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Strategic manipulability without resoluteness or shared beliefs: Gibbard-Satterthwaite generalized

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Abstract. The Gibbard-Satterthwaite Theorem on the manipulability of social-choice rules assumes *resoluteness:* there are no ties, no multi-member choice sets. Generalizations based on a familiar lottery idea allow ties but assume perfectly shared probabilistic beliefs about their resolution. We prove a more straightforward generalization that assumes almost no limit on ties or beliefs about them.

Introduction

The social-choice procedures found in practice all allow strategic manipulation, the profitable misrepresentation of someone's preference, as when a voter pivotal under plurality rule votes for his second-favorite of three candidates because his favorite would not win anyway. The Gibbard-Satterthwaite Theorem (GS) is often said to show that manipulability is inescapable.[1] It does not. Take a social choice function that picks one alternative $C(P_1, \ldots, P_n)$ from a fixed set A of three or more, given any orderings P_i of A by n individuals i. Assuming that C can pick any $x \in A$ (Citizens' Sovereignty) and thwart any i (Nondictatorship), GS says C must be manipulable. Remarkable for the sweep of those two assumptions, GS also assumes *resoluteness:* unlike ordinary voting rules, C always picks single members of A, not subsets, leaving no room for ties. Ties may be rare, but GS assumes they are

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[1] Independently proved by Gibbard (1973) and Satterthwaite (1975).

impossible, not rare, leaving open the possibility that some reasonable procedures escape manipulability by allowing rare ties. True, resolute C might combine a nonresolute C' with a tie-break rule: ties do get resolved. Often, however, we cannot predict how. That dulls the bite of GS by opening the possibility that those who can manipulate do not know they can, even if they know all preferences. Often, moreover, the tie-break rule is normatively arbitrary or otherwise uninteresting. Then the procedure we care about is not C but C', of which GS says nothing.

So let C pick subsets of A, not necessarily singletons. Then C is manipulable if two conditions are met by some $P_1, \ldots, P_n$, i, and sets X and Y: (1) A solo change in P_i changes $C(P_1, \ldots, P_n)$ from X to Y. (2) It is possible for i, with true preference ordering P_i, to profit from this change. What does (2) mean? Since P_i orders alternatives, not sets of them, we cannot have YP_iX. But since manipulability requires only the *possibility* of profitable misrepresentation, it is enough that a Y-to-X preference be "compatible" with P_i. What does *that* mean?

Some answers have spawned manipulability theorems that allow ties but otherwise assume much more than GS.[2] Zeckhauser (1973), Gibbard (1977), and Feldman (1980) base GS-like theorems on another answer: socially chosen along with X is a single X-*lottery* (a lottery with support X) and likewise Y, and some utility representative of P_i gives the latter lottery a greater expected utility than the former. Then, however, i might manipulate though $X = Y$: he might change the lottery but not the set. Then, moreover, all individuals must see the same X- and Y-lotteries, sharing beliefs about tie resolution. That is plausible if ties are resolved by chance, but often they are not. For example, in 1824 the U.S. House of Representatives resolved a presidential-election tie, and the outcome surprised some voters more than others. If, as in that case, the C of interest is part of a longer procedure whose later steps resolve its ties, participants may well have diverse beliefs about later outcomes.

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[2] Gärdenfors (1976), Kelly (1977), and Barberá (1977a,b) construe (2) to mean that a Y-to-X preference is *compelled* by P_i – as, e.g., "xP_iy" compels "i prefers $\{x\}$ to $\{x, y\}$ and $\{x, y\}$ to $\{y\}$." But Gärdenfors assumes "democratic" conditions (anonymity, neutrality, Condorcet); Barberá (a), strict monotonicity; Barberá (b), acyclicity of strict social preference; and Kelly, transitivity. Also Barberá (b) and Kelly let the feasible set vary: manipulability is not proved for an arbitrary but fixed set. Feldman (1979) and MacIntyre and Pattanaik (1981) offer results in a similar vein.

