Struct Multidisc Optim 22, 307–321

Development of simplified models for design and optimization of automotive structures for crashworthiness

C.H. Kim, A.R. Mijar, J.S. Arora

Abstract Simplified models can be useful for up-front design of automotive structures for passenger safety during crash. Formulations based on the system identification approach are presented for development of simplified models for simulation and design for automotive crash environment. Numerical crash data available from experiments or simulations are used in the development of such models. Parametric as well as nonparametric formulations of the problem are investigated. Standard nonlinear programming optimality conditions and methods are used to solve the resulting nonlinear identification problem. Simple numerical examples are solved to illustrate the proposed formulations and methodologies. As a practical example, the front horn of an automotive structure is replaced by a single degree of freedom system (SDOF). Two basis functions that identify the given target data are studied: Hat functions (piecewise linear) and Chebyshev polynomials. Effects of the number of design variables on the final solution to the problem are investigated. In addition, using the identified SDOF model, redesign of the front horn to improve its performance is discussed.

Key words automotive structures, crashworthiness, simplified models, optimization, system identification

1 Introduction

Numerical methods for simulation of automotive crash events have been developed for the last about twenty years. Many finite element programs, such as DYNA3D and several of its variants, have become available. These programs have greatly enhanced the state-of-the-art for simulation of various automotive crash events, such as frontal crash, side impact, rear crash, offset crash, etc. These programs use explicit methods for numerical integration of the equations of motion, which require a very small time increment to obtain realistic simulations. In addition, the finite element models are usually very large having tens of thousands of degrees of freedom. As a result, the number of calculations for a realistic simulation is extremely large requiring enormous computing time on supercomputers.

Design of automotive structures for passenger safety during crash uses an iterative process where design changes are made and the structure is re-analyzed for its response. An evaluation of any design change requires simulation of the system, which is a nonlinear transient dynamic problem. The redesign process can be quite tedious and time-consuming when full-scale finite element models are used and the design process is carried out manually. Such detailed models and simulations are not useful at the conceptual design stage where quick design decisions need to be made. Therefore it is desirable to develop simplified models that can be used at the conceptual design development stage for quick analysis and redesign. The design obtained using the simplified models can then be cascaded to detailed design of the components.

The purpose of this paper is to explore various methods and formulations for development of simplified models for automotive structure and its components. Considerable data are available from laboratory/field experiments for structural components and the system. These data can be exploited in the development of the simplified models. To explore various methods and formulations, development of single degree of freedom models is considered first. The methods to be explored are generally classified as system identification techniques. The basic idea of these approaches is to determine the system properties, given the input to the system and output from it. For linear systems, these techniques have been developed and used for many years. However, very little has been done for nonlinear systems, especially under dynamic loading environment (Hollowell 1986). Nonlinear systems are highly complex, requiring systematic ap-

Received March 2, 2000
Revised manuscript received November 14, 2000

C.H. Kim, A.R. Mijar, J.S. Arora

Optimal Design Laboratory, College of Engineering, The University of Iowa, Iowa City, Iowa 52242 USA
e-mail: `Jasbir-Arora@uiowa.edu`

proaches to be developed for their identification. In this paper, optimization-based approaches are investigated for identification of nonlinear dissipative dynamic systems. Different formulations of the problem are proposed and evaluated.

In Sect. 2, a brief overview of the literature on the subject is presented. Section 3 defines a general system identification problem. Section 4 presents parametric identification of a single degree of freedom system. Section 5 presents nonparametric identification of the single degree of freedom system. A quadratic programming formulation for the single degree of freedom system is discussed in Sect. 6. Section 7 considers identification of the front horn as a single degree of freedom system. Data from simulation with DYNA3D are used in the identification process.Hat functions and Chebyshev polynomials are used to analytically represent the resisting force in the front horn. Section 8 considers a redesign of the front horn to improve its energy absorption capability. Finally, Sect. 9 contains discussion and concluding remarks.

2 Overview of literature

2.1 Crashworthiness analysis

Lumped mass models have been used since early 1970's for analysis and design of automotive structures for safety during crash (Kamal 1970; Kamal and Wolf Jr. 1982). In these models, major nonstructural components are represented by lumped masses and major deformable structural components are modelled as nonlinear spring elements, typically represented with the force-displacement data obtained from tubes. Lumped parameter models have been employed for dynamic simulations and occupant analysis (Bennett *et al.* (1991). Car to barrier, car to car, side impact, off-set impact and other crash events have been investigated. More recently, finite element models and a combination of them with lumped mass models have been utilized for crash simulation (Hollowell 1986).

Ni and Song (1986) described three methods for simulation of automotive structures for crash environment. The first method was called the hybrid method, which used a lumped mass model for the structure. The structural components were represented by nonlinear spring elements whose force-displacement characteristics were obtained by static crush tests in the laboratory. Dynamic correction or amplification factors were used to convert these curves for the dynamic environment of the crash. Applications of the method were limited to mostly one-dimensional problems. The second method was called the analytical method, which was based on either the limit analysis of the automotive structure modelled as a space frame, or the finite element analysis of the structure using beam and shell elements. The third method was called the mixed method, which combined the hybrid and the analytical methods. That approach was demonstrated on two example problems where some structural components were modelled as nonlinear spring elements and others were modelled using the finite elements. The mixed method was found to give adequate simulation capability for design of automotive structures for crash environment. Ni and Song (1986) also formulated and solved an optimization problem using the mixed simulation method. Some of the beam components were modelled using finite elements. Mass of these elements was taken as the cost function that was minimized. There were two design variables: wall thickness of a beam component. Constraints were imposed on Vehicle Crash Severity Index (VCSI), Windshield Residual Crush distance, and the A-pillar deformation, which was represented by the front tunnel deformation (this indirectly assesses the steering column displacement regulated by Federal Motor Vehicle Safety Standards).

Mahmood *et al.* (1993) have described in detail a procedure for rapid simulation and design of the frame of an automotive structure. They developed a simplified program, called V-CRUSH, for rapid simulation of the structure. The program used special collapsible 3-D thin-wall beam elements and was used to design full front-end frame for a light truck. The frame was divided into several substructures that were designed and tested. Experiments were also performed on the structures. Correlation between the experimental and simulation results was very good.

Huang *et al.* (1995) described Ford's Energy Management System that used CRUSH (Crash Reconstruction Using Static History) lumped mass modelling capability. In that system, the energy absorbing (EA) structural components were represented by nonlinear springs. Force-displacement characteristics of the EAs were obtained through static crush tests. Those were input directly to the program. Dynamic environment of the crash event was treated by the velocity sensitivity factors (dynamic amplification factors). Using the system, barrier loads and passenger compartment loads were calculated and compared to the test results in a frontal crash. Cheva *et al.* (1996) have also used lumped mass models for frontal/offset crash studies. They also used design of experiments approach to perform parameter optimization for crash environment.

Yamazaki and Han (1998) studied crushing energy absorption of circular and square tubes. Four node shell and solid finite elements in DYNA3D program were used to model the tubes. The tubes had a rigid mass attached to one end. They were crashed into a rigid wall at some initial velocity. Several cases were solved with different thickness and radius (width) of the tubes. Initial imperfections were introduced in some of the cases. Depending on the dimensions, axisymmetric, nonaxisymmetric, or column buckling modes got generated during crushing. Some of the results were compared with the available experimental data. Maximum energy was absorbed for the axisymmetric crush mode. An approximate response surface as a function of the radius (width) and thickness of the tube was generated using the "design of experiments" approach. The tube was optimized to maximize the energy absorption during impact with side constraints on the dimensions. It was found that the energy absorption increased with a decrease of the radius (width) and an increase of the thickness. However, if the radius was reduced too much, the tube could go into a column buckling mode, which had a dramatic decrease in the energy absorption capacity. It was also found that the mean impact force increased linearly with the radius (width) multiplied by the thickness squared. It was also observed that the energy absorption capacity of the cylindrical tube was slightly better than that for the square tube for the same mass of the tubes.

Lust (1992) considered optimization of automotive structures with constraints based on both linear elastic and crashworthiness loading conditions. A two phase crashworthiness analysis approach was used. In phase one, the structural components were analyzed to develop their force deformation characteristics. These force-deformation characteristics were assumed to be scalable with respect to the wall thickness of the structural elements, which were treated as design variables in the optimization process. A nonlinear approximation was developed for the crashworthiness constraints. In the second phase of crashworthiness analysis, a lumped mass model was used to determine response of the system. Stress, displacement, and frequency constraints were imposed for linear elastic response. For crashworthiness, constraints were imposed on VCSI (Vehicle Crash Severity Index) and total crush distance of the front end. It was found that a lighter design was obtained with the consideration of the simultaneous requirement of performance under linearly elastic condition and the crash condition.

