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Spin-transport in multi-terminal normal metal-ferromagnet systems with non-collinear magnetizations

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Abstract. A theory of spin-transport in hybrid normal metal-ferromagnetic electronic circuits is developed, taking into account non-collinear spin-accumulation. Spin-transport through resistive elements is described by 4 conductance parameters. Microscopic expression for these conductances are derived in terms of scattering matrices and explicitly calculated for simple models. The circuit theory is applied to 2-terminal and 3-terminal devices attached to ferromagnetic reservoirs.

PACS. 72.10.Bg General formulation of transport theory – 72.10.-d Theory of electronic transport; scattering mechanisms – 75.70.-i Magnetic properties of thin films, surfaces, and interfaces – 75.70.Pa Giant magnetoresistance

1 Introduction

Spin-injection from a ferromagnetic metal into a non-magnetic metal was first realized by Tedrow and Meservey in the seventies [1]. The discovery of the giant magnetoresistance (GMR) in metallic magnetic multilayers has led to an explosion of interest into spin-transport in hybrid normal metal-ferromagnetic metal systems in the nineties (for reviews see Ref. [2]). Although discovered only ten years ago, the GMR is commercially utilized in high-end magnetic recording media. Spin-transport between ferromagnets through tunnel junctions has also attracted renewed interest (for a review see Ref. [3]). Recently a magnetic double barrier tunnel device was fabricated which enables the study of the interplay between spin-polarized tunneling and Coulomb charging effects [4–6].

Magneto-electronic principles have until now led to magnetic-field sensor devices. Future applications might include non-volatile memory cells or even transistors, the latter being a three-terminal device. Johnson realized that novel long-range effects in transport between ferromagnets and normal metals can occur in multi-terminal systems [7]. In Johnson's spin-transistor different ferromagnetic (Ohmic) contacts provide information about the amount of spin-accumulation in a normal metal film over distances much larger than the mean-free path. A transistor-like effect was observed on switching the configuration of the magnetizations of the ferromagnetic contacts from parallel to anti-parallel.

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Spin-transport in systems comprising ferromagnets with non-collinear magnetization directions has attracted less attention. Moodera *et al.* measured the dependence of the current through magnetic tunnel junctions on the relative angle between the magnetization directions of the electrodes [8]. The results were in agreement with the theoretical predictions based on the spin-torque model introduced by Slonczewski [9]. Non-collinear spin transport has been addressed by a number of theoretical papers [10].

The above examples do not exhaust the novel phenomena that can be anticipated for multi-terminal hybrid normal metal-ferromagnet circuits and devices. The question of the most appropriate theoretical approach to the field arises. If the possibilities of macroscopic quantum coherence and its application to quantum computing are contemplated, a fully quantum mechanical treatment of the many-body system is required, of course. However, for contact with the experiments mentioned above and probably also with most to be realized in the near future a full quantum mechanical treatment is unnecessary and unrealistic. When at least a part of the device is diffusive simplified approaches are called for. The situation is similar to that in the field of inhomogeneous superconductors which has recently been reviewed by Belzig *et al.* [11]. The theoretical framework of choice for transport in superconducting and/or magnetic "dirty" systems is the non-equilibrium Keldysh Green functions formalism in the quasiclassical approximation. However, it is technically difficult and physically not very transparent for all but the devoted specialist. This led one of us to simplify equations for complicated hybrid superconductor-normal

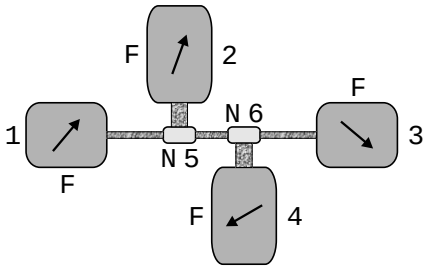

Fig. 1. A many-terminal circuit consisting of normal metals (N) and ferromagnets (F) with arbitrary magnetization directions. The normal metals (ferromagnets) are coupled to adjacent nodes by contacts which determine the resistance of the system.

metal systems to a handful of easily accessible rules, the circuit theory of Andreev reflection [12]. We recently introduced a circuit (or finite-element) theory of spin and charge transport in hybrid ferromagnet-normal metal systems which provides parametric dependencies of the electron transport properties as well as microscopic expressions for the parameters and illustrated its appeal [15]. Figure 1 shows an example of a typical many-terminal configuration of present interest. The main physical parameters of such a circuit are the magnetization directions, the chemical potentials of the leads, and the contact conductances between the normal and ferromagnetic metals. In this paper we introduce the spin-polarized kinetic equations based on the Keldysh Green function technique in the quasi-classical approximation. When solved with judiciously chosen boundary conditions the basic equations of the circuit theory emerge.

Before turning to the technical details let us first discuss the conditions under which long-range spin effects are observable in normal metals. Spins injected into a normal metal node relax due to unavoidable spin-flip processes. Naturally the dwell time on the node must be shorter than the spin-flip relaxation time in order to observe non-locality in the electron transport. For a simple ferromagnet (F) normal metal (N) double heterostructure (F-N-F) with anti parallel magnetizations the condition can be quantified following [5]. The spin-current into the normal metal node is roughly proportional to the particle current, $e(\mathrm{d}s/\mathrm{d}t)_{\mathrm{tr}} \sim I = V/R$, where s is the number of excess spins on the normal metal node, V is the voltage difference between the two reservoirs coupled to the normal metal node, and R is the F-N contact resistance. When the node is smaller than the spin-diffusion length, the spin-relaxation rate is $e(\mathrm{d}s/\mathrm{d}t)_{\mathrm{rel}} = -s/\tau_{\mathrm{sf}}$, where τ_{sf} denotes the spin-relaxation time on the node. (Otherwise this simple approach breaks down since the spatial dependence of the spin-distribution in the normal metal should be taken into account [13]). The number of spins on the normal metal node is equivalent to a non-equilibrium chemical potential difference $\Delta\mu = s\delta$ in terms of the energy level spacing δ (the inverse density of states) (more generally the relation between $\Delta\mu$ and s is determined by the spin-susceptibility [5,14]). The spin-accumulation on the normal metal node significantly affects the transport properties when the non-equilibrium chemical potential difference is of the same order of magnitude or larger than the applied source-drain voltage, $\Delta\mu > eV$ or $\delta\tau_{\mathrm{sf}}/h > R/R_K$, where $R_K = e^2/h$ is the quantum resistance. We see that spin-accumulation is only relevant for sufficiently small normal metal nodes and/or sufficiently long spin-accumulation times and/or good contact conductances. The implications of this relation have been discussed in [5] for different materials.

In the present article we explain in more detail the foundations of the circuit theory of spin-transport [15] and present more applications; a general result for two-terminal devices and results for the spin-resistance in three-terminal devices similar to the set-up of Johnson [7]. The manuscript is organized in the following way. In Section 2 we describe the basic entities in a circuit theory, the nodes, the contacts and the reservoirs. The contact conductances are computed in Section 3 for a diffusive, a ballistic and a tunnel contact. The circuit theory is employed in Section 4 to find the current trough two-terminal systems and the 'spin-resistance' of a three terminal device. Our conclusions can be found in Section 5. Appendix A gives the detailed derivation of the transformation from a non-collinear to a collinear two-terminal system.

2 Circuit theory

A typical (magneto)electronic circuit as schematically shown in Figure 1 can be divided into contacts (resistive elements), nodes (low impedance interconnectors) and reservoirs (voltage sources). The present theory is applicable when the contacts limit the electric current and the nodes are characterized by a distribution function which is constant in position and isotropic in momentum space. The latter condition justifies a diffusion approximation and requires that the nodes are either irregular in shape or contain a sufficient number of randomly distributed scatterers. State-of-the art magneto-electronic devices are rather dirty, so this does not appear to be a major restriction for the applicability of the theory. Because the spin-accumulation is not necessarily parallel to the spin-quantization axis, the electron distribution at each node is described by a 2×2 matrix in spin-space. The current through each contact can be calculated as a function of the distribution matrices on the adjacent nodes. The spin-current conservation law then allows computation of the circuit properties as a function of the applied voltages. The recipe for calculating the current-voltage characteristics can be summarized as:

- Divide the circuit into nodes, contacts, and reservoirs.
- Specify the 2×2 distribution matrix in spin-space for each node and reservoir.
- Compute the current through a contact and the distribution matrices in the adjacent nodes, which are related by the spin-charge conductances specified below.
- Make use of the spin-current conservation law at each node, stating that the difference between in and outgoing spin-currents equals the spin-relaxation rate.

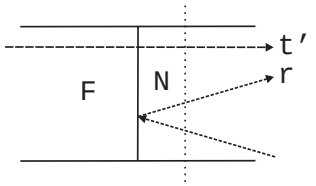

Fig. 2. A contact between a ferromagnetic node and a normal metal node. The current is evaluated at the normal metal side (dotted line). The transmission coefficient from the ferromagnet to the normal metal is t' and the reflection matrix from the normal metal to the normal metal is r.

