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Exposita Notes

Growth and equilibrium indeterminacy: the role of capital mobility*

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Summary. The paper presents a human capital driven endogenous growth model which, in general, permits a multiplicity of equilibrium balanced growth paths. It is shown that allowing for perfect capital mobility across countries *increases* the range of parameter values for which the model permits equilibrium indeterminacy. As opposed to the closed capital markets case, simple restrictions on preferences are no longer sufficient to eliminate the indeterminacy. Intuitively, under perfect capital mobility agents are able to smooth consumption completely. This induces an economy with open capital markets to behave like a closed economy with *linear* preferences thereby increasing the possibility of equilibrium indeterminacy.

Keywords and Phrases: Growth – Human capital – Equilibrium indeterminacy – Capital mobility.

JEL Classification Numbers: F4, O4.

1 Introduction

Over the last decade or more the literature on the economic growth has focused a lot of attention on models of endogenous growth. One particular focus in this area has been on the properties of the growth paths that are implied by these models. A number of authors have shown that endogenous growth models with external

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effects often give rise to equilibrium growth paths which are indeterminate.[1] A separate issue which has also attracted attention in recent years is the effect of international integration on economic growth. While work in this area has mostly concentrated on the growth effects of goods market integration, lately there has also been some work on the effect of financial market integration on economic growth.[2]

This paper attempts to bring together the main themes of the growth and indeterminacy literature with the work on financial integration and growth. I present a simple model of human capital driven endogenous growth which, in general, exhibits an indeterminacy of equilibrium growth paths. The first result of interest is that reasonable restrictions on preferences are sufficient to rule out the possibility of equilibrium indeterminacy in a closed economy. It is then demonstrated that for a small open economy with perfect capital mobility, the possibility of equilibrium indeterminacy is much greater. In particular, simple restrictions on preferences are no longer sufficient to rule out the indeterminacy of growth paths.[3,4]

The intuition for this result is simple. For economies with *closed* capital accounts there is a direct, one-for-one link between production decisions and consumption decisions. Under a concave utility function, fluctuating consumption extracts utility costs. Hence, introducing sufficient curvature into the utility function (an intertemporal elasticity of consumption substitution which is less than unity) is sufficient to rule out the possibility of equilibrium indeterminacy since the utility costs become too high.[5] On the other hand, a small *open* economy which faces perfect world capital markets can borrow and lend freely at a given world interest rate and, hence, smooth consumption completely. The consumption smoothing causes a delinking of the production time path from the consumption time path. This, in turn, implies that alternative paths for schooling and output no longer have an associated utility cost through the intertemporal elasticity of substitution parameter. In effect, the economy behaves like a closed economy with *linear* preferences. This feature increases the range in which equilibrium indeterminacy may arise in the model.

As stated earlier the question of indeterminacy in endogenous growth models is not new to this paper. A number of authors have constructed examples where the balanced growth path is indeterminate. Thus, Benhabib and Perli (1994) show

[1] Three representative papers which formalize this are Benhabib and Perli (1994), Boldrin and Rustichini (1994), and Xie (1994).

[2] Research on this theme can be found in Devereaux and Smith (1994) and Obstfeld (1994).

[3] The focus of the paper is on capital flows. Hence, throughout the paper the distinction between a closed economy and an open economy pertains to the capital account. In any case, the one good structure of the model makes this the only relevant distinction.

[4] The model is along the same lines as the last model studied by Benhabib and Perli (1994). A two good growth model with the same features for human capital accumulation can be found in Asea and Lahiri (1999). However, that paper studies the issue of natural resources and their impact on growth.

[5] Benhabib and Perli (1994) make exactly this point and then show how introducing an endogenous labor-leisure choice can reduce the required intertemporal elasticity of consumption by facilitating investment fluctuations through changes in leisure rather than changes in consumption.

that equilibrium indeterminacy may arise quite easily in the Lucas (1988) model. Similarly, Boldrin and Rustichini (1994) have shown that by moving from one sector to two sector models it becomes remarkably easy to generate indeterminacy. However, the focus of almost all that work is on closed economy models as opposed to the open economy focus of this paper. The key contribution of this paper is in analyzing the effect of international capital mobility on equilibrium indeterminacy.

