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Railway network design with multiple project stages and time sequencing

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Abstract. This paper presents a spatial decision support system for network design problems in which different kinds of projects can be built in stages over time. It was developed by the World Bank and China's Ministry of Railways to plan investment strategies for China's overburdened railway system. We first present a mixed-integer program for the single-period network design problem with project choices such as single or multiple tracks and/or electrification with economies of scale. Then, because such projects can be built all at once or in stages, we developed a heuristic backwards time sequencing procedure with a cost adjustment factor to solve the "project staging" problem. Other innovations include a preloading routine; coordinated modeling of arcs, paths, and corridors; and a custom-built GIS.

Key words: Railway transportation, network design model, economies of scale, time sequencing, China, project staging

JEL classification: C61, F12, L92, N75, O53, R41

1 Introduction

Since 1988, the World Bank and the Chinese Ministry of Railways (MOR) have been building, using, and improving a spatial decision support system (SDSS) for railway investment planning. The goal of the Railway Investment Study (RIS) is to evaluate capacity expansion strategies and prioritize investments for medium term (15 years) planning. At the heart of the SDSS is a network design model which simultaneously constructs new links and assigns traffic to the network in a cost-minimizing fashion. The model is a mixed-integer program solved using branch-and-bound. The SDSS includes a database, several analysis modules, and a geographic information system (GIS).

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The RIS has helped evaluate several important investment decisions involving hundreds of millions of dollars.

The Chinese railway system is one of the largest, fastest-growing, and yet most congested in the world. With 57,583 km of track, the Chinese railway network is the 3rd largest in the world. In 1997, it carried 1697 million tons for an average distance of 772 km. In addition, China's railways are still the major form of long-distance passenger transport, carrying 925 million passengers an average of 383 km. In all, the railway system carries 34% of the country's freight traffic and 35% of passenger traffic in terms of ton-km and passenger-km. China's economy has been growing at 9.7% per year since 1978, and railway passenger and freight traffic have been trying to keep pace, growing at 6.4% and 4.8% respectively.

As a result of this heavy usage and rapid growth, the system has been near the saturation point for decades. Despite adding 9000 km of track since 1978, the density of traffic (ton-km or passenger-km per km of track) is twice as high as India's, the next busiest rail system. Chinese railway planners struggle to do more with less, as capital remains scarce. Double tracking or electrification of existing rail lines can provide more new capacity per unit investment, but there is also a pressing need to build new single-track railways in areas still unserved. Currently, 36% of railways have multiple tracks (up from 16% in 1978), and 21% are electrified (2% in 1978).

The RIS-SDSS is important for MOR management because it can not only optimize network expansion but it can also estimate the potential economic benefits of an investment program. The model analyzes investment decisions in the context of the rail system as a whole, and is designed to assist planners in answering a range of policy questions, such as:

(a) How should available railway funds be allocated among various capacity-expansion investment categories, including new line construction, line electrification, or extra tracks?
(b) What would be the most cost-effective strategy for staging the capacity expansion of the railway network over the next 10–15 years?
(c) What is the tradeoff between building many dispersed small projects (with higher unit investment costs but more direct routing alternatives) versus fewer but larger projects (with lower unit investment costs but less direct routing)?
(d) What are the tradeoffs between building a single large project in an early time period (which costs more up front but is less expensive over time) versus building it in stages (which costs more over time but is better scaled to the growth of demand)?
(e) What is the economically optimum level of investment for capacity expansion, and what would be the economic losses to the economy from under-investment?
(f) What are the system-wide cost effects of allowing local traffic to grow and take up line capacity which would otherwise be allocated to through traffic?

A network design model was developed to answer these complex questions faced by Chinese railway planners. Four main improvements set the model apart from other linear programming (LP) network design models. First, the model chooses endogenously among different types of network improvements:

single track, multiple track, electrification, or other. There are significant economies of scale associated with both additional tracks and electrification. Second, the model employs various strategies in order to be as realistic as possible and still remain solvable on a practical basis. These include preloading some traffic, and meshing three kinds of spatial units – links, paths, and corridors. The third improvement stems from the fact that a second track or electrification may be built all at once or added at a later stage. This flexibility creates a "project staging" problem. Fourth, the LP model is integrated as part of a SDSS with various other analysis modules and a Chinese-built GIS for data manipulation, analysis, and visualization.

2 Literature review

Network design represents a broad class of models for network planning (Magnanti and Wong, 1984). We use the term here in a more restrictive sense to refer to fixed charge capacitated models that add arcs (links) to a network from a set of discrete choices while simultaneously assigning traffic to the expanded network (LeBlanc 1975). Both cost minimization (Billheimer and Gray 1973; Boyce et al. 1973) and equilibrium (LeBlanc et al. 1975) versions of the problem are widely used.

For modeling multiple capacity levels (e.g., different project types), Magnanti and Wong (1984) suggest using parallel artificial arcs with different capacities and investment and operating costs. Parallel arcs, however, are not well suited to models such as ours that choose among a limited set of path choices because they create a proliferation of effectively identical paths. Parallel arcs would work better with heuristic column-generating techniques that include shortest path models as endogenous parts of the solution algorithm (Desrosiers et al. 1984).

Economies of scale in transport network design have been incorporated using concave cost functions (Yaged 1971; Abdulaal 1989; Guisewite and Pardalos 1990). Magnanti and Wong (1984) state that concave costs arise in many applied contexts and are difficult to solve. Ben-Ayed et al. (1992) developed a model for rural roads in developing countries with continuous link capacities based on incremental improvements, solved via bi-level linear programming. Osleeb et al. (1986) used mixed-integer programming to choose among several projects at each coal port, each with unique fixed and variable costs that imply a piecewise linear cost function with economies of scale. Different link capacities also arises in the context of airline networks, with economies of scale according to plane size (Kuby and Gray 1993; Branhart and Schneur 1996).

In the dynamic facility location literature, a number of authors have modeled the problem of phasing facilities in and out in response to changing conditions (Wesolowsky 1973; Roodman and Schwarz 1975). Van Roy and Erlenkotter (1982) developed a dual-based procedure for dynamic facility location. Kuby et al. (1993) developed a model of the Chinese coal mining, transport, and power systems that built capacity endogenously over time. Capacity constraints for each location and time period summed capacity additions for that time period and all previous time periods. However, not until time sequencing is combined with the choice of multiple and separable project types does the issue of building a larger project in stages arise. This problem appears not to have been treated in the literature prior to this paper.