– Solve the resulting system of linear equations to obtain all currents as a function of the reservoir chemical potentials.

2.1 Node

We denote the 2×2 distribution matrix at a given energy ϵ in the node by $\hat{f}(\epsilon)$, where hat '$\hat{}$' denotes a 2×2 matrix in spin-space. The external reservoirs are assumed to be in local equilibrium so that the distribution matrix is diagonal in spin-space and attains its local equilibrium value $\hat{f}=\hat{1}f(\epsilon,\mu_\alpha)$, $\hat{1}$ is the unit matrix, $f(\epsilon,\mu_\alpha)$ is the Fermi-Dirac distribution function and μ_α is the local chemical potential in reservoir α. The direction of the magnetization of the ferromagnetic nodes is denoted by the unit vector $\mathbf{m}_\alpha$.

The 2×2 non-equilibrium distribution matrices in the nodes in the stationary state are uniquely determined by current conservation

$$\sum_\alpha \hat{I}_{\alpha\beta}=\left(\frac{\partial \hat{f}_\beta}{\partial t}\right)_{\rm rel}, \qquad (1)$$

where $\hat{I}_{\alpha\beta}$ denotes the 2×2 current in spin-space from node (or reservoir) α to node (or reservoir) β and the term on the right hand side describes spin-relaxation in the normal node. The right hand side of equation (1) can be set to zero when the spin-current in the node is conserved, *i.e.* when an electron spends much less time on the node than the spin-flip relaxation time $\tau_{\rm sf}$. If the size of the node in the transport direction is smaller than the spin-flip diffusion length $l_{\rm sf}=\sqrt{D\tau_{\rm sf}}$, where D is the diffusion coefficient then the spin-relaxation in the node can be introduced as $(\partial\hat{f}^{\rm N}/\partial t)_{\rm rel}=(\hat{1}{\rm Tr}(\hat{f}^{\rm N})/2-\hat{f}^{\rm N})/\tau_{\rm sf}$. If the size of the node in the transport direction is larger than $l_{\rm sf}$ the simplest circuit theory fails and we have to use a more complicated description with a spatially dependent spin-distribution function [13].

2.2 Current through a contact

A schematic picture of a contact between a normal metal and a ferromagnetic node is shown in Figure 2. The current is evaluated on the normal side of the contact (dotted line). The current through the contact is

$$\hat{I}=\frac{e}{h}\Big\{\sum_{nm}[\hat{t}'^{nm}\hat{f}^F\left(\hat{t}'^{mn}\right)^\dagger-(M\hat{f}^N-\hat{r}^{nm}\hat{f}^N\left(\hat{r}^{mn}\right)^\dagger)]\Big\}, \qquad (2)$$

where $r^{nm}_{ss'}$ is the reflection coefficient for electrons from transverse mode m with spin s' incoming from the normal metal side reflected to transverse mode n with spin s on the normal metal side, and $t'^{nm}_{ss'}$ is the transmission coefficient for electrons from transverse mode m with spin s' incoming from the ferromagnet transmitted to transverse mode n with spin s on the normal metal side. (Note that the Hermitian conjugate in (2) operates in the spin-space and the space spanned by the transverse modes, *e.g.* $(\hat{r}^{mn})^\dagger_{ss'}=(\hat{r}^{nm}_{s's})^*$.) The relation (2) can be found intuitively in the spirit of the Landauer-Büttiker formalism [16], but can also be derived more rigorously by using the Keldysh formalism for non-equilibrium transport. We use below the latter approach and clarify the order of the spin-indices. Implicitly included in (2) are also effects related to the precession of spins non-collinear to the magnetization direction in ferromagnets which will be made more explicit below.

The relation (2) between the current and the distributions has a simple form after transforming the spin-quantization axis. The detailed calculation of this transformation is shown in Appendix A. Disregarding spin-flip processes in the contacts, the reflection matrix for an incoming electron from the normal metal transforms as

$$\hat{r}^{nm}=\sum_s \hat{u}^s r^{nm}_s\,,$$

where $r^{nm}_\uparrow$ $(s=\uparrow)$ and $r^{nm}_\downarrow$ $(s=\downarrow)$ are the spin-dependent reflection coefficients in the basis where the spin-quantization axis is parallel to the magnetization in the ferromagnet, the spin-projection matrices are

$$\hat{u}^\uparrow=(\hat{1}+\hat{\sigma}\cdot\mathbf{m})/2\,, \qquad (3)$$

$$\hat{u}^\downarrow=(\hat{1}-\hat{\sigma}\cdot\mathbf{m})/2 \qquad (4)$$

and $\hat{\sigma}$ is a vector of Pauli matrices. Similarly for the transmission matrix

$$\hat{t}'^{nm}(\hat{t}'^{mn})^\dagger=\sum_s \hat{u}^s|t'^{nm}_s|^2\,,$$

where $t^{nm}_\uparrow$ and $t^{nm}_\downarrow$ are the spin-dependent transmission coefficients in the basis where the spin-quantization axis is parallel to the magnetization in the ferromagnet. Using the unitarity of the scattering matrix, we find that the general form of the relation (2) reads

$$e\hat{I}=G^\uparrow\hat{u}^\uparrow\left(\hat{f}^{\rm F}-\hat{f}^{\rm N}\right)\hat{u}^\uparrow+G^\downarrow\hat{u}^\downarrow\left(\hat{f}^{\rm F}-\hat{f}^{\rm N}\right)\hat{u}^\downarrow$$
$$-G^{\uparrow\downarrow}\hat{u}^\uparrow\hat{f}^{\rm N}\hat{u}^\downarrow-(G^{\uparrow\downarrow})^*\hat{u}^\downarrow\hat{f}^{\rm N}\hat{u}^\uparrow\,, \qquad (5)$$

where we have introduced the spin-dependent conductances $G^\uparrow$ and $G^\downarrow$

$$G^\uparrow=\frac{e^2}{h}\left[M-\sum_{nm}|r^{nm}_\uparrow|^2\right]=\frac{e^2}{h}\sum_{nm}|t^{nm}_\uparrow|^2\,, \qquad (6)$$

$$G^{\downarrow} = \frac{e^2}{h}\left[M - \sum_{nm}|r_{\downarrow}^{nm}|^2\right] = \frac{e^2}{h}\sum_{nm}|t_{\downarrow}^{nm}|^2 \qquad (7)$$

and the mixing conductance

$$G^{\uparrow\downarrow} = \frac{e^2}{h}\left[M - \sum_{nm} r_{\uparrow}^{nm}(r_{\downarrow}^{nm})^*\right]. \qquad (8)$$

The precession of spins leads to an effective relaxation of spins non-collinear to the local magnetization in ferromagnets and consequently the distribution function is limited to the form $\hat{f}^{\mathrm{F}} = \hat{1}f_0^{\mathrm{F}} + \hat{\sigma}\cdot\mathbf{m}f_s^{\mathrm{F}}$. Such a restriction does not appear in the normal metal node and $\hat{f}^{\mathrm{N}}$ can be any Hermitian 2×2 matrix.

We thus see how the relation between the current through a contact and the distributions in the ferromagnetic node and the normal metal node are determined by 4 parameters, the two real spin-dependent conductances ($G^{\uparrow}$, $G^{\downarrow}$) and the real and imaginary parts of the mixing conductance $G^{\uparrow\downarrow}$. These contact-specific parameters can be obtained by microscopic theory or from experiments. The spin-conductances $G^{\uparrow}$ and $G^{\downarrow}$ have been used in descriptions of spin-transport for a long time [2]. The *mixing conductance* is a new concept which is relevant for transport between non-collinear ferromagnets. Note that although the mixing conductance is a complex number the 2×2 current in spin-space is Hermitian and consequently the current and the spin-current in any direction given by equation (5) are real numbers. From the definitions of the spin-dependent conductances (6), (7) and the 'mixing' conductance (8) we find

$$2\mathrm{Re}G^{\uparrow\downarrow} = G^{\uparrow} + G^{\downarrow} + \frac{e^2}{h}\sum_{nm}|r_{\uparrow}^{nm} - r_{\downarrow}^{nm}|^2$$

and consequently the conductances should satisfy

$$2\mathrm{Re}G^{\uparrow\downarrow} \geq G^{\uparrow} + G^{\downarrow}. \qquad (9)$$

A physical interpretation of this result is given below.

