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Spatial dispersion in cournot competition

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Abstract. This paper considers the spatial model used by Anderson and Neven (1991) to study firms' decisions on locations without restricting the consumers' reservation price. We note that the pattern of locations varies as the reservation price for a fixed transportation rate decreases. For a high enough reservation price, we find Anderson and Neven (1991)'s result where firms group at the center of the market and serve all consumers. As the reservation price falls, firms start to move away from each other, increasing the quantities shipped to the consumers close to their locations.

JEL classification: L13

Key words: Price discrimination, cournot competition, spatial dispersion, duopoly, subgame perfect equilibrium

1 Introduction

Casual observation shows industries, mainly producing intermediate goods, with firms using a distribution network to deliver their products. These networks allow consumers to have the consumption goods closer entailing low costs of transport. The behavior of this type of markets has been studied applying a type of models called "shipping models".

Anderson and Neven (1991) propose a shipping model to analyze the behavior of price discriminating firms in a spatial model à la Hotelling, and address the

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question of firms location in a Cournot-type oligopoly. A conclusion of their paper shows that competition leads to spatial agglomeration. Nevertheless, their analysis is incomplete in that there is a restriction on the feasible values of the reservation price ensuring that both firms supply all consumers at the same time. Consequently, the analysis does not capture those market structures where a monopoly emerges.

Intuition tells us that when reservation prices are sufficiently small with respect to the size of the market, firms may prefer to give up some market segments instead of competing with another firm in locations sufficiently far away. In other words, it looks as if firms had given up the opportunity to compete by implicitly sharing the market in an adequate way. The aim of this paper is to give a tentative answer to these situations.

The literature that refers to this type of model can be divided into two groups: The works of Greenhut and Greenhut (1975), Norman (1981), Phlips (1983) and Anderson and Thisse (1988) concerned with the equilibrium prices over space resulting from Cournot competition, and the works of Hamilton et al. (1989) and Anderson and Neven (1991) concerned with firms' location.

The solution shows that the pattern of locations varies as the reservation price for a fixed transportation rate decreases. For a high enough reservation price, we find Anderson and Neven's (1991) result with clustering of firms. As the reservation price falls, firms start to move away from each other. Finally, we identify a critical value of the reservation price such that below this price firms start monopolizing segments of the market.

The paper is organized as follows: the model is presented in Sect. 2. Section 3 solves the model and, finally, a section with conclusions closes the paper.

2 The model

Consumers are uniformly distributed with the unit density over $[0, 1]$. The economy has an oligopolistic sector producing a homogeneous good and a competitive sector producing a composite good, which is considered as the *numeraire*.

At each location there is a representative consumer whose preferences are represented by the "quasi-linear" utility function $U(q_0, q) = q_0 + \alpha q - \beta \frac{q^2}{2}$, where q and q_0 are the quantities of the homogeneous and the composite goods respectively, and parameters α and β are positive. There are two firms, A and B, selling the homogeneous good and sited at locations $x_A \in [0, 1]$ and $x_B \in [0, 1]$ respectively. Without loss of generality, we assume $x_A \leq x_B$. There are no production costs and transport costs are assumed to be linear, i.e., $t\,|x - x_j|$, where $x \in [0, 1]$ is a consumer's location and $j = A$ and B.

Maximizing the utility function subject to the budget constraint $q_0 + pq = I$ yields the linear inverse demand function $p = \alpha - \beta q$, where p is the mill price of the homogeneous good. Therefore, firm j's profit at location x is given by

$$\Pi^j(x) = p(x)\, q^j(x) - t\,|x_j - x|\, q^j(x). \qquad (1)$$

We consider the following two-stage game: in the first stage, both firms simultaneously choose their locations, and in the second stage, firms compete in quantities given these locations. We assume that arbitrage is not feasible,[1] therefore, quantities set at different locations by the same firm are strategically independent.

3 The resolution of the model

Anderson and Neven (1991) restricted the analysis to $\alpha > 2t$ avoiding the presence of a monopoly. We now extend the analysis of the quantity game to $\alpha \leq 2t$ allowing for different market configurations. We identify market patterns where firms compete over the whole market as well as patterns where a firm behaves as a monopoly in a market segment.

Let $x \in [0, 1]$ be the location of a consumer. Each firm maximizes its profit given the quantity chosen by the other firm. Therefore, the equilibrium quantities are either the Cournot equilibrium $q_C^A(x)$ and $q_C^B(x)$, provided they are positive, or the monopoly equilibrium $q_M^j(x)$ provided $q_C^{-j}(x)$ is negative where $-j$ denotes firm j's competitor.