Increasingly, transport optimization models are being integrated into spatial decision support systems. SDSS facilitate a flexible decision research process, and empower analysts and decision makers by enabling them to explore and refine the problem and to generate and evaluate alternative solutions (Densham 1994). Armstrong, Densham, and Rushton (1986) stated that an SDSS should include a database management system, a suite of integrated and/or alternative analytical models, cartographic display and other typical GIS functions, report and graphics generators, and a user interface. Transportation GIS, or GIS-T, has incorporated many of the characteristics of SDSS in one prepackaged system. Waters (1999) lists the commonly included modules in GIS-T's analytic toolkit as shortest paths, vehicle routing, arc routing, network flow, partitioning, location-allocation, spatial interaction, and the four standard UTMS models. Network design is notably absent, although it can be viewed as an extension of traffic assignment. One of the most fruitful applications of SDSS has been to hierarchical multi-scale problems where the optimal solution is solved at one level of aggregation but must be translated to another for implementation, e.g., forest management (Church et al. 2000). Another is multiobjective optimization, in which cartographic *and* tabular and graphic visualization are important to the decisionmakers' understanding (Coutinho-Rodrigues et al. 1994, 1999). A third application is to ill-defined problems requiring frequent modification and experimentation (Brill et al. 1990).

3 Brief history of the RIS

Since the early 1990s, two versions of the RIS SDSS have been used by the MOR to analyze railway investment packages for the same basic purpose. Both versions have been used to identify an investment program for the upcoming three Five Year Plans that minimizes the cost of satisfying railway demands. Under communism, the Five Year Plans were the central government's attempt to integrate the production plans and investment requests submitted by all regions and all ministries. Now, in the reform era, although the Five Year Plans are no longer mandatory, planning is still done in five-year increments, and central government approval is still required for the largest investments projects such as railroads.

The original RIS optimization used a traffic assignment model iteratively to do network design (Cook et al. 1994). A set of pre-selected new projects (known as the "investment package") conforming to budgetary restrictions would be created outside of the model. The forecasted O-D demand would be assigned to the network by a traffic assignment model that minimized operating costs plus the costs of unsatisfied demands. These steps would be repeated for each of three 5-year periods. Analysts would study the network's performance and then propose and analyze a modified investment package. Through repeated analysis, analysts would converge on a better-performing "preferred" investment package. The benefit of each project would then be estimated by running the traffic assignment model many times, with a different rail line removed each time. The cost difference between the "with" and "without" runs (including their investment costs) would be used to estimate the economic benefit of each project.

This process was clearly cumbersome, time-consuming, and suboptimal, and estimated economic benefits inaccurately. In the new model, analysts input several potential projects for each rail line, and the network design model simultaneously chooses the projects and assigns the traffic to the expanded network. Whereas the old RIS traffic assignment model minimized the sum of operating costs and unsatisfied demand costs, the new model minimizes the sum of those two costs plus annualized investment costs. An important improvement is that the new network design model evaluates all possible combinations of projects. The new method for ranking the projects is by gradually tightening the budget constraint. Those projects chosen under the smallest budget are the highest priority. Economic benefits indicators such as net present value (NPV) are calculated for the group of projects that are added under each budget increment. Although the new method does not estimate benefits for individual projects, it is still an improvement because the old "with/without" method would overestimate a project's benefit by creating unrealistic bottlenecks. Below, we begin by describing the single-period network design model with different project types. We then place it within the context of the other programs in the SDSS, and then describe the time sequencing heuristic and GIS functions.

4 The network design model with different project types and economies of scale

Magnanti and Wong (1984) identified several different variations of network design problems, features of which are combined here into a single model. The RIS-ND model is a system-optimizing, capacitated, static, fixed charge mixed-integer program with budget constraints and, indirectly, economies of scale. The model also allows for some demand to go unsatisfied – a common condition that the MOR cannot ignore but which was not reviewed by Magnanti and Wong.

Network design models can be formulated using either link (arc) flow variables or path flow variables. Although our model includes both kinds of flow variables, it is best understood as a path flow formulation. Link flows are calculated as the sum of path flows on a link, for the purpose of accurate and flexible cost computations as we explain later, but the transportation primitives are paths. Our choice of a path flow formulation is largely dictated by the nature of the demand, which is defined for O-D pairs. Demand for freight or passengers at a destination cannot be satisfied from any origin, but rather from particular origins. Although it is possible to preserve origin-specificity in a link flow formulation, it leads to an unmanageable proliferation of node-specific, commodity-specific, and O-D-specific balance constraints. A path variable formulation naturally preserves O-D specificity and avoids balance constraints altogether. The drawback of an LP path formulation is that only those paths that are input as potential variables are possible, whereas all paths would be possible in a link variable formulation.

Minimize:

$$Z = \sum_{l}\sum_{m}\sum_{f} c_{lmf} A_{lmf} + \sum_{c}\sum_{f} F^{a}_{cf} Y_{cf} + \sum_{i}\sum_{j}\sum_{m} p_{ijm} U_{ijm} \tag{1}$$

Subject to:

Demand Constraint:

$$\sum_p \delta_{ijpm} h_{pm} + U_{ijm} = T_{ijm} \quad \forall i, j, m \tag{2}$$

Link Flow Definition Constraint:

$$\sum_p \delta_{lp} h_{pm} - \sum_f A_{lmf} = -L_{lm} \quad \forall l, m \tag{3}$$

Capacity Constraint:

$$\sum_m \beta_{lmf} A_{lmf} - K_{clf} Y_{cf} \leq 0 \quad \forall l, f \tag{4}$$

Special-Ordered Set Constraint:

$$\sum_f Y_{cf} = 1 \quad \forall c \tag{5}$$

Budget Constraint:

$$\sum_c \sum_f F^b_{cf} Y_{cf} \leq B \tag{6}$$

Upper Bound on Unsatisfied Demands:

$$U_{ijm} \leq Q_{ijm} \quad \forall i, j, m \tag{7}$$

Non-negativity Constraints

$$A_{lmf}, h_{pm}, U_{ijm} \leq 0, \quad Y_{cf} \in 0, 1 \tag{8}$$

where:

Subscripts
p = path
i = origin
j = destination
m = commodity
l = link (or arc)
c = corridor
f = project (including the existing project, which may have zero capacity in the case of a brand new line)