Pattanaik (1978) takes (2) to mean that yP_ix when y is the P_i-worst member of Y and x that of X. This is less demanding than it looks, as in effect our $\mathbb{M}$-Lemma (1) below shows. But Pattanaik uses strong democratic conditions and lets the feasible set vary.

Barberá, Sonnenschein, and Zhou (1991) let individuals order *sets* of alternatives, restricted by a "separability" condition but still including, e.g., $\{x\}P_i\{y\}P_i\{x, y\}$. Schwartz (1982) restricts preferences between sets to avoid such anomalies. But he uses a host of opaque preferential axioms, one found questionable by Martin van Hees (personal communication), and he too lets the feasible set vary.

Manipulability is little affected by ties or beliefs about them. For we can generalize GS by allowing almost unlimited ties and, when (1) holds, counting C as manipulable only if a Y-to-X preference would be P_i-compatible *whatever i*'s relevant beliefs happened to be: for *every* X-lottery and *every* Y-lottery, some utility representative of P_i gives the latter lottery a greater expected utility than the former – now impossible if $X = Y$. Beyond that we require linear P_i (no two alternatives at the same level), countable $C(P_1, \dots, P_n)$, and this residuum of resoluteness: there is no tie if everyone professes the same ordering with (say) x first, y second, and there is still no tie if one i moves y above x.

2 Theorem

Formally, the theorem is about an integer n, set A, and function C. Denote $1, 2, \dots, n$ by i, j, elements of A by x, y, z, nonempty countable subsets of A by X, Y. A *profile* is an ordered n-tuple of linear orderings of A (asymmetric, transitive, connected in A), denoted $\mathbf{P} = (P_1, \dots, P_n)$, $\mathbf{P}' = (P'_1, \dots, P'_n)$, etc. An *i-variant* of $\mathbf{P}$ is any $\mathbf{P}'$ with $P'_j = P_j$ for all $j \neq i$. An *X-lottery* is any $\lambda : X \to (0, 1]$ with $\sum_{x \in X} \lambda(x) = 1$. A *representative of* P_i *in* X is any $u : X \to \Re$ with $u(x) > u(y) \Leftrightarrow xP_iy$ for all $x, y \in X$.

Theorem. *Assuming that* $|A| \geq 3$ *and that* C *turns every* $\mathbf{P}$ *into a nonempty countable* $C(\mathbf{P}) \subseteq A$, *four conditions are inconsistent:*

- $\not\mathbf{M}$ *For no* $\mathbf{P}$, *i, and i-variant* $\mathbf{P}'$ *of* $\mathbf{P}$ *is this true: for every* $C(\mathbf{P})$*-lottery* λ *and every* $C(\mathbf{P}')$*-lottery* λ', *some representative u of* P_i *in* $C(\mathbf{P}) \cup C(\mathbf{P}')$ *has* $\sum_{x \in C(\mathbf{P}')} \lambda'(x)u(x) > \sum_{x \in C(\mathbf{P})} \lambda(x)u(x)$ *(Nonmanipulability).*
- **CS** *For all x, some* $\mathbf{P}$ *has* $x \in C(\mathbf{P})$ *(Citizens' Sovereignty).*
- $\not\mathbf{D}$ *No i is such that, for all x and* $\mathbf{P}$, $\{x\} = C(\mathbf{P})$ *if x is atop* P_i *(Nondictatorship).*
- **RR** *If all* $P_{j \neq i}$ *are the same, with x first and y second, and if* P_i *is either the same as them or else the same but with y first and x second, then* $C(\mathbf{P})$ *is a singleton (Residual Resoluteness).*