Yang *et al.* (1994) presented a feasibility study of using numerical optimization methods to design structural components for crash. The presented procedure required several software, which included parametric modelling (Pro/ENGINEER), automatic mesh generation (PDA PATRAN3), nonlinear finite element analysis (RADIOSS), and optimization programs. Both single and multiple objective formulations were used for numerical optimization, which resulted in better designs. It was found that crash optimization was feasible but costly and that finite element mesh quality was essential for successful crash analysis and optimization. To demonstrate this work a simplified front horn was used that is also discussed later in this paper.

The nonlinear springs represented by static test data with some dynamic factors in the forgoing lumped mass models are no longer accurate for many current configurations. More accurate descriptions of the structural characteristics in a vehicle are required for acceptable predictions. In addition, data observed in a static crush test are generally different from those observed in a crash event. Therefore, it is desired to drive the structural characteristics from the data given during a structure crash test (Hollowell 1986). This can be posed as a system identification problem for the nonlinear dynamic structure. Therefore in the next two subsections, a survey of literature for system identification problems for linear or nonlinear structures is presented. Two types of formulations are discussed: parametric system identification and nonparametric system identification.

2.2 Parametric system identification

System identification methods can be classified on the basis of their search space:

– parametric methods that search in parameter spaces, and
– nonparametric methods that search in function spaces.

Basically, parametric methods seek to determine value of the parameters for an assumed model of the system to be identified. Schwibinger and Nordmann (1988) presented a procedure to identify a reduced order model for torsional vibration of large steam turbine generators. A 250 DOF finite element model was successfully reduced to a 13 degree of freedom model. The basic idea was to minimize the residuals between the eigenvalues of the finite element model and the reduced order model for the lowest few modes of vibrations. A weighted least squares approach was used, and an iterative numerical algorithm was used to identify stiffness parameters for the reduced model.

Lingener and Doege (1988) used the system identification approach in the time domain to identify stiffness and damping properties for the supports of a nuclear reactor. The vessel was modelled as a rigid body supported by a six DOF spring-damper system with three translational and three rotational degrees of freedom. Experimental data about the support system were collected during construction of the reactor. Two objective functions were tried: weighted sum of the squared errors, and an absolute sum of the errors. The second objective function performed better, and so results with only that function were presented.

Udwadia and Shah (1976) presented an approach to identify structures using records obtained during strong earthquake ground motion. The history-matching problem was expressed as a minimization of the error between the measured response and the model response. The building was modelled as a shear beam, and stiffness as a function of the height. Any constraints on the problem were handled using penalty parameters. The steepest descent and conjugate gradient methods were used to solve the unconstrained problem. The sensitivity analysis of the objective functional was performed using the adjoint variable method, which was judged to be quite efficient compared to the direct differentiation approach. The solution was found to be highly dependent on the starting values for the unknown variables (the stiffness). It was noted that the nonuniqueness of the solution could result in stiffness that was 20% to 30% different from the actual stiffness.

Hollowell (1986) attempted an identification of lumped mass models in the time domain by modelling the structural components as nonlinear springs. In the approach developed, the stiffness and damping characteristics of an element were assumed to be separable. In addition, they were assumed to be piecewise linear over certain time intervals. The coefficients of the piecewise linear stiffness and damping characteristics of an element were treated as the parameters to be identified. Linear and quadratic programming techniques were used to solve for these unknown parameters. Three error norms were tried as cost functions of the optimization problem: L_1, L_2 and L_∞. In the most solution cases, the error function based on the L_2 norm gave better results. The methodology was applied to a three DOF model having six nonlinear springs and two damping elements. Five time increments were used in the identification process. Constraints were imposed on the maximum and minimum values of the damping force. An adaptive approach was developed to allow for variation in the number of time steps during each increment of the identification time interval.

2.3
Nonparametric system identification

Nonparametric methods produce the best functional representation for structural elements of the system without a priori assumptions about the deformable elements of the model. Therefore, these methods make it possible to eliminate the restriction of forcing the system characteristics to fit an assumed form such that the system can not be changed (Masri and Caughey 1979). Automotive structures and components must be designed to operate in the linear elastic range during normal operations; however, they must deform inelastically during crash to dissipate a large amount of energy. Therefore, nonlinear systems must be treated. Masri and Caughey (1979) approximated the unknown restoring force for a single degree of freedom system in terms of Chebyschev polynomials. Then regression analysis was used to identify the coefficient of the polynomials. Orthogonality property of these polynomials gave closed form expressions for the unknown coefficients. The restoring force was assumed to depend on the deformation (displacements) as well as deformation rate (velocities). Two approximations for the restoring force were tried:

– Chebyshev polynomials in terms of two variables (displacement and velocity), and
– sum of two terms, one depending on displacement and the other dependent on velocity.

The second approximation worked well when there was weak coupling between the displacement and velocity. When there is strong coupling, the first approximation worked better. Four example problems were presented: simple damped linear system, Duffing Oscillator, Van der Pol Oscillator, and Hysteretic Oscillator. For all examples, the system was successfully identified using a known solution, and then tested using another input function. The approach was also used for multi-degree of freedom systems where Chebyschev polynomials were replaced by arbitrary orthogonal functions (Udwadia and Shah 1976). A method to identify viscoelastic materials characterized by Volterra integral equations was presented by Distefano and Todeschini (1973). An optimization based approach was used by Jao *et al.* (1991) to identify plastic, viscoelastic and viscoplastic materials modeled using the endochronic constitutive theory. The problem had several nonlinear constraints. These material identification problems are highly nonlinear.

A detailed overview of the literature presented by Hollowell (1986) revealed that many techniques had been investigated for identification of linear systems. This was reasonable at that time since most of the systems were designed to operate in the linear regime. The review was divided into two parts: frequency domain approach, and time domain approach. Much of the literature was devoted to the frequency domain approach because frequency related data could be measured more accurately. Sophisticated test equipment and procedures were available for measuring natural frequencies and mode shapes. The identification approach determines the mass, stiffness and/or damping matrices for the system. The time domain data for impulsive loads, resonant testing or free decay response could also be utilized to identify the modal parameters. The methods include the least squares complex exponential method, the polyreference method, and the Ibrahim time domain method (Leuridan *et al.* 1985). To identify damping and stiffness of a building subjected to earthquake excitation, a time domain recursive least square technique that required no matrix inversion was used by Caravani *et al.* (1977). The proposed algorithm provided means to account for both the model uncertainty and the investigators' confidence in the initial guess of the parameters.

Masri *et al.* (1993) introduced a procedure based on the use of artificial neural networks for the identification of nonlinear dynamic systems. The proposed method was applied to the damped Duffing oscillator under deterministic excitation. The representation of the restoring force was given by the neural network topology with its weights instead of Chebyshev polynomials. Therefore identification was to find optimum weights to reduce least-squares error of the restoring force. It was shown that employing a three-layer neural net with inputs, one output, and 15 and 10 nodes in the first and second hidden layers, respectively, was adequate to characterize the internal force in the damped Duffing oscillator. The comparison between neural networks and another nonparametric identification method in a previous paper (Masri and Caughey 1979) was discussed.

Argoul and Jezequel (1989) proposed a new interpolation procedure which improves identification of the nonlinear part of lumped parametric systems in areas of the state space fields where experimental data are insufficient. A modal representation was used and was approximated by two-dimensional Chebyshev polynomials while in Masri and Caughey (1979), Masri *et al.* (1993) the restoring force was approximated by Chebyshev polynomials. Two examples were presented: Van der Pol oscillator and a two degree of freedom model presenting a Duffing type nonlinear oscillator. The proposed method requires information on the identification of the linear behavior of the structure – its pertinent mode shapes – which allows change to the modal representation and its dynamic nonlinear response.

Ni *et al.* (1999) presented a nonparametric identification method for nonlinear hysteretic systems. Making use of the Duhem hysteresis operator, the multi-valued relationship of hysteretic restoring force in terms of the displacement and velocity of the phase plane, is mapped onto two single valued surfaces in an appropriate subspace in terms of the state variables–displacement and hysteretic restoring force. Fitting the surfaces with the generalized orthogonal or nonorthogonal polynomials in terms of the displacement and the hysteretic restoring force identified the functions describing the surfaces. As an example, a wire-cable isolation system was analyzed with Taylor series expansion.

3 Formulation of the system identification problem

The basic system identification problem is to determine properties of the system using some known data. The data could be available from laboratory or field experiments, or from computer simulations. For *dynamic system identification*, the system parameters to be identified could be mass, stiffness, and damping properties. If the system properties are represented by parameters, such as stiffness parameters, damping parameters, mass, etc., then the problem is called *parametric identification*; otherwise it is called *nonparametric system identification.* In either case, the system identification problem is defined as follows.

Problem 1. Minimize a measure of the error between the known data and the analytical data generated by a functional representation of the system properties subject to constraints on the response of the system and/or response related quantities.

