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# Two-loop application of the Breitenlohner-Maison/'t Hooft-Veltman scheme with non-anticommuting $\gamma_5$ : full renormalization and symmetry-restoring counterterms in an abelian chiral gauge theory

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**ABSTRACT:** We apply the BMHV scheme for non-anticommuting  $\gamma_5$  to an abelian chiral gauge theory at the two-loop level. As our main result, we determine the full structure of symmetry-restoring counterterms up to the two-loop level. These counterterms turn out to have the same structure as at the one-loop level and a simple interpretation in terms of restoration of well-known Ward identities. In addition, we show that the ultraviolet divergences cannot be canceled completely by counterterms generated by field and parameter renormalization, and we determine needed UV divergent evanescent counterterms. The paper establishes the two-loop methodology based on the quantum action principle and direct computations of Slavnov-Taylor identity breakings. The same method will be applicable to nonabelian gauge theories.

**KEYWORDS:** Anomalies in Field and String Theories, BRST Quantization, Renormalization Regularization and Renormalons

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**1 Introduction**

Dimensional Regularization (DReg) [1–4] is one of the most commonly employed schemes for practical calculations in perturbative quantum field theories. However, in this scheme, the  $\gamma_5$  Dirac matrix requires a special treatment since not all its 4-dimensional properties have straightforward  $d$ -dimensional extensions. This fact complicates calculations, and various alternative treatments have been proposed; the issue of  $\gamma_5$  has been known for a long time [5–12], but is of increasing importance in current investigations, see refs. [13–26] and ref. [12] for an extensive overview of the situation and further references.

In this paper, we follow the “*Breitenlohner-Maison-’t Hooft-Veltman*” (BMHV) scheme [4, 27–29] of Dimensional Regularization. In this scheme  $\gamma_5$  is non-anticommuting in  $d$  dimensions, but the scheme is rigorously established at all orders [30–33]. Gauge invariance and the related BRST symmetry are broken in intermediate steps by the modified algebraic relations.

For these reasons, the renormalization and counterterm structure in the BMHV scheme involves several BMHV-specific complications: the ultraviolet (UV) divergences cannot be cancelled by counterterms generated by field and parameter renormalization; additional, UV divergent evanescent counterterms (corresponding to operators which vanish in strictly 4 dimensions) are needed; and the breaking of BRST symmetry needs to be repaired by adding finite, symmetry-restoring counterterms.

In a previous paper [34] we have started a research programme on the rigorous practical application of this BMHV scheme to chiral gauge theories. In that reference we treated a general non-abelian massless gauge theory with fermionic and scalar matter fields, and we determined the full BMHV counterterm structure at the one-loop level. In particular we established a method to determine the required symmetry-restoring counterterms which compensate the breaking of BRST symmetry. The same method was developed and applied earlier in ref. [35] to study one-loop symmetry breakings in the BMHV scheme, and refs. [36–38] used the same strategy to study supersymmetry in the context of dimensional reduction up to the 3-loop level.

In the present work we present the first extension of this programme to the two-loop level. For the sake of clarity and to highlight conceptual and methodological issues, we particularize our calculations to the case of an abelian gauge theory with chiral fermions, a chiral QED (“ $\chi$ QED”) model. Our goal is to determine the full two-loop structure of the special counterterms in the BMHV scheme, i.e. the determine evanescent UV divergences, the deviations from parameter and field renormalization, and ultimately the symmetry-restoring counterterms.

The outline of the paper is as follows. After a brief reminder of notation and properties of the scheme in section 2 we define the abelian model in section 3. In this section we also set up the Slavnov-Taylor identity corresponding to BRST invariance and show that it is already broken at tree-level in the BMHV scheme. Section 4 summarizes the general strategy of renormalization and lays out the general procedure for finding UV divergent and finite symmetry-restoring counterterms. Sections 5, and 6 contain the one-loop counterterm results for this model. Both the singular, including the evanescent ones, as well as the BRST-restoring finite counterterms can also be derived by particularizing our previously obtained generic results [34] to this model.

Section 7 begins the two-loop analysis. It presents detailed results for the UV divergences of subrenormalized two-loop Green functions, and determines the required singular two-loop counterterms and their relationship to field and parameter renormalization. Subsection 8 presents first the evaluation of the two-loop breaking of the Slavnov-Taylor identity by the regularization, using the method described in section 4 and ref. [34]. It then presents the required symmetry-restoring two-loop counterterms. We also provide a consistency check by explicitly evaluating the analog of the usual QED Ward identities for two-, three- and four-point functions and checking that they are correctly restored as well.

## 2 Generalities and notations

We re-employ the notations and conventions of [34]. The  $d$ -dimensional space is split into a direct sum of 4-dimensional and  $d - 4 = -2\epsilon$ -dimensional subspaces. Lorentz covariants are extended into this  $d$ -dimensional space and consist of 4-dimensional (denoted by bars:  $\bar{\cdot}$ ) and  $(-2\epsilon)$ -dimensional (also called “evanescent”, denoted by hats:  $\hat{\cdot}$ ) components.

This split is performed for any tensorial quantity; this includes the definition and properties of metric tensors<sup>1</sup>  $g_{\mu\nu}$ ,  $\bar{g}_{\mu\nu}$  and  $\hat{g}_{\mu\nu}$ , of vectors  $k_\mu$ ,  $\bar{k}_\mu$  and  $\hat{k}_\mu$ , and of the  $\gamma$ -matrices  $\gamma^\mu$ ,  $\bar{\gamma}^\mu$  and  $\hat{\gamma}^\mu$ . In addition two intrinsically 4-dimensional objects are defined and appropriate properties for them are given. The first of them is the Levi-Civita symbol  $\epsilon_{\mu\nu\rho\sigma}$ . The second one is the  $\gamma_5$  matrix. They are related by  $\gamma_5 = \frac{-i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$ . In the BMHV scheme, the most important properties of the  $\gamma_5$  matrix are its commutation and anticommutation relations with the other  $\gamma$ -matrices, in particular

$$\{\gamma_5, \bar{\gamma}^\mu\} = 0, \quad \{\gamma_5, \gamma^\mu\} = \{\gamma_5, \hat{\gamma}^\mu\} = 2\gamma_5\hat{\gamma}^\mu, \quad (2.1a)$$

$$[\gamma_5, \hat{\gamma}^\mu] = 0, \quad [\gamma_5, \gamma^\mu] = [\gamma_5, \bar{\gamma}^\mu] = 2\gamma_5\bar{\gamma}^\mu. \quad (2.1b)$$

These relations will be used throughout all calculations in the present paper. They are the root of the breaking of symmetries and the appearance of UV divergences associated with purely evanescent operators.

## 3 Right-handed chiral QED ( $\chi$ QED) and its extension to $d$ dimensions

The present paper is devoted to the first 2-loop application of the method described in ref. [34]. We restrict ourselves to the abelian U(1) case without scalar fields and denote the corresponding model as  $\chi$ QED. The model may be viewed either as a chiral version of QED with purely right-handed fermion couplings, or as a variant of the U(1) part of the electroweak Standard Model. Since the adjoint representation is trivial, trilinear and quartic gauge boson interactions as well as ghost-gauge interaction are absent. The fermionic generators are reduced to the hypercharge, with opposite sign for the conjugate representation. The model is first defined in 4 dimensions, then extended to  $d$  dimensions, providing the respective Lagrangian, BRST transformations and Slavnov-Taylor identities, with motivation for particular choice for the evanescent part of the fermion kinetic term and for the fermionic interaction term. We then specify the BRST breaking of the model at tree-level. Finally we collect the symmetry identities defining the model at higher orders.