Every condition of nonmanipulability rests on a test of manipulability. The stronger the test, the weaker the condition – and the stronger any theorem like ours. $\not\mathbf{M}$'s test follows the colon and begins with three quantifiers: $\forall\lambda\forall\lambda'\exists u$. The nonmanipulability condition of Zeckhauser (1973), Gibbard (1977), and Feldman (1980) weakens that test, strengthening $\not\mathbf{M}$, by weakening the universal quantification to an instance of it: for us manipulability requires that a certain relation hold for *every* pair of lotteries with supports $C(\mathbf{P})$ and $C(\mathbf{P}')$, but for them the relation need hold only for *one* pair, socially chosen along with $C(\mathbf{P})$ and $C(\mathbf{P}')$. An even stronger nonmanipulability condition weakens the "all" of our test to "some," $\forall\lambda\forall\lambda'\exists u$ to $\exists\lambda\exists\lambda'\exists u$. But that condition is preposterously strong: it counts C as manipulable so long as $C(\mathbf{P}) = C(\mathbf{P}')$

and $x \neq y$ (whence xP_iy or yP_ix) for some x and y therein.[3] Instead of weakening $\forall\lambda\forall\lambda'\exists u$, one might strengthen it to $\exists u\forall\lambda\forall\lambda'$ or even $\forall u\forall\lambda\forall\lambda'$. But that makes $\not\mathbf{M}$ so weak that our conditions are all satisfied when $n = |A| = 3$ and C picks just the Condorcet winner when there is one but otherwise all of A.[4]

Drop resoluteness and you allow two versions each of Citizens' Sovereignty and Nondictatorship, ours and these:

CS+ For all x, some $\mathbf{P}$ has $\{x\} = C(\mathbf{P})$.
$\not\mathbf{D}$+ No i is such that, for all x and $\mathbf{P}$, $x \in C(\mathbf{P})$ if x is atop P_i.

Obviously **CS** and $\not\mathbf{D}$ are weaker.

RR is quite restrictive if $n = 2$, but otherwise we can imagine no objection. Zeckhauser (1973), Gibbard (1977), and Feldman (1980) do not assume even that much resoluteness, but Zeckhauser adds *ex ante* Pareto optimality, Gibbard bans random dictators and Feldman dual dictators, and all three assume commonly perceived lotteries – uniform ones in Feldman's case. We can avoid those extra assumptions and still drop **RR** by strengthening **CS** and $\not\mathbf{D}$ to **CS+** and $\not\mathbf{D}$+.[5] We cannot strengthen **CS** alone because, for finite A, our remaining conditions would then be met by $C(\mathbf{P}) \equiv \{x \mid x \text{ is atop } P_1 \text{ or } P_2\}$, nor $\not\mathbf{D}$ alone because, for finite A and subset B such that $|B| \geq 2$ and $|A \backslash B| \geq 2$, our remaining conditions would then be met by $C(\mathbf{P}) \equiv \{x \mid x \text{ is atop } P_1 \text{ in } B \text{ or atop } P_2 \text{ in } A \backslash B\}$. *A fortiori* we cannot simply drop **RR**. However, **CS+** obviously incorporates a degree of resoluteness, and so, more subtly, does $\not\mathbf{D}$+: it bans those exceedingly *ir*resolute procedures that always pick every alternative ranked first by anyone. We gladly add a separate bit of resoluteness as the price of keeping our other conditions as few, as weak, and as free of implicit resoluteness as possible.

3 Proof

Gibbard and Satterthwaite prove their theorem by defining a "social preference" function that must meet Arrow's (1963) inconsistent conditions if C

[3] In effect, Ching and Zhou (1998) also have $\exists\lambda\exists\lambda'\exists u$, but they ingeniously avoid the anomalous consequence by following $\exists\lambda\exists\lambda'$ with an added constraint on λ and λ': those lotteries come from some one lottery on A, λ by conditioning on $C(\mathbf{P})$, λ' by conditioning on $C(\mathbf{P}')$. That is weaker than the Zeckhauser-Gibbard-Feldman (ZGF) test when $C(\mathbf{P})$ and $C(\mathbf{P}')$ are disjoint because the constraint is then vacuous, stronger when they are identical because, unlike the ZGF test, it bans manipulability in that case. So the Ching-Zhou test neither implies nor follows from ZGF's. Obviously it is much weaker than ours, making their nonmanipulability condition much stronger: for us manipulability requires that a certain relation hold for *every* pair of lotteries, but for them the relation need hold only for *one* pair, suitably constrained. (That allows them to drop **RR** in their theorem.)