Variables
Y_{cf} = 1, if project f is built on corridor c
0, otherwise

A_{lmf} = traffic flow of commodity m on link l with project f (tons or passengers per year)
U_{ijm} = unsatisfied demand from i to j of commodity m (tons or passengers per year)
h_{pm} = flow of commodity m on path p (tons or passengers per year)

Coefficients and sets
c_{lmf} = operating cost for shipping commodity m on link l with project f (yuan per ton or passenger)
F^a_{cf} = annualized fixed cost of project f on corridor c (yuan per year)
ρ_{ijm} = shadow price of unsatisfied demand from i to j of commodity m (yuan per ton or yuan per passenger)
δ_{ijpm} = 1, if path p can carry commodity m from i to j
0, otherwise
T_{ijm} = railway demand from i to j for commodity m (tons or passengers per year)
K_{clf} = capacity of link l on corridor c if project f is built (ton-equivalents per year)
L_{lm} = pre-loaded flow of commodity m on link l, composed of small inter-zonal flows (tons or passengers per year)
β_{lmf} = capacity factor of commodity m on link l with project f (tons of freight displaced by one unit of flow of commodity m on link l with project f)
δ_{lp} = 1, if path p uses link l
0, otherwise
B = capital budget (yuan per budget period)
F^b_{cf} = total financial investment cost of project f on corridor c (yuan per budget period)
Q_{ijm} = upper bound on unsatisfied demand for commodity m from i to j (tons or passengers per year)

The objective function (1) of this problem is to minimize the total yearly cost of the railway system. Included are three components. The first component, $\sum_l \sum_m \sum_f c_{lmf} A_{lmf}$, is the operating cost of the traffic, summed over all links l, all types of facilities (projects) f on those links, and all commodities m. The purpose of making the operating costs a function of link flows rather than path flows is to capture the effect economies of scale have on *operating* costs, separate from the investment cost savings. For instance, electrification not only provides more unit capacity per yuan (or dollar) of investment, but generally lowers the operating cost per ton-mile by increasing train speed (though this is partially offset by higher fuel costs). In this formulation, operating costs c_{lmf} will depend on which type of project f is built on a link l. The second component, $\sum_c \sum_f F^a_{cf} Y_{cf}$ is the cost of railway corridor investment. Because Y_{cf} is a zero-one variable, F^a_{cf} must be the annualized total investment cost, not a unit cost. This is the second place where economies of scale are captured, as represented by the ratio of fixed costs to capacity (F^a_{cf}/K_{clf}) for the different project types f. Figure 1 shows how the investment cost, operating cost, and capacity of each project define a total cost curve for that link. Taken together, the various project types may form a piecewise linear concave cost function exhibiting economies of scale.

The third component in the objective function, the cost of unsatisfied demand, $\sum_i \sum_j \sum_m \rho_{ijm} U_{ijm}$, is summed over all i,j pairs and all commodities

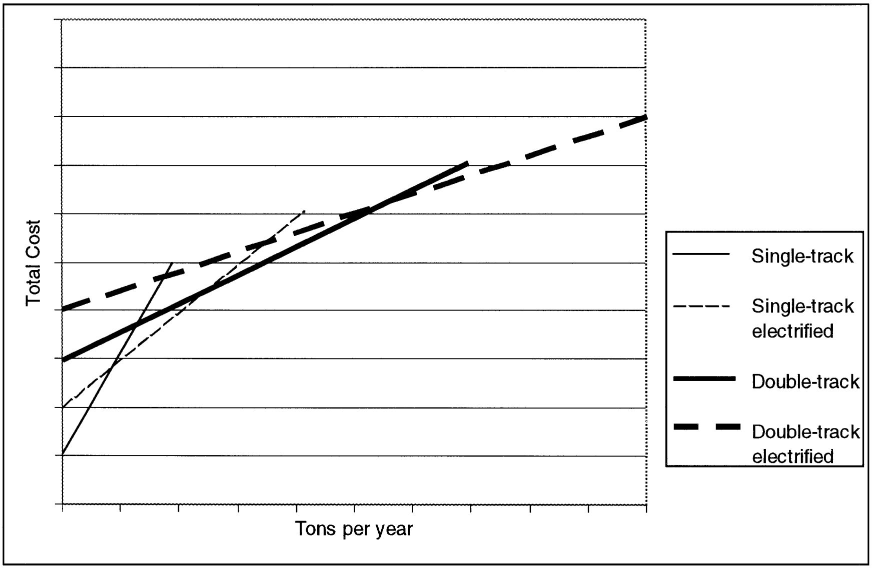

Fig. 1. Sample cost curves for multiple project types. The Y-intercept represents the annualized fixed cost, the slope represents the operating cost per ton or passenger, and the endpoint of the line represents the project's capacity

m. These costs are assumed to be linear with respect to the size of the shortage, which is consistent with measuring that cost as either lost revenue to the MOR (i.e., tariffs) or the cost of an alternative mode. Linear costs would not be a good assumption, however, if the model were measuring the economic cost of shortages to the industries that must do without.

The demand constraint (2) requires that the total shipments of each commodity between each origin and destination, summed over all paths p between that origin and destination, plus the unsatisfied demand from origin i to destination j for commodity m, must equal the desired demand for shipments, T_{ijm}. In other words, all demand must be accounted for, either by being satisfied or unsatisfied. The term δ_{ijpm} is an incidence matrix whose values are 1 if path p can carry commodity m from i to j, and 0 otherwise. Therefore, $\sum_p \delta_{ijpm} h_{pm}$ is equal to the annual flow of commodity m from i to j.

The link flow definition constraint (3) sets the annual link flow of commodity m, $\sum_f A_{lmf}$, equal to the sum of the flows of commodity m of all paths using link l, plus the preloaded flows L_{lm} of commodity m on link l. The summation of A_{lmf} over f is necessary because it is not known ahead of time which project f will be built on link l.

Preloading reduces the size of the O-D matrix while minimizing any resulting suboptimality. It removes all small traffic demands below a certain cutoff level from the O-D matrix, instead preloading them onto their shortest path. The preloaded flows in (3) effectively reduce the capacity available to the flows above the cutoff level. Thus, all traffic flows follow realistic routes, but small flows are forced onto their shortest path. As a test, we took a 150 × 150 railway demand O-D matrix, and cut off all freight demands less than 1,000 tons per year. This eliminated 17.5% of O-D pairs, but only .08% of total traffic. A cutoff level of 5,000 tons would eliminate 40% of the O-D pairs but still less than 1% of the traffic. This is a large benefit for model size and performance, while very little is lost in the way of realism or optimality.