### 3.1 $\chi$ QED in 4 dimensions

In  $\chi$ QED, the only generator is the hypercharge, which we can assume to be diagonal,

$$\mathcal{Y}_{Rij} \equiv (\text{diag}\{\mathcal{Y}_R^1, \dots, \mathcal{Y}_R^{N_f}\})_{ij},$$

where  $N_f$  is the number of fermion flavours. The 4-dimensional classical Lagrangian of the model reads:

$$\mathcal{L} = i\bar{\psi}_{Ri}\not{D}_{ij}\psi_{Rj} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \bar{c}\partial^2 c + \rho^\mu sA_\mu + \bar{R}^i s\psi_{Ri} + R^i s\bar{\psi}_{Ri}, \quad (3.1)$$

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<sup>1</sup>Our convention for the 4-dimensional metric signature is mostly minus, i.e.  $(+1, -1, -1, -1)$ .

where only purely right-handed fermions  $\psi_R$  appear. In general, we use the standard right/left chirality projectors  $\mathbb{P}_R = (\mathbb{1} + \gamma_5)/2$  and  $\mathbb{P}_L = (\mathbb{1} - \gamma_5)/2$  and abbreviate  $\psi_{R/L} = \mathbb{P}_{R/L}\psi$ . The left-handed fermions  $\psi_L$  are thus decoupled from the theory. The covariant derivative acting on the fermion field is defined in the diagonal basis for couplings by

$$D_{ij}^\mu = \partial^\mu \delta_{ij} - ieA^\mu \mathcal{Y}_{Rij}, \quad (3.2)$$

and the field strength tensor is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (3.3)$$

In order not to have anomalies in  $\chi$ QED, the following *anomaly cancellation condition* is imposed on the hypercharge couplings,

$$\text{Tr}(\mathcal{Y}_R^3) = 0. \quad (3.4)$$

Next, the Lagrangian contains an  $R_\xi$  gauge fixing term with gauge parameter  $\xi$  and a corresponding Faddeev-Popov ghost kinetic term. The last three terms of eq. (3.1) are the BRST transformations of the physical fields, coupled to external sources (or Batalin-Vilkovisky “anti-fields”, [39–41]), where the external sources do not transform under BRST transformations. The non-vanishing BRST transformations are

$$sA_\mu = \partial_\mu c, \quad (3.5a)$$

$$s\psi_i = s\psi_{Ri} = ie c \mathcal{Y}_{Rij} \psi_{Rj}, \quad (3.5b)$$

$$s\bar{\psi}_i = s\bar{\psi}_{Ri} = ie \bar{\psi}_{Rj} c \mathcal{Y}_{Rji} \quad (3.5c)$$

$$s\bar{c} = B \equiv -\frac{1}{\xi} \partial A, \quad (3.5d)$$

where “ $s$ ” is the generator of the BRST transformation, which acts as a fermionic differential operator and is nilpotent for any linear combination of fields. The last of these equations also introduces the auxiliary Nakanishi-Lautrup field  $B$ , which is integrated out from the action in eq. (3.1) and in the rest of this paper. The 4-dimensional tree-level action

$$S_0^{(4D)} = \int d^4x \mathcal{L} \quad (3.6)$$

satisfies the following Slavnov-Taylor identity

$$\mathcal{S}(S_0^{(4D)}) = 0, \quad (3.7)$$

where the Slavnov-Taylor operation is given for a general functional  $\mathcal{F}$  as

$$\mathcal{S}(\mathcal{F}) = \int d^4x \left( \frac{\delta \mathcal{F}}{\delta \rho^\mu} \frac{\delta \mathcal{F}}{\delta A_\mu} + \frac{\delta \mathcal{F}}{\delta R^i} \frac{\delta \mathcal{F}}{\delta \psi_i} + \frac{\delta \mathcal{F}}{\delta R^i} \frac{\delta \mathcal{F}}{\delta \bar{\psi}_i} + B \frac{\delta \mathcal{F}}{\delta \bar{c}} \right), \quad (3.8)$$

where again  $B$  is treated as an abbreviation to its value given in eq. (3.5d). As usual in the context of algebraic renormalization, several additional functional identities hold. In particular all functional derivatives of  $S_0^{(4D)}$  with respect to the fields  $c$ ,  $\bar{c}$  or  $\rho^\mu$

	$A_\mu$	$\bar{\psi}_i, \psi_i$	$c$	$\bar{c}$	$B$	$\rho^\mu$	$R^i, \bar{R}^i$	$\partial_\mu$	$s$
mass dimension	1	3/2	0	2	2	3	5/2	1	0
ghost number	0	0	1	-1	0	-1	-1	0	1
(anti)commutativity	+1	-1	-1	-1	+1	-1	+1	+1	-1

**Table 1.** List of fields, external sources and operators, and their quantum numbers.

are linear in the propagating fields, and one may write down identities of the form  $\delta S_0^{(4D)}/\delta c(x) = (\text{linear expression})$ . Such identities may be required to hold at all orders as part of the definition of the theory.<sup>2</sup> We highlight first the so-called ghost equation

$$\left(\frac{\partial}{\partial \bar{c}} + \partial_\mu \frac{\partial}{\partial \rho_\mu}\right) S_0^{(4D)} = 0, \quad (3.9)$$

which is a linear combination which has analogues also in the non-abelian case.<sup>3</sup> Second, the so-called antighost equation, based on  $\delta S_0^{(4D)}/\delta c(x)$ , contains the essence of the original gauge transformations. Combining it with the Slavnov-Taylor identity yields the functional form of the abelian Ward identity (for extensive discussions of the more general case and the importance to the Standard Model and extensions see e.g. [42–44]). Here this functional Ward identity reads

$$\left(\partial^\mu \frac{\delta}{\delta A^\mu(x)} - ie \mathcal{Y}_R^j \sum_\phi (\pm) \phi(x) \frac{\delta}{\delta \phi(x)}\right) S_0^{(4D)} = -\partial^2 B(x). \quad (3.10)$$

The summation extends over the charged fermions and their sources,  $\phi \in \{\psi_{R_j}, \bar{\psi}_{R_j}, R^j, \bar{R}^j\}$ , and the signs are  $+, -, +, -$ , respectively. Finally, we summarize in table 1 a list of the quantum numbers (mass dimension, ghost number and (anti)commutativity) of the fields and the external sources of the theory, that are necessary for building the whole set of all possible renormalizable mass-dimension  $\leq 4$  field-monomial operators with a given ghost number.

### 3.2 The $\chi$ QED in $d$ dimensions and its BRST breaking

The extension of the  $\chi$ QED model eq. (3.1) to  $d$  dimensions is not unique, due to the fermionic kinetic and interaction terms. Here we follow the procedure used in ref. [34]. The extension to  $d$  dimensions requires fully  $d$ -dimensional fermion propagators, so as to ensure that Feynman diagrams involving fermions can be regularized. This is achieved by introducing a left-chiral U(1)-singlet fermion into the kinetic part of the Lagrangian, thus promoting the intrinsically 4-dimensional  $\chi$ QED fermionic kinetic term to a full  $d$ -dimensional one. On the other side the fermion-gauge boson interaction is chosen to be fully chiral-projected, with right-handed fermions only. This procedure, together with the straightforward extension of the other terms in eq. (3.1) to  $d$  dimensions, leads to the

<sup>2</sup>If  $B$  is not integrated out, the same is true for the functional derivative  $\delta S_0^{(4D)}/\delta B(x)$ .

<sup>3</sup>It can be obtained in general from evaluating  $\delta S(S_0^{(4D)})/\delta B$  if the field  $B$  is not eliminated.

tree-level action  $S_0$ ,

$$\begin{aligned}
 S_0 = \int d^d x \left( i\bar{\psi}_i \hat{\partial} \psi_i + e \mathcal{Y}_{Rij} \bar{\psi}_{Ri} A \psi_{Rj} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 \right. \\
 \left. - \bar{c} \partial^2 c + \rho^\mu (\partial_\mu c) + i e \bar{R}^i c \mathcal{Y}_{Rij} \psi_{Rj} + i e \bar{\psi}_{Ri} c \mathcal{Y}_{Rij} R^j \right) \quad (3.11) \\
 \equiv \sum_i S_{\bar{\psi}\psi}^i + \sum_i \overline{S_{\bar{\psi}_R A \psi_R}^i} + S_{AA} + S_{\text{g-fix}} + S_{\bar{c}c} + S_{\rho c} + S_{\bar{R}c\psi_R} + S_{Rc\bar{\psi}_R},
 \end{aligned}$$

where the last line introduces explicit abbreviations for each of the eight terms in the action.