This paper also provides an interesting contrast to the results obtained in a related paper by Boyd and Smith (1997). Using an overlapping generations model with a neoclassical technology, they show that under credit market imperfections capital mobility across countries can give rise to multiple steady states and equilibrium cycles. Moreover, the asymmetric steady states are characterized by perverse capital flows from the poorer to the richer country. Their results are complementary to the results of this paper in that both suggest reasons for caution regarding international capital flows. While this paper shows that perfect financial integration in the absence of any credit market imperfections could open the door to extrinsic uncertainty and, hence, output volatility, Boyd and Smith show that financial flows in the presence of credit market frictions could lead to cyclical fluctuations and perverse capital flows.

The next section presents the model and studies the closed economy case. Section 3 presents the case of a small open economy with perfect capital mobility while Section 4 presents an example where the equilibrium path is unique for a closed economy but is indeterminate for an open economy. The last section contains concluding remarks.

2 The model

Consider an economy which is inhabited by an infinitely lived representative agent who consumes and produces a single good called y. The agent maximizes lifetime welfare which is given by

$$V = \int_{t=0}^{\infty} \exp(-\rho t) u(c_t) dt \tag{1}$$

where ρ is the rate of time preference which is exogenous and constant, c denotes consumption of good y, and where u is assumed to be a homothetic, twice differentiable, and strictly concave function. The agent is endowed with one unit of labor time at every instant which can be freely allocated between working as unskilled labor and schooling time. Thus,

$$n_t + s_t = 1 \tag{2}$$

where n denotes labor supply while s denotes schooling time. Importantly, equation (2) has to hold at all points in time.

Time devoted to schooling enables the agent to accumulate human capital. The evolution equation for human capital is assumed to be given by

$$\dot{e}_t = h(s_t)e_t \ , \quad h' > 0 \ , \ h'' < 0 \tag{3}$$

where it is assumed that e_0 is known. e denotes the stock of human capital. The salient feature of (3) is that the rate of growth of human capital is assumed to be increasing and concave in schooling time. Further, I assume that $h'(0) = \infty$ and that $h(0) = 0$ which implies that human capital accumulation requires a strictly positive input of schooling time.

The good is produced by two inputs – unskilled labor, n, and human capital (or skilled labor), e. The production technology is given by

$$y_t = f(n_t, e_t, E_t) \tag{4}$$

where E denotes the *average* per capita stock of human capital. The representative agent takes E as exogenously given. Of course, in equilibrium we must have $e_t = E_t$ for all t. I assume that f is strictly concave and increasing in both n and e and that it exhibits constant returns to scale in n and e. In order to guarantee constant steady state growth I further assume that there are constant returns overall to human capital so that $ef_e + Ef_E = f$. Lastly, I assume that $f_n(0, ., .) = \infty$ which in combination with the assumption $h'(0) = \infty$ ensures that the model generates an interior solution for n. Note that the engine of growth in this economy is human capital since it is the only factor which can be accumulated over time.

2.1 The closed economy case

I start by studying the case of a closed economy. For an economy with closed capital markets consumption has to equal output at all points in time. Hence, we must have

$$c_t = f(n_t, e_t, E_t) \tag{5}$$

The agent maximizes lifetime welfare (1) by choosing c, n, and s subject to the constraints (2), (3), and (5).[6] The first order conditions for optimization are given by

$$u'(c_t)f_n(n_t, e_t, E_t) = \lambda_t e_t h'(s_t) \tag{6}$$

$$\dot{\lambda}_t = \left[\rho - h(s_t) - \frac{e_t f_e(n_t, e_t, E_t)h'(s_t)}{f_n(n_t, e_t, E_t)}\right]\lambda_t \tag{7}$$

$$\lim_{t \longrightarrow \infty} \exp(-\rho t)\lambda_t e_t = 0 \tag{8}$$

where a prime on a single variable function indicates the derivative with respect to the argument, while a dot over a variable indicates a time derivative. λ is the shadow value of human capital (the Pontryagin multiplier associated with constraint (3)).