3.1 Definition of an error function

The error between the given data and the analytically generated data can be written in several different ways. To define this, let $s^{\text{given}}(t)$ be the given data and $s^{\text{appr}}(\mathbf{b},t)$ be the analytically generated response data, where $\mathbf{b}$ is a vector representing system properties, such as stiffness, damping and mass. The vector $\mathbf{b}$ could also contain coefficients of the expansion of the system properties in terms of some known functions; e.g., in nonparametric system identification, use of sine and cosine functions, Chebyshev polynomials, splines, etc. Now the error function between the given data and analytical representation can be defined as

$$e(\mathbf{b},t) = s^{\text{given}}(t) - s^{\text{appr}}(\mathbf{b},t)\,. \tag{1}$$

The system identification problem is to reduce this error and at the same time satisfy all the constraints on performance of the system. This can be stated as a problem to minimize a scalar measure of the error function over the given time interval, as

$$\min_{\mathbf{b}} E(\mathbf{b})\,, \tag{2}$$

where $E(\mathbf{b})$ can be defined using L_1, L_2, or L_∞ norms of the error function. Using L_1 norm of the error function, $E(\mathbf{b})$ is defined as

$$E(\mathbf{b}) = \int_0^T |e(\mathbf{b},t)|\,\mathrm{d}t\,, \tag{3}$$

where $[0,T]$ is the time interval of interest. This will be called L_1 cost function. Using L_2 norm of the error function, $E(\mathbf{b})$ is defined as

$$E(\mathbf{b}) = \int_0^T [e(\mathbf{b},t)]^2\,\mathrm{d}t\,. \tag{4}$$

This will be called L_2 cost function. Using L_∞ norm of the error function, $E(\mathbf{b})$ is defined as

$$E(\mathbf{b}) = \max_{t\in[0,T]} |e(\mathbf{b},t)|\,. \tag{5}$$

This will be called L_∞ cost function. The problem defined in (4) is usually called the *least squares problem.* A preliminary investigation for the error representation showed that the L_2 cost functional worked better than those with L_1 and L_∞ cost functions. It is shown later that the L_2 cost function has a positive definite Hessian. If the constraints are linear, then a global minimum point is obtained for the problem. Therefore, only the formulation and results with the L_2 cost function are presented later in the paper.

3.2 Constraints

Some constraints may need to be imposed while solving the system identification problem. For example, the following constraints may be imposed.

– Final displacement must be equal to some specified displacement.
– Maximum force should not exceed a specified value.
– Energy absorbed must be greater than some specified value.
– Acceleration should not exceed a specified value at any time.
– HIC (Head Injury Criterion) needs to be bounded.
– VCSI (Vehicle Crash Severity Index) needs to be bounded.

Note that some of the constraints are time dependent, and so they must be treated appropriately in the optimization algorithm.

3.3 Nonparametric identification

For nonparametric system identification, the unknown quantity must be represented analytically. For this purpose some basis functions can be used as follows:

$$s^{\text{appr}}(\mathbf{b},t) = \sum_{i=0}^{m} b_i \Phi_i(t)\,, \tag{6}$$

where b_i are the coefficients of the expansion, $m+1$ is the number of terms, and $\Phi_i(t)$ are the basis functions (time-dependent).

Several basis functions can be used. For example if one uses Fourier cosine functions, then the basis functions are given as

$$\Phi_i(t) = \cos\frac{i\pi t}{T}\,. \tag{7}$$

Chebyshev polynomials as basis functions are given as

$$\Phi_i(\tau) = \cos(i\cos^{-1}\tau)\,, \quad \tau\in[-1,1]\,. \tag{8}$$

Chebyshev polynomials given in (8) satisfy the weighted orthogonality property as

$$\int_{-1}^{1} \frac{\Phi_i(\tau)\Phi_j(\tau)}{\sqrt{1-\tau^2}}\,\mathrm{d}\tau = \begin{cases} 0 & i\neq j\,,\\ \frac{\pi}{2} & i=j\neq 0\,,\\ \pi & i=j=0\,, \end{cases} \tag{9}$$

where the weighting function $w(\tau)$ is $(1-\tau^2)^{-1/2}$. According to the definition of Chebyshev polynomials, the normalized time parameter τ $(-1\leq\tau\leq 1)$is defined as

$$\tau = \frac{t-\frac{T}{2}}{\frac{T}{2}} = \frac{2t-T}{T}\,. \tag{10}$$

From this equation, $\tau=-1$ when $t=0$ and $\tau=1$ when $t=T$. In numerical calculations, the following equations can be used to generate Chebyshev polynomials:

$$\Phi_i(\tau) = 2\tau\Phi_{i-1}(\tau) - \Phi_{i-2}(\tau)\,, \quad \text{for } i\geq 2\,,$$

$$\Phi_0(\tau)=1\,, \quad \Phi_1(\tau)=\tau\,. \tag{11}$$

Another popular representation is the piece-wise linear representation. This can be represented in terms of the Hat function over a subinterval $t_{i-1}(=t_i-\Delta t)$ and $t_{i+1}(=t_i+\Delta t)$ as

$$\Phi_i(t) = \begin{cases} 0 & 0\leq t\leq t_{i-1}\,,\\ \frac{t-t_{i-1}}{t_i-t_{i-1}} & t_{i-1}\leq t\leq t_i\,,\\ \frac{t_{i+1}-t}{t_{i+1}-t_i} & t_i\leq t\leq t_{i+1}\,,\\ 0 & t_{i+1}\leq t\leq T, \end{cases} \tag{12}$$

where Δt is a time increment between control points. Many other basis functions can be used.

In the remaining sections, some of the foregoing formulations will be implemented and evaluated for identification of nonlinear dynamic systems.

4 Parametric system identification

4.1 Introduction

To introduce the system identification problem, a linear damped single degree of freedom system, shown in Fig. 1, is considered initially. The system is at rest at time $t=0$ and is subjected to an initial velocity v_0. The equation of motion and the initial conditions are

$$M\ddot{x}+C\dot{x}+Kx(t)=0\,,$$

$$\text{for} \quad 0\leq t\leq T\,, \quad x(0)=0\,, \quad \dot{x}(0)=v_0\,. \tag{13}$$

The response for the system can be written in the closed form for under-damped, critically damped, or over-damped cases (Hibbeler 1978). The following data are used to generate the response that is used as the given information in the system identification process: $M=31.6$ kg, $K=20\,000$ N/m, $v_0=-2.0$ m/s, $T=0.4$ s,

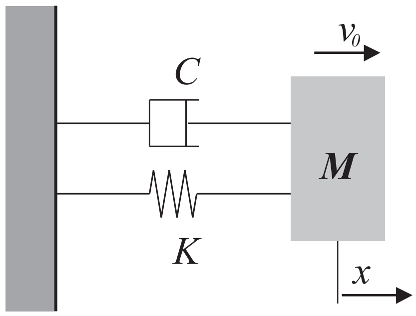

Fig. 1 Single degree of freedom system

$C = 700\,\mathrm{N\,s/m}$. With these data the system is underdamped, with undamped natural frequency of 4 Hz (25.16 radians/s).

4.2 System identification problem

The system identification problem is to determine the system parameters, stiffness and damping coefficients, to match the approximated response to the given analytical response. Let the approximate stiffness and damping be represented as

$$K^{\mathrm{appr}}[x^{\mathrm{given}}(t)] = [b_1 + b_2 x^{\mathrm{given}}(t)]K' ,$$

$$C^{\mathrm{appr}}[\dot{x}^{\mathrm{given}}(t)] = [b_3 + b_4 \dot{x}^{\mathrm{given}}(t)]C' , \tag{14}$$

where K' and C' are given constants, and b_1, b_2, b_3 and b_4 are the unknown parameters to be determined. x^{given} and $\dot{x}^{\mathrm{given}}$ are the given displacement and the given velocity data, respectively. The cost function for the system identification problem is defined as the integral of the square of L_2 norm of the error function between the approximated acceleration and the known acceleration of the mass over the time interval T. This error function depends on the parameters b_i in Eqs. (14) as

$$E(\mathbf{b}) = \int_0^T \left[a^{\mathrm{given}}(t) - a^{\mathrm{appr}}(\mathbf{b}, t)\right]^2 \mathrm{d}t , \tag{15}$$

where $a^{\mathrm{appr}}(\mathbf{b}, t)$ is the approximated acceleration, which is computed by integrating (13) with the nonlinear stiffness, the nonlinear damping coefficient in (14), and the initial conditions in (13). $a^{\mathrm{given}}(t)$ is the given acceleration (calculated using the analytical solution for single degree of freedom system for demonstration purposes).

Constraints are imposed on the explicit lower and upper bounds for stiffness and damping parameters as follows:

$$K^L \le K^{\mathrm{appr}}(t_i) \le K^U ,$$

$$C^L \le C^{\mathrm{appr}}(t_i) \le C^U , \quad i = 1, 2, \dots, n , \tag{16}$$

where the subscript i represents a time grid point except the initial point. In the actual computation, these constraints produce 1600 inequalities, since n is 400.

