

Exploring the minimal 4D $\mathcal{N} = 1$ SCFT

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ABSTRACT: We study the conformal bootstrap constraints for 4D $\mathcal{N} = 1$ superconformal field theories containing a chiral operator ϕ and the chiral ring relation $\phi^2 = 0$. Hints for a minimal interacting SCFT in this class have appeared in previous numerical bootstrap studies. We perform a detailed study of the properties of this conjectured theory, establishing that the corresponding solution to the bootstrap constraints contains a $U(1)_R$ current multiplet and estimating the central charge and low-lying operator spectrum of this theory.

KEYWORDS: Conformal and W Symmetry, Renormalization Group

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1 Introduction

The conformal bootstrap has emerged as a powerful tool for studying conformal field theories (CFTs) in $D > 2$, with numerous applications. In recent years it has allowed us to learn precise quantitative information about known strongly-interacting CFTs, such as the 3D Ising [1–5], $O(N)$ vector [6–13], and Gross-Neveu models [14], known 3D $\mathcal{N} = 2$ [15–17] and $\mathcal{N} = 8$ [18, 19] SCFTs, 4D $\mathcal{N} = 2$ [20–22] and $\mathcal{N} = 4$ SCFTs [23–26], and the mysterious 6D (2, 0) SCFTs [27, 28]. Moreover, because it probes the full space of CFTs without reference to any particular microscopic description, the conformal bootstrap is also a powerful tool for discovering new, previously unknown theories.

Hints for a possible new 4D CFT with $\mathcal{N} = 1$ supersymmetry appeared in [29] (building on earlier studies [30–36]), manifesting as a kink in general bounds on the scaling dimension of the leading non-chiral scalar in the OPE $\bar{\phi} \times \phi$, where ϕ is a chiral operator. This coincided with the disappearance of a lower bound on the chiral operator OPE coefficient $\phi \times \phi \sim \lambda_{\phi^2} \phi^2$, allowing this coefficient to vanish precisely at this dimension. Moreover, it was established in [16] that a similar feature appears at all $2 \leq D \leq 4$ in SCFTs with four supercharges, where as $D \rightarrow 2$ it merges with the 2D $\mathcal{N} = 2$ minimal model. The absence of the ϕ^2 operator in $D = 3, 4$ could also be seen more directly in the approximate solutions to crossing symmetry reconstructed in [16].

However, the correct interpretation of these features in both $D = 3, 4$ is not yet understood. Based on their similarity to features that are known to coincide with the 3D Ising [1–5] and 3D $O(N)$ vector models [6, 13], it is tempting to conjecture the existence of a family of new SCFTs. In this work we study the 4D $\mathcal{N} = 1$ version of these kinks in greater detail, exploring the properties of the theory that we conjecture to live there.

We will establish several properties of this conjectured theory using the conformal bootstrap conditions for the correlator $\langle \bar{\phi} \phi \bar{\phi} \phi \rangle$, building on the earlier results of [16, 29]. First we establish directly that assuming the chiral ring condition $\phi^2 = 0$ imposes a sharp lower bound $\Delta_\phi \geq 1.415$. In particular we exclude the possibility that $\Delta_\phi = \sqrt{2}$. Second, after imposing the chiral ring condition we place a bound on the leading spin-1 superconformal primary and find that it forces the existence of a $U(1)_R$ current multiplet when the lower bound on Δ_ϕ is saturated.

Having established that this putative theory contains a $U(1)_R$ current multiplet (whose descendant is the stress-energy tensor), we proceed to compute general lower and upper bounds on the conformal central charge for SCFTs with $\phi^2 = 0$. The upper bounds are somewhat dependent on the gap until the next spin-1 primary, but for all gaps the lower

and upper bounds merge at the minimal value of Δ_ϕ . We estimate that this minimal theory has $c/c_{\text{free}} \simeq 8/3$ where c_{free} is the central charge of a free chiral multiplet. We also make preliminary determinations of the OPE coefficient of the $\bar{\phi}\phi$ operator, the dimensions of the second scalar and spin-1 superconformal primaries, and the dimension of the leading spin-2 superconformal primary.

In the present work we have not yet found a set of gap assumptions that isolate this solution, i.e. we do not yet see islands analogous to what was found in [4, 5, 13]. For this we anticipate that we will need to consider a larger system of correlators containing both the ϕ and $\bar{\phi}\phi$ operators. However, in our current setup we can already uncover a lot of information about this theory and we hope that the results of this paper are a useful step towards identifying the nature of this mysterious 4D $\mathcal{N} = 1$ SCFT.