The preloaded amount, L_{lm}, also includes intrazonal flows within each of the 483 zones, i.e., the traffic in the diagonal of the O-D matrix. These short, intrazonal flows are important to account for because, in China, local rail traffic often uses up capacity that could otherwise be available for long-distance flows. The existence of intrazonal flows is due to the aggregation of thousands of railway stations into 483 zones. Were a station-to-station O-D matrix available instead of a zone-to-zone matrix, we would eliminate the intrazonal flow problem but we would enormously aggravate the large O-D matrix problem. Likewise, had we used the 60 railway subadministrations as our centroids, the flows would have been targeted on too few points, which might have the effect of overstating the severity of certain bottlenecks. The intrazonal flow issue is related to the more generic modifiable areal unit problem that occurs in many GIS applications, in that if the zones were defined differently, the intrazonal flow pattern would change. However, it is important to note that although the 483 zones are theoretically modifiable, they are defined by the MOR for the entire MOR, not just for this model.

To reduce the number of integer variables, several contiguous links in the same section of a railway line are combined into corridors and treated as a single unit for investment decisions. In the current version there are 77 corridors. This simplification is made solely to make the model more computationally tractable, since the number of zero-one variables is the primary determinant of the model's solution time. The use of corridors as a decision-making unit is common in railway planning in most countries because (a) it makes train scheduling easier, and (b) the preponderance of through traffic on many lines means that similar capacities are needed on adjacent links. If, however, analysis indicates that part of a corridor is being grossly underutilized, it can be split into two corridors for further analysis. This could be considered a modifiable *linear* unit problem.

Generally, a corridor's investment choices will range from single-track to double-track electrified, with certain exceptions. Some corridors will not require all these choices because some are already built. Other corridors may be physically restricted from double tracking or electrification, or may lack the demand to even consider such options.

The link capacity constraint (4) states that the total flow on each link must not exceed the open capacity of that link. The "open" capacity depends on which of the projects f is built on corridor c which contains link l. Constraint (5) is a special-ordered set constraint that guarantees that only one of the investment variables Y_{cf} on each corridor will be equal to one – that is, only one project can be built on a corridor. Constraints (4) and (5) thus work in tandem to control the capacity logic. In this formulation, even existing project types, representing the do-nothing choice, must be represented by a zero-one Y_{cf} variable. This project will have an investment cost of zero and a capacity equal to the existing capacity. In the case of existing lines, constraint (5) requires that either the existing project is kept at no investment cost, or one of several new projects be built. Brand new lines must have a 0-1 variable with nonzero fixed cost and nonzero capacity. Note also that in constraint (4), capacity is measured in ton-equivalent units. The total link flow, $\sum_m \beta_{mlf} A_{lmf}$, is the sum of the tons plus the ton-equivalent of the passengers. For $m =$

passengers, the parameter β_{mlf} is the number of tons of freight displaced by each passenger on link l with project type f, which is calculated uniquely for each link.

The budget constraint (6) requires that the spending on construction must not exceed the upper limit B. Here, the investment costs are superscripted by a "b" in order to signify financial cost, not annualized cost. The entire investment cost of a project is attributed to the budget of the single Five Year Plan in which it becomes operational, even if, because of a long construction lead time, some of the expenditures actually would have taken place during an earlier Five Year Plan. The budget itself is for the whole Five Year Plan, not only for the key year.

Constraint (7) provides optional upper bounds, Q_{ijm}, on unsatisfied demands. If these constraints are not used, or the upper bounds are set very high, the budgets, capacities, and shadow prices will determine which demands are unsatisfied, and by how much. However, these constraints may be used to force a more equal sharing of unsatisfied demands across i,j pairs and across commodities. If they are used to limit unsatisfied demand to a number less than the demand, it will effectively force at least some of the demand for each i,j pair to be satisfied. Care should be taken in their use, however, because in cases where no existing railway route exists to handle the demand of an i,j pair, use of these upper bounds could force an entire project to be built just to avoid that one i,j pair from having some unsatisfied demand. Conceivably, overuse of these upper bounds in scenarios with tight budget constraints could lead to an infeasible solution.

As solved for the Chinese railways, the single-period model typically has around 40,000 constraints and 108,000 variables, of which 77 are special-order sets containing several 0-1 variables from which to choose. It is solved with XPRESS-MP using a combination of the Simplex algorithm, the Newton-Barrier interior point method, and branch-and-bound. The LP relaxation usually solves on a 500 MHz PC with 128 MB of RAM in around 30 minutes. The MIP is allowed to run overnight and is usually stopped the following morning, typically after it has found 3–10 solutions. The large size of the model necessitates the use of preloading and corridors and path variables that we have described, and the iterative heuristic solution of three separate time periods, described later.

5 RIS analysis procedure

The network design model just described is generated and solved within a larger system of supporting software modules, as presented in Fig. 2. The RIS SDSS is structured as a series of two optimization modules (MathPro and XPress-MP), six input processing submodels (top half of chart), three output processing submodels (bottom half of chart), and a control interface for time sequencing (dashed arrows). Figure 2 also shows the main database files (DBFs) that store the inputs and outputs of each submodel.

The key inputs to the model fall into four categories. First are traffic data, specifically an origin-destination (O-D) matrix which at present is for 483 by 483 nodes for two commodities (freight and passengers) and three key years (2005, 2010, and 2015). These forecasts are generated exogenously by a different branch of the MOR. Second, network data include the network struc-

