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Profitability of price and quantity strategies in a duopoly with vertical product differentiation*

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Summary. Using a model according to Mussa and Rosen (1978) and Bonanno and Haworth (1998) we consider a sub-game perfect equilibrium of a two-stage game in a duopolistic industry in which the products of the firms are vertically differentiated. In the industry, there are a high quality firm and a low quality firm. In the first stage of the game, the firms choose their strategic variables, price or quantity. In the second stage, they determine the levels of their strategic variables. We will show that, under an assumption about distribution of consumers' preference, we obtain the result that is similar to Singh and Vives (1984)' proposition (their Proposition 3) in the case of substitutes with nonlinear demand functions. That is, in the first stage of the game, a quantity strategy dominates a price strategy for both firms.

Keywords and Phrases: Price and quantity strategies, Duopoly, Vertical product differentiation.

JEL Classification Numbers: L13.

1 Introduction

Singh and Vives (1984) showed the following result. In a duopoly with (horizontally) differentiated products in which firms can choose a quantity or price strategy, if the products are substitutes and the firms' reaction functions in a Cournot game (a quantity game) are downward sloping and those in a Bertrand game (a price game) are upward sloping, and some assumptions which ensure the existence of unique Cournot and Bertrand equilibria are satisfied, a quantity

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strategy dominates a price strategy, and the Cournot equilibrium constitutes the sub-game perfect equilibrium of the two-stage game[1].

In this paper, we consider a sub-game perfect equilibrium of a two-stage game in a duopolistic industry with vertical product differentiation. In the industry, there are a high quality firm and a low quality firm. In the first stage of the game, the firms choose their strategic variables, price or quantity. In the second stage, they determine the levels of their strategic variables.

In the next section, we present the model of this paper. In Section 3 and 4 we investigate the conditions for our model to satisfy the requirements for Singh and Vives' proposition in the case of substitutes with nonlinear demand functions (their Proposition 3), and analyze a subgame perfect equilibrium of the game. We will show that, under an assumption about distribution of consumers' preference, we obtain the result that is similar to Singh and Vives' Proposition 3. That is, in the first stage of the game, a quantity strategy dominates a price strategy for both firms.

2 The model

We use a model of vertical product differentiation according to Mussa and Rosen (1978) and Bonanno and Haworth (1998). There is a continuum of consumers with the same income, denoted by y, but different values of the taste parameter θ. Each consumer buys at most one unit of a product. If a consumer with parameter θ buys one unit of a product of quality k at price p, his utility is equal to $y-p+\theta k$. If a consumer does not buy the product, his utility is equal to his income y. The parameter θ is distributed according to a smooth distribution function $\rho = F(\theta)$ in the interval $0 < \theta \leq 1$[2]. ρ denotes the probability that the taste parameter is smaller than θ. The size of consumers is normalized as one. There are two firms, Firm H (the high-quality firm) and Firm L (the low-quality firm). Firm H sells a product of quality k_H, and Firm L sells a product of quality k_L, with $k_H > k_L > 0$. k_H and k_L are fixed. Let p_i be the price charged by Firm i (i=H, L) and q_i be the output of Firm i.

Let θ_0 be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality product. Then

$$\theta_0 = \frac{p_L}{k_L}.$$

[1] Cheng (1985) presented a geometric analysis, and Jéhiel and Walliser (1995) generalized an analysis by Singh and Vives (1984) to a general two person game. Klemperer (1986) analyzed Nash equilibria of a one-stage game, not two-stage game, in which strategic variables are endogenously determined. Qin and Stuart (1997) considered a choice of strategic variables in a homogeneous oligopoly.

[2] If we assume a uniform distribution like Bonanno and Haworth (1998), demand functions are linear.

Let θ_1 be the value of θ for which the corresponding consumer is indifferent between buying the low-quality product and the high-quality one. Then

$$\theta_1 = \frac{p_H - p_L}{k_H - k_L}.$$

We assume $0 < \theta_0 < \theta_1 < 1$.

Accordingly, the direct demand functions are given by

$$q_H = h_H(p_H, p_L) = 1 - F\left(\frac{p_H - p_L}{k_H - k_L}\right), \tag{1}$$

and

$$q_L = h_L(p_H, p_L) = F\left(\frac{p_H - p_L}{k_H - k_L}\right) - F\left(\frac{p_L}{k_L}\right). \tag{2}$$

We have $0 < q_L < 1$ and $0 < q_H < 1$.

The unit cost for Firm H is c_H and that for Firm L is c_L, with $c_H > c_L > 0$. There is no fixed cost.