[4] We thank an anonymous referee for that example.

[5] We used **RR** only to prove Topset and $\not\mathbf{B}$, of Section 2 below, where **CS+** and $\not\mathbf{D}$+ would suffice. The inconsistency of $\not\mathbf{M}$ with these stronger conditions is proved in an earlier version of this paper, Duggan and Schwartz (1993).

meets theirs. Resoluteness helps by erasing "social indifference," social-preference gaps. We likewise exploit a variant of Arrow; it uses transitivity of social preference but not of indifference, now unerasable.

Call X a *top set* in $\mathbf{P}$ if xP_iy for all i, all $x \in X$, and all $y \notin X$. Call $\mathbf{P}'$ an xy-twin of $\mathbf{P}$ if $xP'_iy \Leftrightarrow xP_iy$ for all i. Define function F (our "social preference" function) from all profiles $\mathbf{P}$ to relations $F(\mathbf{P}) \subseteq A^2$:

$xF(\mathbf{P})y$ iff $x \neq y$ and $\{x\} = C(\mathbf{P}')$ for every
xy-twin $\mathbf{P}'$ of $\mathbf{P}$ with top set $\{x, y\}$.

It follows that F turns every $\mathbf{P}$ into an asymmetric $F(\mathbf{P}) \subseteq A^2$ (Asymmetry, or $\not{\mathbf{S}}$), and $xF(\mathbf{P})y \Rightarrow xF(\mathbf{P}')y$ whenever $\mathbf{P}'$ is an xy-twin of $\mathbf{P}$ (Independence of Irrelevant Alternatives, or **IIA**).

To the assumptions of the theorem, add $\not{\mathbf{M}}$, **CS**, $\not{\mathbf{D}}$, and **RR**. We shall deduce a contradiction by way of six consequences. The first alone makes use of $\not{\mathbf{M}}$.

$\not{\mathbf{M}}$-Lemma. *If* $\mathbf{P}'$ *is an i-variant of* $\mathbf{P}$ *and* $x \in C(\mathbf{P}')$, *then*

(1) *x or some P'_i-worse y belongs to $C(\mathbf{P})$, and*
(2) *x or some P_i-better y belongs to $C(\mathbf{P})$.*

Proof. Let λ be any $C(\mathbf{P})$-lottery and λ' any $C(\mathbf{P}')$-lottery. If (1) is false then zP'_ix for all $z \in C(\mathbf{P})$, and some representative u of P'_i in $C(\mathbf{P}) \cup C(\mathbf{P}')$ must make $u(x)$ low enough that $\sum_{z \in C(\mathbf{P})} \lambda(z)u(z) > \sum_{z \in C(\mathbf{P}')} \lambda'(z)u(z)$, contrary to $\not{\mathbf{M}}$. Or if (2) is false then xP_iz for all $z \in C(\mathbf{P})$, and some representative u of P_i in $C(\mathbf{P}) \cup C(\mathbf{P}')$ must make $u(x)$ great enough that $\sum_{z \in C(\mathbf{P}')} \lambda'(z)u(z) > \sum_{z \in C(\mathbf{P})} \lambda(z)u(z)$, again contrary to $\not{\mathbf{M}}$.

Topset. *If X is a top set in* $\mathbf{P}$ *then* $C(\mathbf{P}) \subseteq X$.