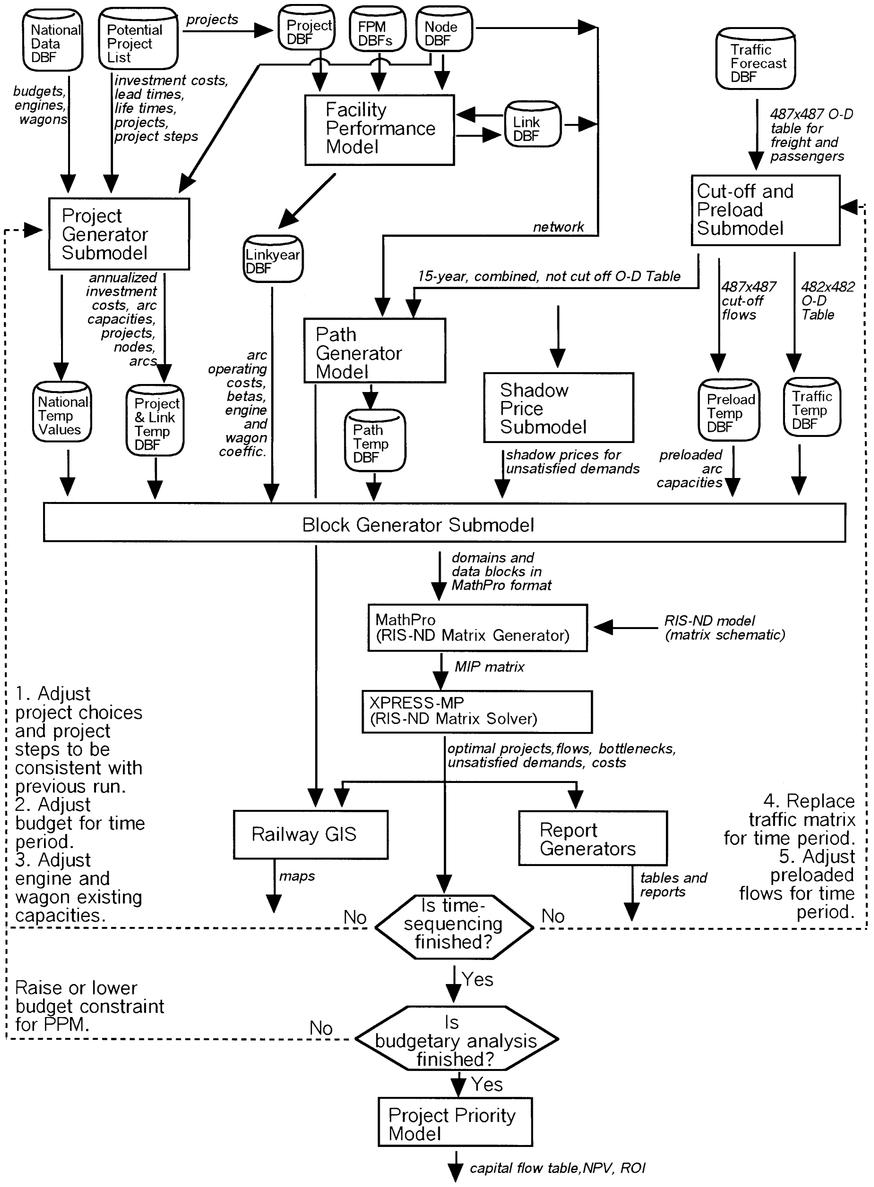

Fig. 2. Flowchart of RIS network design analysis steps. (Temp) DBF refers to (temporary) database files

ture, the types of facilities and rolling stock, and their capacities. Currently there are more than 700 separate links in the network. Another type of network data are several possible paths for each O-D pair. The third kind of data are project data that describe the possible new railway facilities that can be built. Fourth are economic data, which include operating costs, investment costs, the cost of unsatisfied demand (i.e., shadow prices), and budget data.

The RIS model uses a backwards time sequencing heuristic to solve for the best 15-year investment program. The data above are first generated for the last key year, e.g., 2015. The RIS network design (RIS-ND) model is then solved, and maps and tables of results can be generated for that key year. The SDSS then continues on to the previous key year (e.g., 2010), and builds new projects in 2010 chosen from among those built in 2015 in order to satisfy the lower demand for the earlier year. The process is then repeated for the first year, e.g., 2005. Then, when time sequencing is finished, the entire process is repeated for different budget levels in order to prioritize the projects and estimate their economic benefits, as explained earlier. As the budget is increased step by step, the model will add more and more projects. These additional projects are not as good as the ones chosen with lower budgets (or they would have been chosen instead), but they are still profitable (or they would not have been chosen at all because either all demand was satisfied or the cost of unsatisfied demand was less).

The key outputs mirror the four categories of input data. Traffic-related outputs are the amount of demand that is satisfied (and unsatisfied), by node or by region. Network outputs include routes followed, link flows of passengers and freight, percentage of capacity utilized, bottleneck links (at or near capacity), and underutilized links (at or near zero flow). Project outputs describe the projects built by corridor, time period, and type. Economic outputs include the total cost separated into investment costs, operating costs, and unsatisfied demand costs, plus return on investment and NPV for the new projects selected under each budget constraint.

The function of each of the main submodels in Fig. 2 is summarized below.

1. The *Project Generator Submodel* calculates annualized investment cost, operating cost, and capacity data for all potential new projects. The list of projects is nominated by MOR staff and local railway administrations.
2. The *Facility Performance Submodel* estimates the capacity, costs, and transit times for existing and proposed railway links.
3. The *Cut-off and Preload Submodel* reduces the size of the O-D matrix by removing all small O-D demands below a certain "cutoff" volume and preloading them onto their shortest path. It also preloads intrazonal flows within the 483 zones.
4. The *Path Generator Submodel* generates the *k* best "differentiated" routes from each origin to each destination. The network design model may choose only among these pre-generated paths. Differentiated paths are defined as paths that are as short as possible and also as different from each other as possible, in the sense of the routes being nonoverlapping, and are generated heuristically as described in Kuby et al. (1997). The shortest path is always included in this set.
5. The *Shadow Price Submodel* generates shadow prices representing the cost of unsatisfied railway demand. The submodel currently generates financial shadow prices based on revenue lost to MOR, i.e. railway tariffs, but plans are to also generate economic shadow prices based on the cost of an alternative mode of transportation, i.e., roads.
6. The *Block Generator Submodel* contains software programs that process input data into *MathPro* format for "blocks" of coefficients for each type of variable in each constraint.

7. The *RIS-ND Model* is a mixed-integer programming formulation of the network design model in Eqs. (1)–(8).
8. The *MathPro Matrix Generator* creates the RIS-ND model in standard MPS format.
9. The *XPRESS-MP* MIP Solver solves the RIS-ND model using a combination of the Simplex algorithm, the Newton-Barrier interior point method, and branch-and-bound.
10. The *Time Sequencing Control Interface* sets up the RIS-ND model and guides it through three time periods. Based on the results from solving one time period, it passes the investment decisions from one time period to the next, adjusts some other data such as traffic forecasts, and runs the submodels and RIS-ND model again.
11. *Report Generators* create aggregated and disaggregated summary reports on model outputs.
12. The *Railway GIS* was developed specially by and for the Chinese MOR. It displays both input data and model results such as link flows, bottlenecks, and new projects in graphical, tabular, or cartographic form.
13. The *Project Priority Submodel* calculates a capital flow table, NPV, and return on investment for groups of projects with similar priority.