The rest of this subsection is devoted to the discussion of BRST symmetry of the tree level action which follows the corresponding discussion in ref. [34] for a generic non-Abelian model. First we can define  $d$ -dimensional BRST transformations and a  $d$ -dimensional Slavnov-Taylor operation  $\mathcal{S}_d$  by straightforward extensions of the 4-dimensional versions. Then it is elementary to see that the  $d$ -dimensional action may be written as the sum of two parts, an “invariant” and an “evanescent” part,

$$S_0 = S_{0,\text{inv}} + S_{0,\text{evan}}, \quad (3.12a)$$

$$S_{0,\text{evan}} = \int d^d x i\bar{\psi}_i \hat{\partial} \psi_i. \quad (3.12b)$$

The first part is BRST invariant even in  $d$  dimensions, i.e. it satisfies

$$s_d S_{0,\text{inv}} = 0, \quad (3.13a)$$

$$\mathcal{S}_d(S_{0,\text{inv}}) = 0. \quad (3.13b)$$

The second part  $S_{0,\text{evan}}$  consists solely of one single, evanescent fermion kinetic term, but it breaks  $d$ -dimensional BRST invariance and the tree-level Slavnov-Taylor identity,

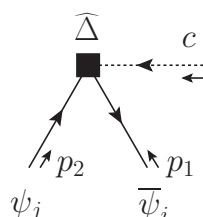
$$s_d S_0 = s_d S_{0,\text{evan}} \equiv \hat{\Delta}, \quad (3.14a)$$

$$\mathcal{S}_d(S_0) = \hat{\Delta}. \quad (3.14b)$$

The breaking  $\hat{\Delta}$  is an integrated evanescent operator, comprised of one ghost and two fermions,

$$\hat{\Delta} = \int d^d x e \mathcal{Y}_{Rij} c \left\{ \bar{\psi}_i \left( \overleftarrow{\hat{\partial}}_{\mathbb{P}_R} + \overrightarrow{\hat{\partial}}_{\mathbb{P}_L} \right) \psi_j \right\} \equiv \int d^d x \hat{\Delta}(x), \quad (3.15)$$

and generates an interaction vertex whose Feynman rule (with all momenta incoming) is



$$= e \mathcal{Y}_{Rij} \left( \hat{p}_1^{\mathbb{P}_R} + \hat{p}_2^{\mathbb{P}_L} \right). \quad (3.16)$$

For later use we also need the expression for the linearized Slavnov-Taylor operator [45],  $b_d$ , defined such that

$$\mathcal{S}_d(S_0 + \hbar\mathcal{F}) = \mathcal{S}_d(S_0) + \hbar b_d\mathcal{F} + \mathcal{O}(\hbar^2). \quad (3.17)$$

Its functional definition in  $\chi$ QED and its relation to  $s_d$  are:

$$\begin{aligned} b_d &= \int d^d x \left( \frac{\delta S_0}{\delta \rho^\mu} \frac{\delta}{\delta A_\mu} + \frac{\delta S_0}{\delta A_\mu} \frac{\delta}{\delta \rho^\mu} + \frac{\delta S_0}{\delta \bar{R}^i} \frac{\delta}{\delta \psi_i} + \frac{\delta S_0}{\delta \psi_i} \frac{\delta}{\delta \bar{R}^i} + \frac{\delta S_0}{\delta R^i} \frac{\delta}{\delta \bar{\psi}_i} + \frac{\delta S_0}{\delta \bar{\psi}_i} \frac{\delta}{\delta R^i} + B \frac{\delta}{\delta \bar{c}} \right) \\ &= s_d + \int d^d x \left( \frac{\delta S_0}{\delta A_\mu} \frac{\delta}{\delta \rho^\mu} + \frac{\delta S_0}{\delta \psi_i} \frac{\delta}{\delta \bar{R}^i} + \frac{\delta S_0}{\delta \bar{\psi}_i} \frac{\delta}{\delta R^i} \right). \end{aligned} \quad (3.18)$$

### 3.3 Defining symmetry relations for the renormalized theory

At higher orders a set of symmetry identities can be imposed on the finite, renormalized theory. These identities may be viewed as part of the definition of the model; they constrain the regularization/renormalization procedure and particularly determine the symmetry-restoring counterterms. Here we collect the relevant symmetry identities which are the basis of the subsequent sections. All following identities are valid at tree level by construction. In principle, it is crucial to establish that they can be fulfilled also at higher orders. For the present model this is clear from the general analysis of algebraic renormalization of gauge theories<sup>4</sup> and the anomaly condition eq. (3.4).

All identities are formulated as functional identities for the fully renormalized, finite and 4-dimensional effective action  $\Gamma_{\text{ren}}$ , which formally satisfies  $\Gamma_{\text{ren}} = S_0^{(4D)} + \mathcal{O}(\hbar)$ . The first and most important symmetry is BRST invariance, which is expressed as the Slavnov-Taylor identity

$$\mathcal{S}(\Gamma_{\text{ren}}) = 0 \quad (3.19)$$

for the renormalized theory. In addition, we require a set of more trivial relations

$$\frac{\delta \Gamma_{\text{ren}}}{\delta c(x)} = \frac{\delta S_0^{(4D)}}{\delta c(x)}, \quad \frac{\delta \Gamma_{\text{ren}}}{\delta \bar{c}(x)} = \frac{\delta S_0^{(4D)}}{\delta \bar{c}(x)}, \quad \frac{\delta \Gamma_{\text{ren}}}{\delta \rho^\mu(x)} = \frac{\delta S_0^{(4D)}}{\delta \rho^\mu(x)}. \quad (3.20)$$

These identities correspond to the absence of higher-order corrections involving the fields  $c$ ,  $\bar{c}$ ,  $\rho^\mu$  (a similar identity for the  $B$ -field is valid in case  $B$  is not yet eliminated). They can be imposed since the respective derivatives of the tree-level action are linear in the dynamical fields as described between eqs. (3.8) and (3.10).

Like at tree level, the Ward identity

$$\left( \partial^\mu \frac{\delta}{\delta A^\mu(x)} - ie \mathcal{Y}_R^j \sum_\phi (\pm) \phi(x) \frac{\delta}{\delta \phi(x)} \right) \Gamma_{\text{ren}} = -\partial^2 B(x), \quad (3.21)$$

is an automatic consequence of the Slavnov-Taylor identity eq. (3.19) combined with the antighost equation in eq. (3.20). It is not manifestly valid at higher orders but it will be

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<sup>4</sup>See refs. [46–49] for important treatments of abelian theories in such contexts and refs. [45, 47] for general overviews.



automatically valid once the Slavnov-Taylor identity holds. In fact we will see that the breaking and restoration of the Slavnov-Taylor identity can be well interpreted in terms of the Ward identity.

In what follows we will only refer to BRST invariance and the Slavnov-Taylor identity, which are the most important symmetry requirements. The requirements eq. (3.20) are manifestly valid at all steps and individually for the regularized Green functions and for the counterterms.

## 4 Multiloop regularization and renormalization formulae

Dimensional regularization using the BMHV scheme inevitably breaks BRST symmetry, which therefore has to be restored at any order in perturbation for ensuring the consistency of the theory. In this section we collect the general formulae governing the construction of the renormalized theory and the procedure for finding singular (i.e. UV divergent) and finite symmetry-restoring counterterms. The calculational details at the one-loop and two-loop level will be given in the following sections.