Also, a constraint is imposed on the dissipated energy as

$$D \le D_I , \tag{17}$$

where D_I is the input kinetic energy given as $0.5mv_0^2$ (63.2 Nm), and D is the energy dissipated by the damper given as

$$D = \int_0^{x_f} F_{\mathrm{damping}}\,\mathrm{d}x = \int_0^{x_f} C^{\mathrm{appr}} \dot{x}^{\mathrm{given}}\,\mathrm{d}x = \int_0^T C^{\mathrm{appr}} \dot{x}^{\mathrm{given}2}\,\mathrm{d}t , \tag{18}$$

where x_f is the final displacement at $t = T$. The data for the problem is taken as $K^L = 18\,000$, $K^U = 22\,000$, $C^L = 500$, $C^U = 800$. The analytical solution for the identification problem is $b_1 = 1$, $b_2 = 0$, $b_3 = 1$ and $b_4 = 0$.

The SQP algorithm in IDESIGN software is used to solve the optimization problem (Arora 1989). Note that with the stiffness and damping representations given in (14), the equations of motion become nonlinear. DYNA3D program is called directly through the USER routines of IDESIGN to integrate the equation of motion. Starting from the following initial values for the design variables:

$$b_1 = 15 , \quad b_2 = 10 , \quad b_3 = 10 , \quad b_4 = 5 ,$$

with

$$E = 9661.7 ,$$

the algorithm converges to almost the exact solution in 13 iterations:

$$b_1 = 1.00189 , \quad b_2 = 0 , \quad b_3 = 1.00004 , \quad b_4 = 0 ,$$

with

$$E = 9.0277 \times 10^{-5} .$$

At the starting point some constraints are violated by 1273%; however, at the optimum point, no constraint is violated. Figure 2 shows the history of the cost function [E, error in (15)] for the optimization process. Note that the optimal point is obtained after 13 iterations and as a stopping criterion, the length of the search direction is required to be less than 0.01. Figure 3 shows the exact and approximated displacement histories. There is practically no difference between the analytical and the

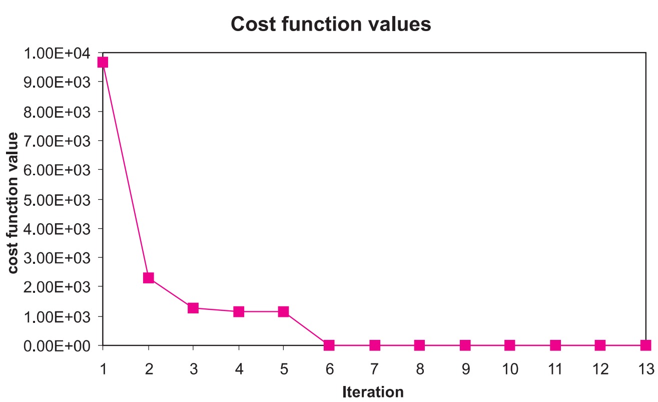

Fig. 2 Cost function history

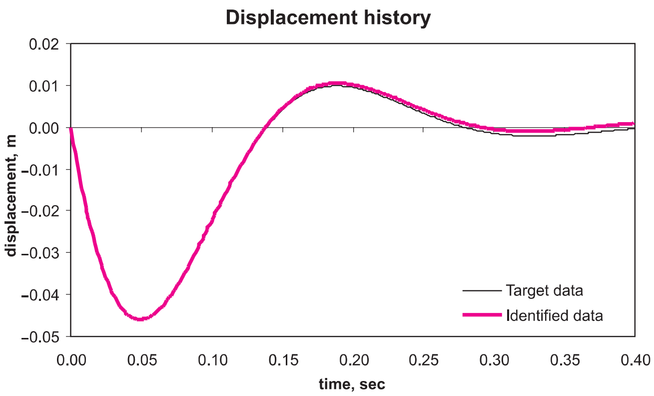

Fig. 3 Displacement–time histories for target data and identified data

approximated histories. Similar matches are obtained for velocity and acceleration. The active constraints at optimum point are the upper limits for stiffness. Although the formulation identifies the linear system well, it needs displacement, velocity and acceleration data. This may not be desirable in some practical applications. It is possible to formulate and solve the identification problem when only the acceleration data are known.

5
Nonparametric system identification

5.1
Introduction

In this section, a nonparametric representation for the resisting force of the structural element is considered. This representation is more general than the parametric form. The system is shown in Fig. 4. The equation of motion for the system is written as

$$M\ddot{x} + FS(t) = 0\,,$$

for

$$0 \le t \le T\,, \quad x(0) = 0\,, \quad \dot{x}(0) = v_0\,. \tag{19}$$

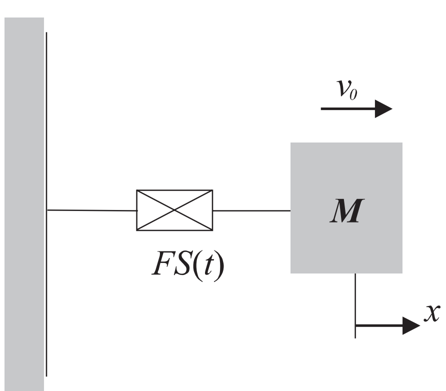

Fig. 4 Nonparametric single degree of freedom system

5.2
System identification problem

The system identification problem is to determine an analytical representation for the force $FS(t)$ using the given data for the force or the acceleration of the mass. This data could be available from experiments or computer simulations. Let the acceleration data be known. The cost function for the problem is defined as the minimization of

$$E(\mathbf{b}) = \int_0^T [e(\mathbf{b}, t)]^2 \, \mathrm{d}t\,, \tag{20}$$

where $e(\mathbf{b}, t)$ is given as

$$e(\mathbf{b}, t) = a^{\text{given}}(t) - a^{\text{appr}}(\mathbf{b}, t)\,. \tag{21}$$

Here $a^{\text{given}}(t)$ is the given acceleration data, $a^{\text{appr}}(\mathbf{b}, t)$ is the approximate representation of the acceleration, and $\mathbf{b}$ is a vector of unknown parameters treated as design variables of the system identification problem. To calculate the approximate acceleration from (19), the force $FS(t)$ is approximated as

$$FS^{\text{appr}}(\mathbf{b}, t) = \sum_{i=0}^{m} b_i \Phi_i(t)\,, \tag{22}$$

where b_i are the parameters of the expansion, $\Phi_i(t)$ for $0 \le t \le T$ are some known shape functions, and $m+1$ is the number of terms used to represent the force. Now the system identification problem is to determine the parameters b_i to minimize the error function defined in (20). Constraints may be imposed on the parameters themselves, and/or other quantities such as displacements. The constraint on the force is expressed as

$$FS^L \le FS^{\text{appr}}(\mathbf{b}, t) \le FS^U\,, \tag{23}$$

where FS^L and FS^U are the lower and upper limits on the force.

To solve the foregoing optimization problem, an SQP algorithm available in the program IDESIGN is used. The program requires evaluation of problem functions and their gradients. The program DYNA3D is coupled directly to IDESIGN to evaluate response of the system for any values of the design variables. The gradients of the cost function and the constraints are evaluated using the central finite difference option available in IDESIGN. The following data for the system are used: $M = 31.6$ kg, $v_0 = -2.0$ m/s, $T = 0.1$ s, $FS^L = -1600$ N, $FS^U = 200$ N, spring constant $k = 20\,000$ N/m, and the damping coefficient $c = 700$ N s/m. These data are used to calculate the acceleration of the mass that is taken as the given data in (21). Five points corresponding to the time 0 s, 0.025 s, 0.050 s 0.075 s, and 0.1 s, are used to represent the force; i.e., $m = 4$ in (22). In the present example, the Hat function defined in (12) is used. Therefore the parameter b_i

becomes the force at the i-th point. In the numerical solution process, 101 grid points (100 intervals) are used in the simulation and the integral evaluations. The bounds on the force shown in (23) are imposed at each time grid point, and so there are 202 constraints for the problem.

Figure 5 shows estimated acceleration and the target acceleration curves. It is seen that there is practically no difference between the two curves. Since the restoring forces are proportional to acceleration from (19), their histories match quite well also. The displacement and velocity histories also matched very well. The cost function, calculated from (20), is almost zero. The values for the 5 design variables are

$$b_0 = -1449.2\,, \quad b_1 = -1350.5\,, \quad b_2 = -890.4\,,$$

$$b_3 = -417.7\,, \quad b_4 = 50.2\,.$$

At the optimal point, a few of the largest values for the approximated force are active constraints.