Of all these supporting software programs, the time sequencing heuristic has presented the greatest challenge.

6 Time sequencing and project staging

Difficult modeling issues arise when time sequencing is undertaken with projects that can be built in steps or stages. To begin, we explain how time is treated in this model. As a static, single-period model, a multi-period investment plan can be created only by solving separate model runs for each time period. Thus, despite the fact that many parameters – including O-D traffic demands, network structure, and existing link capacities – may change over time, they are not indexed by t in the mathematical formulation. The unit of time in the RIS-ND model is a "key year" (e.g., 2005) which is the final year of a Five-Year Plan (e.g., 2001–2005). Investment projects for each 5-year period are assumed to be completed by the start of the key year (e.g., January 1, 2005). Operating variables such as traffic flow represent the activities of just the single key year. These two different conceptions of time can be meshed together because amortized investment costs, demands, and capacities are all measured on an annual basis.

Although the RIS-ND model could potentially be formulated as an exact multi-time period model following Kuby et al. (1993), a heuristic for time sequencing is necessary because the problem is simply too large to solve three time periods simultaneously without grossly oversimplifying the network. Solving three time periods sequentially (heuristically) instead of simultaneously (exactly), however, will tend to make the solution suboptimal, and this is especially true when projects can be built in stages. In this section of the paper we introduce adjustments to make the sequential solution as optimal as possible.

In theory, time sequencing can be done forwards or backwards. Both introduce suboptimality, but in different ways. With forwards time sequencing, the model would be solved for $t = 1$ first. The chosen projects in $t = 1$ would then defined as existing projects for $t = 2$. Project choices, budgets, existing capacities, traffic demands, and preloaded flows would all be updated for $t = 2$. Then the model would be solved, and additional projects built for $t = 2$. This process would then be repeated for $t = 3$.

With backwards time sequencing, the model is first solved for $t = 3$. The solution is interpreted as the cumulative projects that are to be built between $t = 1$ and $t = 3$, not as the projects built in the $t = 3$ key year only. The projects chosen for $t = 3$ will logically eliminate some of the choices for $t = 2$. For instance, if a single-track project is built in $t = 3$, we can eliminate the possibility of building a double-track project in $t = 2$ because it would imply the destruction of one track from $t = 2$ to $t = 3$.

There are advantages and disadvantages of either method. The RIS model uses backwards time sequencing because it is more far-sighted: it optimizes for the long-term, and then, for earlier time periods, it tries to figure out the best development path for reaching the $t = 3$ optimum. Forwards time sequencing, however, has the advantage that the first optimization – which constrains the following optimizations – is done using an earlier and thus more reliable traffic forecast (long-term forecasts are less reliable than short-term). Forwards time sequencing is also more easily understood by decision-makers.

Time sequencing would be a theoretically simple though computationally slow process were it not for the project staging problem. The problem can be illustrated with the following example for three possible projects on a corridor:

Project	*Investment cost (mil. Y)*	*Capacity (mil. tons/yr)*
None	0	0
Single track (ST)	6,000	20
Single track Electric (STE)	8,000	30
Double track (DT)	10,000	80

If either of the latter two projects is built in stages, the cost of upgrading in a later year is typically more than the original cost differential. Building in stages means remobilizing capital, labor, and materials, disrupting service and local communities, widening tunnels, bridges, and rights-of-way, and other extra costs. In the illustrative example below, adding electrification to an existing single track line might cost Y3,000 million, which is Y1,000 million more than the original differential between a single track line (Y6,000 million) and a single track electrified line (Y8,000 million). Similarly, coming back and building a second track at a later stage might increase the investment cost of the second track from Y4,000 million to Y5,500 million:

Stage	*Investment cost (mil. Y)*
Adding electrification to a single track line	+3,000
Adding a double track to a single track line	+5,500

Suppose the model has chosen to build a multistage project, e.g., a double-track line, in $t = 3$ on a corridor that had no existing rail line in $t = 0$. Sequencing backwards, the model will have three choices in $t = 2$:

Case 1 – build nothing in $t = 2$, which implies the entire project f^{12} consisting of both tracks would be built later in $t = 3$;
Case 2 – build the first (intermediate) stage f^1 consisting of a single track line in $t = 2$, which implies that the second stage f^2 consisting of a second track would be added in $t = 3$; or
Case 3 – build the entire project f^{12} consisting of a double track line in $t = 2$, which implies nothing new would be built in $t = 3$.

Case 2 is the problematic case, for if the model chooses to build just the intermediate stage in $t = 2$ it creates added cost implications for the $t = 3$ solution that was already solved. Using the hypothetical cost data above, in Case 2 the model would have chosen to build the double-track line in $t = 3$ at a fixed cost of Y10,000 million, but the decision in $t = 2$ would increase the fixed cost to Y11,500 million, with the "extra staging cost" of Y1,500 million being owed in the next time period. If we do not account for the extra staging cost, the backwards time-sequencing solution would be biased in favor of construction in stages.

The method for overcoming this bias is to include the extra staging cost in the fixed cost of the 0-1 decision variable representing Case 2 when the earlier time periods are solved. This enables the model to "see into the future" and consider the future implications of current decisions. However, two adjustments have to be made to the extra staging cost in order not to bias the earlier solutions *against* construction in stages. First, because the extra staging cost is not owed until $t = 3$, it must be discounted to present value before it is added to the investment cost of the first stage in the objective function of the earlier time period. Second, the extra staging cost must be scaled down to the smaller capacity of the first stage project. In our example, the double-track project has a capacity four times larger than a single-track line. In the real world, the extra staging cost would be spread over as much as 80 million tons per year in $t = 3$. However, with backwards time sequencing, the entire extra staging cost would be placed on the 0-1 decision variable for the single-track first stage in $t = 1$ or $t = 2$, spreading that cost over at most 20 million tons per year. On a cost-per-unit-of-traffic basis, this would create a large bias against building the project in stages. Our idea is not to create a bias one way or another, but to force the model to consider the future cost implications of building a project in stages. To do so, the extra staging cost is discounted to present value and scaled down by the ratio of the first stage capacity to the final project capacity, and then added to the investment cost of the first stage f^1, which is then annualized and used as the fixed cost coefficient F_{cf} on the Y_{cf} (0-1) variable in the objective function. No cost adjustments are made to the fixed costs of building nothing or of building the entire project f.