2 Results

In this work we study the correlator $\langle \bar{\phi}\phi\bar{\phi}\phi \rangle$ where ϕ is a chiral operator in a 4D $\mathcal{N} = 1$ SCFT, similar to what was done in [29]. Crossing symmetry of this correlator leads to the sum rules

$$\sum_{\mathcal{O} \in \bar{\phi} \times \phi} |\lambda_{\mathcal{O}}|^2 \begin{pmatrix} \mathcal{F}_{\Delta, \ell}(z, \bar{z}) \\ \tilde{\mathcal{F}}_{\Delta, \ell}(z, \bar{z}) \\ \tilde{\mathcal{H}}_{\Delta, \ell}(z, \bar{z}) \end{pmatrix} + \sum_{\mathcal{O} \in \phi \times \phi} |\lambda_{\mathcal{O}}|^2 \begin{pmatrix} 0 \\ F_{\Delta, \ell}(z, \bar{z}) \\ -H_{\Delta, \ell}(z, \bar{z}) \end{pmatrix} = 0, \quad (2.1)$$

where the functions of the conformal cross-ratios z and \bar{z} that appear are related to conformal and superconformal blocks and defined in [29].¹ In general we assume that the superconformal primary operators \mathcal{O} in the first sum satisfy the unitarity bound $\Delta_{\mathcal{O}} \geq \ell + 2$ [48, 49], while the even-spin operators in the second sum may either be conformal primaries in BPS multiplets with $\Delta_{\mathcal{O}} = 2\Delta_\phi + \ell$, or conformal primaries in unprotected multiplets satisfying the unitarity bound $\Delta_{\mathcal{O}} \geq |2\Delta_\phi - 3| + 3 + \ell$ [33, 36].

As described in [29], in order to rule out assumptions about the spectrum we can look for a 3-vector of functionals $\vec{\alpha}$ that when applied this sum rule leads to a contradiction. In particular, if the functional is > 0 on the identity operator contribution and ≥ 0 on all other possible contributions, then the sum rule can never be satisfied. Alternatively, by normalizing the functional on the contribution of a particular operator \mathcal{O}_0 and extremizing the action on the identity operator, we can obtain upper or lower bounds on the OPE coefficient of \mathcal{O}_0 . We apply this logic below to obtain bounds on operator dimensions and OPE coefficients, using SDPB [5] to solve the relevant optimization problem after phrasing it in the language of semi-definite programming. The functional search space is governed by the parameter Λ , where each component α_i is a linear combination of $\frac{1}{2} \lfloor \frac{\Lambda+2}{2} \rfloor (\lfloor \frac{\Lambda+2}{2} \rfloor + 1)$ independent nonvanishing derivatives $\alpha_i \propto \sum_{m,n} a_{mn} \partial_z^m \partial_{\bar{z}}^n|_{1/2, 1/2}$ with $m+n \leq \Lambda$.

First, we reproduce the general upper bound on the dimension of the leading unprotected scalar operator $\Delta_{\bar{\phi}\phi}$, finding precise agreement with [29]. This bound is shown in figure 1. At $\Lambda = 21$ there is a mild kink in this upper bound around $\Delta_\phi \simeq 1.407$. We show

¹Additional results and formalism for setting up the 4D $\mathcal{N} = 1$ superconformal bootstrap has e.g. been developed in [16, 33, 36–47].