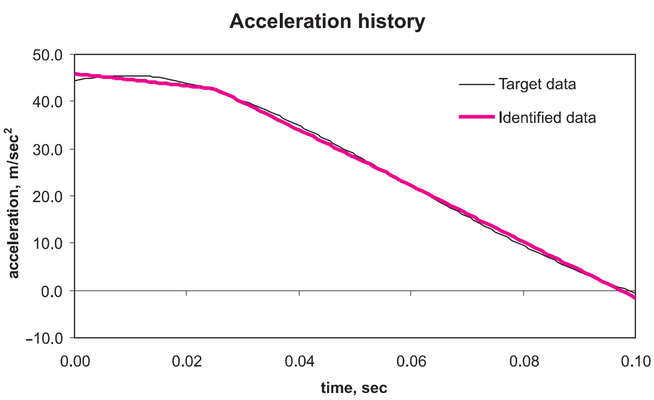

Fig. 5 Acceleration–time histories for target data and identified data

5.3 Redesign of the system

The system identified in the previous section is redesigned to meet the desired system response. It is required that change of kinetic energy (D) during the time interval T be at least 10% more than that of the base system (D_b). The increase in the kinetic energy change yields more energy absorption for the system. This constraint is expressed as

$$D \geq 1.1 D_b\,, \tag{24}$$

where D and D_b are given as

$$D = \frac{1}{2} M[v_0^2 - v^2(T)]\,, \quad D_b = \frac{1}{2} M[v_0^2 - v_b^2(T)]\,. \tag{25}$$

Here $v_b(T)$ is the base model velocity at time $T\,(= 0.66983\,\text{m/s})$, and $v(T)$ is the final velocity of the redesigned system. Note that (24) implies that the final velocity of the redesigned system is reduced compared to the base model. Other data are the same except that FS^L is changed to -1800 N.

The cost function is changed to minimize the maximum acceleration of the system over the time interval 0 to T,

$$\min \left\{ \max_{0 \leq t \leq T} |a(t)| \right\}\,. \tag{26}$$

This min-max problem is converted to the standard one by introducing an artificial design variable $a_{\max}$ and an additional constraint as follows:

$$\min a_{\max}\,, \tag{27}$$

subject to an additional constraint

$$|a(t)| \leq a_{\max}\,, \quad 0 \leq t \leq T\,. \tag{28}$$

This constraint is also imposed at each time grid point, just as the force constraints in (23). Thus there are 304 constraints for the problem including the constraint of (24).

Five terms are used for Hat functions in (22) to represent the force-time curve as in the previous section. All other data are the same as for the original case. Figures 6–10 show various response histories for the re-designed system. It is seen that the objective of

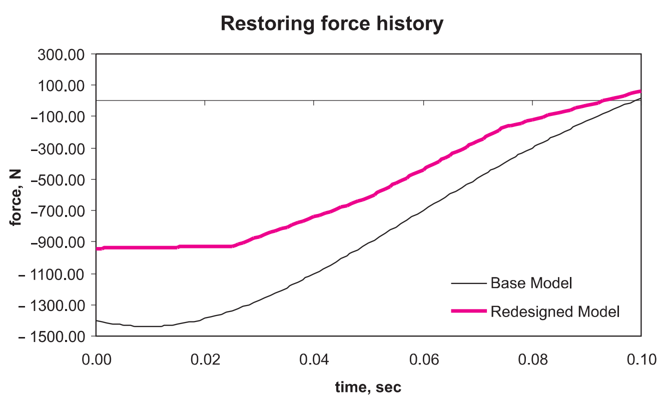

Fig. 6 Restoring force–time histories for redesign problem

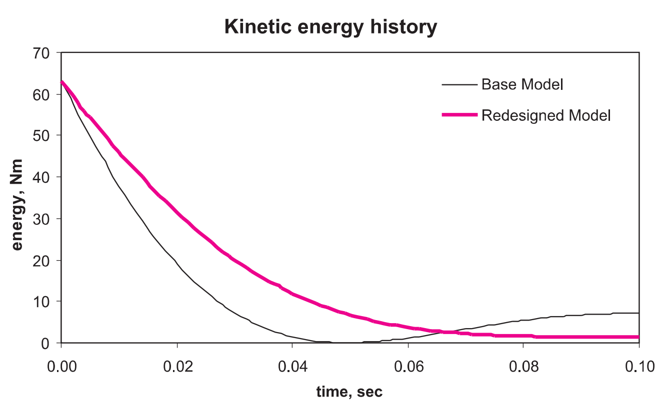

Fig. 7 Kinetic energy–time histories for redesign problem

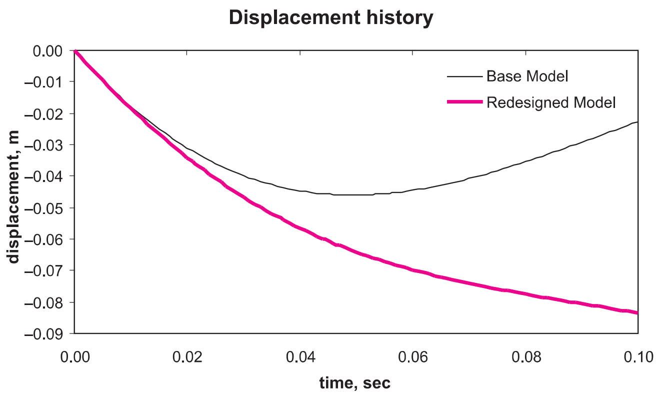

Fig. 8 Displacement–time histories for redesign problem

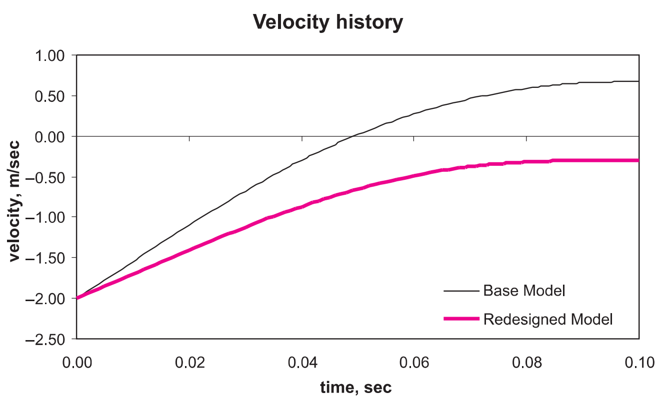

Fig. 9 Velocity–time histories for redesign problem

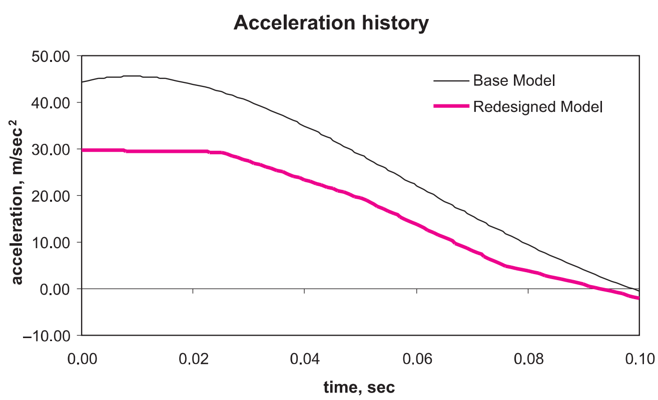

Fig. 10 Acceleration–time histories for redesign problem

10% increase in the kinetic energy change is achieved ($D = 61.75\,\mathrm{kg\,m^2/s^2}$; $D_b = 56.11\,\mathrm{kg\,m^2/s^2}$). This implies that the absolute velocity of the mass is reduced from 0.6683 m/s to 0.302577 m/s compared to the base system giving more absorption of energy. Note that the force-time curve for the re-designed system is below that for the base model. The reason for this behavior is that no constraint is imposed on the displacement of the system, and so the displacement of the redesigned system has increased from about 0.023 m to 0.082 m. The constraint in (24) is active at the optimal point.