Before outlining the derivation of (2) leading to (5), let us discuss the physics in simple terms. Some insight can be gained by re-writing the current and the distribution function in terms of a scalar particle and a vector spin-contribution, $\hat{I} = (\hat{1}I_0 + \hat{\sigma}\cdot\mathbf{I}_s)/2$, $\hat{f}^{\mathrm{N}} = \hat{1}f_0^{\mathrm{N}} + \hat{\sigma}\cdot\mathbf{s}f_s^{\mathrm{N}}$ and $\hat{f}^{\mathrm{F}} = \hat{1}f_0^{\mathrm{F}} + \hat{\sigma}\cdot\mathbf{m}f_s^{\mathrm{F}}$. The particle current can then be written as

$$I_0 = (G^{\uparrow} + G^{\downarrow})(f_0^{\mathrm{F}} - f_0^{\mathrm{N}}) + (G^{\uparrow} - G^{\downarrow})(f_s^{\mathrm{F}} - \mathbf{m}\cdot\mathbf{s}f_s^{\mathrm{N}}). \qquad (10)$$

The familiar expressions for collinear transport are recovered when $\mathbf{m}\cdot\mathbf{s} = \pm 1$. The spin-current is

$$\begin{aligned}\mathbf{I}_s = \mathbf{m}[&(G^{\uparrow} - G^{\downarrow})(f_0^{\mathrm{F}} - f_0^{\mathrm{N}}) \\ &+(G^{\uparrow} + G^{\downarrow})f_s^{\mathrm{F}} + (2\mathrm{Re}G^{\uparrow\downarrow} - G^{\uparrow} - G^{\downarrow})\mathbf{s}\cdot\mathbf{m}f_s^{\mathrm{N}}] \\ &-\mathbf{s}2\mathrm{Re}G^{\uparrow\downarrow}f_s^{\mathrm{N}} + (\mathbf{s}\times\mathbf{m})2\mathrm{Im}G^{\uparrow\downarrow}f_s^{\mathrm{N}}. \qquad (11)\end{aligned}$$

The first three terms point in the direction of the magnetization of the ferromagnet $\mathbf{m}$, the fourth term is in the direction of the non-equilibrium spin-distribution $\mathbf{s}$, and the last term is perpendicular to both $\mathbf{s}$ and $\mathbf{m}$. The last contribution solely depends on the imaginary part of the mixing conductance. We can interpret this term by considering how the direction of the spin on the normal metal node $\mathbf{s}$ would change in time keeping, all other parameters constant. The cross product creates a precession of $\mathbf{s}$ around the magnetization direction $\mathbf{m}$ of the ferromagnet similar to a classical torque while keeping the magnitude of the spin-accumulation constant. In contrast, the first four terms represent diffusion-like processes which decrease the magnitude of the spin-accumulation. We now understand condition (9) since (11) implies that the non-equilibrium spin-distribution f_s^{N} propagates easier into a configuration parallel to $\mathbf{s}$ than parallel to $\mathbf{m}$, since these processes are governed by *positive* diffusion-like constants $2\mathrm{Re}G^{\uparrow\downarrow}$ and $2\mathrm{Re}G^{\uparrow\downarrow} - G^{\uparrow} - G^{\downarrow}$, respectively.

2.3 Derivation of the current

In this section the relation between the current through a contact and the adjacent distribution functions (2) is derived. This derivation is not crucial for the understanding of Sections 3–5 and can be skipped by readers mainly interested in the physical implications of (2) and (5). We follow the lines of the derivation of the contact current between a normal metal and a superconductor in reference [12]. We consider a F-N system comprising of a ferromagnet with arbitrary magnetization direction and a normal metal node separated by a contact, as shown in Figure 2. The Stoner Hamiltonian is

$$\hat{H} = \left[-\frac{1}{2m}\nabla^2 + V^{\mathrm{p}}(\mathbf{r})\right]\hat{1} + \hat{V}^{\mathrm{s}}(\mathbf{r}), \qquad (12)$$

where $V^{\mathrm{p}}(\mathbf{r})$ is the spin-independent potential and $\hat{V}^{\mathrm{s}}(\mathbf{r})$ is the spin-dependent potential. The latter vanishes in the normal metal node and has an arbitrary, but spatially independent direction in spin-space in the ferromagnetic reservoir, $\hat{V}^{\mathrm{s}}(\mathbf{r}) = (\boldsymbol{\sigma}\cdot\mathbf{m})V^{s}(\mathbf{r})$. Non-equilibrium transport properties are most conveniently discussed in the framework of the Keldysh formalism. The Keldysh Green function is given by the (4×4) matrix

$$\check{G} = \begin{pmatrix}\hat{G}^R & \hat{G}^K \\ \hat{0} & \hat{G}^A\end{pmatrix},$$

where $\hat{G}^R$, $\hat{G}^K$ and $\hat{G}^A$ are the retarded, Keldysh and advanced Green functions respectively, which are (2×2) matrices in spin-space and $\widehat{0}$ is the (2×2) zero matrix. The retarded Green function in spin-space is

$$\hat{G}^R(1,1') = \begin{pmatrix}G^R_{\uparrow\uparrow}(1,1') & G^R_{\uparrow\downarrow}(1,1') \\ G^R_{\downarrow\uparrow}(1,1') & G^R_{\downarrow\downarrow}(1,1')\end{pmatrix}$$

and there are analogous expressions for $\hat{G}^K$ and $\hat{G}^A$. Here 1 denotes the spatial and the time coordinates, $1 = \mathbf{r}_1 t_1$.

The symbol "check" ($\check{}$) denotes (4×4) matrices in Keldysh space and the symbol "hat" ($\hat{}$) denotes (2×2) matrices in spin-space. The spin-components of the Green functions are

$$G^R_{\sigma s}(1,1') = -\mathrm{i}\theta(t_1 - t_{1'}) \left\langle \left[\psi_\sigma(1), \psi^\dagger_s(1')\right]\right\rangle_+ , \quad (13)$$

$$G^A_{\sigma s}(1,1') = \mathrm{i}\theta(t_{1'} - t_1) \left\langle \left[\psi_\sigma(1), \psi^\dagger_s(1')\right]\right\rangle_+ , \quad (14)$$

$$G^K_{\sigma s}(1,1') = -\mathrm{i} \left\langle \left[\psi_\sigma(1), \psi^\dagger_s(1')\right]\right\rangle , \quad (15)$$

where $\psi^\dagger_s(1)$ is the electron field operator for an electron with spin s in the z-direction. The Keldysh Green functions is determined by the equation

$$\left(\hat{H} - \mathrm{i}\hbar\frac{\partial}{\partial t}\right)\check{G}(\mathbf{r}t, \mathbf{r}'t') = \check{1}\delta(\mathbf{r}t - \mathbf{r}'t'),$$

and the boundary conditions to be discussed below ($\check{1}$ is a 4×4 unit matrix). In the stationary situation $\check{G}(1,1') = \int \mathrm{d}(E/2\pi)\exp(\mathrm{i}E\,(t_1 - t_{1'}))\check{G}_E(r,r')$ and the Green function on a given energy shell is determined from

$$\left(\hat{H} - E\right)\check{G}_E(\mathbf{r},\mathbf{r}') = \check{1}\delta(\mathbf{r} - \mathbf{r}')\,.$$

We will in the following omit the index E in denoting the Green function at a given energy. The Keldysh Green function can be decomposed into quasi-one-dimensional modes as

$$\check{G}_{ss'}(\mathbf{r},\mathbf{r}') = \sum_{nm,\alpha\beta} \tilde{G}^{\alpha\beta}_{nsms'}(x,x') \times \chi^n_s(\boldsymbol{\rho};x)\chi^{m*}_{s'}(\boldsymbol{\rho}';x')\mathrm{e}^{\mathrm{i}\alpha k^n_s x - \mathrm{i}\beta k^m_{s'} x'} , \quad (16)$$

where $\chi^n_s(\boldsymbol{\rho};x)$ is the transverse wave function and k^n_s denotes the longitudinal wave-vector for an electron in transverse mode m with spin s. The indices α and β denote right-going (+) and left-going (−) modes. The symbol 'tilde' ($\tilde{}$) denotes matrices in Keldysh space, spin-space, ant the space spanned by the transverse modes and the directions of propagation.

The current operator can be found from the continuity relation for the electron density. The spin-density matrix is

$$\rho_{\sigma s}(1) = \left\langle \psi^\dagger_s(1)\psi_\sigma(1)\right\rangle .$$

The time-evolution of the spin-density matrix reads

$$\frac{\partial}{\partial t_1}\rho_{\sigma s} = -\frac{\partial}{\partial \mathbf{r}_1}\mathbf{J}^{\mathrm{p}}_{\sigma s} + \left(\frac{\partial \rho_{\sigma s}}{\partial t_1}\right)_{\mathrm{prec.}} ,$$

where we have inserted the Hamiltonian (12) and found the spin-current

$$\mathbf{J}^{\mathrm{p}}_{\sigma s} = \frac{\hbar\mathrm{i}}{2m}\left\langle \frac{\partial \psi^\dagger_s}{\partial \mathbf{r}_1}\psi_\sigma - \psi^\dagger_s\frac{\partial \psi_\sigma}{\partial \mathbf{r}_1}\right\rangle ,$$

and the spin-precession

$$\left(\frac{\partial \rho_{\sigma s}}{\partial t_1}\right)_{\mathrm{prec.}} = \frac{1}{\mathrm{i}\hbar}\sum_\alpha \left[V^{\mathrm{s}}_{\sigma\alpha}\rho_{\alpha s} - \rho_{\sigma\alpha}V^{\mathrm{s}}_{\alpha s}\right] . \quad (17)$$

In three dimensions the spin-precession is an average over many states with different Larmor frequencies which average out very quickly in a ferromagnet, leading to an efficient relaxation of the non-diagonal terms in the spin-density matrix that represent a spin-accumulation non-collinear to the magnetization in the ferromagnet. This spin-relaxation mechanism does not exist in normal metals where in the absence of spin-flip scattering the spin-wave functions remain coherent.