From (1) and (2) we obtain the inverse demand functions as follows,

$$p_H = f_H(q_H, q_L) = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L),$$

and

$$p_L = f_L(q_H, q_L) = k_L G(1 - q_H - q_L),$$

where $G(\rho)$ is the inverse function of $F(\theta)$. We have

$$G'(\rho) = \frac{1}{F'(\theta)} > 0, \text{ and } G''(\rho) = -\frac{F''(\theta)}{[F'(\theta)]^2}.$$

Since $0 < G(1 - q_H - q_L) < 1$ and $G(1 - q_H - q_L) < G(1 - q_H)$, we have $0 < p_H < k_H$ and $0 < p_L < k_L$.

We assume

Assumption 1. *$F(\theta)$ satisfies the following relation for $0 < \theta \leq 1$,*

$$|F''(\theta)| < \frac{k_H - k_L}{k_H} F'(\theta),$$

or equivalently

$$|G''(\rho)| < \frac{k_H - k_L}{k_H} G'(\rho).$$

This means that $F(\theta)$ is not so concave or convex.

3 The Singh and Vives' proposition and the equilibria in the second stage

The Singh and Vives' proposition in the case of substitutes is stated as follows.

The Singh and Vives' Proposition In a duopoly with differentiated products in which firms can choose a quantity or price strategy, if the following conditions are satisfied, a quantity strategy dominates a price strategy.

1. The products are substitutes, and the reaction functions in a Cournot game (a quantity game) are downward sloping and the reaction functions in a Bertrand game (a price game) are upward sloping.
2. Some assumptions (their Assumption 1 and 2) which ensure the uniqueness of the Cournot equilibrium and the Bertrand equilibrium are satisfied.

From the demand functions we obtain

$$\frac{\partial h_H}{\partial p_L} = \frac{\partial h_L}{\partial p_H} = \frac{1}{k_H - k_L} F'\left(\frac{p_H - p_L}{k_H - k_L}\right) > 0.$$

Also from the inverse demand functions we obtain

$$\frac{\partial f_H}{\partial q_L} = \frac{\partial f_L}{\partial q_H} = -k_L G'(1 - q_H - q_L) = -\frac{k_L}{F'\left(\frac{p_L}{k_L}\right)} < 0.$$

These mean that the products of Firm H and Firm L are substitutes.

Next, we consider the conditions for profit maximization for the firms. When one of the firms chooses a price (respectively quantity) strategy, the other firm determines its price or quantity *given* the rival firm's price (respectively quantity). We call the latter firm a *price taking (respectively quantity taking) firm* or a *price taker (respectively quantity taker)*.

When Firm L chooses a price strategy, Firm H is a price taker and its profit is

$$\pi_H = \left[1 - F\left(\frac{p_H - p_L}{k_H - k_L}\right)\right](p_H - c_H).$$

The first order and second order conditions for Firm H are

$$\frac{\partial \pi_H}{\partial p_H} = 1 - F\left(\frac{p_H - p_L}{k_H - k_L}\right) - \frac{p_H - c_H}{k_H - k_L} F'\left(\frac{p_H - p_L}{k_H - k_L}\right) = 0, \tag{3}$$

and

$$\frac{\partial^2 \pi_H}{\partial p_H^2} = -\frac{1}{k_H - k_L}\left[2F'\left(\frac{p_H - p_L}{k_H - k_L}\right) + \frac{p_H - c_H}{k_H - k_L} F''\left(\frac{p_H - p_L}{k_H - k_L}\right)\right] < 0. \tag{4}$$

When Firm H chooses a price strategy, Firm L is a price taker and its profit is

$$\pi_L = \left[F\left(\frac{p_H - p_L}{k_H - k_L}\right) - F\left(\frac{p_L}{k_L}\right)\right](p_L - c_L).$$

The first order and second order conditions for Firm L are

$$\frac{\partial \pi_L}{\partial p_L} = F\left(\frac{p_H - p_L}{k_H - k_L}\right) - F\left(\frac{p_L}{k_L}\right) - (p_L - c_L)\left[\frac{1}{k_H - k_L}F'\left(\frac{p_H - p_L}{k_H - k_L}\right) + \frac{1}{k_L}F'\left(\frac{p_L}{k_L}\right)\right] = 0, \quad (5)$$

and

$$\frac{\partial^2 \pi_L}{\partial p_L^2} = -2\left[\frac{1}{k_H - k_L}F'\left(\frac{p_H - p_L}{k_H - k_L}\right) + \frac{1}{k_L}F'\left(\frac{p_L}{k_L}\right)\right] - (p_L - c_L)\left[-\frac{1}{(k_H - k_L)^2}F''\left(\frac{p_H - p_L}{k_H - k_L}\right) + \frac{1}{k_L^2}F''\left(\frac{p_L}{k_L}\right)\right] < 0. \quad (6)$$

