pattern, which has half as many deteriorating connections. An increase in the number of deteriorating connections adversely affects behavior because it greatly increases the sensitivity to post-yield tangent stiffness. The analytical results in Fig. 13 demonstrate this effect. The C pattern case displays large maximum drifts and/or collapse once the post-yield tangent stiffness ratio K_{py}/K_i becomes more strongly negative than -0.03 (negative three-percent). The B pattern has only half of its connections softening, and is relatively insensitive to the post-yield tangent stiffness. The B pattern's two ductile connections provide reserve capacity, which reduces global strength loss and prevents dynamic instability. Tests with fracturing connections showed that the number of fractures has very similar effects on sensitivity to post-fracture tangent stiffness (Rodgers and Mahin, 2006).

4.4. Effects of deterioration severity

Figure 13 shows that the response of the specimen is not very sensitive to the severity of the deterioration unless all connections are softening. The sharp increase in story drift for the C pattern case when K_{py}/K_i is more strongly negative than three-percent (-0.03) suggests that there may be a threshold level of negative tangent stiffness, beyond which the specimen becomes unstable (if all

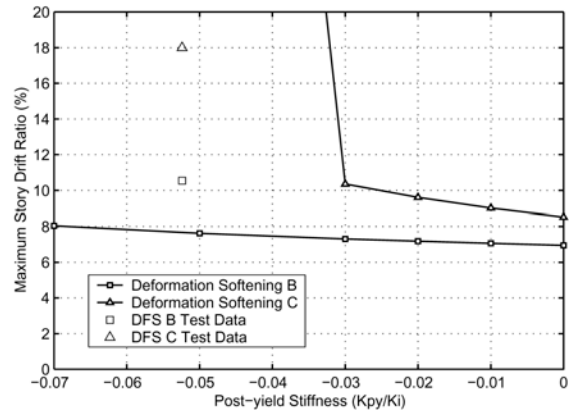


Figure 13. Effects number of softening connections and deterioration severity measured by negative post-yield stiffness ratio.

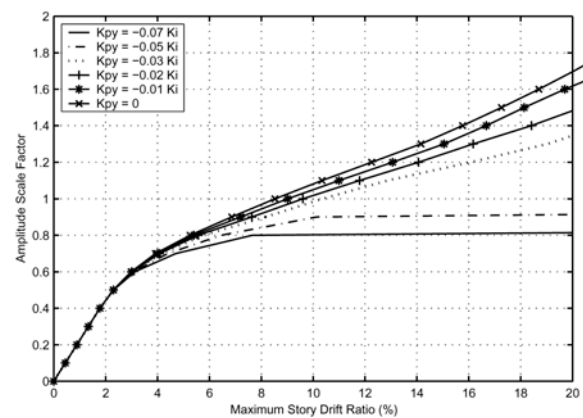


Figure 14. Incremental dynamic analysis of deformation softening C pattern with variable K_{py} , 1.2 second cosine pulse, P- Δ effects included.

connections have the same tangent stiffness) during the pulse excitations employed. IDAs using the 1.2 second cosine pulse excitation, shown in Fig. 14, indicate that potential threshold levels are dependent on excitation amplitude.

4.5. Combined effects of deformation softening and P- Δ

The potential combination of the negative tangent stiffness in the connection hysteresis and the globally negative tangent stiffness contributed by P- Δ effects was examined analytically. IDAs with P- Δ effects included via the corotational formulation (used to accurately model large deformations) were compared to those without P- Δ effects. Comparison of Fig. 14 (P- Δ effects included) and Fig. 15 (P- Δ effects not included), shows the increase in peak drifts caused by P- Δ effects. In particular, the cases with larger negative values of K_{py} were prone to instability at lower levels of excitation amplitude. In contrast, the ideally ductile case in Fig. 12 does not reach a similar instability point, though P- Δ effects are included (also via

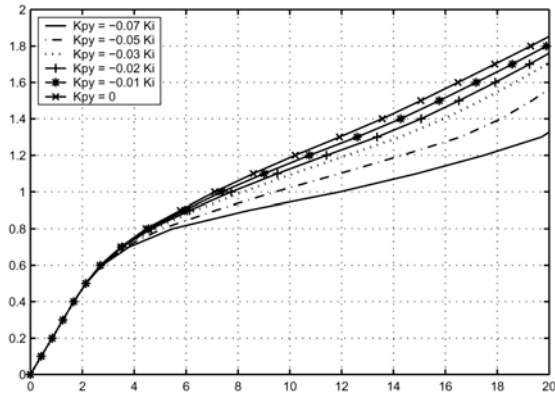


Figure 15. Incremental dynamic analysis of deformation softening C pattern with variable K_{py} , 1.2 second cosine pulse, P- Δ effects not included.