The necessary cost adjustment is as follows. Let

f^{12} = a project built in time t that could possibly have been built in stages $f^{12} = f^1 + f^2$
f^1 = the project consisting of building only the first or intermediate stage
f^2 = the project consisting of building from the intermediate stage to the final stage
F^b_{cf} = total investment cost (i.e., not annualized) of project f on corridor c
$F^{b^*}_{cf^1}$ = adjusted investment cost of project f^1 on corridor c (this cost will then be annualized and put into the objective function)

t = time period, $t = 1, 2, 3$

t^* = the time period in question (Note: this is not necessarily $t-1$. The intermediate stage could be built in 2005, and the second stage in 2015, skipping 10 years).

r = interest rate in decimal form

Then, the adjusted fixed cost for the first stage f^1 is as follows:

$$F^{b^*}_{cf^1} = F^b_{cf^1} + (K_{cf^1}/K_{cf^{12}})(F^b_{cf^1} + F^b_{cf^2} - F^b_{cf^{12}})/(1+r)^{5(t-t^*)} \tag{9}$$

The first term, $F^b_{cf^1}$, is the cost of building the project in time period t^*. The second term, $(K_{cf^1}/K_{cf^{12}})(F^b_{cf^1} + F^b_{cf^2} - F^b_{cf^{12}})/(1+r)^{5(t-t^*)}$, adds the capacity-scaled present value of the extra staging cost. The term $(F^b_{cf^1} + F^b_{cf^2} - F^b_{cf^{12}})$ represents the extra staging cost that would have to be paid at a later date. The denominator reduces that extra cost by a factor of $(1+r)$ for each year the extra cost will be delayed. The ratio $(K_{cf^1}/K_{cf^{12}})$ is the capacity scaling factor.

The net effect of these cost adjustments depends on the interest rate, the cost differentials, and the capacity ratios. Below we illustrate the calculation of the modified fixed cost for the construction of the intermediate stage single track project in $t = 2$ assuming that a double-track line was built in the model in $t = 3$:

$$\begin{aligned} F^{b^*}_{cf^1} &= 6000 + (20/80)(6000 + 5500 - 10000)/(1+.08)^{5(3-2)} \\ &= 6000 + (.25)1021 \\ &= 6{,}255 \end{aligned}$$

In this example, the model in $t = 2$ would be faced with 3 choices:

Case 1 – build nothing, which costs nothing and provides no capacity;
Case 2 – build the single-track line, which costs Y6,255 million and provides 20 million tons per year capacity for the next 5 years (after which it is scheduled by the $t = 3$ model run to be expanded to 80 million tons per year by adding a second track at a cost of Y5,500);
Case 3 – build the double-track line which costs Y10,000 and provides 80 million tons per year of capacity.

Table 1 shows the effects of length of time, interest rates, and capacity ratios on the different steps of the cost adjustment. In these 8 examples, the adjustment ranges from +3.8% to +8.7%. The adjustment factor will be larger for smaller staging costs, lower interest rates, shorter time spans, and smaller capacity ratios.

Now that the cost adjustment has been specified, the complete backwards time sequencing methodology for the network design project staging model can be stated as follows:

Table 1. Effects of assumptions on the intermediate stage project cost adjustment*

Intermediate stage project	Time period of project staging (t^*)	Interest rate	Final project in $t = 3$	Capacity ratio	First stage investment cost ($F^b_{cf^1}$)	$F^b_{cf^1}$ + extra staging cost	$F^b_{cf^1}$ + discounted extra staging cost	$F^b_{cf^1}$ + capacity-scaled discounted extra staging cost
ST	$t = 2$	8%	STE	2/3	6,000	7,000	6,681	6,454
ST	$t = 1$	8%	STE	2/3	6,000	7,000	6,463	6,309
ST	$t = 2$	5%	STE	2/3	6,000	7,000	6,784	6,522
ST	$t = 1$	5%	STE	2/3	6,000	7,000	6,614	6,409
ST	$t = 2$	8%	DT	2/8	6,000	7,500	7,021	6,255
ST	$t = 1$	8%	DT	2/8	6,000	7,500	6,695	6,174
ST	$t = 2$	5%	DT	2/8	6,000	7,500	7,175	6,294
ST	$t = 1$	5%	DT	2/8	6,000	7,500	6,921	6,230

* All cost and capacity data are hypothetical.

6.1 Steps for setting up the time sequence run for $t = T$:

1. Run all submodels for time t.
2. Run CUT-OFF/PRELOAD/UNCUT module for time t using existing network. (Some cut-off flows may not be able to be preloaded because there is no valid path on the existing network between the origin and destination. These cut-off flows must be added back into O-D matrix by an "UNCUT" submodel.)
3. For budget value, use $\sum_{t=1,\dots,T} B_t$.
4. Generate and solve the network design model for time t.

6.2 Steps for backwards time sequencing for time periods $t = 1$ to $T - 1$

5. Run CUTOFF/PRELOAD/UNCUT submodel for $t - 1$.
6. For budget value, use $\sum_{t=1,\dots t-1} B_t$.
7. Read Y_{cf} solution file for key year t. Determine which projects were built in all periods leading up to time t.
8. Eliminate all projects that are no longer possible to build in time $t - 1$ (because it would imply removing part of the project in time t).
9. Eliminate all corridors with no projects built.
10. Adjust the cost of intermediate stage projects based on equation (9).
11. Run the annualizing submodel.
12. Generate all blocks with new data.
13. Solve the network design model for $t - 1$.
14. If $t = 1$, STOP. Else, let $t = t - 1$ and return to Step 5.

One last important detail is to remember that the adjusted fixed costs are for use in the objective function only, for the purpose of sending the correct economic signals. The unadjusted figures should be used in reporting total investment costs or total annual costs to decision-makers.

It should be noted that this time sequencing methodology is not solved via dynamic programming. While dynamic programming decomposes a problem into stages which are usually solved backwards, it also relies on solving the nth stage for all possible solutions of the $(n - 1)$th stage. Given the astronomical number of solutions to each time stage, the dynamic programming approach would not have worked in this case.