Proof. Suppose not; say $y \in C(\mathbf{P})\backslash X$. Take any $x \in X$ and construct $\mathbf{P}^x$ so all P_i^x are the same with x first and something second. By **CS**, $x \in C(\mathbf{P}^*)$ for some $\mathbf{P}^*$. Change $\mathbf{P}^*$ to $\mathbf{P}^x$ one i at a time. Since x is atop every P_i^x, each P_i^*-to-P_i^x change keeps x in the choice set (value of C) by $\not{\mathbf{M}}$-Lemma (2). So $x \in C(\mathbf{P}^x)$, whence $\{x\} = C(\mathbf{P}^x)$ by **RR**. Now starting from $\mathbf{P}$, a P_1-to-P_1^x change keeps y in the choice set or includes some P_1-worse y', by $\not{\mathbf{M}}$-Lemma (1). Either way, since X is a top set in $\mathbf{P}$, the new choice set contains some $z \notin X$. Repeating this argument $n - 1$ times, we have $z \in C(\mathbf{P}^x) = \{x\}$ for some $z \notin X$, impossible since $x \in X$.

Dominance. *If* $C(\mathbf{P}) = \{x\}$ *and* $x \neq y$ *then* $xF(\mathbf{P})y$.

Proof. Suppose not: $C(\mathbf{P}') \neq \{x\}$ for some xy-twin $\mathbf{P}'$ of $\mathbf{P}$ with top set $\{x, y\}$. Change $\mathbf{P}'$ to $\mathbf{P}$ one i at a time. Some ith change must change the choice set from some $Y \neq \{x\}$ to $\{x\}$. Say $x \neq z \in Y$. By $\not{\mathbf{M}}$-Lemma (1), z or some P'_i-worse z' must belong to $\{x\}$, so $x = z'$ and zP'_ix. Since $\{x, y\}$ is a top set in $\mathbf{P}'$, $z = yP'_ix$ by Topset. But by $\not{\mathbf{M}}$-Lemma (2), y or some P_i-better y' belongs to $\{x\} \neq \{y\}$, so $y' = xP_iy$, impossible since yP'_ix and $\mathbf{P}'$ is an xy-twin of $\mathbf{P}$.

3-Undomination: *If* $x \in C(\mathbf{P})$ *and* $\{x, y, z\}$ *is a 3-member top set in* $\mathbf{P}$, *then not* $yF(\mathbf{P})x$.

Proof. Change $\mathbf{P}$ to $\mathbf{P}^{xy}$ by moving z just below x and y in every ordering, leaving all else the same. For every i, if $xP_i y$ then x is atop P_i^{xy}, so the P_i-to-P_i^{xy} change keeps x in the choice set by $\not\mathbf{M}$-Lemma (2). And if $yP_i x$ then $\not\mathbf{M}$-Lemma (1) implies that x or some P_i-worse w belongs to the choice set. But since $\{x, y, z\}$ remains a top set, such a w must be z by Topset, so $P_i^{xy} = P_i$ and x again remains in the choice set. Hence, $x \in C(\mathbf{P}^{xy}) \neq \{y\}$. But $\mathbf{P}^{xy}$ is an xy-twin of $\mathbf{P}$ with top set $\{x, y\}$, so not $yF(\mathbf{P})x$.

Unanimity (U): *If* $xP_i y$ *for all* i *then* $xF(\mathbf{P})y$.

Proof. If $\mathbf{P}'$ is any xy-twin of $\mathbf{P}$ with top set $\{x, y\}$ then $\{x\}$ too is a top set in $\mathbf{P}'$, so $C(\mathbf{P}') = \{x\}$ by Topset; i.e., $xF(\mathbf{P})y$.

Nonblocker ($\not\mathbf{B}$): *No* i *has this property: for all* x, y, $\mathbf{P}$, *if* $xP_i y$ *but* $yP_j x$ *for all* $j \neq i$ *then not* $yF(\mathbf{P})x$.