Similarly, the first order and second order conditions for Firm H as a quantity taker are

$$\frac{\partial \pi_H}{\partial q_H} = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L) - [(k_H - k_L)G'(1 - q_H) + k_L G'(1 - q_H - q_L)]q_H - c_H = 0, \quad (7)$$

and

$$\frac{\partial^2 \pi_H}{\partial q_H^2} = -2[(k_H - k_L)G'(1 - q_H) + k_L G'(1 - q_H - q_L)] + [(k_H - k_L)G''(1 - q_H) + k_L G''(1 - q_H - q_L)]q_H < 0. \quad (8)$$

The first order and second order conditions for Firm L as a quantity taker are

$$\frac{\partial \pi_L}{\partial q_L} = k_L G(1 - q_H - q_L) - k_L G'(1 - q_H - q_L)q_L - c_L = 0, \quad (9)$$

and

$$\frac{\partial^2 \pi_L}{\partial q_L^2} = -k_L[2G'(1 - q_H - q_L) - G''(1 - q_H - q_L)q_L] < 0. \quad (10)$$

From Assumption 1 we find that (4), (6), (8) and (10) globally (for $0 < p_L < k_L$, $0 < p_H < k_H$, $0 < q_L < 1$ and $0 < q_H < 1$) hold.

Now we can show

Lemma 1. *The Bertrand reaction functions are upward sloping, and the Cournot reaction functions are downward sloping.*

Proof. See Appendix A.

And

Lemma 2.

$$\frac{\partial^2 \pi_i}{\partial p_i^2} + \left|\frac{\partial^2 \pi_i}{\partial p_i p_j}\right| < 0 \text{ for } 0 < p_L < k_L,\ 0 < p_H < k_H,\ i = H, L,\ j \neq i, \quad (11)$$

and

$$\frac{\partial^2 \pi_i}{\partial q_i^2} + \left|\frac{\partial^2 \pi_i}{\partial q_i q_j}\right| < 0 \text{ for } 0 < q_L < 1,\ 0 < q_H < 1,\ i = H, L,\ j \neq i. \quad (12)$$

		Firm L	
		Price	Quantity
Firm H	Price	π_H^B, π_L^B	π_H^P, π_L^Q
	Quantity	π_H^Q, π_L^P	π_H^C, π_L^C

Table 1 The first stage game

Proof. See Appendix B.

(11) and (12) are similar to Assumption 1 and 2 in Singh and Vives (1984). They ensure that the Bertrand reaction functions and the Cournot reaction functions are well behaved, the abolute values of whose slopes are less than 1, and there exist unique Bertrand and Cournot equilibria (Friedman, 1977, 1983).

The four equilibrium configurations in the second stage of the game are as follows.

1. The Cournot equilibrium. Both firms are quantity takers.
2. The Bertrand equilibrium. Both firms are price takers.
3. Firm H chooses a price strategy, and Firm L chooses a quantity strategy. In this case Firm H is a quantity taker, and Firm L is a price taker.
4. Firm H chooses a quantity strategy, and Firm L chooses a price strategy. In this case Firm H is a price taker, and Firm L is a quantity taker.

Denote the profit of Firm H in these four cases by, respectively, π_H^C, π_H^B, π_H^P and π_H^Q, and denote the profit of Firm L in these four cases by, respectively, π_L^C, π_L^B, π_L^Q and π_L^P. Then we can show

Proposition 1.

$$\pi_H^P < \pi_H^C,\ \pi_L^P < \pi_L^C,\ \pi_H^Q > \pi_H^B,\ \text{and}\ \pi_L^Q > \pi_L^B.$$

Proof. Similar to the proof of Proposition 3 in Singh and Vives (1984).

4 Price or quantity: The first stage

Next we consider the firms' choice of strategic variables in the first stage of the game. The game is depicted in Table 1.

From Proposition 1 we have $\pi_H^P < \pi_H^C,\ \pi_L^P < \pi_L^C,\ \pi_H^Q > \pi_H^B,\ \text{and}\ \pi_L^Q > \pi_L^B$. Then we obtain the following result.

Proposition 2. *A quantity strategy is dominant for both firms, and both firms choose a quantity strategy in the first stage of the game.*

Therefore the Cournot equilibrium constitutes the subgame perfect equilibrium of the two-stage game.