6 Quadratic programming formulation with L_2 cost function

System identification problem defined in Sect. 5.2 is formulated as a quadratic optimization problem. The cost function for the optimization is given from (20) and (21) as

$$\min E(\mathbf{b}) = \int_0^T \left[a^{\text{given}}(t) - a^{\text{appr}}(\mathbf{b}, t)\right]^2 \,\mathrm{d}t\,, \tag{29}$$

where $a^{\text{given}}(t)$ is the given acceleration, T is a termination time for the simulation, $a^{\text{appr}}(\mathbf{b}, t)$ is the approximated acceleration obtained from the basis function representation of the force element. Substituting the basis function representation for the force element in (22) into the equation of motion (19), the approximated acceleration is given as

$$a^{\text{appr}}(\mathbf{b}, t) = -\frac{1}{M} FS^{\text{appr}}(\mathbf{b}, t) = -\frac{1}{M}\sum_{i=0}^{m} \Phi_i b_i = -\frac{1}{M}\mathbf{\Phi}^T\mathbf{b}\,. \tag{30}$$

Substitute (30) into the cost function in (29) and simplify it to obtain

$$E(\mathbf{b}) = \frac{1}{2}\mathbf{b}^T\mathbf{Q}\mathbf{b} + \mathbf{c}^T\mathbf{b} + \text{constant}\,, \tag{31}$$

where

$$Q_{ij} = \frac{2}{M^2}\int_0^T \Phi_i\Phi_j \,\mathrm{d}t\,, \quad i, j = 0, 1, 2, \ldots, m\,, \tag{32}$$

$$c_i = \frac{2}{M}\int_0^T a^{\text{given}}(t)\Phi_i(t)\,\mathrm{d}t\,, \quad i = 0, 1, 2, \ldots, m\,, \tag{33}$$

$$\text{constant} = \int_0^T [a^{\text{given}}(t)]^2 \,\mathrm{d}t\,. \tag{34}$$

Equality constraints are imposed on the final displacement and/or the final velocity:

$$g_d(\mathbf{b}) = \frac{x^{\text{appr}}(\mathbf{b}, T)}{x_f} - 1.0 = \mathbf{a}^T\mathbf{b} + d = 0\,, \tag{35}$$

$$g_v(\mathbf{b}) = \frac{\dot{x}^{\text{appr}}(\mathbf{b}, T)}{v_f} - 1.0 = \mathbf{s}^T\mathbf{b} + r = 0\,, \tag{36}$$

where x_f and v_f are the given final displacement and velocity, and $x^{\text{appr}}(\mathbf{b}, T)$ and $\dot{x}^{\text{appr}}(\mathbf{b}, T)$ are the approximated final displacement and velocity obtained by integrating the equation of motion. Equations (35) and (36) can be written as

$$\mathbf{g}(\mathbf{b}) = \begin{bmatrix} g_d(\mathbf{b}) \\ g_v(\mathbf{b}) \end{bmatrix} = \begin{bmatrix} \mathbf{a}^T\mathbf{b} + d \\ \mathbf{s}^T\mathbf{b} + r \end{bmatrix} = \mathbf{A}^T\mathbf{b} + \mathbf{d} = \mathbf{0}\,, \tag{37}$$

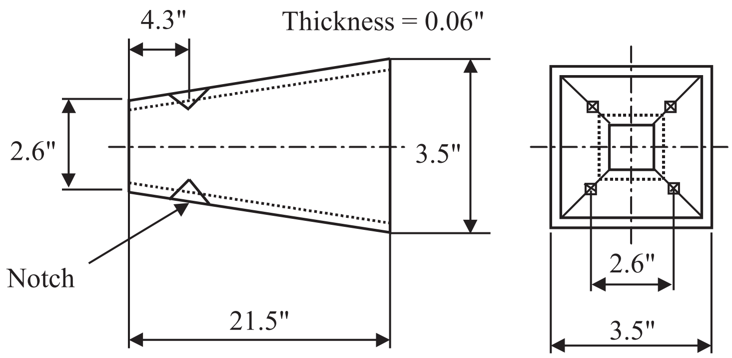

Fig. 11 Shape of front horn with notches

where $\mathbf{A} = [\mathbf{a} \quad \mathbf{s}]_{(m+1)\times 2}$ and $\mathbf{d}^T = [d \quad r]$. Detailed expressions for matrix $\mathbf{Q}$, and vectors $\mathbf{c}$, $\mathbf{a}$, $\mathbf{s}$, etc., with Hat and Chebyshev basis functions are given by Kim *et al.* (1999).

The identification problem defined in (31) and (37) can be solved in a closed form. The Lagrangian function for this QP problem is written as

$$L = \frac{1}{2}\mathbf{b}^T\mathbf{Q}\mathbf{b} + \mathbf{c}^T\mathbf{b} + \text{constant} + \mathbf{v}^T(\mathbf{A}^T\mathbf{b} + \mathbf{d}), \qquad (38)$$

where $\mathbf{v}$ is the Lagrange multiplier vector. The optimality condition gives

$$\mathbf{Q}\mathbf{b} + \mathbf{c} + \mathbf{A}\mathbf{v} = \mathbf{0}. \qquad (39)$$

In addition, the equality constraints given in (37) must be satisfied at the optimal point. Substituting (39) into (37), the Lagrange multiplier vector is given as

$$\mathbf{v} = (\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{A})^{-1}(-\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{c} + \mathbf{d}). \qquad (40)$$

The optimal values of $\mathbf{b}$ are computed by substituting (40) into (39). It can be shown that the Hessian matrix is positive definite. Since the constraints in (37) are linear, the QP problem is convex. Thus the solution obtained for the problem is a global minimum.

The forgoing quadratic formulation with Hat functions and Chebyshev polynomials is applied to the example problems for identification and redesign in the next two sections.

7 Numerical examples: Front horn system

Among many components, the front horn is an important part of the automotive structure. Design of this component can affect performance of the vehicle during a crash. Therefore it is important to have a good design for this component. In this section, the system identification problem for the front horn to reduce it to a single degree of freedom system as shown in Fig. 4 is addressed. First a finite element simulation of the front horn is performed. Then the simulation data are used to identify the system. Three objective functions based on L_2, L_1 and L_∞ norms of the error function introduced in Sect. 3 are investigated for the identification problem; however, results with only L_2 cost function are presented.

Figure 11 shows overall dimensions of the front horn used as the sample problem. Figure 12 shows the crash environment for the system. The data for the problem are as follows: initial velocity is $v_0 = 525$ in/s (approximately 30 mph), mass of the horn is 0.0114 lbf·s^2/in, attached mass is 3.69 lbf·s^2/in (corresponds to half of 1300 kg, the vehicle weight). The system is travelling from right to left and crashes into a fixed barrier. To obtain the response histories, a finite element model for the horn is developed and analyzed with DYNA3D program. The finite element model consists of 1184, 4-node quadrilateral shell elements.

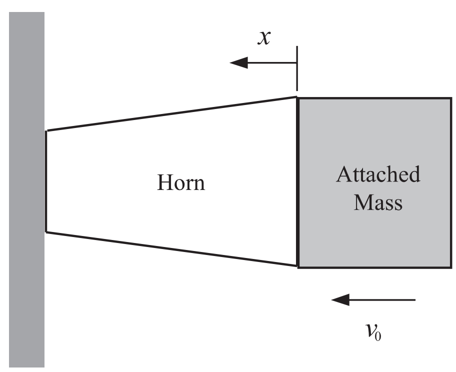

Fig. 12 Crash environment for front horn

The model, shown in Fig. 13, has 7104 degrees of freedom. A 40 ms simulation is performed. The following data are used: material is elastic-plastic strain hardening, yield stress is 30 000 lb/in^2, density is 7.1×10^{-4} (lbf $\cdot$ s^2/in) /in^3, tangent modulus is 60 000 lb/in^2, Young's modulus is 3.0×10^7 lb/in^2), Poisson's ratio is 0.3. Figure 14 shows the deformed configuration of the component. At $T = 0.04$ s, $x(T) = 19.6$ in, and $v(T) = 456.7$ in/s. The system has not come to rest yet. The kinetic energy of the system has reduced from 510 000 lb $\cdot$ in to 386 000 lb $\cdot$ in.

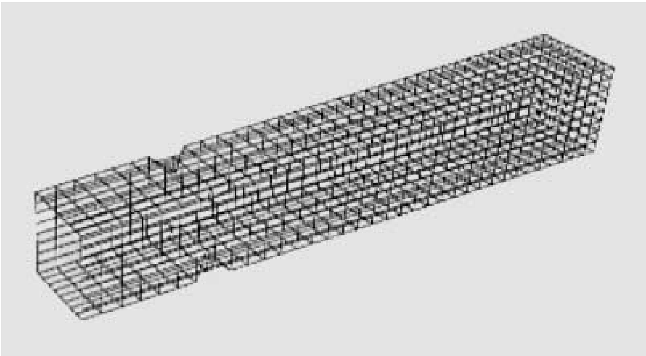

Fig. 13 Finite element model of front horn

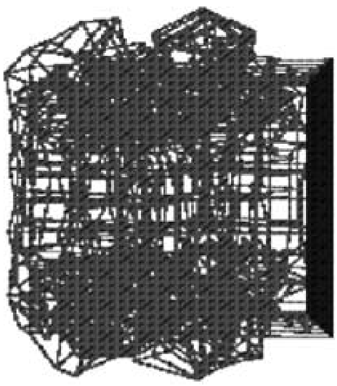

Fig. 14 Deformed shape of front horn at $t = 0.038$ s

All the responses are obtained using the 100 Hz low pass filter options in LS-TAURUS, the post-processor for DYNA3D.