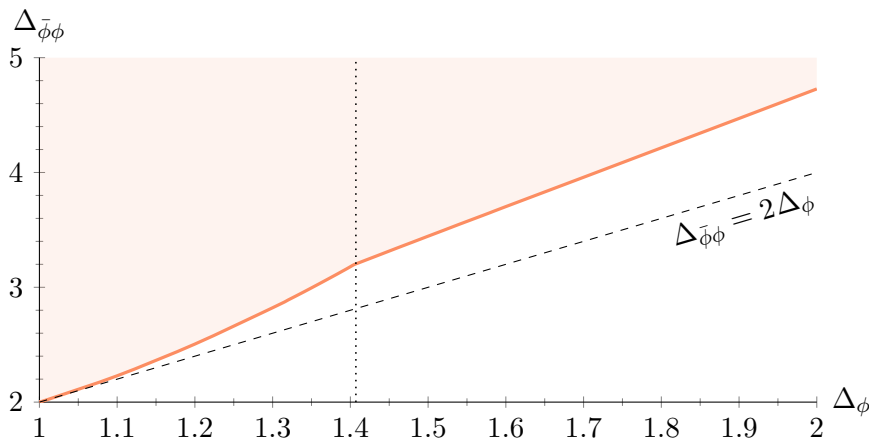


Figure 1. Upper bound on the allowed dimension of the operator $\bar{\phi}\phi$ (the leading relevant nonchiral scalar singlet) as a function of the dimension of ϕ . The generalized free theory dashed line $\Delta_{\bar{\phi}\phi} = 2\Delta_{\phi}$ is also shown. The shaded area is excluded. If we assume that ϕ^2 is not in the spectrum then everything to the left of the dotted line at $\Delta_{\phi} = 1.407$, which is the position of the kink, is excluded. Here we use $\Lambda = 21$.

how this position changes as we increase the search space of the functional in a later plot. Note that any theory saturating this bound necessarily does not contain any scalar superconformal primaries of dimension 2, i.e. ϕ cannot be charged under any global symmetries.

Next we recompute this bound imposing the additional condition that the chiral ϕ^2 operator does not appear in the $\phi \times \phi$ OPE. This condition has the effect of excluding all points to the left of the dotted vertical line in figure 1. The region to the right remains the same. In other words, it imposes the strict lower bound $\Delta_{\phi} \geq 1.407$, causing the mild kink to turn into a sharp corner.

One can see that this had to be the case by considering bounds on the OPE coefficient of the operator ϕ^2 , shown in figure 2. The lower bound on λ_{ϕ^2} disappears exactly at $\Delta_{\phi} = 1.407$. Thus, figure 2 makes it clear that if we demand $\lambda_{\phi^2} = 0$, implying that ϕ^2 is not in the spectrum, then all points to the left of $\Delta_{\phi} = 1.407$ must be excluded. Our general bound is also compatible with the results of [16], which found that the ϕ^2 operator was absent in approximate solutions to crossing symmetry living on the boundary of the allowed region to the right of the kink.

In figure 3 we show an upper bound on the OPE coefficient $\lambda_{\bar{\phi}\phi}$ of an operator whose dimension saturates the bound in figure 1. Without any additional assumptions the upper bound attains a minimum at precisely the location of the kink, occurring at $\lambda_{\bar{\phi}\phi} \simeq 0.905$. If we further impose the absence of ϕ^2 , then all points to the left of the dotted vertical line in figure 3 are excluded.

Next we would like to ask the question: if there is an SCFT living near the kink with the chiral ring relation $\phi^2 = 0$, does it contain a stress-energy tensor? In other words, could it correspond to a local SCFT? In figure 4 we assume $\phi^2 = 0$ and place an upper bound on the leading spin-1 superconformal primary V in the $\bar{\phi} \times \phi$ OPE, again at $\Lambda = 21$. We see that the bound on Δ_V approaches 3 as Δ_{ϕ} approaches its minimum value. Thus, the $U(1)_R$ current multiplet V_R is required to be in the spectrum at this point.

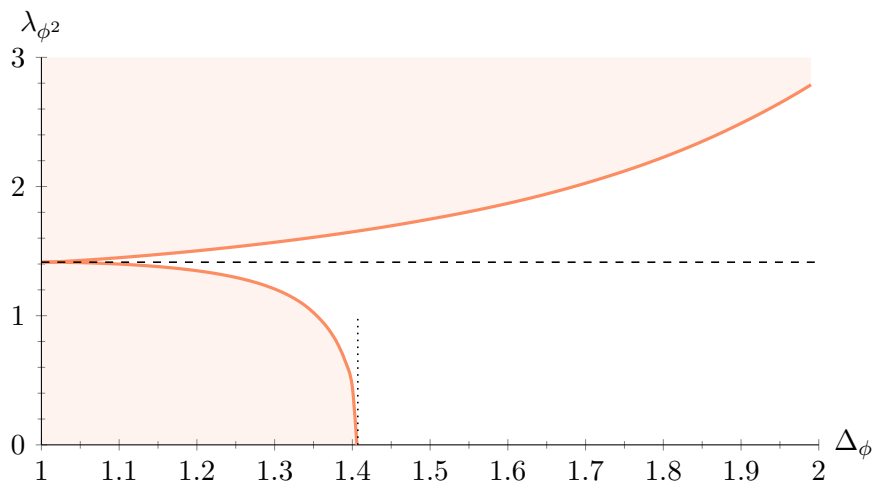


Figure 2. Lower and upper bounds on the OPE coefficient of the operator ϕ^2 in the $\phi \times \phi$ OPE. The vertical dotted line is at $\Delta_\phi = 1.407$ and the horizontal dashed line is at the free theory value $\lambda_{\phi^2} = \sqrt{2}$. The shaded area is excluded. Here we use $\Lambda = 21$.