In general, counterterms contain UV divergent (“singular”) and finite contributions. As noted in our previous paper [34], in dimensional regularization it is useful to further subdivide the counterterms into five types: singular BRST invariant and noninvariant (evanescent or non-evanescent) counterterms, finite BRST invariant and noninvariant (BRST restoring) counterterms as well as finite evanescent counterterms,

$$S_{\text{ct}} = S_{\text{sct,inv}} + S_{\text{sct,noninv}} + S_{\text{fct,inv}} + S_{\text{fct,restore}} + S_{\text{fct,evan}} \equiv S_{\text{sct}} + S_{\text{fct}}. \quad (4.1)$$

In this and the following equations, symbols with no index denote all-order quantities. For the following perturbative expressions we will also use an upper index  $i$  for quantities of precisely order  $i$ , an upper index  $(i)$  for quantities up to and including order  $i$ . For example, the counterterm action and the bare action can be split as

$$S_{\text{bare}} = S_0 + S_{\text{ct}}, \quad S_{\text{ct}} = \sum_{i=1}^{\infty} S_{\text{ct}}^i, \quad S_{\text{ct}}^{(i)} = \sum_{j=1}^i S_{\text{ct}}^j. \quad (4.2)$$

The perturbative construction of the effective action in dimensional regularization and renormalization is performed iteratively at each order of  $\hbar$  (or loops), starting from the tree-level action  $S_0$  of order  $\hbar^0$ . Then, at each higher loop order  $i \geq 1$  a counterterm action  $S_{\text{ct}}^i$  has to be constructed. The counterterms are subject to the two conditions that the renormalized theory is UV finite and in agreement with all required symmetries listed in section 3.3.<sup>5</sup>

In general, at each order  $i$  one may distinguish Green functions at various levels of regularization, partial or full renormalization. Of particular importance are “subloop-renormalized” Green functions and the corresponding effective action. To keep the notation

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<sup>5</sup>As mentioned in section 3.3, in what follows we will only refer to BRST invariance and the Slavnov-Taylor identity, which are the most important symmetry requirements. The other related symmetry requirements eq. (3.20) are manifestly valid at all steps.

simple in the present paper we use the symbol  $\Gamma^i$  for this subloop-renormalized effective action of order  $i$ . By definition this is obtained at order  $i$  by using Feynman rules from the tree-level action and counterterms up to order  $i - 1$ . By constructing and including singular counterterms of the order  $i$  we obtain the quantity

$$\Gamma^i + S_{\text{sct}}^i = \text{finite for } \epsilon \rightarrow 0. \tag{4.3}$$

This equation determines the singular counterterms unambiguously, including their evanescent parts, introduced in eq. (4.1). By also including additional, finite counterterms of the order  $i$  we obtain

$$\Gamma_{\text{DReg}}^i := \Gamma^i + S_{\text{sct}}^i + S_{\text{fct}}^i. \tag{4.4}$$

This resulting effective action is finite at this order and essentially renormalized but still contains the variable  $\epsilon$  and evanescent quantities. The fully renormalized effective action is given by taking the limit  $d \rightarrow 4$  and by setting all evanescent quantities to zero. This operation is denoted as

$$\Gamma_{\text{ren}}^i := \text{LIM}_{d \rightarrow 4} \Gamma_{\text{DReg}}^i. \tag{4.5}$$

The basic procedure to determine the finite counterterms  $S_{\text{fct}}^i$ , specifically their symmetry-restoring part, is as follows. The ultimate symmetry requirement is the Slavnov-Taylor identity expressing BRST invariance for the fully renormalized theory, which can be written as

$$\text{LIM}_{d \rightarrow 4} (\mathcal{S}_d(\Gamma_{\text{DReg}})) = 0. \tag{4.6}$$

As discussed in detail in ref. [34] there are several possibilities to extract the symmetry-restoring counterterms from this equation. Like in that reference, we choose again to use the regularized quantum action principle [31], which allows to rewrite<sup>6</sup>

$$\mathcal{S}_d(\Gamma_{\text{DReg}}) = (\widehat{\Delta} + \Delta_{\text{ct}}) \cdot \Gamma_{\text{DReg}}, \tag{4.7}$$

where the insertions  $\widehat{\Delta}$  and  $\Delta_{\text{ct}}$  in the present abelian theory are given as

$$\widehat{\Delta} = \mathcal{S}_d(S_0) = s_d S_0, \tag{4.8a}$$

$$\widehat{\Delta} + \Delta_{\text{ct}} = \mathcal{S}_d(S_0 + S_{\text{ct}}), \tag{4.8b}$$

$$\Delta_{\text{ct}} \equiv s_d S_{\text{ct}}. \tag{4.8c}$$

The first two equations are valid in general, the third one is valid in the present context because, as we will see in the concrete calculations, there will be no counterterms involving external fields. The previous equations can be plugged into eq. (4.6) and perturbatively expanded at the order  $i$ . This leads to

$$\text{LIM}_{d \rightarrow 4} \left( \widehat{\Delta} \cdot \Gamma_{\text{DReg}}^i + \sum_{k=1}^{i-1} \Delta_{\text{ct}}^k \cdot \Gamma_{\text{DReg}}^{i-k} + \Delta_{\text{ct}}^i \right) = 0, \quad \text{for } i \geq 1, \tag{4.9}$$

---

<sup>6</sup>The same equation has been presented specifically for the one-loop case in ref. [34] and for the general case in ref. [36]. Ref. [35] presents a slightly different version. All versions of the equation become equal in the present context of an abelian gauge theory where there are no counterterms involving external fields.

which explicitly exhibits the genuine  $i$ -loop counterterm via  $\Delta_{\text{ct}}^i$  (see eq. (4.8)). The fact that the limit  $d \rightarrow 4$  exists provides a consistency check on the divergent part of  $\Delta_{\text{ct}}^i$ , which contains the singular counterterms  $S_{\text{sct}}^i$ . The finite part of the equation determines the finite part of  $\Delta_{\text{ct}}^i$ . This determines the desired finite counterterms not unambiguously. Rather, their symmetry-restoring parts are fixed, while it remains possible to add finite symmetric counterterms (to adjust renormalization conditions) and finite evanescent counterterms, which are not needed in the following.

## 5 Evaluation of the one-loop singular counterterm action $S_{\text{sct}}^{(1)}$ in $\chi$ QED versus QED

We start by evaluating the one-loop (order  $\hbar^1$ ) singular counterterm action  $S_{\text{sct}}^{(1)}$ , defined from the divergent parts of the one-loop diagrams constructed with the Feynman rules of the tree-level action  $S_0$ . These counterterms are basically determined by eq. (4.3) at the one-loop level, and they will be part of the dimensionally-regularized one-loop effective action  $\Gamma_{\text{DReg}}$ .

The calculations are performed in  $d = 4 - 2\epsilon$  dimensions. We use notational conventions from [34]. Here and in the rest of the paper, the necessary Feynman diagrams have been computed using the `Mathematica` packages `FeynArts` [50] and `FeynCalc` [51, 52]; the  $\epsilon$ -expansion of the amplitudes has been cross-checked using the `FeynCalc`'s interface `FeynHelpers` [53] to `Package-X` [54].