Proof. Take any i. By $\not D$, for some $\mathbf{P}$, x, and y, x is atop P_i but $x \neq y \in C(\mathbf{P})$. Make $\mathbf{P}'$ so P_i' has x first and y second and every $P_{j \neq i}'$ is the same but with y first and x second. Each $P_{j \neq i}$-to-$P_{j \neq i}'$ change keeps y in the choice set by $\not\mathbf{M}$-Lemma (2). And the P_i-to-P_i' change then puts y or some P_i-worse $z \neq x$ in the new choice set by $\not\mathbf{M}$-Lemma (1). Since $C(\mathbf{P}') \subseteq \{x, y\}$ by Topset, $y \in C(\mathbf{P}')$. So $C(\mathbf{P}') = \{y\}$ by **RR**, so $yF(\mathbf{P}')x$ by Dominance: i lacks the property.

Transitivity (T): *If* $xF(\mathbf{P})yF(\mathbf{P})z$ *then* $xF(\mathbf{P})z$.

Proof. Let $\mathbf{P}'$ be an xy-, yz-, and xz-twin of $\mathbf{P}$ with top set $\{x, y, z\}$. Then $C(\mathbf{P}') \subseteq \{x, y, z\}$ by Topset, and $xF(\mathbf{P}')yF(\mathbf{P}')z$ by IIA. So $C(\mathbf{P}') = \{x\}$ by 3-Undomination, whence $xF(\mathbf{P}')z$ by Dominance, and thus $xF(\mathbf{P})z$ by IIA.

Contradiction: Elsewhere it has been proved that $|A| \geq 3$, $\not\mathbf{S}$ **IIA**, **U**, $\not\mathbf{B}$, and **T** are inconsistent.[6]

4 Relaxations

We allowed diverse beliefs, but only up to a point: given $\mathbf{P}$, different individuals might see different lotteries, but $C(\mathbf{P})$ is the support set for all of them. However, a close reading of the proof of $\not\mathbf{M}$-Lemma shows we could have relaxed $\not\mathbf{M}$ by allowing different individuals to see different support sets – to give positive probabilities to different alternatives – within limits. Letting λ and λ' denote arbitrary lotteries with support $X, X' \subseteq A$, $\not\mathbf{M}$ amounts to the following:

[6] Mas-Colell and Sonnenschein (1972), Fishburn (1973 : 128), Schwartz (1986 : 59).

For no $\mathbf{P}$, i, and i-variant $\mathbf{P}'$ of $\mathbf{P}$ is this true: for every λ and every λ' with $(X, X') = (C(\mathbf{P}), C(\mathbf{P}'))$, some representative u of P_i in $X \cup X'$ has $\sum_{x \in X'} \lambda'(x)u(x) > \sum_{x \in X} \lambda(x)u(x)$.

To relax $\not\mathbf{M}$ as a whole, we relax "$(X, X') = (C(\mathbf{P}), C(\mathbf{P}'))$" to this: if either some element of $C(\mathbf{P})$ is P_i-worse than every element of $C(\mathbf{P}')$ or some element of $C(\mathbf{P}')$ is P_i-better than every element of $C(\mathbf{P})$, then either some element of X is P_i-worse than every element of X' or some element of X' is P_i-better than every element of X. Far from assuming $(X, X') = (C(\mathbf{P}), C(\mathbf{P}'))$, we no longer assume any connection at all between subjective support sets X and X' and objective choice sets $C(\mathbf{P})$ and $C(\mathbf{P}')$ except in this special case: $C(\mathbf{P}')$ is either maximin or maximax better than $C(\mathbf{P})$ according to P_i. Even then not much of a connection is assumed, merely that X' is better than X in one of those two ways – not necessarily the same way.

This relaxation lets us drop another assumption. We left the cardinality of A unrestricted but assumed that choice sets are countable. Drop that assumption and $\not\mathbf{M}$-Lemma is blocked: any lottery over uncountable $C(\mathbf{P}')$ gives probability zero to some members, possibly x. But all we really need is that *support sets* be countable. If choice sets do not have to be support sets, they do not have to be countable.