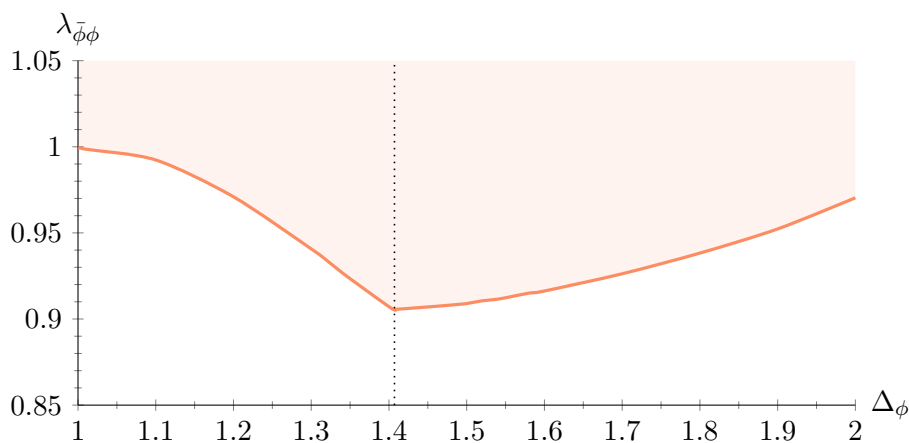


Figure 3. Upper bound on the OPE coefficient of an operator $\bar{\phi}\phi$ with dimension $\Delta_{\bar{\phi}\phi}^{(\text{bound})}$ as a function of the dimension of ϕ . Here we do not assume that $\bar{\phi}\phi$ is the scalar with the lowest dimension in the OPE $\bar{\phi} \times \phi$. The shaded area is excluded. In this plot we use $\Lambda = 21$.

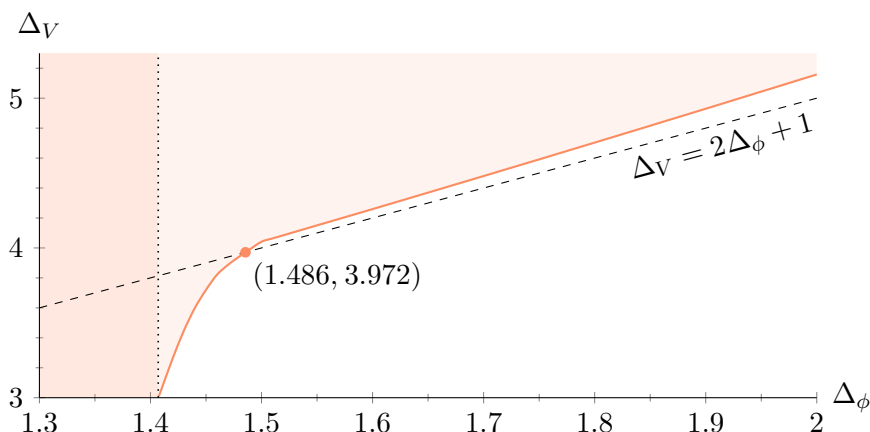


Figure 4. Upper bound on the dimension of the leading superconformal primary vector operator in the OPE $\bar{\phi} \times \phi$ as a function of the dimension of ϕ . The shaded area is excluded. Everything to the left of the vertical dotted line at $\Delta_\phi = 1.407$ is excluded due to the assumption that there is no ϕ^2 operator. The generalized free theory dashed line $\Delta_V = 2\Delta_\phi + 1$ as well as its intersection with the bound are also shown. In this plot we use $\Lambda = 21$.