Since intermediate results can be obtained from the presentation of ref. [34], we immediately provide the full result for the singular one-loop counterterm action. It reads

$$S_{\text{sct},\chi\text{QED}}^1 = \frac{-\hbar e^2}{16\pi^2\epsilon} \left( \frac{2 \text{Tr}(\mathcal{Y}_R^2)}{3} \overline{S_{AA}} + \xi \sum_j (\mathcal{Y}_R^j)^2 \left( \overline{S_{\psi\psi_R}^j} + \overline{S_{\psi_R A \psi_R}^j} \right) + \frac{\text{Tr}(\mathcal{Y}_R^2)}{3} \int d^d x \frac{1}{2} \bar{A}_\mu \hat{\partial}^2 \bar{A}^\mu \right), \quad (5.1)$$

and it may be compared to the corresponding result of ordinary QED with Dirac fermions of charges  $\mathcal{Y}$ ,

$$S_{\text{sct},\text{QED}}^1 = \frac{-\hbar e^2}{16\pi^2\epsilon} \left( \frac{4 \text{Tr}(\mathcal{Y}^2)}{3} S_{AA} + \xi \sum_j (\mathcal{Y}^j)^2 \left( S_{\psi\psi}^j + S_{\psi A \psi}^j \right) \right). \quad (5.2)$$

Notice that we restore explicit  $\hbar$  order for every final result of the counterterm action from now on. Most of the monomials have already been introduced; the bar in  $\overline{S_{AA}}$  designates the fully 4-dimensional version of  $S_{AA}$ , and the additional terms  $\overline{S_{\psi\psi_R}^i}$ ,  $\overline{S_{\psi_R A \psi_R}^i}$  are the fully right-chiral-projected equivalents to their usual  $d$ -dimensional versions,

$$\overline{S_{\psi\psi_R}^i} = \int d^d x i \bar{\psi}_i \overleftrightarrow{\not{\partial}} \mathbb{P}_R \psi_i \equiv \int d^d x \frac{i}{2} \bar{\psi}_i \overleftrightarrow{\not{\partial}} \mathbb{P}_R \psi_i, \quad (5.3a)$$

$$\overline{S_{\psi_R A \psi_R}^i} = \int d^d x e \mathcal{Y}_R^i \bar{\psi}_i \overleftrightarrow{\not{A}} \mathbb{P}_R \psi_i. \quad (5.3b)$$

In both models there are three kinds of UV divergent Green functions, corresponding to the photon self energy, the fermion self energy and the fermion-photon interaction. The results eqs. (5.1) and (5.2) differ in three characteristic ways. Clearly, in  $\chi$ QED there are half as many fermionic degrees of freedom, hence the fermion loop contributions to the photon self energy generate the prefactor 2/3 instead of 4/3. In addition, the purely right-handed nature of the interaction leads to a purely evanescent divergent non-transverse contribution to the photon self energy in the second line of eq. (5.1). Finally, the fermion self energy and the fermion-photon interaction receive only purely 4-dimensional right-handed corrections in  $\chi$ QED, while in (non-chiral) QED these contributions remain  $d$ -dimensional.

Both  $\chi$ QED and ordinary QED models are abelian, and as a result there are no loop corrections involving ghosts or external BRST source fields. This property reflects the identities eq. (3.20) and persists at all orders. This implies in particular that the linearized Slavnov-Taylor operator  $b_d$  reduces to the BRST operator  $s_d$  when acting on the loop contributions of the effective action.

As in ref. [34] we can re-express the result for the singular one-loop counterterms  $S_{\text{sct}}^1$  in a structure reminiscent of the one appearing in the usual renormalization transformations, where fields renormalize multiplicatively as  $\varphi \rightarrow \sqrt{Z_\varphi} \varphi$ ,  $Z_\varphi \equiv 1 + \delta Z_\varphi$ , and the coupling constant renormalizes additively as  $e \rightarrow e + \delta e$ . The sum of the singular counterterms can be written as

$$S_{\text{sct}}^1 = S_{\text{sct,inv}}^1 + S_{\text{sct,evan}}^1, \tag{5.4}$$

where the first term arises in the usual way from a renormalization transformation, while the second term has a different structure. In detail, the first term can be obtained by applying the renormalization transformation  $S_{0,\text{inv}} \rightarrow S_{0,\text{inv}} + S_{\text{ct,inv}}$ , and it is given by

$$S_{\text{ct,inv}}^1 = \frac{\delta Z_A^1}{2} L_A + \frac{\delta Z_c^1}{2} L_c + \frac{\delta Z_{\psi_{Rj}}^1}{2} \overline{L_{\psi_{Rj}}} + \frac{\delta e^1}{e} L_e. \tag{5.5}$$

The one-loop renormalization constants  $\delta Z_\varphi$ ,  $\delta e$  agree with the usual ones (see e.g. [55–57]) and read

$$\delta Z_A^1 = \delta Z_c^1 = -2 \frac{\delta e^1}{e}, \tag{5.6a}$$

$$\delta Z_A^1 = \frac{-\hbar e^2}{16\pi^2 \epsilon} \frac{2 \text{Tr}(\mathcal{Y}_R^2)}{3}, \tag{5.6b}$$

$$\delta Z_{\psi_{Rj}}^1 = \frac{-\hbar e^2}{16\pi^2 \epsilon} \xi(\mathcal{Y}_R^j)^2. \tag{5.6c}$$

The first of these relations again reflects eq. (3.20) as in ordinary QED. The  $L_\varphi$  functionals corresponding to field renormalizations can be written in various ways, either as a field-numbering operators acting on the tree-level action or as total  $b_d$ -variations or in terms of the monomials of eq. (3.11). Here we provide the results in the form

$$L_A = b_d \int d^d x \tilde{\rho}^\mu A_\mu = 2S_{AA} + \overline{S_{\psi_A \psi_R}} - S_{\bar{c}c} - S_{\rho c}, \tag{5.7a}$$

where  $\tilde{\rho}^\mu = \rho^\mu + \partial^\mu \bar{c}$  is the natural combination arising from the ghost equation (3.9);

$$L_c = \int d^d x c(x) \frac{\delta S_0}{\delta c(x)} = S_{\bar{c}c} + S_{\rho c} + S_{\bar{R}c\psi_R} + S_{Rc\bar{\psi}_R}, \quad (5.7b)$$

$$\begin{aligned} L_{\psi_R} &= -b_d \int d^d x (\bar{R}^i \mathbb{P}_R \psi_i + \bar{\psi}_i \mathbb{P}_L R^i) \\ &= 2 \left( \int d^d x i \bar{\psi}_i \hat{\partial} \mathbb{P}_R \psi_i + \overline{S_{\bar{\psi}_A \psi_R}} \right) + i \bar{\psi}_i \hat{\partial} \psi_i \equiv \overline{L_{\psi_R}} + S_{0,\text{evan}} = \sum_i L_{\psi_{Ri}}. \end{aligned} \quad (5.7c)$$

The  $L_e$  functional corresponding to renormalization of the physical coupling can be expressed in terms of the monomials of eq. (3.11) or related to the field renormalization functionals as

$$L_e = e \frac{\partial S_0}{\partial e} = \overline{S_{\bar{\psi}_A \psi_R}} + S_{\bar{R}c\psi_R} + S_{Rc\bar{\psi}_R} = L_c + L_A - 2S_{AA}. \quad (5.8)$$

Despite the non-nilpotency of  $b_d$ , several of the  $L_\varphi$  are actually  $b_d$ -invariant in the following sense:

$$b_d L_A = 0, \quad b_d \overline{L_{\psi_R}} = 0. \quad (5.9)$$

In contrast,  $L_c$  is not  $b_d$ -invariant in this sense;<sup>7</sup> instead, it is easy to see that

$$b_d L_c = \hat{\Delta}, \quad (5.10)$$

with the same breaking as in eq. (3.15). As a result, also  $L_e$ , corresponding to gauge coupling renormalization, is not  $b_d$ -invariant. Note, however, that in the limit  $d \rightarrow 4$  and evanescent terms vanishing, all the  $L_\varphi$  functionals presented here become invariant under the linear  $b$  transformation in 4 dimensions.