In the stationary situation the spin-current is

$$\mathbf{J}_{\sigma s} = \left(\frac{\partial}{\partial \mathbf{r}_1} - \frac{\partial}{\partial \mathbf{r}_{1'}}\right)\frac{\hbar\mathrm{i}}{2m}\int\frac{\mathrm{d}E}{2\pi}\int \mathrm{d}(t_1 - t_{1'}) \times \exp(\mathrm{i}E(t_1 - t_{1'}))\left\langle \psi^\dagger_s(1)\psi_\sigma(1')\right\rangle |_{\mathbf{r}_{1'}=\mathbf{r}_1} . \quad (18)$$

We now define the extended 4×4 current matrix in Keldysh and spin-space as

$$\check{I}(x) = \int \mathrm{d}\boldsymbol{\rho}\frac{e\hbar}{m}\left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)\check{G}(\mathbf{r},\mathbf{r}')|_{\mathbf{r}'=\mathbf{r}}.$$

The transverse wave function $\chi^n_s(\boldsymbol{\rho};x)$ is spatially independent in the leads and we find

$$\check{I}_{ss'}(x) = \mathrm{i}e\sum_{n\alpha\beta}(\alpha v^n_s - \beta v^m_{s'})\,\tilde{G}^{\alpha\beta}_{nsms'}(x,x) \times \int \mathrm{d}\boldsymbol{\rho}\chi^n_s(\boldsymbol{\rho};x)\chi^{m*}_{s'}(\boldsymbol{\rho};x)\,, \quad (19)$$

where $v^n_s = \hbar k^n_s/m$ is the longitudinal velocity for an electron in transverse mode n with spin s. In a normal metal, the transverse states and the longitudinal momentum are spin-independent and the Keldysh current simplifies to

$$\check{I}_{ss'}(x) = 2\mathrm{i}e\sum_{n\alpha}\alpha v^n\tilde{G}^{\alpha\alpha}_{nsns'}(x,x)\,, \quad (20)$$

which we will use to calculate the spin-current on the normal side of the contact. We use the representation

$$\mathrm{i}\tilde{G}^{\alpha\beta}_{nsms'}(x,x') = \frac{\tilde{g}^{\alpha\beta}_{nsms'}(x,x')}{\sqrt{v^n_s v^m_{s'}}} + \check{1}\delta_{ss'}\frac{\alpha\delta_{\alpha,\beta}\mathrm{sign}(x - x\prime)}{v^n_s}, \quad (21)$$

where the latter term does not contribute to the current on the normal side, and we have on the normal metal side

$$\check{I}_{ss'}(x) = 2e\sum_{n\alpha}\alpha\tilde{g}^{\alpha\alpha}_{nsns'}(x,x)\,. \quad (22)$$

We now introduce the transfer matrix M between waves propagating to the right (left) on the right hand side of the contact Ψ^+_R (Ψ^-_R) and waves propagating to the right (left) on the left hand side of the contact Ψ^+_L (Ψ^-_L)

$$\begin{pmatrix}\Psi^+_R\\ \Psi^-_R\end{pmatrix} = M\begin{pmatrix}\Psi^+_L\\ \Psi^-_L\end{pmatrix} .$$

The elements of the transfer matrix are related to the reflection and transmission coefficients by

$$\begin{pmatrix} m^{++} & m^{+-} \\ m^{-+} & m^{--} \end{pmatrix} = \begin{pmatrix} t - r'(t')^{-1}r & r'(t')^{-1} \\ -(t')^{-1}r & (t')^{-1} \end{pmatrix},$$

Similarly we can introduce the scattering matrix

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

so that

$$S = \begin{pmatrix} -(m^{--})^{-1}m^{-+} & (m^{--})^{-1} \\ m^{++} - m^{+-}(m^{--})^{-1}m^{-+} & m^{+-}(m^{--})^{-1} \end{pmatrix},$$

where $r^{s\sigma}_{nm}$ is the reflection matrix for incoming states from the left in mode m and spin σ to mode n with spin s, $t^{s\sigma}_{nm}$ is the transmission matrix for incoming states from the left transmitted to outgoing states to the right, r' is the reflection matrix for incoming states from the right reflected to the right, and t' is the transmission matrix for incoming states from the right transmitted to the left. Unitarity of the S-matrix requires $S^\dagger S = 1$ and $SS^\dagger =$ and implies that the transfer matrix satisfies

$$\bar{M}^\dagger \bar{\Sigma}_z \bar{M} = \bar{\Sigma}_z\,, \tag{23}$$

$$\bar{M} \bar{\Sigma}_z \bar{M}^\dagger = \bar{\Sigma}_z\,. \tag{24}$$

where $\left(\bar{\Sigma}_z\right)^{\alpha\beta}_{nsms'} = \alpha\delta_{\alpha,\beta}\delta_{ns,ms'}$ is a Pauli matrix with respect to the direction of propagation. In order to connect the Green function to the left and to the right of the contact, we use the transfer matrix of the contact $\tilde{g}^{\sigma\sigma'}_{nsms'}(x = x_2, x') = \sum_{ls'',\sigma''} M^{\sigma\sigma''}_{nsls''}\tilde{g}^{\sigma''\sigma'}_{ls''ms'}(x = x_1, x')$ and similarly for the x'-coordinate. Hence

$$\tilde{g}_2 = M\tilde{g}_1 M^\dagger\,, \tag{25}$$

where $\tilde{g}_{2(1)} = \tilde{g}(x = x_{2(1)}, x' = x_{2(1)})$. Up to this point the Keldysh Green functions have been obtained exactly for the Hamiltonian (12). Now we should include the proper boundary conditions to uniquely define the Green functions. To this end we introduce the assumptions of isotropizations at the nodes. The incoming modes are assumed to take their quasi-classical values described by the quasi-classical Green functions $\bar{G}$, whereas the outgoing modes are determined by the properties of the contact [12]:

$$\left(\bar{\Sigma}_z + \bar{G}_1\right)\left(\bar{\Sigma}_z - \bar{g}_1\right) = 0 \tag{26}$$

$$\left(\bar{\Sigma}_z + \tilde{g}_1\right)\left(\bar{\Sigma}_z - \bar{G}_1\right) = 0 \tag{27}$$

$$\left(\bar{\Sigma}_z - \bar{G}_2\right)\left(\bar{\Sigma}_z + \tilde{g}_2\right) = 0 \tag{28}$$

$$\left(\bar{\Sigma}_z - \tilde{g}_2\right)\left(\bar{\Sigma}_z + \bar{G}_2\right) = 0\,, \tag{29}$$

where $\bar{G}_1$ ($\bar{G}_2$) is the isotropic Green function in reservoir 1 (2):

$$\left(\bar{G}_1\right)^{\alpha\beta}_{nsms'} = \delta_{n,m}\delta^{\alpha\beta}(\check{G}_1)_{ss'}\,.$$

In a normal metal, the homogeneous retarded quasi-classical Green function is

$$\bar{G}_R = \begin{pmatrix} G_R^{++} & G_R^{+-} \\ G_R^{-+} & G_R^{--} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

and the advanced quasi-classical Green function is $\bar{G}_A = -\bar{G}_R$. (Note that here 1 means a unit matrix in the basis of the transverse modes and spin, $1 \to \delta_{ns,ms'}\hat{1}$.) The Keldysh component of the Green function is

$$\bar{G}_{K,1(2)} = \hat{h}_{1(2)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

where the 2×2 distribution matrix $\hat{h}$ is related to the (non-equilibrium) distribution functions $\hat{f}(\epsilon)_{1(2)}$ in the nodes by

$$\hat{h}_{1(2)} = 2(2\hat{f}(\epsilon)_{1(2)} - 1)\,.$$

The isotropization conditions (26, 27, 28) and (29) relate the retarded and advanced Green function on the left and right hand side of the contact:

$$\tilde{g}_{R,1} = \begin{pmatrix} 1 & 0 \\ \tilde{g}^{-+}_{R,1} & 1 \end{pmatrix}$$

$$\tilde{g}_{R,2} = \begin{pmatrix} 1 & \tilde{g}^{+-}_{R,2} \\ 0 & 1 \end{pmatrix}$$

where $\tilde{g}^{-+}_{R,1}$ and $\tilde{g}^{+-}_{R,2}$ are determined by the Green function on the right and the scattering properties of the contact. The advanced Green function is related to the retarded Green function by

$$\tilde{g}_A = -\tilde{g}^\dagger_R\,.$$

The Keldysh component on the left side is determined by (26, 27, 28, 29) and (30) and is dictated by the boundary conditions given by the isotropization process to be

$$\tilde{g}_{K,1} = \begin{pmatrix} \hat{h}_1 1 & \hat{h}_1 r^\dagger \\ r\hat{h}_1 & \tilde{g}^{--}_{K,1} \end{pmatrix}, \tag{30}$$

$$\tilde{g}_{K,2} = \begin{pmatrix} \tilde{g}^{++}_{K,2} & r'\hat{h}_2 \\ \hat{h}_2 r'^\dagger & \hat{h}_2 1 \end{pmatrix}. \tag{31}$$