From (1) and following direct computations, the Cournot equilibrium at location x is

$$q_C^j(x) = \frac{1}{3\beta}\left(\alpha - 2t\left|x_j - x\right| + t\left|x_{-j} - x\right|\right) \tag{2}$$

and the monopoly equilibrium at location x is

$$q_M^j(x) = \frac{1}{2\beta}\left[\alpha - t\left|x_j - x\right|\right]. \tag{3}$$

From (2) and (3), the conditions under which Cournot equilibrium (2) is positive over $[0, 1]$ are

$$q_C^j(0) \quad \geq \quad 0 \text{ and } q_C^j(1) \geq 0, \tag{4}$$

whereas, the conditions under which monopoly equilibrium (3) is positive over $[0, 1]$ are

$$q_M^j(0) \geq 0 \text{ and } q_M^j(1) \geq 0 \tag{5}$$

Conditions (4) and (5) follow from the fact that (2) and (3) decrease as we approach to the edge of the market.

As a consequence of (5), notice that as long as $\alpha < t$, there are some locations from which no firm would provide consumers sited on the market boundaries. We avoid this situation and focus the analysis on $\alpha \geq t$.

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[1] If the transaction costs between two consumers are low, any attempt to sell a given good to two consumers at different prices runs into the problem that the low-price consumer buys the good to resell it to the high-price one. What happens if the transaction cost is the same as for the firms? If the marginal equilibrium price with respect to distance is smaller than the marginal transportation cost, it is not profitable for consumers to resell the product to the neighbors. Consequently, it is not necessary to assume the possibility of arbitrage. However, it is not clear if firms would choose this equilibrium price knowing there are identical transaction costs for firms and consumers.

From (4) and (5), the set of feasible locations can be divided into four regions that we denote as R_k for $k = 1, 2, 3$ and 4. If firms choose any pair of locations in R_1, both firms provide the homogeneous good for the whole market; in R_2, firm B operates as a monopoly in a market segment; in region R_3, firm A is a monopoly in a market segment; and finally, in region R_4, each firm monopolizes a market segment. Notice that market segments which are not monopolized are supplied by both firms at the same time. Region R_4 is also divided into two subsets, depending on whether firms set the monopoly quantities at their respective locations or not (see Fig. 1).

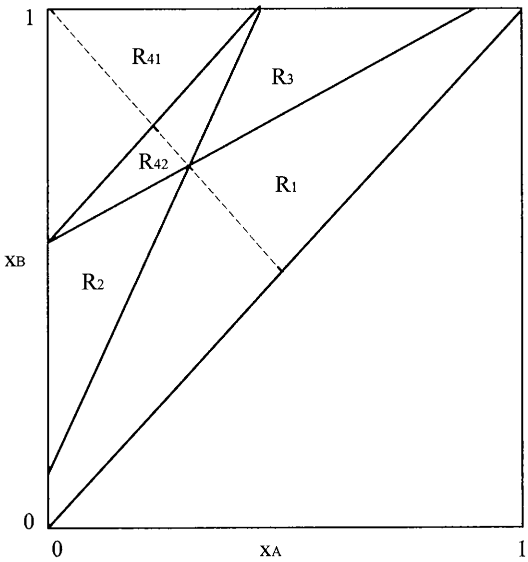

Fig. 1.

At the first stage, firms simultaneously choose locations. To obtain the optimal location for each firm, we compute the total profit of a firm, setting the equilibrium quantities already obtained at the second stage in the profit function (1) and summing up over $[0, 1]$. At the solution, there is either uniqueness or multiplicity of equilibria depending on the rate between α and t. Anderson and Neven (1991) find agglomeration of firms at the market center assuming $\alpha > 2t$. Relaxing the previous restriction, we obtain the following result.

Proposition 1. *Suppose that* $\alpha \in [t, 2t]$. *Then*

(a) There is a unique equilibrium location, $x_A^* = x_B^* = \frac{1}{2}$, *when* $\frac{3}{2}t \leq \alpha \leq 2t$.
(b) There are two equilibrium locations, $x_A^* = x_B^* = \frac{1}{2}$ *and* $x_A^* = \frac{2\alpha - t}{4t}$, $x_B^* = 1 - x_A^*$, *when* $\frac{11}{10}t \leq \alpha \leq \frac{3}{2}t$.
(c) There are two equilibrium locations, $x_A^* = x_B^* = \frac{1}{2}$ *and*
$x_A^* = \frac{1}{434t}\left(208t - 46\alpha - 4\sqrt{-117t^2 + 540\alpha t - 356\alpha^2}\right)$, $x_B^* = 1 - x_A^*$, *when* $t \leq \alpha \leq \frac{11}{10}t$.