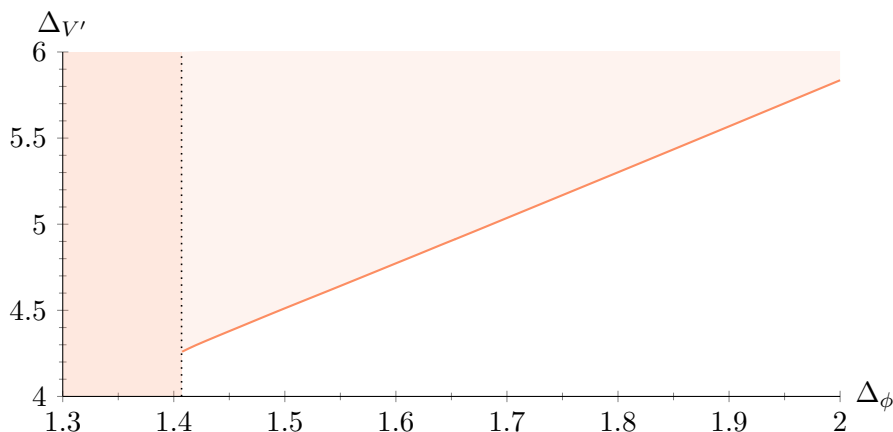


Figure 5. Upper bound on the dimension of the second superconformal primary vector operator in the OPE $\bar{\phi} \times \phi$ as a function of the dimension of ϕ , assuming that the first vector has dimension 3. the shaded area is excluded. everything to the left of the vertical dotted line at $\Delta_\phi = 1.407$ is excluded due to the assumption that there is no ϕ^2 operator. In this plot we use $\Lambda = 21$.

Note that for sufficiently small Δ_ϕ the bound excludes the line that would correspond to a generalized free theory with $\Delta_V = 2\Delta_\phi + 1$. This is natural, as our assumption that ϕ^2 is absent is not true in a generalized free theory. On the other hand, when $\Delta_\phi \geq 3/2$, the contribution in the sum rule corresponding to the chiral ϕ^2 operator is identical to one contained in the unprotected scalar contributions in $\phi \times \phi$. Thus, we expect the generalized free line should be allowed for $\Delta_\phi \geq 3/2$. Here we see that it crosses this line at $\Delta_\phi \sim 1.486$, compatible with this expectation.

Now that we have established the existence of a $U(1)_R$ current multiplet, we can assume it to be in the spectrum and place an upper bound on the second spin-1 operator V' . The result is shown in figure 5. We see that $\Delta_{V'} \lesssim 4.25$ at the minimum value of Δ_ϕ .

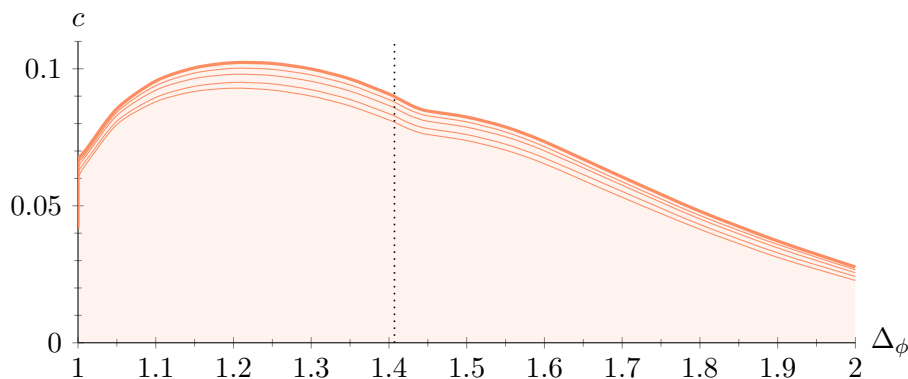


Figure 6. Lower bound on the central charge as a function of the dimension of ϕ . The shaded area is excluded. For the strongest bound (thick line) we use $\Lambda = 29$, while for the weaker bounds (thin lines) we use $\Lambda = 21, 23, 25, 27$ (from bottom to top). The dotted line is at $\Delta_\phi = 1.407$.