We now use the relation between the Green function on the left hand side and the right hand side of the contact (25) to obtain the retarded Green functions

$$\tilde{g}^{-+}_{R,1} = 2r$$

$$\tilde{g}^{+-}_{R,2} = 2r'\,,$$

and the Keldysh Green function

$$\tilde{g}^{--}_{K,1} = t'\hat{h}_2 t'^\dagger + \hat{h}_1 r^\dagger \tag{32}$$

$$\tilde{g}^{++}_{K,2} = t\hat{h}_1 t^\dagger + r'\hat{h}_2 r'^\dagger\,. \tag{33}$$

Inserting the expression the Keldysh component (33) into (22) and using (2.3) we finally find the expression for the current through the contact (2).

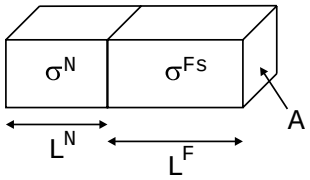

Fig. 3. A diffusive contact between a ferromagnetic node and a normal metal node. The length of the ferromagnetic (normal metal) part of the contact is $L^{\rm F}$ ($L^{\rm N}$), and the conductivity of the ferromagnetic (normal metal) is $\sigma^{\rm F}$ ($\sigma^{\rm N}$). The cross-section of the contact is A.

3 Contact conductances

The four conductance parameters $G^{\uparrow}$, $G^{\downarrow}$, ${\rm Re}G^{\uparrow\downarrow}$ and ${\rm Im}G^{\uparrow\downarrow}$ depend on the microscopic details as illustrated below for 3 elementary model contacts: a diffusive, a ballistic, and a tunnel contact.

3.1 Diffusive contact

We consider first a diffusive contact between a normal metal node and a ferromagnetic node and show the relations between the conductances (see Fig. 3). The cross-section of the contact is A, the length of the normal metal part of the contact is L^N, the length of the ferromagnetic part of the contact is L^F, the conductivity on the normal side σ^N and the spin-dependent conductivities on the ferromagnetic side σ^{Fs}. The conductance of the normal part is $G_D^N = A\sigma^N/L^N$ and the spin-dependent conductances of the ferromagnetic part are $G_D^{Fs} = A\sigma^{Fs}/L^F$. The spin-dependent conductances of the whole contact are obtained simply as the diffusive ferromagnetic and normal metal regions in series:

$$G_D^{\uparrow} = \frac{G_D^{F\uparrow} G_D^N}{G_D^{F\uparrow} + G_D^N}\,, \tag{34}$$

$$G_D^{\downarrow} = \frac{G_D^{F\downarrow} G_D^N}{G_D^{F\downarrow} + G_D^N}\cdot \tag{35}$$

These spin-dependent conductances ($G_D^{\uparrow}$ and $G_D^{\downarrow}$) fully describe collinear transport (in the absence of spin-flip scattering). For non-collinear magnetizations the mixing conductance is also needed. It can be derived from the scattering matrix, *e.g.* with the method developed in reference [17]. Here we use a much simpler approach based on the diffusion equation, describing the scattering properties of the contact by a spatially dependent distribution matrix. The current density on the normal side of the contact ($x<0$) is $\hat{\imath}(x<0) = \sigma^N \partial_x \hat{f}$ and consequently the total current is

$$\hat{I}(x<0) = G_D^N (L^{\rm N}\partial_x)\hat{f}\,,$$

where $\hat{f}$ is the spatially dependent distribution matrix on the normal side in the contact. In the normal metal node the boundary condition is

$$\hat{f}(x=-L^N) = \hat{f}^N. \tag{36}$$

In a ferromagnet spin-up and spin-down states are incoherent, and hence spins non-collinear to the magnetization direction relax according to (17) and only spins collinear with the magnetization will propagate sufficiently far away from the NF-interface. We assume that the ferromagnet is sufficiently strong and that the contact is longer than the ferromagnetic decoherence length $\xi = \sqrt{D/h_{\rm ex}}$, where D is the diffusion constant and $h_{\rm ex}$ is the exchange splitting. The decoherence length is typically very short in ferromagnets, $\xi = 2$ nm in Ni wires [18]. The distribution function on the ferromagnetic side can then be represented by a 2-component distribution function

$$\hat{f}(x>0) = \hat{u}^{\uparrow} f^{\uparrow} + \hat{u}^{\downarrow} f^{\downarrow}\,,$$

where $\hat{u}^{\uparrow}$ and $\hat{u}^{\downarrow}$ are the spin-projection matrices (3) and (4).

We allow for a spin-accumulation collinear to the magnetization direction in the ferromagnet. The boundary condition determined by the distribution function in the ferromagnetic node is thus

$$f^{\uparrow}(x=L^F) = f^{F\uparrow}\,, \tag{37}$$

$$f^{\downarrow}(x=L^F) = f^{F\downarrow}\,. \tag{38}$$

The total current in the ferromagnet is

$$\hat{I}(x>0) = G_D^{F\uparrow}\hat{u}^{\uparrow}\partial_x f^{\uparrow} + G_D^{F\downarrow}\hat{u}^{\downarrow}\partial_x f^{\downarrow}\,.$$

We assume that the resistance of the diffusive region of the contacts is much larger than the contact resistance between the normal and the ferromagnetic metal. The distribution function is in this limit continuous across the normal metal-ferromagnetic interface,

$$\hat{f}(0^+) = \hat{f}(0^-)\,. \tag{39}$$

Current conservation on the left ($x<0$) and on the right ($x>0$) of the normal metal-ferromagnet interface dictates

$$\partial_x \hat{I} = 0\,. \tag{40}$$

Note that the component of the spin-current that is non-collinear to the magnetization direction in the ferromagnet is not conserved on going through the interface due to strong relaxation induced by (17). The first order differential equation (40) and the boundary conditions (36–38) and (39) uniquely determine the distribution functions and hence the conductance in the diffusive contact. The current on the normal side of the contact becomes

$$\begin{aligned} e\hat{I} = {} & G_D^{\uparrow}\hat{u}^{\uparrow}(\hat{f}^{\rm F} - \hat{f}^{\rm N})\hat{u}^{\downarrow} + G_D^{\downarrow}\hat{u}^{\downarrow}(\hat{f}^{\rm F} - \hat{f}^{\rm N})\hat{u}^{\downarrow} \\ & + G_D^N\left[\hat{u}^{\uparrow}(\hat{f}^{\rm F} - \hat{f}^{\rm N})\hat{u}^{\downarrow} + \hat{u}^{\uparrow}(\hat{f}^{\rm F} - \hat{f}^{\rm N})\hat{u}^{\downarrow}\right]. \end{aligned} \tag{41}$$

The current in a diffusive contact thus takes the generic form (5) with $G^{\uparrow} = G_D^{\uparrow}$, $G^{\downarrow} = G_D^{\downarrow}$ and $G^{\uparrow\downarrow} = G_D^N$. The mixing conductance is thus real and only depends on the normal conductance. The latter results can be understood as a consequence of the effective spin-relaxation of spins non-collinear to the local magnetization direction. Those spins cannot propagate in the ferromagnet, and consequently the effective conductance can only depend on the conductance in the normal metal as (41) explicitly demonstrates.

3.2 Ballistic contact

A simplified expression for the conductances can be found for a ballistic contact. Firstly, the reflection and transmission coefficients appearing in (6, 7) and (8) are diagonal in the space of the transverse channels since the transverse momentum is conserved. In a simplified model [19] the transmission channels are either closed $t = 0$ or open $t = 1$. The conductances (6, 7) and (8) can then be found by simply counting the number of propagating modes. We obtain the spin-dependent conductances

$$G_B^{\uparrow} = \frac{e^2}{h} N^{\uparrow} \tag{42}$$

$$G_B^{\downarrow} = \frac{e^2}{h} N^{\downarrow}\,, \tag{43}$$

where $N^{\uparrow}$ is the number of spin-up propagating channels and $N^{\downarrow}$ is the number of spin-down propagating channels. The mixing conductance is determined by

$$G_B^{\uparrow\downarrow} = \max(G_B^{\uparrow}, G_B^{\downarrow})$$

and is real. In a quantum mechanical calculation the channels just above the potential step are only partially transmitting and the channels below a potential step can have a finite transmission probability due to tunneling. Furthermore, the band structure of ferromagnetic metals is usually complicated and interband scattering exists even at ideal interfaces. We may therefore expect that in general the phase of the scattered wave will be relevant giving a non-vanishing imaginary part of the mixing conductance. First-principles calculations of the complete conductance matrix are therefore highly desirable.