Finally, the evanescent counterterms appearing in eq. (5.4) can be written as

$$S_{\text{sct,evan}}^1 = \frac{-\hbar e^2 \text{Tr}(\mathcal{Y}_R^2)}{16\pi^2 \epsilon} \frac{1}{3} \left( 2(\overline{S_{AA}} - S_{AA}) + \int d^d x \frac{1}{2} \bar{A}^\mu \hat{\partial}^2 \bar{A}_\mu \right). \quad (5.11)$$

For later use we record the corresponding BRST breaking of the singular one-loop counterterms. This breaking originates solely from the evanescent non-invariant second term of  $S_{\text{sct,evan}}^1$  and is given by

$$\Delta_{\text{sct}}^1 = s_d S_{\text{sct}}^1 = -\frac{\hbar e^2 \text{Tr}(\mathcal{Y}_R^2)}{16\pi^2 \epsilon} \frac{1}{3} \int d^d x (\bar{\partial}_\mu c) (\hat{\partial}^2 \bar{A}^\mu). \quad (5.12)$$

## 6 BRST symmetry breaking and its restoration; evaluation of the one-loop finite counterterm action $S_{\text{fct}}^1$

In the previous section we determined the singular counterterms action  $S_{\text{sct}}^1$ , eq. (5.4). Here we discuss the determination of symmetry-restoring counterterms  $S_{\text{fct}}^1$  at the one-loop level. We follow the general procedure outlined in section 4 but will be brief since the

<sup>7</sup>This fact appears to be in contradiction with a claim made in [35].

computation is essentially a special case of the one presented in ref. [34]. We begin by specializing the general formulae of section 4 to the one-loop case, then we present the results and a brief discussion.

At the one-loop level the structure of renormalization can be written as

$$\Gamma_{\text{DReg}}^{(1)} = \Gamma^{(1)} + S_{\text{sct}}^1 + S_{\text{fct}}^1, \tag{6.1a}$$

$$\Delta_{\text{ct}}^1 = \mathcal{S}_d(S_0 + S_{\text{ct}})^1, \tag{6.1b}$$

where the counterterms are subject to the conditions discussed in section 4, which here simplify to

$$S_{\text{sct}}^1 + \Gamma_{\text{div}}^1 = 0, \tag{6.2a}$$

$$(\widehat{\Delta} \cdot \Gamma^1 + \Delta_{\text{ct}}^1)_{\text{div}} = 0, \tag{6.2b}$$

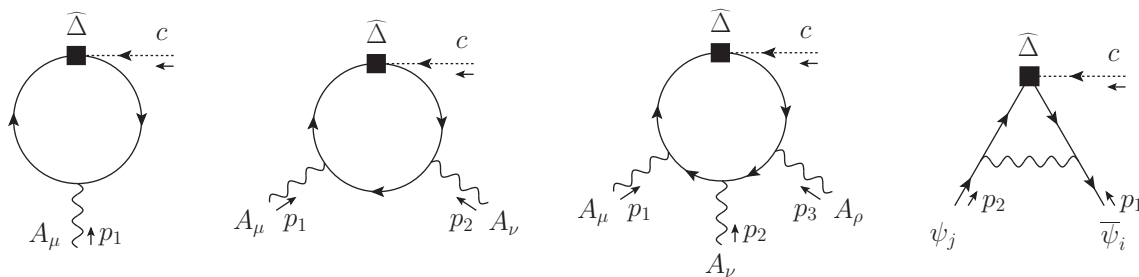
$$\text{LIM}_{d \rightarrow 4}(\widehat{\Delta} \cdot \Gamma^1 + \Delta_{\text{ct}}^1)_{\text{fin}} = 0. \tag{6.2c}$$

Here the subscripts ‘div,fin’ refer to the pure  $1/\epsilon$  pole part and the  $\epsilon$ -independent part, respectively. Compared to eq. (4.9) we dropped the index ‘DReg’ because the one-loop insertions arise from genuine one-loop diagrams and not from one-loop counterterms. Equation (6.2a) has already been satisfied in the previous section, and eq. (6.2b) must automatically hold by construction, providing a consistency check. The last equation determines the finite symmetry-restoring counterterms, with a remaining ambiguity of changing finite symmetric or evanescent counterterms. The equation can also be written as

$$N[\widehat{\Delta} \cdot \Gamma^1] + \Delta_{\text{fct}}^1 = 0, \tag{6.3}$$

which implicitly fixes the choice of the finite, evanescent counterterms. This version of the equation uses the result (5.12) that the BRST variation of the one-loop singular counterterms contains no finite term (which could in principle arise from the evaluation of  $s_d$ ), hence  $\Delta_{\text{ct}}|_{\text{fin}} = \Delta_{\text{fct}}$ . The symbol  $N[\mathcal{O}]$  denotes the Zimmermann-like definition [47, 58–60] of a renormalized local operator (also called “*normal product*”), defined as an insertion of a local operator  $\mathcal{O}$  and followed, in the context of Dimensional Regularization and Renormalization, by a minimal subtraction prescription [61].

In order to determine the finite counterterms we need to compute the quantity  $\widehat{\Delta} \cdot \Gamma^1$ , corresponding to the breaking of the Slavnov-Taylor identity or BRST symmetry by one-loop regularized Green functions. This is given by one-loop Feynman diagrams with one insertion of the vertex  $\widehat{\Delta}$ , the BRST breaking of the  $d$ -dimensional action given in eq. (4.8). In principle, infinitely many Feynman diagrams can give a nonzero result. However in most cases the result is purely evanescent or of order  $\epsilon$ . Only power-counting divergent diagrams can lead to a result which contributes to the above equations, i.e. which contains either a  $1/\epsilon$  pole or which is finite and survives in the  $\text{LIM}_{d \rightarrow 4}$ . The four contributing diagrams are shown in figure 1.



**Figure 1.** The four one-loop diagrams contributing to  $\widehat{\Delta} \cdot \Gamma^1$  in a relevant way. Only the first diagram provides UV divergent contributions, which are evanescent. All diagrams provide UV finite non-evanescent contributions, i.e. contributions which remain in the  $\text{LIM}_{d \rightarrow 4}$ .

The result of all these diagrams can be compactly written as an insertion in the effective action in terms of field monomials of ghost number one, as

$$\begin{aligned} \widehat{\Delta} \cdot \Gamma^1 = \frac{\hbar}{16\pi^2} \int d^d x \left[ \frac{e^2 \text{Tr}(\mathcal{Y}_R^2)}{3} \left( \frac{1}{\epsilon} (\bar{\partial}_\mu c) (\widehat{\partial}^2 \bar{A}^\mu) + (\bar{\partial}_\mu c) (\bar{\partial}^2 \bar{A}^\mu) \right) \right. & (6.4) \\ + \frac{e^4 \text{Tr}(\mathcal{Y}_R^4)}{3} c \bar{\partial}_\mu (\bar{A}^\mu \bar{A}^2) - \frac{5 + \xi}{6} e^3 \sum_j (\mathcal{Y}_R^j)^3 c \bar{\partial}^\mu (\bar{\psi}_j \bar{\gamma}_\mu \mathbb{P}_R \psi_j) & \\ \left. - \frac{2 e^3 \text{Tr}(\mathcal{Y}_R^3)}{3} \epsilon^{\mu\nu\rho\sigma} c (\bar{\partial}_\rho \bar{A}_\mu) (\bar{\partial}_\sigma \bar{A}_\nu) \right]. & \end{aligned}$$

In this equation, further terms of order  $\epsilon$  and evanescent terms of order  $\epsilon^0$  have been omitted. The first two terms correspond to the first diagram of figure 1; they involve an evanescent UV divergence and a UV finite, non-evanescent term. Their interpretation is the violation of the Slavnov-Taylor identity for the photon self energy (describing essentially its transversality). The other terms are UV finite and non-evanescent. They correspond in an obvious way to the third and fourth diagrams of figure 1, and they correspond to the violation of the Slavnov-Taylor identities involving the photon 4-point function and the fermion-photon interaction, respectively.