Equation (6) determines the optimal labor supply-schooling decision by equating the future utility value of an incremental unit of current schooling ($\lambda eh'$) with

[6] In modelling the households problem I have combined the consumption and production decisions just for convenience. Separating the two decision making units would leave the results unchanged.

the utility cost of foregone current output due to reduced labor supply $(u'f_n)$.[7] Equation (7) is the equation of motion for the shadow value of human capital while (8) is the transversality condition for human capital.

Totally differentiating equation (6) and substituting equations (3) and (7) into the result yields

$$\chi \dot{n} = -hu'f_n \frac{u''}{u'} f - hu'f_n + \left(\rho - \frac{ef_e h'}{f_n} \right) u'f_n \tag{9}$$

where $\chi = \left[f_n^2 u'' + f_{nn} u' + u'f_n (h''/h') \right] < 0$, and where I have substituted in the equilibrium relationship $e = E$, and have also used the fact that f is homogenous of degree one in both e and E and in n and e. Note that the time subscripts in equation (9) have been suppressed for notational convenience.

In order to rule out inessential dynamics I now specialize the problem by assuming that the representative agent's preferences belong to the Constant Relative Risk Aversion (CRRA) class so that $-cu''/u' = \sigma$ where σ is a positive constant. Noting that in the closed economy case output has to equal consumption, we can rewrite equation (9) as

$$\frac{\dot{n}}{n} = \frac{u'f_n}{n\chi} \left[\rho + (\sigma - 1)h - \frac{ef_e h'}{f_n} \right] \tag{10}$$

Equation (10) is the fundamental differential equation driving the equilibrium dynamics of this economy.

In steady state we must have $\dot{n} = 0$.[8] Hence, the steady state level of unskilled labor is implicitly given by the equation

$$\rho + (\sigma - 1)h(1 - n^c) = \frac{ef_e(n^c, e, E)h'(1 - n^c)}{f_n(n^c, e, E)} \tag{11}$$

where n^c denotes the steady state labor supply. Note that the growth rate of the economy is given by $\dot{e}/e = h(1 - n)$ and $h' > 0$. Hence, the higher the labor supply the lower is the schooling time and, hence, the lower the growth rate.

It is easy to show that the steady state growth rate is unique for $\sigma \geq 1$. However, the economy could have multiple steady states for the case $\sigma < 1$. Inspite of the fact that the possibility of multiple steady states does exist in the model, I shall assume throughout the rest of the paper that the technology is such that the steady state is *unique*.[9]

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[7] In general, the first order condition for optimal work is given by $u'f_n \geq \lambda e$. However, the assumptions $f_n(0, ., .) = \infty$ and $h'(0) = \infty$ allow us to express this condition as an equality since none of the two corner solutions for schooling can be sustained as an equilibrium.

[8] It is easy to check that the model is consistent with steady state growth since ef_e/f_n is independent of e under the assumption of linear homogeneity of f in e and E.

[9] A simple restriction which ensures a unique steady state is to assume that the human capital accumulation technology is linear in schooling so that $h(s) = s$ and the production function to be Cobb-Douglas with $f(n, e, E) = n^\alpha e^{1-\alpha} E^\alpha$. In this event $ef_e h'/f_n = n(1 - \alpha)/\alpha$ and the steady state is unique.