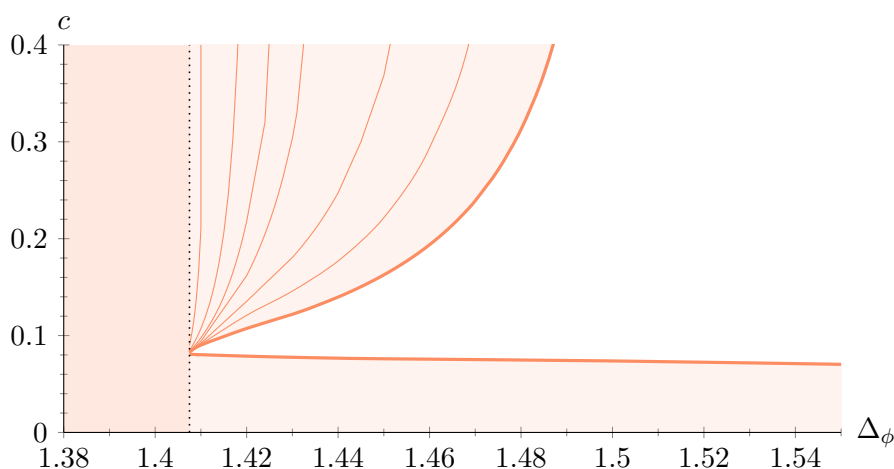


Figure 7. Lower and upper bounds on the central charge as a function of the dimension of ϕ , with the assumption that there is no ϕ^2 operator and all vector operators but the first one obey $\Delta_{V_{\text{other}}} \geq 4.1$ (thick upper bound line). The thinner upper bound lines correspond to $\Delta_{V_{\text{other}}} \geq 3.1, 3.3, 3.5, 3.7, 3.9, 4$ (from left to right). The shaded area is excluded. In this plot we use $\Lambda = 21$.

We can also compute general lower bounds on the central charge c , using that the OPE coefficient $\lambda_{V_R}^2 = \Delta_\phi^2/72c$. Here our normalization is such that $c_{\text{free}} = 1/24$ for a free chiral multiplet. Similar bounds were computed in [29]. Here these bounds are shown in figure 6 for $\Lambda = 21, 23, \dots, 29$. As in [29], these bounds drop very sharply as $\Delta_\phi \rightarrow 1$ so as to be compatible with the free theory value $c_{\text{free}} = 1/24$.

We can also impose a gap until the second spin-1 dimension $\Delta_{V'}$ and find upper bounds on c for each value of the gap. These bounds are shown in figure 7 at $\Lambda = 21$, where we have also imposed that there is no ϕ^2 operator. We see that the upper and lower bounds meet at the minimum value of Δ_ϕ , essentially uniquely fixing the central charge at this point, with $c \simeq .081$ at $\Lambda = 21$.

On the other hand, as seen in figure 6, our bounds have not yet converged, so the location of this unique point in $\{\Delta_\phi, c\}$ space will change somewhat at larger values of Λ . We have explored the location of this point up to $\Lambda = 35$, shown in figure 8. Our

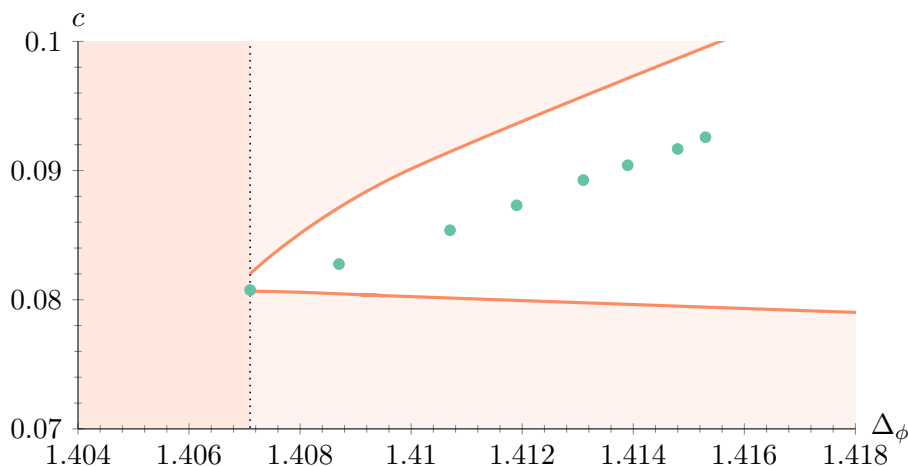


Figure 8. Lower and upper bounds on the central charge as a function of the dimension of ϕ , with the assumptions that there is no ϕ^2 operator and that all vector operators but the first one obey $\Delta_{V_{\text{other}}} \geq 4.1$. The shaded area is excluded. Here we use $\Lambda = 21$ for the bounds. The green points are allowed points closest to the corresponding lower bound for $\Lambda = 21, 23, \dots, 35$ (from left to right).