the corotational formulation). These results indicate that the presence of negative post-yield tangent stiffness interacts with P- Δ and reduces the story drift at which the structure becomes unstable, in some cases substantially. For this flexible specimen with stiff columns, the interaction between negative post-yield stiffness and P- Δ effects is observed at large drifts. Such effects are expected to occur at significantly smaller drifts in structures more susceptible to P- Δ effects, such as tall structures and those with heavy gravity loads.

4.6. Effects of location of specimen period on response spectrum

The location of the fundamental period of the specimen on the response spectrum, as expressed for pulses by the ratio $T_{specimen}/T_{pulse}$ of the specimen fundamental period $T_{specimen}$ and the pulse period T_{pulse} influences the severity of the effects of softening-induced strength loss. A low ratio (between approximately 0.3 and 1) results in increases in drift over the ductile baseline case for the same amount of strength loss due to deformation softening. This trend is apparent in Fig. 16, in which analytical results show that deformation softening causes significant amplification of the peak story drift response in the short and intermediate period range when compared to elastic and ductile baseline cases.

5. Conclusions and Recommendations

The effects of connection deformation softening on frame behavior under simple pulse loading depend on several factors. These factors include the amount of negative post-yield stiffness, the number of deteriorating connections, the excitation amplitude, and the specimen's position on the response spectrum. Both displacement and force response quantities were sensitive to the combination of the negative post-yield tangent stiffness ratio and the number of deteriorating connections. Specifically, in the case where all connections deteriorated with the same

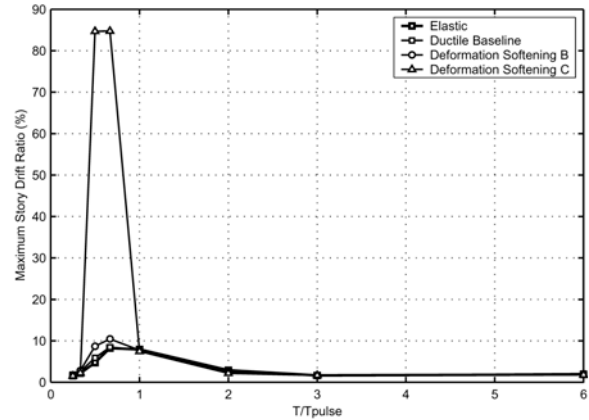


Figure 16. Shock spectra for deformation softening, ideally ductile, and elastic cases.

negative post-yield stiffness, the response was very sensitive to the amount of post-yield stiffness. A combination of a sufficiently severe pulse excitation and a large negative post-yield stiffness ratio in all connections caused collapse of the experimental specimen. When only two of the four connections deteriorated, the response was not sensitive to the post-yield stiffness ratio and no collapses of the experimental specimen occurred. Further analyses indicate that the negative post-yield stiffness ratio necessary for collapse varies with the excitation amplitude. These analyses also show that a negative slope in the connection hysteresis exacerbates the effects of P- Δ at large deformations.

Future research should examine full-scale moment frame structures with connections undergoing deformation softening, as well as the effects of different excitation characteristics. Researchers should to pay particular attention to structures that are more susceptible to P- Δ effects, such as taller structures and those with heavy gravity loads. The authors anticipate that the presence of softening connections in such structures will have significant adverse effects on system performance at much lower drift levels than observed for the experimental specimen used in this research. Until further research makes it possible to develop detailed design recommendations, the authors have two interim recommendations. Engineers assessing existing buildings with connection types that have exhibited negative-post yield stiffness in laboratory tests should explicitly model the expected negative slope in their nonlinear analyses. Engineers designing new buildings should try to minimize the adverse effects caused by deformation softening in connections by selecting connection designs that are less susceptible to such softening.

Acknowledgments

The authors gratefully acknowledge the support of the National Science Foundation under Grant No. CMS-

9807069, and the assistance of Mauricio van Dam, Wesley Neighbor, Patxi Uriz, Don Clyde, Chunho Chang, and David MacLam. The findings and conclusions in this work are those of the authors alone.