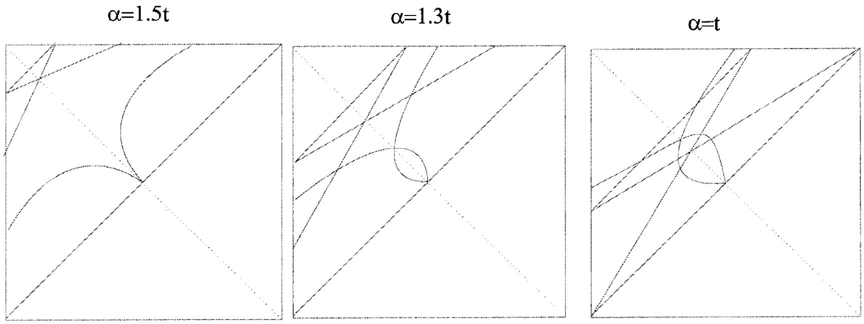

Fig. 2.

The proof of Proposition 1 is relegated to the Appendix.

Concerning the properties of location equilibria, when there are multiplicities of equilibria, items (b) and (c) of Proposition 1, the agglomerated equilibrium location is stable and the dispersed one is unstable (see reaction functions in Fig. 2).[2] Furthermore, total profits in the dispersed equilibrium are higher than in the agglomerated one.[3]

4 Conclusions

We generalize Anderson and Neven's analysis by considering a broader interval of the reservation price α-parameter. The solution says that when $1.5t < \alpha \leq 2t$ we still replicate the previous result. That is, firms locate at the market center. More interestingly, when $\alpha \leq 1.5t$, we obtain that a dispersed equilibrium exists together with the agglomerated one obtained before. These equilibria arise from different market patterns. In particular, when $1.1t \leq \alpha \leq 1.5t$, both firms supply the whole market and move away from each other to acquire more influence over different market segments. However, when $t \leq \alpha < 1.1t$, each firm monopolizes a segment on the boundaries of the market and competes with the rival firm in the rest choosing separated locations.

In this model, different effects come into play. When we are in a dispersed equilibrium, firms minimize the costs of transport and the degree of competition

[2] Formally, the condition of local stability is $\Delta \equiv \Pi^A_{AA}\Pi^B_{BB} - \Pi^A_{AB}\Pi^B_{AB} > 0$ and this comes from the comparison of the total profit functions' slopes at the equilibrium locations. Consider region R_1: At the agglomeration equilibrium we obtain $\Delta = \frac{32t^2}{81\beta^2}\left(\alpha - \frac{3}{2}t\right)(2\alpha - t)$. This expression is positive as long as there is uniqueness of equilibria, and negative as long as there is multiplicity of equilibria. At the dispersed equilibrium, $\Delta = \frac{48t^2}{81\beta^2}(2\alpha - 3t)(2\alpha - t)$. This expression is positive when a dispersed equilibrium exists. Analogous procedure can be made when equilibria lay in region R_{42} (see Tirole, 1992).

[3] Let Π^A_{agg} denotes firm A's agglomeration profit and Π^A_{dis} is the dispersed one. When $1.1t \leq \alpha \leq 1.5t$, $f(\alpha) \equiv \Pi^A_{dis} - \Pi^A_{agg} = 10\alpha^3 - 27\alpha^2 t + \frac{27}{2}\alpha t^2 + \frac{27}{4}$. $f(\alpha) \geq 0$ since $f'(\alpha) = 15(\alpha - 0.3t)(2\alpha - 3t)$ and $f(1.5t) = 0$. When $t \leq \alpha \leq 1.1t$, $f(\alpha)$ no explicit solution to this problem can be obtained. Solution is based on a computer simulation. A parallel argument also holds for firm B.

between them. The reason is that the transport costs are high compared with the demand size. Therefore, they prefer to divide the influence over the consumers choosing separate locations in the market. Conversely, if the demand size is high enough, firms cluster at the market center because the competition is not very strong and makes it possible to reduce transport costs.

A Appendix: Proof of Proposition 1

The proof follows two steps: At the first step, we show that reaction functions are symmetrical and continuously differentiable, and at the second step, we compute the equilibrium locations.

Step 1

The symmetry of the reaction functions, i.e., $\Pi^A(x_A, x_B) = \Pi^B(1 - x_B, 1 - x_A)$, directly follows from the symmetry of the model. Furthermore, reaction functions of firm A and firm B are differentiable in $[0, 1]$. Since profits are represented by cubic functions, it could only be a discontinuity at locations on the frontier between two regions. However, it is straightforward to check that profits are continuously differentiable at those locations. As is common in this type of models, the given specifications of demand function and transport costs permit deriving the reaction functions from the first order condition of the optimization problem.