strongest bound is $\Delta_\phi \geq 1.415$ at $\Lambda = 35$. In this plot we also compare these points to the upper and lower bounds on c computed at $\Lambda = 21$ and $\Delta_{V'} \geq 4.1$ ($\Delta_{V'} \geq 4.2$ seems to be excluded at $\Lambda = 35$). Unfortunately, the location has not yet completely converged at $\Lambda = 35$, but there is a striking linear relation between Δ_ϕ and c , given approximately by $c \approx 1.454\Delta_\phi - 1.965$. Moreover, as we increase Λ the rate of convergence appears to be well-described by a fit that is linear in $1/\Lambda$ (similar to the fit done in [28]),

$$\{\Delta_\phi(\Lambda), c(\Lambda)\} \approx \left\{ 1.428 - \frac{0.441}{\Lambda}, 0.111 - \frac{0.642}{\Lambda} \right\}. \quad (2.2)$$

These fits are shown in figure 9. While these extrapolations should be taken with a grain of salt, it is intriguing that the minimal point may be converging to $c(\infty) = 1/9$ or $c(\infty)/c_{\text{free}} = 8/3$. If the minimal 4D $\mathcal{N} = 1$ SCFT exists and has a simple rational central charge, this is our current best conjecture.² It is also possible that Δ_ϕ is converging to the rational value $\Delta_\phi(\infty) = 10/7$.

We finish with some preliminary explorations of the higher spectrum. In figure 10 we show the upper bound on the dimension of the second nonchiral scalar in $\bar{\phi} \times \phi$, assuming that the first saturates its upper bound and also assuming the chiral ring relation $\phi^2 = 0$. Based on this we obtain the estimate $\Delta_{R'} \lesssim 7.2$.

In figure 11 we show an upper bound on the leading spin-2 superconformal primary in $\bar{\phi} \times \phi$ assuming $\phi^2 = 0$ in the chiral ring. At least at $\Lambda = 21$, this bound is very close to the generalized free value when Δ_ϕ attains its minimal value, $\Delta_S \lesssim 4.82$. We do not know why this is the case, given that the chiral ring relation does not hold in the generalized free solution and we could potentially exclude this line for $\Delta_\phi < 3/2$.

²If this conjecture is true, the bounds of [50] would then imply that $\frac{1}{18} \leq a \leq \frac{1}{6}$.

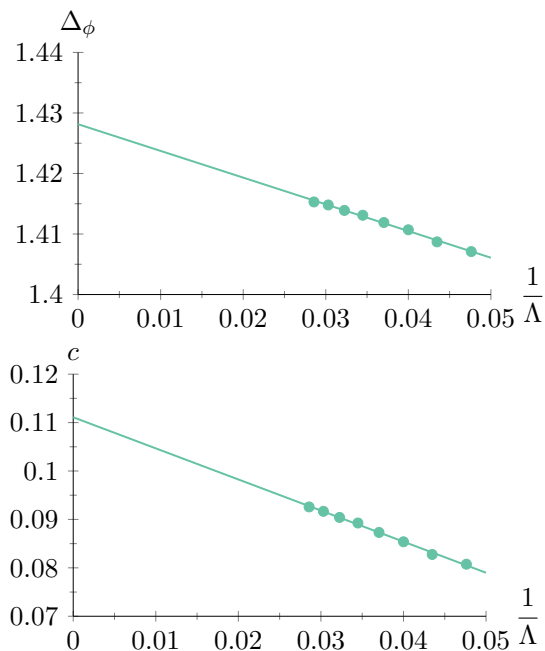


Figure 9. Linear extrapolations of the position of the minimal value of Δ_ϕ (assuming ϕ^2 is absent) and the corresponding value of c as a function of the inverse cutoff $1/\Lambda$.