Note that in the simulation process, energy absorption by other components of the vehicle is ignored. Thus all the input energy is being absorbed due to crushing of the front horn. This is not a realistic assumption, practically. However, the generated simulation data are used in the identification process to develop and demonstrate the procedure. The error function $e(\mathbf{b}, t)$ of (21) is defined using the force data and an analytical force representation of (22). m is taken as 25 and the basis functions are taken the Hat functions and Chebyshev polynomials in (22). Thus there are 26 design variables. A constraint is imposed on the final displacement to obtain the desired response for the identified system, as $x(T) = x_f$, where $x(T)$ is the final displacement of the identified system and x_f is the specified final displacement, which is given from the simulation results as 19.6 in.

The problem is solved using the SQP algorithm in IDESIGN program Arora (1989). The starting value for the force is set to 6000 lbs for the entire time interval. An optimal solution is obtained after 30 iterations. In addition, the analytical solution of the QP problem is obtained using a program written in MATLAB with 25 design variables. The target force and two identified force data set using Hat basis functions, are shown in Fig. 15. It is seen that the three force curves match quite well. The

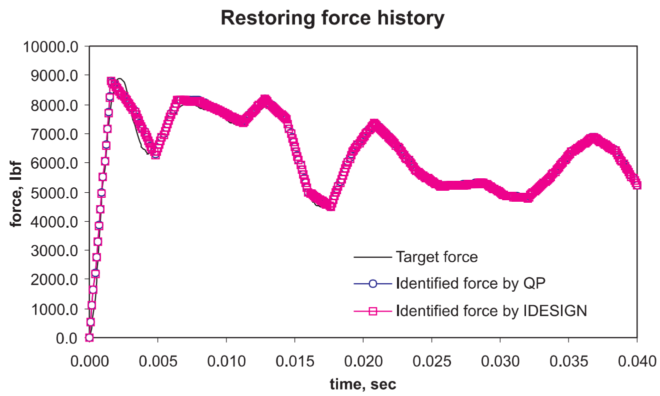

Fig. 15 Comparison of target data and identified data using Hat basis functions

results from IDESIGN and the QP problem match perfectly. In the QP problem, 25 design variables are used, since the first time control point [t_0 in (12)] does not start from zero.

As another basis function to approximate the given force data Chebyshev polynomials are used. Numerical results from IDESIGN and QP formulation are given in Fig. 16. It is seen that Chebyshev polynomials give

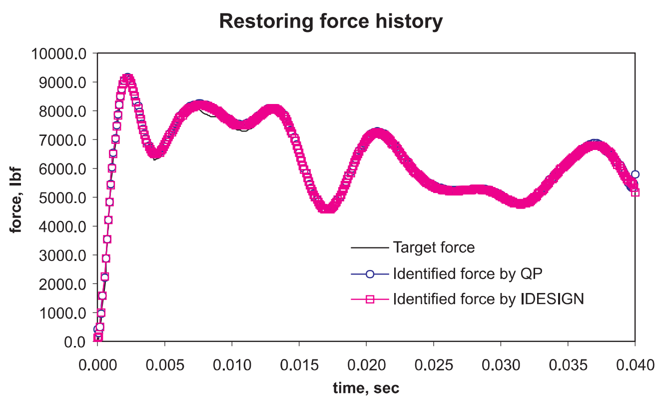

Fig. 16 Comparison of target data and identified data using Chebyshev polynomials

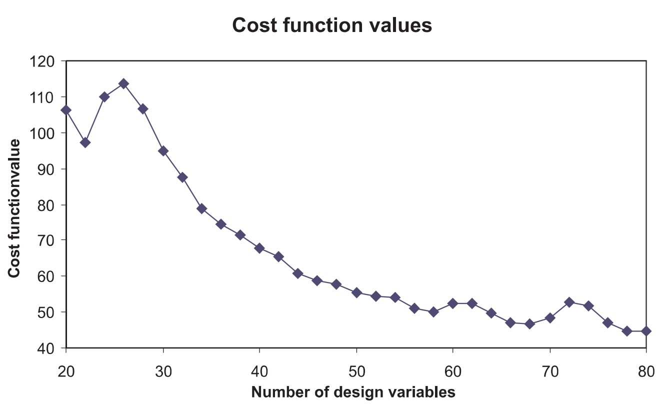

Fig. 17 Minimum cost value vs. number of design variables with Hat functions

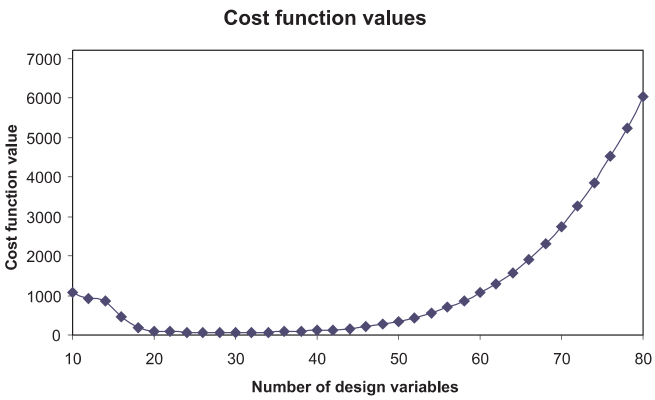

Fig. 18 Minimum cost value vs. number of design variables with Chebyshev polynomials

a little bit smoother curves. However, as the number of design variables (i.e., the number of terms in the basis function expansion) is increased, the optimum cost function value increases with Chebyshev polynomials compared to those with the Hat function. This is seen in Figs. 17 and 18. This is due to the known characteristics of the Chebyshev polynomials that give relatively large errors at the end points. The Hat function representation does not have this limitation. Thus, it appears that Hat function representation will work better for practical applications.

8 Redesign with simplified models

So far system identification has been discussed, which was to obtain mathematical expressions to represent the given tabular target data. Once the system is identified, it is usually required to redesign it to improve its performance. Therefore, a redesign of the horn structure is considered next. The procedure for redesign of the system is the same as for the identification problem. It is required to reduce the final velocity from 456.7 in/s to 430 in/s keeping the final displacement same as before, i.e., 19.6 in. The reduced velocity yields smaller kinetic energy at the