Notation

- K_i : Initial elastic slope of the connection moment-rotation relationship
 K_{py} : Post-yield tangent slope of the connection moment-rotation relationship
 M_y : Moment at which connection yields
 $T_{specimen}$: fundamental period of test specimen
 T_{pulse} : period of idealized fault normal, near-field pulse excitation

References

- Aschheim, M. and Black, E. (1999). "Effects of prior earthquake damage on seismic response of simple stiffness-degrading structures." *Earthquake Spectra*, 15(1), pp. 1-24.
- Bernal, D. (1992). "Instability of buildings subjected to earthquakes." *Journal of Structural Engineering*, ASCE, 118(8), pp. 2239-2260.
- Bertero, V. V., Anderson, J. C., and Krawinkler, H. (1994). Performance of steel building structures during the Northridge earthquake. *Rep. No. UCB/EERC-94/09*, Earthquake Engrg. Res. Ctr., University of California, Berkeley, Calif.
- Federal Emergency Management Agency (FEMA) (2000a). Recommended seismic design criteria for new steel moment-frame Buildings. *FEMA-350*, prepared by the SEAOC-ATC-CUREE (SAC) Joint Venture for FEMA, Washington, DC.
- FEMA (2000b). State of the art report on connection performance. *FEMA-355D*, prepared by the SAC Joint Venture for FEMA, Washington, DC.
- Ibarra, L.F. and Krawinkler, H. (2005). Global collapse of frame structures under seismic excitations. *Rep. No. PEER/2005/06*, Pacific Earthquake Engrg. Res. Ctr., Richmond, Calif.
- Lignos, D. and Krawinkler, H. (2007). Contributions to collapse prediction for frame structures. *Final Report*, Kajima- CUREE Joint Research Program, Phase VI: Investigation of Factors Leading to Progressive Collapse of Structures, Category 2: Analysis of Structural Component Failure, Stanford Univ., Stanford, Calif.
- Mahin, S. A. and Morishita, K. (1998). "The effect of strength deterioration on the dynamic response of simple systems subjected to impulsive loading." *Proc. U.S.-Japan Workshop on Near-field Earthquake Damage in Urban Areas*, published as *Rep. EERL 98-01*, Earthquake Engrg. Res. Lab., California Inst. of Tech., Pasadena, Calif.
- Mazzoni, S., McKenna, F., Scott, M. H., Fenves, G. L., and Jeremic, B. (2003). *Open system for earthquake engineering simulation (OpenSees) command language manual*, Online publication <www.opensees.berkeley.edu> (April 14, 2004).
- Miranda, E., and Akkar, S. D. (2003). "Dynamic instability of simple structural systems." *Journal of Structural Engineering*, ASCE, 129(12), pp. 1722-1726.
- Rahnama, M. and Krawinkler, H. (1993). "Effects of soft soil and hysteresis model on seismic demands." *Rep. BLUME-108*, John A. Blume Earthquake Engrg. Ctr., Stanford Univ., Stanford, Calif.
- Rodgers, J. E. and Mahin, S. A. (2004). Effects of connection hysteretic degradation on the seismic behavior of steel moment-resisting frames. *Rep. No. PEER/2003/13*, Pacific Earthquake Engrg. Res. Ctr., Richmond, Calif.
- Rodgers, J. E., and Mahin, S. A. (2006). "Effects of connection fractures on global behavior of steel moment frames subjected to earthquakes." *Journal of Structural Engineering*, ASCE, 132(1), pp. 78-88.
- Rodgers, J. E., Mahin, S. A., and van Dam, M. (2006). "Versatile mechanical connections for use in research and education related to the inelastic seismic behavior of steel frames." *Proc., 4th Int. Symp. on Steel Struct.*, Seoul, Korea, pp. 1214-1224.
- Vamvatsikos, D. and Cornell, C. A. (2002). "Incremental dynamic analysis." *Earthquake Engineering and Structural Dynamics*, 31(3), pp. 491-514.
- van Dam, M. (2000). *Effect of hysteretic degradation on seismic response of steel structures*. M.Eng. thesis, Univ. of California, Berkeley, Calif.
- Vian, D. and Bruneau, M. (2003). "Tests to collapse of single degree of freedom frames subjected to earthquake excitations." *Journal of Structural Engineering*, ASCE, 129(12), pp. 1676-1785.