3.3 Tunnel contact

For a tunneling contact the transmission coefficients are exponentially small and the reflection coefficients have a magnitude close to one. The spin-dependent conductances are

$$G_T^s = \frac{e^2}{h} \sum_{nm} |t_s^{nm}|^2\,. \tag{44}$$

For simple models of tunnel barriers $r_s^{nm} = \delta^{nm} \exp \mathrm{i}\phi^n - \delta r_s^{nm}$, where the phase-shift ϕ^n is spin-independent. We expand (8) in terms of the small correction δr_s^{nm} and find that

$$\mathrm{Re} G_T^{\uparrow\downarrow} = (G_T^{\uparrow} + G_T^{\downarrow})/2\,,$$

where $G_T^{\uparrow}$ and $G_T^{\downarrow}$ are the spin-dependent tunneling conductances (44). Since the transmission coefficients in a tunnel contact are all exponentially small, the imaginary part of $G^{T\uparrow\downarrow}$ is of the same order of magnitude as $G^{T\uparrow}$ and $G^{T\downarrow}$ but it is not universal and depends on the details of the contact.

4 Illustrations of the theory

We will in this chapter illustrate the appeal of the circuit theory of spin-transport by computing the transport properties of two-terminal devices and a three terminal device. It is assumed that the normal metal node in these devices is smaller than the spin-diffusion length so that the spatial distribution function is homogeneous within the node.

4.1 Two terminals

First we consider a normal metal node attached to two ferromagnetic reservoirs with identical contacts, *e.g.* $G_1^{\uparrow} = G_2^{\uparrow} = G^{\uparrow}$, $G_1^{\downarrow} = G_2^{\downarrow} = G^{\downarrow}$ and $G_1^{\uparrow\downarrow} = G_2^{\uparrow\downarrow} = G^{\uparrow\downarrow}$ as shown in Figure 4. The relative angle between the magnetization in the two ferromagnetic reservoirs is θ. With the aid of (1) and (5) we find the current

$$I(\theta) = \frac{G}{2} V \left(1 - \frac{P^2}{1 + g_{\mathrm{sf}}} \frac{\tan^2 \theta/2}{\tan^2 \theta/2 + \alpha}\right)\,, \tag{45}$$

where

$$\alpha = \frac{|\eta|^2 + g_{\mathrm{sf}}(3\eta_R + 2g_{\mathrm{sf}})}{(\eta_R + 2g_{\mathrm{sf}})(1 + g_{\mathrm{sf}})}\,. \tag{46}$$

Here, we have introduced the total conductance of one contact $G = G^{\uparrow} + G^{\downarrow}$, the polarization $P = (G^{\uparrow} - G^{\downarrow})/(G^{\uparrow} + G^{\downarrow})$, the relative mixing conductance $\eta = 2G^{\uparrow\downarrow}/(G^{\uparrow} + G^{\downarrow})$ and the ratio of the 'spin-flip conductance' [5] $G_{\mathrm{sf}} = e^2/(2\delta\tau_{\mathrm{sf}})$ (δ is the energy level spacing) to the conductance of the whole device in the parallel configuration $g_{\mathrm{sf}} = G_{\mathrm{sf}}/(G/2)$). The current is an even function of θ. Note that $\alpha > 1$. When the magnetizations are parallel ($\theta = 0$), there is no spin-accumulation on the normal metal node and the current is given by Ohm's law $I_{\mathrm{P}} = I(\theta = 0) = GV/2$. The anti-parallel magnetization configuration ($\theta = \pi$) generates the largest spin-accumulation, reducing the particle current to $I_{\mathrm{AP}} = I(\theta = \pi) = G[1 - P^2/(1 + g_{\mathrm{sf}})]V/2$. In this case the magneto-resistance ratio $(I_{\mathrm{P}} - I_{\mathrm{AP}})/I_{\mathrm{P}}$ is $P^2/(1 + g_{\mathrm{sf}})$, irrespective of the relative mixing conductance η. Naturally, the spin-accumulation and consequently the magnetoresistance decreases with spin-flip relaxation time. Any spin flip reduces the effective polarization. The result for long spin-flip relaxation times ($g_{\mathrm{sf}} \ll 1$) was previously obtained for two tunnel junctions [5] and we have thus generalized it to arbitrary contacts. For general θ the current depends on α and thus on the mixing conductance. In Figure 5 we plot the current *vs.* the relative magnetization angle θ for a given effective polarization $P^2/(1 + g_{\mathrm{sf}})$ and a number of values of α ($\alpha = 1$, $\alpha = 10$ and $\alpha = 100$). In general, the current increases with increasing η. As one can see from (5) a large relative mixing conductance means that spins orthogonal to the magnetization in the reservoirs easily can escape from the normal metal node. This suppresses the spin accumulation. Therefore when $\eta \gg 1$ and θ is not close to π the current approaches Ohm's law $I = GV/2$. Except for the anti-parallel magnetizations,

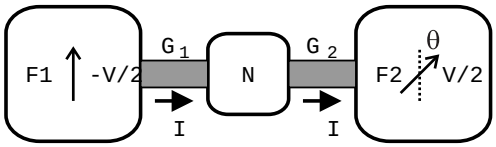

Fig. 4. The two-terminal device comprising a normal metal node attached to two ferromagnetic reservoirs (F1 and F2) with arbitrary relative magnetization direction θ. A source-drain bias V is applied between the ferromagnetic reservoirs and a current I flows between the two reservoirs. The contact between ferromagnetic F1 (F2) and the normal metal node is characterized by the conductances $G_1^{\uparrow}$, $G_1^{\downarrow}$, and $G_1^{\uparrow\downarrow}$ ($G_2^{\uparrow}$, $G_2^{\downarrow}$, and $G_2^{\uparrow\downarrow}$).

the angle dependence vanishes in that limit which explains the sharp dip at θ close to π.

Figure 4 illustrates a universal property of the two-terminal device with non-collinear magnetizations that is independent of the contact conductances and the spin-flip relaxation time. By scaling the total conductance modulation to the difference between the conductance in the parallel and anti-parallel configuration $(G/2)P^2/(1+g_{\rm sf})$ the current change for *any* two terminal device with a diffusive normal metal node should be above the universal curve determined by the minimum value of η, $|\eta| = 1$. Thus, according to our theory the current *vs.* magnetization angle relation for a spin valve must lie above the universal curve obtained for $|\eta| = 1$.

The result (45) has been derived for normal metals islands that can be described as Fermi liquids [20]. It was recently generalized to transport through a Luttinger liquid by Balents and Egger [21]. In a Luttinger liquid the current is non-linear in the applied source-drain bias voltage, and the spin-charge separation reduces the spin-accumulation [21]. Coulomb charging effects can also reduce the spin-accumulation in the linear response regime [5].

4.2 Three terminals

Let us now consider a set-up similar to the Johnson spin-transistor as shown in Figure 6 which was also discussed by Geux *et al.* [22] within the context of a multi-terminal Landauer-Büttiker formalism for collinear magnetization configurations only.

A small normal metal node is attached to two ferromagnetic reservoirs and one normal metal reservoir by three contacts. A voltage bias applied to the ferromagnetic reservoir F1 and the normal metal reservoir N causes a current between the same reservoirs passing through the normal metal node. The spin-accumulation on the normal metal node injected by F1 affects the chemical potential of ferromagnet F2, which is adjusted such that the charge current into F2 vanishes. We characterize the contact between the first (second) ferromagnet and the normal metal node by the total conductance $G_1 = G_1^{\uparrow} + G_1^{\downarrow}$ ($G_2 = G_2^{\uparrow} + G_2^{\downarrow}$), the polarization $P_1 = (G_1^{\uparrow} - G_1^{\downarrow})/G_1$ ($P_2 = (G_2^{\uparrow} - G_2^{\downarrow})/G_2$) and the relative mixing conductance

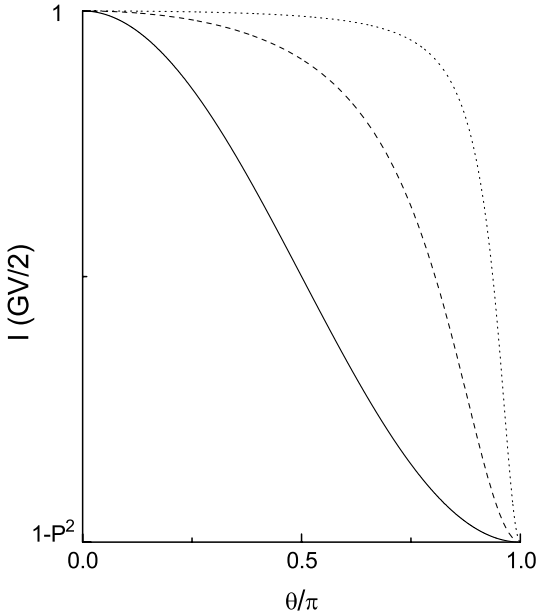

Fig. 5. The current through the two-terminal device as a function of the relative angle θ between the magnetization directions in F1 and F2. The current is normalized by the current in the parallel configuration $I^{\rm P} = GV/2$ and reaches the minimum $I = (1 - P^2/(1+g_{\rm sf}))GV/2 \equiv (1 - P_{\rm eff}^2)GV/2$ when the magnetizations are anti-parallel ($\theta = \pi$). The lowest solid line is for the minimum relative mixing conductance $|\eta = 1|$. This line forms the lowest possible universal curve for the conductance *vs.* relative angle θ, since all other values of the mixing conductance lies above this line.