2.2 Transition dynamics

The transition dynamics of this economy can be studied by differentiating equation (10) with respect to n and evaluating the derivative around the steady state, n^c. This gives

$$\left.\frac{\partial(\dot{n}/n)}{\partial n}\right|_{n=n^c} = \left[(\sigma - 1) + \frac{ef_{en}}{f_n} - \frac{ef_e}{f_n^2}f_{nn} - \frac{ef_e h''}{f_n h'}\right]\frac{u' f_n h'}{-n\chi} \tag{12}$$

Since $\chi < 0$, one can see from (12) that for $\sigma > 1 - \left(\frac{ef_{en}}{f_n} - \frac{ef_e}{f_n^2}f_{nn} - \frac{ef_e h''}{f_n h'}\right)$, we must have $\frac{\partial(\dot{n}/n)}{\partial n} > 0$. Thus, for this case, the differential equation governing the behavior of equilibrium n is *unstable* around a local neighborhood of the steady state. Hence, the economy has to lock into the unique and locally unstable steady state at time 0 itself. If the system doesn't jump to the steady state instantaneously, then n goes to either 0 or 1 over time. $n = 1$ has already been ruled out by the Inada conditions while $n = 0$ would violate both the first order condition (6) as well as the transversality condition (8) since λe would be growing too fast.

On the other hand, when $\sigma < 1 - \frac{ef_{en}}{f_n} - \frac{ef_e}{f_n^2}f_{nn} - \frac{ef_e h''}{f_n h'}$ the differential equation (12) is *stable* around the unique steady state. In this case one cannot tie down the equilibrium dynamics of the economy through standard arguments. Independent of where the system begins all dynamic paths lead to the same unique, locally stable steady state. Hence, the economy exhibits equilibrium indeterminacy.[10] It is important to note that the possible indeterminacy occurs with regard to the initial choice of schooling time and hence, the transition path to the steady state. Once the initial n is given the rest of the dynamic path gets fully determined. These results are collected in the following proposition:

Proposition 1. *The economy exhibits equilibrium indeterminacy for the case* $\sigma < 1 - \left(\frac{ef_{en}}{f_n} - \frac{ef_e}{f_n^2}f_{nn} - \frac{ef_e h''}{f_n h'}\right)$. *For* $\sigma > 1 - \left(\frac{ef_{en}}{f_n} - \frac{ef_e}{f_n^2}f_{nn} - \frac{ef_e h''}{f_n h'}\right)$ *however, the steady state is locally unstable and the economy attains its unique steady state growth rate instantaneously.*

It follows directly from Proposition 2 that a simple method of eliminating the possibility of equilibrium indeterminacy is to restrict preferences such that $\sigma \geq 1$. This restriction is sufficient to ensure that there are no transition dynamics.[11]

In order to understand the intuition for the possibility of indeterminacy, suppose agents expect a high value of human capital tomorrow. Then, it is optimal to increase schooling today which reduces current labor supply and, thus, increases the equilibrium wage. Since, at the optimum, agents equate the marginal cost of schooling (the foregone wage) with the marginal value of schooling, the high

[10] One should note that I am ignoring the special case of $\sigma = 1 - \left(\frac{ef_{en}}{f_n} - \frac{ef_e}{f_n^2}f_{nn}\right)$. In this event, there would be a continuum of steady state growth rates.

[11] The uniqueness of the equilibrium is, of course, contingent on the assumption that the technology is such that the steady state is unique.

equilibrium wage today implies a high equilibrium value of future human capital which rationalizes the initial choice of high schooling. Using a similar argument one can also rationalize a low initial schooling choice.

This, however, is not sufficient for equilibrium indeterminacy. The reduction in current production of the final good that is induced by an increase in current schooling also implies that consumption falls on impact. Hence, this path can only be rationalized if agents are not too averse to letting consumption fluctuate over time. In particular, the elasticity of intertemporal consumption substitution has to be quite high. This result is similar to that obtained by Benhabib and Perli (1994) in the context of the Lucas (1988) model.