Notice that the last term in eq. (6.4), arising from the second diagram in figure 1, cannot be written as the BRST transformation of any local field operator in the action, hence it cannot be removed by any counterterm we can possibly construct, since that counterterm would have to be proportional to a structure like  $\epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu A_\rho A_\sigma$ , which however vanishes. The last term represents the essential anomaly, but since it is proportional to the  $\text{Tr}(\mathcal{Y}_R^3)$ , it will vanish due to the anomaly cancelation condition eq. (3.4), ensuring that the theory is anomaly free. In general, eq. (6.4) reflects important statements established in the context of algebraic renormalization [43–48]. The breaking of the Slavnov-Taylor identity at any order is a local, power-counting renormalizable expression with ghost number one. The term in the last line is the unique and only kind of term that can possibly represent a true anomaly that cannot be canceled by symmetry-restoring counterterms. It is known that if the term vanishes at one-loop order, like here, it vanishes at all orders and the theory is free of anomalies.

We can first use this one-loop result to carry out the check of the divergent contributions eq. (6.2b). The singular counterterms given in eq. (5.4) are BRST invariant up to one evanescent contribution exhibited in eq. (5.11), and the resulting BRST breaking  $\Delta_{\text{sct}}^1$  is given in eq. (5.12). Hence, we see that indeed eq. (6.2b) is fulfilled, as it should be.

Now we turn to the determination of the finite counterterms  $S_{\text{fct}}^{(1)}$ , which serve to restore the Slavnov-Taylor identity in the form of eqs. (6.2c) or (6.3). Explicit inspection of the breaking (6.4) reveals that the following ansatz is correct:

$$S_{\text{fct}}^1 = \frac{\hbar}{16\pi^2} \int d^4 x \left\{ \frac{-e^2 \text{Tr}(\mathcal{Y}_R^2)}{6} \bar{A} \cdot (\bar{\partial}^2 \bar{A}) + \frac{e^4 \text{Tr}(\mathcal{Y}_R^4)}{12} (\bar{A}^2)^2 + \frac{5 + \xi}{6} e^2 \sum_j (\mathcal{Y}_R^j)^2 i \bar{\psi}_j \bar{\gamma}^\mu \bar{\partial}_\mu \mathbb{P}_R \psi_j \right\}. \tag{6.5}$$

Each of the three terms has a clear interpretation. The first restores transversality of the photon self energy, the second restores the Ward identity relation for the quartic photon interaction. The third term restores the Ward identity between the fermion self energy and its photon interaction.

The choice of the symmetry-restoring counterterms is constructed such that it satisfies

$$s_d S_{\text{fct}}^1 = -N[\widehat{\Delta}] \cdot \Gamma^1, \tag{6.6}$$

where the right-hand side corresponds to the purely finite, non-evanescent part of eq. (6.4). Further, the counterterms do not depend on external source fields, which implies the identities

$$s_d S_{\text{fct}}^1 = b_d S_{\text{fct}}^1 = \mathcal{S}_d(S_0 + S_{\text{fct}})^1 = \Delta_{\text{fct}}^1. \tag{6.7}$$

Hence the counterterms  $S_{\text{fct}}^1$  restore the symmetry, and all equations (6.2c), (6.3) and ultimately (4.6) are valid at the one-loop level. As an additional byproduct the simplification announced in eq. (4.8c) is established at this order. As mentioned above, the finite counterterms are not uniquely fixed. One can add any BRST-symmetric term to these finite counterterms without spoiling the restoration of the BRST symmetry. This is required to fulfil specific renormalization conditions but is not further pursued in the present paper.

Further, the finite counterterms are defined as purely four-dimensional quantities. This corresponds to our requirement (6.3). As discussed there, one may change the finite counterterms by evanescent contributions which vanish in the  $\text{LIM}_{d \rightarrow 4}$ . This means that it would be allowed e.g. to change the counterterms by extending them to  $d$  dimensions, i.e. to replace some or all of the  $\bar{A}_\mu$  and  $\bar{\partial}_\mu$  by full  $A_\mu$  and  $\partial_\mu$ . Such changes are irrelevant for pure one-loop discussions, however once the counterterms are inserted into higher-loop diagrams the changes matter and might change the form of two-loop results, corresponding to different renormalization schemes.

As stated around eq. (6.3), in this work we stick to the choice of keeping the finite counterterms in their four-dimensional form when being used in 2-loop calculations, as vertex insertions.



## 7 Evaluation of the two-loop singular counterterm action $S_{\text{sct}}^2$ in $\chi$ QED versus QED

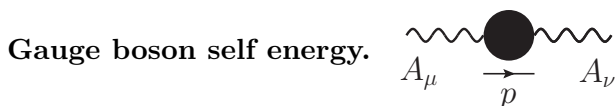
In this section we determine the UV divergences of the subrenormalized two-loop (order  $\hbar^2$ ) Green functions. According to eq. (4.3) these define the singular two-loop counterterm action  $S_{\text{sct}}^2$ . The calculations are performed in  $d = 4 - 2\epsilon$  dimensions, and in the Feynman gauge  $\xi = 1$ , and the results are compared with the corresponding results for ordinary QED.

### 7.1 A comment on the calculation procedure

The calculational procedure uses the same tools already mentioned in section 5. In addition, we compute two-loop self energy integrals using TARCER [62]. Divergences of three-point functions are obtained using two different approaches. In the first approach we reduce the expressions effectively to self energies by setting one external momentum to zero and proceed with TARCER. This is justified since we are interested in the UV divergences which are known to be local and independent of external momenta for the diagrams of our interest (after subrenormalization). This approach fails in case zero external momenta induce infrared divergences. In this case, we use a UV/IR-decomposition [63–65] where effectively all external momenta vanish and propagators become massive. The resulting integrals become massive self energies without external momenta, i.e. massive vacuum integrals. Whenever different approaches can be applied we use both, and the results agree.

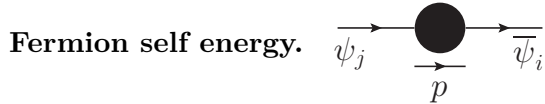
### 7.2 List of divergent two-loop Green functions

Like at the one-loop level there are three kinds of UV divergent Green functions, corresponding to the photon self energy, the fermion self energy and the fermion-photon interaction. Here we first present the explicit results for each subrenormalized two-loop Green function separately and both for  $\chi$ QED and ordinary QED. The blobs shown in the diagrams represent the sum of the all possible subrenormalized two-loop corrections, i.e. two-loop diagrams with tree-level vertices and one-loop diagrams with singular and finite BRST-restoring counterterm insertions.



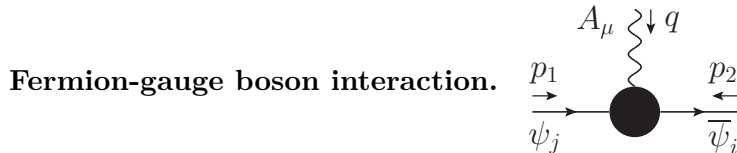
$$i\tilde{\Gamma}_{AA}^{\nu\mu}(p)|_{\text{div}, \chi\text{QED}}^2 = \frac{ie^4}{256\pi^4} \frac{\text{Tr}(\mathcal{Y}_R^4)}{3} \left[ \frac{2}{\epsilon} (\bar{p}^\mu \bar{p}^\nu - \bar{p}^2 \bar{g}^{\mu\nu}) + \left( \frac{17}{24\epsilon} - \frac{1}{2\epsilon^2} \right) \hat{p}^2 \bar{g}^{\mu\nu} \right], \quad (7.1a)$$

$$i\tilde{\Gamma}_{AA}^{\nu\mu}(p)|_{\text{div}, \text{QED}}^2 = \frac{ie^4}{256\pi^4 \epsilon} 2 \text{Tr}(\mathcal{Y}^4) (p^\mu p^\nu - p^2 g^{\mu\nu}). \quad (7.1b)$$



$$i\tilde{\Gamma}_{\psi\bar{\psi}}^{ji}(p)|_{\text{div, } \chi\text{QED}}^2 = \frac{-ie^4}{256\pi^4} \left[ \frac{(\mathcal{Y}_R^2)^{ij} \text{Tr}(\mathcal{Y}_R^2)}{9\epsilon} + (\mathcal{Y}_R^4)^{ij} \left( \frac{7}{12\epsilon} + \frac{1}{2\epsilon^2} \right) \right] \bar{p} \mathbb{P}_R, \quad (7.2a)$$