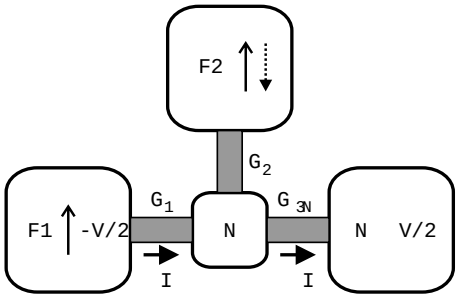

Fig. 6. The three terminal Johnson spin-transistor. A bias voltage is applied between a ferromagnet and a normal metal and a current flows between the same reservoirs through a normal metal node. The potential on another ferromagnet is measured when its magnetizations is parallel or anti-parallel to the first ferromagnet. The contact between the first (second) ferromagnet and the normal metal node is characterized by the total conductance G_1 (G_2), and the polarization P_1 (P_2). The conductance between the normal metal node and the normal metal reservoir is G_{3N}.

$\eta_2 = 2G_2^{\uparrow\downarrow}/(G_2^{\uparrow}+G_2^{\downarrow})$ ($\eta_1 = 2G_1^{\uparrow\downarrow}/(G_1^{\uparrow}+G_1^{\downarrow})$). The contact between the normal metal reservoir and the normal metal node is characterized by a single conductance parameter, G_{3N}. θ is the relative angle between the magnetization of ferromagnet F1 and ferromagnet F2. We assume that the typical rate of spin-injection into the node is faster than

the spin-flip relaxation rate, so that the right hand side of (1) can be set to zero.

The current through the normal metal node is invariant with respect to a flip in the magnetization direction of ferromagnet F1 or F2: $I(\theta) = I(\theta + \pi)$. However the chemical potential of ferromagnet F2 changes during the same process since it is sensitive to the magnitude and direction of the spin-accumulation on the normal metal node: $\mu_2(\theta) \neq \mu_2(\theta + \pi)$. A spin-'resistance' $R_s(\theta)$ can be defined as the ratio between the difference in the chemical potential of ferromagnetic F2 when ferromagnet F1 or ferromagnet F2 is flipped: $R_S = (V_2(\theta + \pi) - V_2(\theta))/I(\theta)$. In the collinear configuration ($\theta = 0$) the spin-resistance $R_s(\theta = 0)$ is thus the ratio between the difference in the chemical potential of ferromagnetic F2 when its magnetization is parallel ($\mu_2^{\rm P}$) and anti-parallel ($\mu_2^{\rm AP}$) to the magnetization of ferromagnet F1 and a current (I) passes from ferromagnet F1 to the normal metal reservoir N. With the aid of the general conductances, valid for arbitrary contacts, we solve for the non-equilibrium distribution function on the normal metal node (1) under the condition that no particle current enters ferromagnet F2. Using the solution for the non-equilibrium distribution function we find the current (5) through the system and subsequently the non-equilibrium chemical potential of ferromagnet F2.

Let us first discuss the results in the collinear configuration, $\theta = 0$ and π. The spin-resistance can be simply expressed as

$$R_S(\theta = 0) = \frac{2P_1P_2}{G_1(1 - P_1^2) + G_{3N} + G_2(1 - P_2^2)}, \quad (47)$$

and is independent of the relative mixing conductances η_1 and η_2 that are only relevant for the transport properties in systems with non-collinear magnetization configurations. The spin-resistance is proportional to the product of the polarizations of the contacts to ferromagnet F1 and ferromagnet F2. In order to measure a large effect of the spin-accumulation, *e.g.* a large spin-resistance, highly resistive contacts should be used. On the other hand, the resistance has to be small enough so that the transport dwell time is shorter than the spin-flip relaxation. The simple result (47) covers a large class of experiments, since we have not specified any details about the contacts between the reservoirs and the normal metal node. It is noted, though, that equation (47) is only valid for a normal metal node that is smaller than the spin-diffusion length and can therefore not be applied directly to Johnson's experiment [7].

We can understand that the present results are quite different from those of reference [22] as follows. The general formulation in terms of transmission probabilities of Geux *et al.* is exact. However, in order to include the effects of spin-relaxation the transmission probabilities were treated as pair-wise resistors between the reservoirs. This corresponds to an equivalent circuit in which resistors connect the three reservoirs in a "ring" topology. The present model, on the other hand, can be described by a "star" configuration circuit, in which all resistors point from the reservoirs to a single node. The present model is more accurate when the contacts dominate the transport properties, whereas Geux's model is preferable when the resistance of the normal metal island is important. Effectively, Johnson's thin film device appears to be closer to the star configuration.

Let us now proceed to discuss the results when the magnetization directions are non-collinear. The analytical expression for the spin-resistance is much simpler when the two contacts F1-N and F2-N are identical, $G_1 = G_2 \equiv G$, $P_1 = P_2 \equiv P$ and $\eta_1 = \eta_2 \equiv \eta$. Furthermore we disregard the imaginary part of the mixing conductance which is very small or zero in the model calculations of tunnel, ballistic and diffusive contacts presented in this paper as well as in recent first-principle band-structure calculations [23]. The spin-resistance then has the simple form

$$R_S = \frac{2(G_{3N} + 2G\eta)P^2\cos(\theta)}{(G_{3N} + G(1 + \eta - P^2))^2 - G^2\cos^2(\theta)(1 - \eta - P^2)^2}. \quad (48)$$

The spin-resistance is an even function of the relative angle between the magnetization directions θ and we recover the result (47) when $\theta = 0$. The spin-resistance vanishes when the magnetizations are perpendicular $\theta = \pi/2$ as expected from the symmetry of the systems. The angular dependence is approximately proportional to $\cos(\theta)$ when the relative mixing conductance is not too large, $\eta \approx 1$. For larger mixing conductances $\eta \gg 1$ the spin-accumulation on the normal metal island is strongly suppressed in the perpendicular configuration $\theta = \pi/2$ due to the large transport rates for spins between the normal metal node and ferromagnet F2. Consequently the spin-resistance is small and only weakly dependent on the relative angle around $\theta = \pi/2$.

Another novel three-terminal device comprising of three ferromagnetic reservoirs (the "spin-flip transistor"), which utilizes the added functionality provided by non-collinear magnetization directions, was introduced in reference [15].

5 Conclusion

We developed a mesoscopic circuit theory of spin-transport in multi-terminal hybrid ferromagnet-normal metal systems starting from microscopic principles. Based on conservation of spin-current on each node the circuit theory is parameterized by the conductances of the contacts, *viz.* two spin-dependent conductances and a (complex) mixing conductance. The latter is a novel concept relevant for transport in systems with non-collinear magnetization configurations. Explicit expressions for the conductances for diffusive, ballistic and tunnel contacts have been derived. The circuit theory leads to simple and quite general results for the conductance of two-terminal and three-terminal devices, like Johnson's spin-transistor. For two-terminal systems a universal lower limit for the current modulation as a function of the relative magnetization has been found.

After completion of this work a complementary approach to spin-transport in ferromagnetic-normal metal systems starting from the scattering matrices was presented in reference [24]. Using random matrices to describe the scattering within disordered normal metal nodes equivalent results to our (2) were obtained.