Step 2

Throughout the following step, let a denotes α/t. First, let x_A and x_B be a pair of locations in region R_1. From the first order condition, equilibrium locations must satisfy one of the following conditions:

$$1 - x_A - x_B = 0 \tag{6}$$

or

$$2a - 1 - x_B + x_A = 0. \tag{7}$$

Assuming that x_A and x_B satisfy Eq. (7). Since $a \in [1, 2]$, we obtain $x_B \geq 1$. Then the unique locations in R_1 satisfying (7) are $x_A = 0$ and $x_B = 1$ as long as $a = 1$. However, these locations do not satisfy the first order condition. Therefore, substituting (6) into the first order condition, we obtain the equation $2(1 - 2x_A)(-2a + 1 + 4x_A) = 0$ whose solutions are $x_A = 0.5$ and $x_A = (2a - 1)/4$. The pair of locations $x_A = 0.5$ and $x_B = 0.5$ are in R_1, whereas $x_A = (2a - 1)/4$ and $x_B = (-2a + 5)/4$ are in R_1 as long as $q_C^A(1) \geq 0$ ($q_C^B(0) \geq 0$ is true just by symmetry), i.e., when $a \geq 1.1$ (notice that $a \leq 1.5$

since it would not give feasible locations). Accordingly, the equilibrium locations in R_1 are $x_A = 0.5$ and $x_B = 0.5$ when $a \in [1, 2]$ and $x_A = (2a - 1)/4$ and $x_B = (-2a + 5)/4$ when $a \in [1.1, 1.5]$.

Equilibrium locations in R_2: from the first order condition, equilibrium locations must satisfy condition (6) or

$$2a - 1 - 3x_B + 3x_A = 0 \tag{8}$$

There are no equilibrium locations satisfying (8) because it gives the condition $x_A > x_B$. Putting (6) into the first order condition, we obtain the unique equilibrium locations in region R_2, $x_A = 0.3$ and $x_B = 0.7$ for $a = 1.1$. Notice that these equilibrium locations were already obtained in region R_1.

The symmetry of the model gives the same solution in region R_3 as in region R_2.

Equilibrium locations in region R_{42} : from the first order condition, equilibrium locations satisfy condition (6) or

$$2a - 1 + x_B - x_A = 0 \tag{9}$$

Following an identical argument as in R_2, solutions cannot satisfy Eq. (9). Substituting (6) into the first order condition gives

$$-\frac{4}{3}(a - 2 + 4x_A)^2 + \frac{4}{3}(a + 1 - 2x_A)^2 - 9(1 - 2x_A)^2 - \frac{9}{4}(a - x_A)^2 = 0. \tag{10}$$

The solutions of (10) are $x_A^{+,-} = \left(208 - 46a \pm 4\sqrt{-117 + 540a - 356a^2}\right)/434$. The pairs of locations $x_A^{+,-}$ and $x_B^{+,-}$ $\left(\equiv 1 - x_A^{+,-}\right)$ are in R_{42} as long as $x_A^{+,-} \in \left[\widehat{x}_A, \overline{x}_A\right]$ ($\overline{x}_A = (2 - a)/3$ divides regions R_1 and R_{42} and $\widehat{x}_A = (2 + a)/4$ divides regions R_{41} and R_{42}). Since $x_A^+ > \overline{x}_A$, x_A^+ and x_B^+ are not in R_{42} whereas x_A^- and x_B^- are in R_{42} for $a \in [1, 1.1]$. Therefore, the equilibrium locations in R_{42} are x_A^- and x_B^- when $a \in [1, 1.1]$.

Equilibrium locations in region R_{41}: From the first order condition, the equilibrium locations must satisfy condition (6) or

$$2a - 1 + x_B - x_A = 0 \tag{11}$$

Equation(11) does not give any solution. Substituting (6) into the first order condition gives $-9(a - x_A)^2/4 + (2a - 1 + 2x_A)^2/3 = 0$. The solutions of this equation are $x_A^{+,-} = \left(\left(43 \pm 24\sqrt{3}\right)a - \left(8 \pm 6\sqrt{3}\right)\right)/11$. Neither x_A^- nor x_A^+ are in R_{41}. Since $a > 1$, $x_A^+ > 1$ and $x_A^- > \widehat{x}_A$ ($\widehat{x}_A$ divides S_{D_1} and S_{D_2}). Therefore, there are no equilibria in R_{41}. □

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