3 The open economy case

In order to analyze the issue of capital mobility for a small open economy I assume that private agents can borrow and lend freely in world capital markets in terms of bonds whose face value is one unit of good y and which pay a fixed interest r in terms of the good at every instant. The agent's flow budget constraint is given by

$$\dot{b} = rb + f(n, e, E) - c \tag{13}$$

where b denotes foreign bonds.

As before, the representative agent maximizes lifetime welfare (1) by choosing c, n, and s subject to the constraints (13), (3), and (2). The first order conditions are given by

$$u'(c) = \mu \tag{14}$$

$$f_n(n, e, E) = \lambda^o e h' \tag{15}$$

$$\dot{\mu} = (\rho - r)\mu \tag{16}$$

$$\dot{\lambda}^o = \left[\rho - h(1-n) - \frac{e f_e(n, e, E) h'(1-n)}{f_n(n, e, E)}\right] \lambda^o \tag{17}$$

and the transversality conditions on b and e. μ denotes the shadow value of foreign bonds (or the Pontryagin multiplier on b) while λ^o denotes the corresponding multiplier for e. I also make the standard small open economy assumption that $\rho = r$. This assumption ensures that μ is a constant. Hence, consumption is constant over time. Access to foreign capital markets enables the consumer to smooth consumption completely.

Differentiating equation (15) with respect to time and substituting (3) and (17) into the result gives

$$\frac{\dot{n}}{n} = \frac{f_n}{n\chi^o}\left[\rho - h - \frac{e f_e h'}{f_n}\right] \tag{18}$$

where $\chi^o = f_{nn} + f_n h''/h' < 0$. Equation (18) is the differential equation which describes the dynamic behavior of the model in the open economy case and is the counterpart of equation (10) for the closed economy case. The steady state growth rate for the economy is given by $1 - n^o$ where n^o solves the equation

$$\rho = h(1 - n^o) + \frac{ef_e(n^o, e, E)h'(1 - n^o)}{f_n(n^o, e, E)} \tag{19}$$

Comparing equations (11) and (19) it is easy to check that $\sigma \geq 1$ is a sufficient condition for the open economy growth rate to unambiguously exceed the steady state growth rate for the closed economy.

3.1 Transition dynamics

The stability properties of the economy can be ascertained by differentiating equation (18) around the steady state. This gives

$$\left.\frac{\partial(\dot{n}/n)}{\partial n}\right|_{n=n^o} = \frac{h'f_n}{-n\chi^o}\left[-1 + \frac{ef_{en}}{f_n} - \frac{ef_e}{f_n^2}f_{nn} - \frac{ef_e h''}{f_n h'}\right] \gtreqless 0 \tag{20}$$

As argued previously, $\frac{\partial(\dot{n}/n)}{\partial n}\big|_{n=n^o} > 0$ implies that the system is unstable around the steady state in which case the system locks into the steady state rate of growth at time 0 itself. $\frac{\partial(\dot{n}/n)}{\partial n}\big|_{n=n^o} < 0$, on the other hand, implies that the steady state is locally stable. In this event one cannot appeal to transversality and optimality conditions to rule out some paths.

It is easy to see that relative to the no capital mobility case, there is now a bigger zone in which equilibrium paths may be indeterminate. Recall that under no capital mobility the equation corresponding to equation (20) was given by $\frac{\partial(\dot{n}/n)}{\partial n}\big|_{n=n^c} = \frac{u'f_n h'}{-n\chi}\left[(\sigma - 1) + \frac{ef_{en}}{f_n} - \frac{ef_e}{f_n^2}f_{nn} - \frac{ef_e h''}{f_n h'}\right]$. Hence, one could rule out equilibrium indeterminacy by restricting preferences such that $\sigma \geq 1$ since $\chi < 0$. But now with perfect capital mobility such restrictions on preferences are insufficient to rule it out. Since the agent is able to smooth consumption perfectly through borrowing and lending, there are no utility costs associated with choosing alternative time paths for output. It is akin to making current and future consumption *perfect substitutes*. Thus, the dynamics under the perfect capital mobility case are similar to the closed economy dynamics under linear utility ($\sigma = 0$).