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Appendix A: Transformation from a non-collinear to a collinear configuration

We consider the transmission and reflection matrices between a normal metal and a ferromagnet. The Schrödinger equation is

$$\hat{H}(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}),$$

where $\psi(\mathbf{r})$ is a two-component spinor. The Hamiltonian is

$$\hat{H}(\mathbf{r}) = \hat{U}\left[-\frac{1}{2m}\nabla^2\hat{1} + V_s(\mathbf{r})\sigma_z + \hat{V}_{\rm c}(\mathbf{r})\right]\hat{U}\,. \qquad (49)$$

We consider transport between a normal metal and a uniform ferromagnet, so that the magnetization direction $\mathbf{m}$ is a spatially independent unit vector. In (49) $V_s(\mathbf{r})$ denotes the spin-dependent potential and $\hat{V}_{\rm c}(\mathbf{r})$ is the scattering potential of the contact. The direction of the magnetization is represented by the angles θ and φ as $\mathbf{m} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$. The Hermitian and unitary matrix $\hat{U}$ that diagonalizes the spin-dependent potential is

$$\hat{U} = \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2)\mathrm{e}^{-\mathrm{i}\varphi} \\ \sin(\theta/2)\mathrm{e}^{\mathrm{i}\varphi} & -\cos(\theta/2) \end{pmatrix}.$$

The spin-dependent potential vanishes in the normal metal, $V_s(\mathbf{r}) = 0$ for $x < x_l$ and attains a constant value in the ferromagnet $V_s(\mathbf{r}) = V_s$ for $x > x_r$. The contact is represented by the scattering potential

$$\hat{V}_{\rm c}(\mathbf{r}) = \begin{pmatrix} V_\uparrow(\mathbf{r}) & V_{\rm sf}(\mathbf{r}) \\ V_{\rm sf}^\dagger(\mathbf{r}) & V_\downarrow(\mathbf{r}) \end{pmatrix} \qquad (50)$$

and attains the bulk values within the normal metal and the ferromagnet for $x < x_l$ and $x > x_r$,respectively. The off-diagonal terms in (50) represent the exchange potentials due to a non-collinear magnetization in the contact, spin-orbit interaction or spin-flip scatterers. The Hamiltonian (49) can be diagonalized in spin-space by

$$\psi(\mathbf{r}) = \hat{U}\phi(\mathbf{r})\,. \qquad (51)$$

The Schrödinger equation for the spinor $\phi(\mathbf{r})$ is

$$\left[-\frac{1}{2m}\nabla^2\hat{1} + V_s(\mathbf{r})\sigma_z + \hat{V}_{\rm c}(\mathbf{r}) - E\right]\phi(\mathbf{r}) = 0\,.$$

Let us now consider an incoming wave from the normal metal in the transverse mode n and with spin s collinear to the magnetization in the ferromagnet. The wave function in the normal metal is

$$\phi_s^n(\mathbf{r}) = \sum_{ms'} \frac{\chi_N^m(\rho)}{\sqrt{k^m}} \times [\delta_{s's}\delta^{mn}\xi_s\mathrm{e}^{\mathrm{i}k^n x} + r_{{\rm c},s's}^{mn}\xi_{s'}\mathrm{e}^{-\mathrm{i}k^m x}], \qquad (52)$$

where $\xi_\uparrow^\dagger = (1,0)$ and $\xi_\downarrow^\dagger = (0,1)$ are the spinors, $\chi_N^m(\rho)$ is the transverse wave function, k^m is the longitudinal wave vector for mode m and $r_{{\rm c},s's}^{mn}$ is the reflection matrix from state ns to state ms'. We would like to transform the result for the reflection matrix into a basis with arbitrary spin quantization axis. To this end we introduce the incoming spinor wave function

$$\psi_s^n(\mathbf{r}) = \sum_{ms'} \frac{\chi_N^m(\boldsymbol{\rho})}{\sqrt{k^m}} \times \left[\delta_{s's}\delta^{mn}\xi_s\mathrm{e}^{\mathrm{i}k^n x} + r_{s's}^{mn}\xi_{s'}\mathrm{e}^{-\mathrm{i}k^m x}\right]. \qquad (53)$$

Using the transformation (51) we can also write the wave function spinor in terms of the basis states $\phi(\mathbf{r})$ as

$$\psi_s^n(\mathbf{r}) = \hat{U}\sum_\sigma \phi_\sigma^n(\mathbf{r})a_{\sigma s}, \qquad (54)$$

where $a_{\sigma s}$ are expansion coefficients to be determined by equating (53) and (54). We thus find that

$$a_{\sigma s} = \xi_\sigma^\dagger U\xi_s = U_{\sigma s}$$

and

$$r_{s's}^{mn} = \sum_{\sigma'\sigma} U_{s'\sigma'}r_{{\rm c},\sigma'\sigma}^{mn}U_{\sigma s}. \qquad (55)$$

Disregarding spin-flip processes in the contact the transformation of the reflection matrix can be written as

$$\hat{r}^{nm} = \hat{u}_\uparrow r_{{\rm c},\uparrow\uparrow}^{mn} + \hat{u}_\downarrow r_{{\rm c},\downarrow\downarrow}^{mn},$$

where the spin-projection matrices $\hat{u}_\uparrow$ and $\hat{u}_\downarrow$ are defined in (3) and (4). Spin-flip processes can also be included by using the general transformation (55), but the reflection matrix $\hat{r}^{nm}$ can then not be expressed in terms of the spin-projection matrices only.

We can perform a similar calculation in order to find the transformation of the transmission coefficients from the ferromagnet into the normal metal. In the basis where

the spin-quantization axis is collinear with the magnetization the incoming wave from the ferromagnet is

$$\phi_s^n(\mathbf{r}) = \sum_{ms'} \frac{\chi_{\mathrm{F}s}^m(\boldsymbol{\rho})}{\sqrt{k_s^m}} \times \left[\delta_{s's}\delta^{mn}\xi_s \mathrm{e}^{\mathrm{i}k_s^n x} + r_{\mathrm{cF},s's}^{mn}\xi_{s'}\mathrm{e}^{-\mathrm{i}k_s^m x}\right], \quad (56)$$

where $\chi_m^{Fs}(\boldsymbol{\rho})$ is the (spin-dependent) transverse wave function and k_s^m is the spin-dependent Fermi wave-vector. The outgoing wave into the normal metal is

$$\phi_s^n(\mathbf{r}) = \sum_{ms'} \frac{\chi_N^m(\boldsymbol{\rho})}{\sqrt{k^m}} t_{\mathrm{cF},s's}^{m,n}\xi_{s'} \exp(\mathrm{i}k_s^m x).$$

By transforming the outgoing wave into an arbitrary magnetization direction according to (51), we see that the transmission coefficient from a state with spin s collinear to the magnetization direction in the ferromagnet to a state with spin s' collinear to the spin-quantization axis along the z-direction is

$$\hat{t}'^{nm} = \hat{U}\hat{t}_{\mathrm{cF}}^{nm}.$$

In the absence of spin-flip scattering in the contact, we have

$$t_{ss'}'^{nm} = U_{ss'}t_{\mathrm{cF},s'}^{nm}.$$

The current in the normal metal is (for a given energy shell) (note that we associate the first index with Ψ and the second index with $\Psi^\dagger$)

$$\frac{h}{e}\hat{I} = M\hat{f}^N - \sum_{nm}\left[\hat{r}^{mn}\hat{f}^N\left(\hat{r}^{nm}\right)^\dagger - \hat{t}'^{mn}\hat{f}^F\left(\hat{t}'^{nm}\right)^\dagger\right]. \quad (57)$$

The contribution from the transmission probability to the spin-current is therefore

$$eI_{\alpha\delta}^F = \frac{e^2}{h}\sum_{nm\beta} t_{\alpha\beta}'^{mn} f_\beta^F \left(t_{\beta\delta}'^{nm}\right)^\dagger = \frac{e^2}{h}\sum_{nm\beta} U_{\alpha\beta} f_\beta^F \left|t_\beta'^{mn}\right|^2 U_{\beta\delta}.$$

Whereas the contribution from the transmission probability becomes

$$e\hat{I}^F = G_\uparrow \hat{u}_\uparrow f_\uparrow^F + G_\downarrow \hat{u}_\downarrow f_\downarrow^F,$$

where the spin-dependent conductance is

$$G_s = \frac{e^2}{h}\sum_{nm}\left|t_s'^{mn}\right|^2.$$

Similarly, the contribution from the normal metal is

$$e\hat{I} = M\hat{f}^N - \sum_{ss'}\hat{u}^s\hat{f}^N\hat{u}^{s'}r_s^{mn}\left(r_{s'}^{mn}\right) = -G_\uparrow\hat{u}_\uparrow\hat{f}^N\hat{u}_\uparrow - G_\downarrow\hat{u}_\downarrow\hat{f}^N\hat{u}_\downarrow - G_{\uparrow\downarrow}\hat{u}_\uparrow\hat{f}^N\hat{u}_\downarrow - G_{\uparrow\downarrow}^*\hat{u}_\downarrow\hat{f}^N\hat{u}_\uparrow,$$

where we have used the unitarity of the scattering matrix so that $M - \sum_{nm}|r_s^{nm}|^2 = \sum_{nm}|t_s'^{mn}|^2$ and the mixing conductance is introduced as

$$G_{\uparrow\downarrow} = \frac{e^2}{h}\left[M - \sum_{nm}\left(r_\uparrow^{mn}\right)^* r_\downarrow^{nm}\right].$$

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