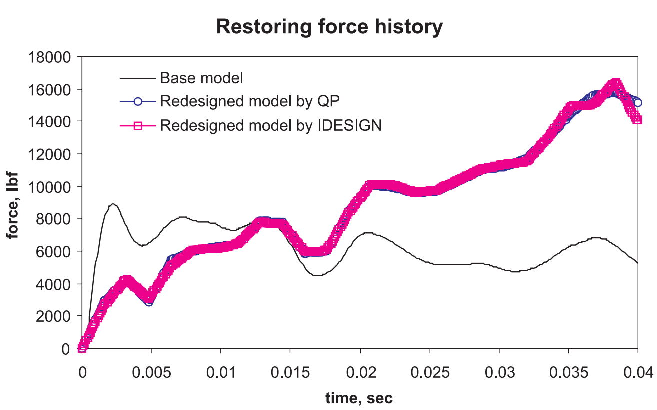

Fig. 19 Redesigned restoring force by Hat functions

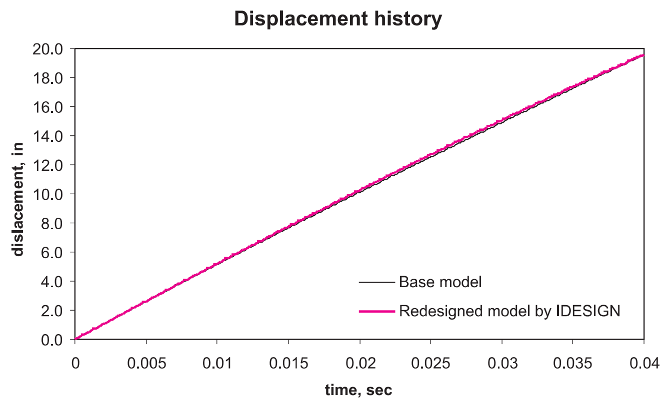

Fig. 20 Redesigned displacement by Hat functions

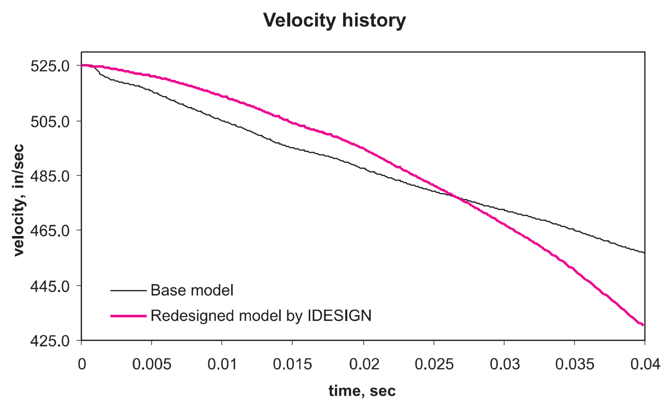

Fig. 21 Redesigned velocity by Hat functions

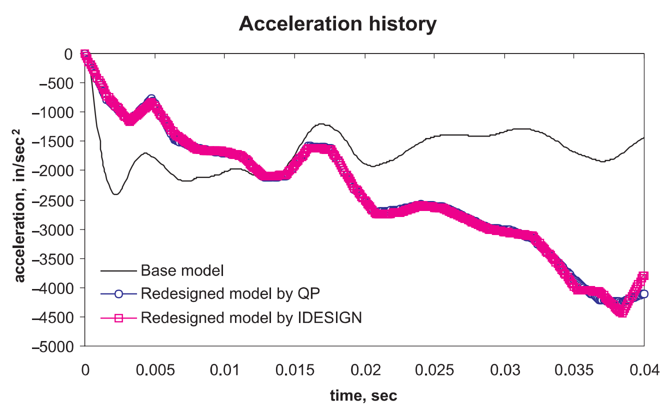

Fig. 22 Redesigned acceleration by Hat functions

final time. Thus more energy absorption would be obtained with crushing of the front horn. It is noted however that this reduced velocity is chosen arbitrarily for demonstration of the methodology. Other more meaningful constraints and data will need to be considered for practical applications. Figure 19 shows that the optimum solutions of the redesign problem from IDESIGN and the QP formulation match each other well. The figure also shows difference between the new force curve and the base data (the target data used in the identification problem). The new force curve can now be used for detailed redesign of the front horn for its improved performance. The predicted displacement, velocity and acceleration using Hat functions and QP formulation are given in Figs. 20–22, with the base data. Figure 21 shows that the final velocity is reduced from 456.7 in/s to the desired value (430 in/s).

9 Discussion and conclusions

The problem of mathematically representing highly complex nonlinear dissipative dynamic systems with simplified models is addressed. Basically two identification methods are discussed: parametric identification and nonparametric identification. In parametric identification, the stiffness and the damping constants that are represented by the linear functions of the displacement and the velocity of a SDOF system are identified using the optimization formulation. DYNA3D is used to perform integration of the nonlinear equation of motion for the system. This formulation needs all displacement, velocity and acceleration data. Secondly, in nonparametric identification, two topics are discussed, system identification and its redesign. Two types of formulations are introduced:

– a general nonlinear programming formulation, and
– a QP formulation.

Hat function and Chebyshev polynomials are used to approximate the given data. First, two formulations using Hat functions and Chebyshev polynomials are demonstrated for a linear system. Then as a practical example, a simplified SDOF system for the horn structure of a vehicle is identified and redesigned using the proposed procedures. Horn structure goes through large displacements and plastic collapse (buckling). It is shown that a single degree of freedom model using appropriate shape functions can capture this complex behavior of the structure quite well. From the study about the effects of the number of design variables on the optimal solution, it is observed that the Hat function representation is superior. Once the system is identified, a redesign of the structure is carried out to improve its energy absorption capacity.

Using the identified simplified model, a new force-displacement curve for the structure can be obtained to improve its performance. The modified force curve can then be used for detailed redesign of the structure to obtain desired response characteristics.

The proposed formulations have also been extended to multi-degree of freedom systems, where several structural elements need to be identified simultaneously.

Acknowledgements This paper is based on a presentation made by the authors at the 3-rd World Congress of Structural and Multidisciplinary Optimization (WCSMO-3), held at the University at Buffalo, Amherst, New York, May 17–21, 1999. Support provided by Ford Motor Company for this research under URP with Dr. C.C. Wu as project monitor is also acknowledged.

References

Arora, J.S. 1989: *IDESIGN user's manual. Version 3.5.2*. Iowa City, IA, USA: Optimal Design Laboratory, College of Engineering, The University of Iowa

Argoul, P.; Jezequel, L. 1989: Improvement of a nonparametric identification procedure used in nonlinear dynamics. *J. Appl. Mech., ASME* **56**, 697–703

Bennett, J.A.; Lust, R.V.; Wang, J.T. 1991: Optimal design strategies in crashworthiness and occupant protection. In: *Crashworthiness and occupant protection in transportation systems*, AMD-Vol. 126/BED-Vol. 19, pp. 51–66. New York: ASME

Caravani, P.; Watson, M.L.; Thompson, W.T. 1977: Recursive least-squares time domain identification of structural parameters. *J. Appl. Mech., ASME* **44**, 135–140

Cheva, W.; Yasuki, T.; Gupta, V.; Mendis, K. 1996: Vehicle development for frontal/offset crash using lumped parameter modeling. *SAE Trans.*, Paper No. 960 437, pp. 53–58

Distefano, N.; Todeschini, R. 1973: Modeling, identification and prediction of a class of nonlinear viscoelastic material (I). *Int. J. Solids Struct.* **9**, 805–818

Hibbeler, R.C. 1978: *Engineering mechanics: dynamics*. New York: Macmillan

Hollowell, W.T. 1986: Adaptive time domain, constrained system identification of nonlinear structures. *Proc. Symp. on Vehicle Crashworthiness Including Impact Biomechanics*, AMD-Vol. 79/BED-Vol. 1, pp. 105–123. ASME

Huang, M.; Chen, R.; Margolin, B. 1995: Use of an advanced CRUSH model in estimating vehicle impact loading and energy absorption. In: *Crashworthiness and occupant protection in transportation systems*, AMD-Vol. 210/BED-Vol. 30, pp. 273–285. New York: ASME

Jao, S.Y.; Arora, J.S.; Wu, H.C. 1991: An optimization approach for material constant determination for the endochronic constitutive model. *Comp. Mech.* **8**, 25–41

Kamal, M.M. 1970: Analysis and simulation of vehicle-to-barrier impact. *SAE Trans.* **79**, 1453–1467, SAE Paper No. 700 414

Kamal, M.M.; Wolf Jr., J.A. (eds.) 1982: *Modern automotive structural analysis*. New York: Van Nostrand Reinhold Co.

Kim, C.H.; Mijar, A.R.; Arora, J.S. 1999: Development of simplified models for design and optimization of automotive structures for crashworthiness. *Technical Report No. ODL-99.12*, Iowa City, IA, USA: Optimal Design Laboratory, Collage of Engineering, The University of Iowa

Leuridan, J.M.; Brown, D.L.; Allemang, R.J. 1985: Time domain parameter identification methods for linear modal analysis: A unifying approach. *Proc. ASME Design Engineering Technical Conf.* (held in Cincinnati, OH). Paper No. 85-DET-90

Lingener, A.; Doege, S. 1988: Estimation of dynamic spring and damping parameters of the supports of a nuclear reactor by means of an adaptive method in time domain. In: Natke, H.G. (ed.) *Application of system identification in engineering*. CISM Course and Lecture No. 296, pp. 511–523. Vienna: Springer

Lust, R. 1992: Structural optimization with crashworthiness constraints. *Struct. Optim.* **4**, 85–89

Mahmood, H.F.; Wheatley, D.G.; El-Bkaily, M.A.; Bak kar, J.Y. 1993: On the front end design of automotive vehicle for crashworthiness. *Crashworthiness and occupant protection in transportation systems*, AMD-Vol. 169/BED-Vol. 25, pp. 311–318. New York: ASME

Masri, S.F.; Caughey, T.K. 1979: A nonparametric identification technique for nonlinear dynamic problems. *J. Appl. Mech., ASME* **46**, 433–447

Masri, S.F.; Chassiakos, A.G.; Caughey, T.K. 1993: Identification of nonlinear dynamic systems using neural networks. *J. Appl. Mech., ASME* **60**, 123–133

Ni, C.-M.; Song, J.O. 1986: Computer-aided design analysis methods for vehicle structural crashworthiness. *Proc. Symp. on Vehicle Crashworthiness Including Impact Biomechanics*, AMD-Vol. 79/BED-Vol. 1, pp. 125–139. ASME

Ni, Y.Q.; Ko, J.M.; Wong, C.W. 1999: Nonparametric identification of nonlinear hysteretic systems. *J. Eng. Mech.* **125**, 206–215

Schwibinger, P.; Nordmann, R. 1988: Optimization of a reduced torsional model using a parameter identification procedure. In: Natke, H.G. (ed.) *Application of system identification in engineering* (CISM Course and Lecture No. 296), pp. 525–542. Berlin, Heidelberg, New York: Springer

Udwadia, F.E.; Shah, P.C. 1976: Identification of structures through records obtained during strong earthquake ground motion. *J. Eng. Industry, ASME*, 1347–1362

Yamazaki, K.; Han, J. 1998: Maximization of the crushing energy absorption of tubes. *Struct. Optim.* **16**, 37–46

Yang, R.J.; Tseng, L.; Nagy, L.; Cheng, J. 1994: Feasibility study of crash optimization. In: Gilmore, B.J.; Hoetzel, D.A.; Dutta, D.; Eschenauer, H.A. (eds.) *Advances in design automation*, DE-Vol. 69-2, pp. 549–556. ASME