4 An example

I now provide a simple example to illustrate the fact that the range for equilibrium indeterminacy is bigger for the open economy than for the closed economy case. Let the production function be given by

$$f(n, e, E) = n^\alpha e^{1-\alpha} E^\alpha \; , \; 0 < \alpha < 1 \tag{21}$$

Further, assume that the technology for human capital accumulation is given by $\dot{e}/e = h(s)$ where

$$h(s) = s^{\beta}\ ,\ 0 < \beta < 1 \tag{22}$$

Under these assumptions it is easy to check that the equilibrium differential equation for the closed economy case (equation (10)) reduces to

$$\frac{\dot{n}}{n} = \frac{1}{\Delta^c}\left[\rho + (\sigma - 1)(1-n)^{\beta} - \frac{(1-\alpha)}{\alpha}\beta n(1-n)^{\beta-1}\right] \tag{23}$$

where $\Delta^c \equiv \left[\alpha - 1 - \alpha\sigma + (\beta - 1)\left(\frac{n}{1-n}\right)\right] < 0$. Two features of this economy are worth noting. First, from equation (23) one can see that $\dot{n}/n$ is non-linear in n. Hence, the local and global dynamics of this economy are not going to be identical. Second, the assumed technology, in general, permits multiple steady states.

Under our assumptions it is easily checked that the equilibrium differential equation for the open economy case (18) reduces to

$$\frac{\dot{n}}{n} = \frac{1}{\Delta^o}\left[\rho - (1-n)^{\beta} - \frac{(1-\alpha)}{\alpha}\beta n(1-n)^{\beta-1}\right] \tag{24}$$

where $\Delta^o \equiv \left[\alpha - 1 + (\beta - 1)\left(\frac{n}{1-n}\right)\right] < 0$. A comparison of equations (23) and (24) reveals that the only difference between the closed and open economies is that the parameter σ is missing from the open economy equation.

Applying our specification to equation (12) it is easy to verify that the condition for local stability of the steady state for the closed economy case reduces to

$$1 - \sigma > \left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^c}{1-n^c}\right)\right]$$

Similarly, from equation (20), the condition for local stability of the open economy steady state growth rate is now given by

$$1 > \left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^o}{1-n^o}\right)\right]$$

Since σ is a positive constant, we have four possible cases of interest:[12]

(i) $1 > 1 - \sigma > \left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^c}{1-n^c}\right)\right] > \left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^o}{1-n^o}\right)\right]$;

(ii) $1 > \left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^c}{1-n^c}\right)\right] > \left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^o}{1-n^o}\right)\right] > 1 - \sigma$;

(iii) $\left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^c}{1-n^c}\right)\right] > \left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^o}{1-n^o}\right)\right] > 1 > 1 - \sigma$;

(iv) $\left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^o}{1-n^o}\right)\right] > 1 > 1 - \sigma > \left(\frac{1-\alpha}{\alpha}\right)\left[1 + (1-\beta)\left(\frac{n^c}{1-n^c}\right)\right]$;

[12] The list of cases below is not exhaustive since there are a few other cases. However, those are not listed here since their stability characteristics are identical to at least one of the cases listed.

Under (i) the steady state is locally *stable* and there is equilibrium indeterminacy for both the closed and the open economies while for case (iii) the steady state is locally *unstable* for both and, hence, there is no indeterminacy in either economy. For case (ii) the closed economy steady state is locally unstable while the open economy steady state is locally stable while case (iv) is the opposite with the open economy steady state being locally unstable while the closed economy steady state is locally stable. It is easy to check that imposing the empirically plausible restriction $\sigma \geq 1$ is sufficient to rule out cases (i) and (iv).[13]

From case (i) one can see that if the equilibrium is indeterminate for the closed economy then it must also be indeterminate for the open economy while case (iii) tells us that a locally unstable steady state for the open economy is a sufficient condition for the equilibrium to be determinate for a closed economy. Importantly, under the restriction $\sigma \geq 1$ it is *not* possible for an economy with closed capital markets to have equilibrium indeterminacy while an otherwise identical economy with open capital markets has a determinate equilibrium. As illustrated by case (ii), the opposite configuration, of course, is possible.