Appendices

A Proof of Lemma 1

This lemma is equivalent to the following inequalities.

$$\frac{\partial^2 \pi_H}{\partial p_H p_L} = \frac{1}{k_H - k_L}\left[F'\left(\frac{p_H - p_L}{k_H - k_L}\right) + \frac{p_H - c_H}{k_H - k_L} F''\left(\frac{p_H - p_L}{k_H - k_L}\right)\right] > 0, \qquad (13)$$

$$\frac{\partial^2 \pi_L}{\partial p_L p_H} = \frac{1}{k_H - k_L}\left[F'\left(\frac{p_H - p_L}{k_H - k_L}\right) - \frac{p_L - c_L}{k_H - k_L} F''\left(\frac{p_H - p_L}{k_H - k_L}\right)\right] > 0, \qquad (14)$$

$$\frac{\partial^2 \pi_H}{\partial q_H q_L} = k_L[-G'(1 - q_H - q_L) + G''(1 - q_H - q_L)q_H] < 0, \qquad (15)$$

and

$$\frac{\partial^2 \pi_L}{\partial q_L q_H} = k_L[-G'(1 - q_H - q_L) + G''(1 - q_H - q_L)q_L] < 0. \qquad (16)$$

(13) and (14) are derived from

$$\frac{p_L - c_L}{k_H - k_L} < \frac{k_L}{k_H - k_L} < \frac{k_H}{k_H - k_L}, \quad \frac{p_H - c_H}{k_H - k_L} < \frac{k_H}{k_H - k_L}$$

and Assumption 1. (15) and (16) are derived from $0 < q_H < 1$, $0 < q_L < 1$ and Assumption 1.

B Proof of Lemma 2

From (4) and (13)

$$\frac{\partial^2 \pi_H}{\partial p_H^2} - \frac{\partial^2 \pi_H}{\partial p_H p_L} < 0,$$

$$\frac{\partial^2 \pi_H}{\partial p_H^2} + \frac{\partial^2 \pi_H}{\partial p_H p_L} = -\frac{1}{k_H - k_L} F'\left(\frac{p_H - p_L}{k_H - k_L}\right) < 0.$$

From (6) and (14)

$$\frac{\partial^2 \pi_L}{\partial p_L^2} - \frac{\partial^2 \pi_L}{\partial p_L p_H} < 0,$$

$$\begin{aligned}
\frac{\partial^2 \pi_L}{\partial p_L^2} + \frac{\partial^2 \pi_L}{\partial p_L p_H} = & -\left[\tfrac{1}{k_H - k_L} F'\left(\tfrac{p_H - p_L}{k_H - k_L}\right) + \tfrac{2}{k_L} F'\left(\tfrac{p_L}{k_L}\right)\right] \\
& -\tfrac{p_L - c_L}{k_L^2} F''\left(\tfrac{p_L}{k_L}\right) \\
= & -\left[\tfrac{1}{k_H - k_L} F'\left(\tfrac{p_H - p_L}{k_H - k_L}\right) + \tfrac{1}{k_L} F'\left(\tfrac{p_L}{k_L}\right)\right] \\
& -\tfrac{1}{k_L}\left[F'\left(\tfrac{p_L}{k_L}\right) + \tfrac{p_L - c_L}{k_L} F''\left(\tfrac{p_L}{k_L}\right)\right] < 0.
\end{aligned}$$

From (8) and (15)

$$\frac{\partial^2 \pi_H}{\partial q_H^2} + \frac{\partial^2 \pi_H}{\partial q_H q_L} < 0,$$

$$\begin{aligned}\frac{\partial^2 \pi_H}{\partial q_H^2} - \frac{\partial^2 \pi_H}{\partial q_H q_L} = & -[2(k_H - k_L)G'(1-q_H) + k_L G'(1-q_H-q_L)] \\ & +(k_H - k_L)G''(1-q_H)q_H \\ = & -[(k_H - k_L)G'(1-q_H) + k_L G'(1-q_H-q_L)] \\ & -(k_H - k_L)[G'(1-q_H) - G''(1-q_H)q_H] < 0.\end{aligned}$$

From (10) and (16)

$$\frac{\partial^2 \pi_L}{\partial q_L^2} + \frac{\partial^2 \pi_L}{\partial q_L q_H} < 0,$$

$$\frac{\partial^2 \pi_L}{\partial q_L^2} - \frac{\partial^2 \pi_L}{\partial q_L q_H} = -k_L G'(1-q_H-q_L) < 0.$$

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