7 Railway geographic information system

The RIS SDSS includes a proprietary GIS system developed in the MOR called the Railway GIS, or RGIS. The RGIS can (1) display data on maps or on pie, bar, or line graphs, (2) edit maps, networks, and databases; (3) query databases, (4) calculate shortest paths and descriptive statistics, and (5) switch between the national map and more detailed maps of the 12 railway administrations and 60 subadministrations. The RGIS includes geo-referenced databases on rail lines, rail stations, commodities, provinces, administrations, subadministrations, and coal, oil, and steel plants. RGIS uses a relational data base structure with a vector data model. Data are in DBase format for interoperability.

Three main links transfer data from the RIS-ND model to be visualized with the RGIS. First, the RGIS is able to display RIS-ND input data, including the base year network, flows, and demands. Second, results from the differentiated path generator can be visualized on the RGIS to make sure they provide reasonable routing alternatives. Third, the RIS-ND model results can be transferred to the RGIS in order to create maps of projects built, arc densities, passenger train pairs, and unsatisfied demands (Fig. 3). Map displays of optimized link density are particularly useful to analysts and decisionmakers. Maps of unused capacity are useful for identifying corridors that should be disaggregated or that do not have enough potential paths traversing them. Maps of bottlenecks and unsatisfied demand are valuable for determining where more paths or more investment options are needed. Automation of these various RIS-RGIS links is ongoing.

8 Conclusions

This paper has introduced a method for network design in which different kinds of capacity expansion projects can be built over time, and more specifically when those projects can be built all at once or in stages. It defined the nature of the project staging problem and explored its major issues and tradeoffs, and offered a first attempt at an approximation method within the boundaries of mixed-integer programming and a one-way pass through the model. A mixed integer programming model was introduced for the single-period, multi-project problem. This formulation captures the economies of scale of double-tracking and electrification projects, including the unit cost savings in both fixed and variable costs. For the project staging problem, a backwards time sequencing algorithm was introduced, with an adjustment to the fixed cost of the intermediate stage project to account for the implications of that decision for later time periods that were already solved. This methodology allows more complex and realistic network design problems to be solved in a more globally optimal fashion.

It is important to understand that, regardless of whether backwards or forwards time sequencing is undertaken, some kind of adjustment factor would be required if the different time periods of the project staging problem are solved independently. The difference is, with forwards time sequencing, the cost adjustment would be necessary for solving the first two time periods rather than the last two. In forwards sequencing, one would want the solution in $t = 1$ to reflect the fact that, if a single track line were built instead of a double, it would cost more money to add the double track at a later time. While the extra cost of adding the double track in $t = 2$ would be assigned as the fixed cost for the later time period solution, the earlier solution would have been solved without any consideration of the future implication. Of course, with forwards time sequencing, one does not even know in $t = 1$ if a larger project will be needed later, which further strengthens the argument for choosing backwards time sequencing. Without such adjustments to the forward method, intermediate stage projects that are scaled to demand would be more likely to be chosen in the earlier time periods. This in turn would constrain the choices for $t = 3$ by precommitting more of its budget. The end result would likely be a rail system that is less well-suited for the higher traffic volumes of the future than the system that would be designed by the backwards time sequencing solution.

Fig. 3. Example of model output from the Railway GIS. This diagram shows two-way freight traffic in 1995. The menu headers are *F* (file), *G* (graphics display), *E* (edit graphics), *R* (railway functions), *T* (tools), and *H* (help)

Several other easy-to-implement innovations were introduced to reduce the size of the problem to be solved by branch-and-bound. These include preloading small flows, which reduces the rows and columns of the LP, and making the spatial unit of investment a corridor of links rather than a single link, which reduces the number of 0-1 variables. While these techniques have been used in other studies before, they often are not published in scientific journals because of their simplicity. Yet their value to modelers may be significant, in that they – along with faster processors and better solver software – enable the use of exact MIP methods.

For future work, several improvements could be made to the model. First, the existing methodology could be applied to additional kinds of track improvements, such as (a) improvement of track components and route geometry to allow faster speeds; (b) lengthening of sidings on single track lines to allow longer trains and reduce delays; (c) improvement of signals to allow less time between trains; (d) rail ferry service; (e) high-speed passenger lines; (f) bridges or tunnels built wider to accommodate additional future tracks; (g) triple or quadruple tracking; and (h) upgrading of the track structure to allow heavier axle loads. High-speed trains and heavy axle loads have been the focus of studies by the World Bank and the MOR. As long as the project can be described by a fixed charge, a variable cost, and a capacity, and it can be slotted into a sequence of project stages, it could be modeled in this framework.

A second direction for future work is to develop heuristic methods such as simulated annealing (Friesz et al. 1993) and genetic algorithms (Xiong and Schneider 1992) that could speed the solution process, perhaps enough to allow for more project types or for all three time periods to be solved simultaneously. A third overlapping area for research would be to adapt column-generating heuristics to this multiple project formulation so that the model would not be limited to just a few path choices for each O-D pair. Fourth would be some consideration of the induced demand of building new lines in unserved areas or in the event of high-speed passenger service. All of these innovations are important to the goal of creating more realistic network design models.

There are several promising directions for improving the functionality of the SDSS surrounding the optimization model. The user interface is relatively undeveloped; it is still largely a system for experts who know how to run each module. The cutoff/preload submodel could be linked to the RGIS so that analysts could visualize which arcs might be filling up with the combined traffic of many small flows and intrazonal traffic. It would be extremely useful for analysts to be able to adjust the cutoff level and see the geographic effects on available link capacity and the effects on model size. The RGIS could also pay dividends in the process of aggregating and disaggregating links into corridors, but it has not been used yet for this task. In the future, more input data could be generated with the RGIS, such as link capacities and tariffs.

Although few new railways have been built in developed countries recently, the need for railway network design is not restricted to less developed countries. Rao et al. (1994) report on a major U.S. carrier that used a network model to analyze changes in line capacity, low density line closures, and asset sharing with foreign carriers. The ongoing merger activity in the U.S. railway industry also creates opportunities for building connectors and/or closing lines, as does the possibility of interconnecting different countries' railway systems in free trade areas. High-speed passenger rail service and double-stacked freight service represent two other types of railway network capacity expansion projects being pursued in developed countries.

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