Two other points are worth noting. First, as $\beta \to 1$ the list of cases above collapses to (i), (ii) and (iii) which implies that it is no longer possible for a closed economy to have equilibrium indeterminacy while the corresponding open economy is determinate for *any value* of σ.[14] Second, even when $\sigma < 1$ case (iv) can be ruled out for σ sufficiently close to either unity or zero. Since, as shown earlier, $n^c > n^o$ for $\sigma = 1$ the result follows for σ sufficiently close to unity simply by arguments of continuity. To check the result for σ close to zero, totally differentiate the equation describing the steady state labor supply for the closed economy case. It is easy to confirm that $\partial n^c/\partial\sigma > 0$ around $\sigma = 0$. The result follows from the fact that $n^c = n^o$ for $\sigma = 0$.

Before closing this section it is worth noting the realism of the required degree of increasing returns for indeterminacy in the open economy case ($\alpha > 1/2$) may be debatable. However, the model is extremely simple. It is deliberately constructed that way to illustrate, as parsimoniously as possible, that the range in which indeterminacy occurs becomes wider for an open economy relative to a closed economy. Introduction of physical capital, an endogenous labor-leisure choice, externalities in the research sector, sector specific external effects, etc., would be simple ways of reducing the required degree of increasing returns. Two recent papers by Meng and Velasco (1998) and Weder (1998) both confirm this point in the context of models with physical capital. In fact, the model of Meng and Velasco has no increasing returns to scale at all.

[13] Available empirical evidence for the intertemporal elasticity of substitution for both developing and developed countries suggest that this elasticity is less than unity which implies that $\sigma \geq 1$. Evidence on this parameter for developing countries can be found in Reinhart and Vegh (1995).

[14] Of course, an interior solution for n is no longer guaranteed when $\beta \to 1$ and one would have to impose parameter restrictions to ensure an interior solution.

5 Conclusion

This paper has studied the effect of capital mobility on the equilibrium growth path of a small open economy. Using a human capital driven endogenous growth model I have shown that opening up the economy to perfect capital flows greatly enhances the range in which the equilibrium growth path may be indeterminate.

The role played by capital mobility in rationalizing equilibrium inderminacy can be understood by noting that under closed capital markets the output and consumption profiles are identical since one cannot borrow or lend. Since the schooling choice today implies an intertemporal substitution of output and consumption, private agents also have to incorporate the utility cost of this substitution into their decision. Perfect capital mobility, on the other hand, implies that agents are able to smooth consumption perfectly. Complete consumption smoothing implies that the intertemporal substitution of output has no utility costs making current and future output perfect substitutes. Hence, the dynamics of the economy resemble those of a closed economy with linear utility and many more paths can be rationalized. However, it is important to note that openness of capital markets by itself is not good enough to generate indeterminacy. The primary message of the paper is that *if* the model permits indeterminacy *then* open capital markets will increase the range of parameter values for which indeterminacy may occur.

It should be noted that while models exhibiting equilibrium indeterminacy could be interpreted as providing a rationalization for the cross-country evidence on growth, the focus on the issue in this paper is also reflective of a desire to study and understand the dynamic implications of financial market integration. Moreover, the results of the paper also raise potentially interesting and important questions regarding the emergence of sophisticated and more complete financial markets and their dynamic implications. This is an area of potentially fruitful future work.

Another question that remains unanswered is how prevalent is this problem of indeterminacy likely to be for open economy growth models? Note that human capital has been modelled in the paper as a non traded intermediate good. Would the possibility of trade in inputs alleviate or worsen the problem? The answer to this question is extremely important since trade in physical capital may be easier than trade in human capital.

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