We can also drop countability by dropping $\not\mathbf{M}$ and *assuming* $\not\mathbf{M}$-Lemma. This nonmanipulability condition captures the idea that a Y-to-X preference is compatible with P_i at least when an extreme optimist or pessimist with preference P_i would prefer Y to X – when some $y \in Y$ is P_i-better than all $x \in X$ or some $x \in X$ is P_i-worse than all $y \in Y$.[7] Our proof of $\not\mathbf{M}$-Lemma gave an expected-utility rationale for that condition, but it could stand alone just as well. In fact, under the assumption of countable choice sets used to prove $\not\mathbf{M}$-Lemma, the two nonmanipulability conditions are equivalent.[8]

Another relaxation of $\not\mathbf{M}$ worth investigating allows manipulations that *contract* $C(\mathbf{P})$ while still forbidding all others. The idea is that if $C(\mathbf{P})$ comprises welfare optima when $P_1, \ldots, P_n$ are true preferences then any contraction of $C(\mathbf{P})$ still ensures an optimal choice. If this relaxation of $\not\mathbf{M}$ is inadequate for a similar result (our proof of Dominance made use of the ban on contractive manipulations), a stronger condition that rules out a sequential form of manipulation merits consideration. Let $x \notin C(\mathbf{P})$ and suppose a series of manipulations, one by i, has shrunk $C(\mathbf{P})$ to $C(\mathbf{P}')$, and i now adds x to $C(\mathbf{P}')$ by changing P_i'. By standard criteria, that change is not a manipulation unless it is P_i'-profitable. But even if it is not, a stronger condition could properly ban it if it were profitable according to i's original P_i.

$\not\mathbf{D}$ is so weak it is no restriction at all unless C is resolute, for it follows

[7] Pattanaik's (1978) maximin condition (note 2) is the pessimism half of $\not\mathbf{M}$-Lemma. But optimism is no less plausible, and as our theorem shows, the combination of the two obviates any need for Pattanaik's strong democracy conditions.

[8] $\not\mathbf{M}$-Lemma is one half of that equivalence. The other is easily verified and probably well-known. A proof may be obtained from the authors.

from "$|C(\mathbf{P})| \neq 1$ for some **P**." But **CS** harbors hidden strength: it makes every $x \in A$ a feasible alternative, yet we assumed C defined for orderings of A, implying that C does not depend on preferences for infeasible alternatives. That was needless. Let $A \subseteq B$ and redefine "profile" so each P_i orders B, not just A. Now C can depend on orderings of infeasible alternatives, those in $B \backslash A$: maybe they are there to infer preference intensities. But the proof is still good: **NM** effectively makes C independent of infeasible alternatives.

What about **RR**? Without *some* limit on ties, all our conditions are met by $C(\mathbf{P}) \equiv A$. True, **RR** bans more than that: it requires one-member choice sets in special cases. But we cannot allow even two-member sets in those cases because, for finite A, $C(\mathbf{P}) \equiv \{x | x \text{ is atop } P_1 \text{ or } P_2\}$ would then meet our conditions. For the same reason we cannot relax **RR** by limiting it to cases where all P_j are the same.

5 Conclusion

Gibbard and Satterthwaite found strategic manipulability to be inescapable in the universe of resoluteness, of agreement and certainty about final outcomes (conditional on preferences). Zeckhauser, Gibbard, and Feldman enlarged the known universe of manipulability to encompass uncertainty, but still no disagreement: everyone assigns probabilities to final outcomes, but all make the same assignment. Our even larger universe accommodates considerable disagreement: all individuals assign positive probabilities to the same outcomes (relaxable, as shown in Section 4), but beyond that they can differ as much as you please in the probabilities they assign.

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