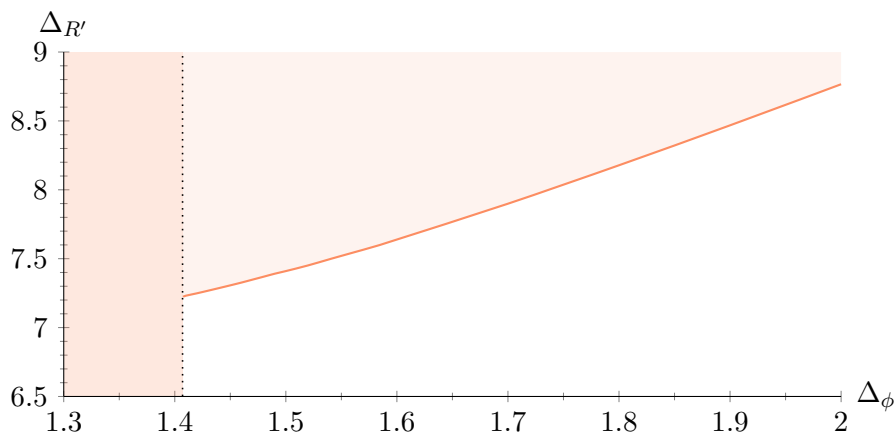


Figure 10. Upper bound on the dimension of the second superconformal primary real scalar in the OPE $\bar{\phi} \times \phi$ as a function of the dimension of ϕ , assuming that the dimension of $\bar{\phi}\phi$ saturates its bound, i.e. $\Delta_{\bar{\phi}\phi} = \Delta_{\bar{\phi}\phi}^{(\text{bound})}$. The shaded area is excluded. Everything to the left of the vertical dotted line at $\Delta_\phi = 1.407$ is excluded due to the assumption that there is no ϕ^2 operator. In this plot we use $\Lambda = 21$.

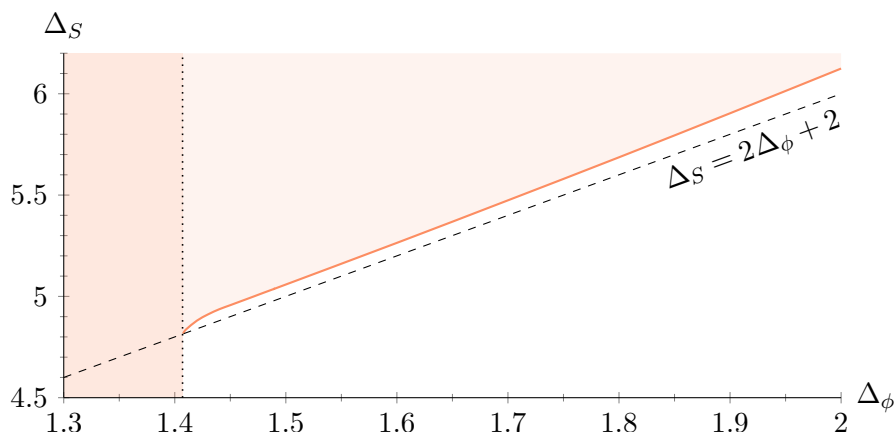


Figure 11. Upper bound on the dimension of the leading superconformal primary spin-2 operator in the OPE $\bar{\phi} \times \phi$ as a function of the dimension of ϕ . The shaded area is excluded. Everything to the left of the vertical dotted line at $\Delta_\phi = 1.407$ is excluded due to the assumption that there is no ϕ^2 operator. The generalized free theory dashed line $\Delta_S = 2\Delta_\phi + 2$ is also shown. In this plot we use $\Lambda = 21$.

It will be interesting in future studies to see how much of these allowed regions are compatible with the conditions of crossing symmetry for larger systems of correlators — in particular we would like to know whether our minimal solution survives and can be isolated e.g. using the condition that the $\phi \times \bar{\phi}\phi$ OPE contains a gap between ϕ and the next scalar operator. We hope that pursuing a mixed correlator study will lead to small islands similar to what was found in [4, 5, 13]. It would also be interesting to see if there are corresponding minimal theories with more general chiral ring relations $\phi^n = 0$. We hope to pursue these directions in a future study.

If this solution survives, the crucial question is to identify the underlying nature of this theory. The small central charge $c \simeq 1/9$ indicates that this theory must have a very small amount of matter and this is not very easy to accommodate in asymptotically-free 4D gauge theories. For example, $\mathcal{N} = 1$ SQCD theories all have central charge larger than 1. The properties of this theory are similar to Wess-Zumino models with a $W = \phi^3$ superpotential, but it has been known for a long time that such theories do not have an interacting fixed point in 4D [51, 52]. Thus, it may be that we have stumbled across a new non-Lagrangian $\mathcal{N} = 1$ SCFT. It would be interesting to better understand if it could arise as a deformation of a known non-Lagrangian theory such as one of the Argyres-Douglas fixed points [53], or perhaps by coupling a known $\mathcal{N} = 1$ SCFT to a topological field theory [54]. We leave further exploration of these possibilities to future work.

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