$$i\tilde{\Gamma}_{\psi\bar{\psi}}^{ji}(p)|_{\text{div, QED}}^2 = \frac{-ie^4}{256\pi^4} \left[ \frac{(\mathcal{Y}^2)^{ij} \text{Tr}(\mathcal{Y}^2)}{\epsilon} + (\mathcal{Y}^4)^{ij} \left( \frac{3}{4\epsilon} + \frac{1}{2\epsilon^2} \right) \right] \not{p}. \quad (7.2b)$$



$$i\tilde{\Gamma}_{\psi\bar{\psi}A}^{ji,\mu}|_{\text{div, } \chi\text{QED}}^2 = \frac{-ie^5}{256\pi^4} \left[ \frac{(\mathcal{Y}_R^2)^{ij} \text{Tr}(\mathcal{Y}_R^3)}{\epsilon} - \frac{(\mathcal{Y}_R^3)^{ij} \text{Tr}(\mathcal{Y}_R^2)}{9\epsilon} + (\mathcal{Y}_R^5)^{ij} \left( \frac{17}{12\epsilon} + \frac{1}{2\epsilon^2} \right) \right] \bar{\gamma}^\mu \mathbb{P}_R, \quad (7.3a)$$

$$i\tilde{\Gamma}_{\psi\bar{\psi}A}^{ji,\mu}|_{\text{div, QED}}^2 = \frac{-ie^5}{256\pi^4} \left[ \frac{(\mathcal{Y}^3)^{ij} \text{Tr}(\mathcal{Y}^2)}{\epsilon} + (\mathcal{Y}^5)^{ij} \left( \frac{3}{4\epsilon} + \frac{1}{2\epsilon^2} \right) \right] \gamma^\mu. \quad (7.3b)$$

The first term with  $\text{Tr}(\mathcal{Y}_R^3) = 0$  does not contribute due to the previously imposed anomaly cancellation condition.

**Three- and four-photon interactions.** The triple-photon interaction amplitude is equal to zero for QED models, while it is finite and purely evanescent for  $\chi\text{QED}$ . The four-photon interaction amplitude is finite and does not provide any singular counterterm.

### 7.3 Singular two-loop counterterms

From the singular part of the two-loop diagrams listed above we obtain the singular counterterm action at the two-loop level, which cancels these divergences,

$$\begin{aligned} S_{\text{sct}}^2 = & - \left( \frac{\hbar e^2}{16\pi^2} \right)^2 \frac{\text{Tr}(\mathcal{Y}_R^4)}{3} \left[ \frac{2}{\epsilon} \overline{S_{AA}} + \left( \frac{1}{4\epsilon^2} - \frac{17}{48\epsilon} \right) \int d^d x \bar{A}_\mu \hat{\partial}^2 \bar{A}^\mu \right] \\ & + \left( \frac{\hbar e^2}{16\pi^2} \right)^2 \sum_j (\mathcal{Y}_R^j)^2 \left[ \left( \frac{1}{2\epsilon^2} + \frac{17}{12\epsilon} \right) (\mathcal{Y}_R^j)^2 - \frac{1}{9\epsilon} \text{Tr}(\mathcal{Y}_R^2) \right] \left( \overline{S_{\psi\psi_R}^j} + \overline{S_{\psi_R A \psi_R}^j} \right) \\ & - \left( \frac{\hbar e^2}{16\pi^2} \right)^2 \sum_j \frac{(\mathcal{Y}_R^j)^2}{3\epsilon} \left( \frac{5}{2} (\mathcal{Y}_R^j)^2 - \frac{2}{3} \text{Tr}(\mathcal{Y}_R^2) \right) \overline{S_{\psi\psi_R}^j}. \end{aligned} \quad (7.4)$$

Its structure is the same as at the one-loop level, see eq. (5.1), corresponding to counterterms to the three divergent kinds of Green functions. Again purely 4-dimensional terms appear, as well as an evanescent contribution to the photon self energy. A conceptually new feature compared to the one-loop case is the term in the last line of  $S_{\text{sct}}^2$ , which breaks BRST invariance by a non-evanescent amount.

In the following we again re-express the result using renormalization transformations. Because of the last term it is not possible to split the two-loop singular counterterms into a BRST invariant plus an evanescent part. We can write

$$S_{\text{sct}}^2 = S_{\text{sct,inv}}^2 + S_{\text{sct,break}}^2. \quad (7.5)$$

The first term is BRST invariant and arises from the renormalization transformation  $S_{0,\text{inv}} \rightarrow S_{0,\text{inv}} + S_{\text{ct,inv}}$ , and is given by

$$S_{\text{sct,inv}}^2 = \frac{\delta Z_A^2}{2} L_A + \frac{\delta Z_c^2}{2} L_c + \frac{\delta Z_{\psi_{Rj}}^2}{2} \overline{L_{\psi_{Rj}}} + \frac{\delta e^2}{e} L_e \quad (7.6)$$

with two-loop renormalization constants  $\delta Z_\varphi^2$ ,  $\delta e^2$ .<sup>8</sup> The split (7.5) is not unique, but we reasonably choose to keep the “trivial” identities (3.20) valid for both  $S_{\text{sct,inv}}^2$  and  $S_{\text{sct,break}}^2$  individually (which simply means the counterterms do not contain the ghost or source fields) and to allow only two-point functions in  $S_{\text{sct,break}}^2$ . Then the renormalization constants are given by

$$\delta Z_A^2 = \delta Z_c^2 = -2 \frac{\delta e^2}{e}, \quad (7.7a)$$

$$\delta Z_A^2 = -\frac{e^4}{256\pi^4\epsilon} \frac{2 \text{Tr}(\mathcal{Y}_R^4)}{3}, \quad (7.7b)$$

$$\delta Z_{\psi_{Rj}}^2 = \frac{e^4}{256\pi^4} (\mathcal{Y}_R^j)^2 \left[ \left( \frac{1}{2\epsilon^2} + \frac{17}{12\epsilon} \right) (\mathcal{Y}_R^j)^2 - \frac{1}{9\epsilon} \text{Tr}(\mathcal{Y}_R^2) \right]. \quad (7.7c)$$

Like at the one-loop level the first equation (7.7a) here reflects the validity of the trivial identities equation (3.20) and the analog of the usual QED Ward identity on the level of  $S_{\text{sct,inv}}^2$ . The results for the other renormalization constants differ from the ones in the literature obtained without the BMHV scheme, see e.g. refs. [55–57]. This difference reflects the modified relationship between the renormalization-group  $\beta$  functions and singular counterterms in the BMHV scheme, see the discussions in refs. [34, 66]. A detailed investigation of this issue will be presented in a forthcoming publication.

The BRST-breaking singular counterterms appearing in eq. (7.5) can be written as

$$S_{\text{sct,break}}^2 = -\left( \frac{\hbar e^2}{16\pi^2} \right)^2 \frac{1}{\epsilon} \frac{\text{Tr}(\mathcal{Y}_R^4)}{3} \left( 2(\overline{S_{AA}} - S_{AA}) + \left( \frac{1}{2\epsilon} - \frac{17}{24} \right) \int d^d x \frac{1}{2} \overline{A^\mu} \widehat{\partial}^2 \overline{A}_\mu \right) \\ - \left( \frac{\hbar e^2}{16\pi^2} \right)^2 \frac{1}{3\epsilon} \sum_j (\mathcal{Y}_R^j)^2 \left( \frac{5}{2} (\mathcal{Y}_R^j)^2 - \frac{2}{3} \text{Tr}(\mathcal{Y}_R^2) \right) \overline{S_{\psi_{Rj}}^j}. \quad (7.8)$$

This counterterm action generates a BRST breaking,

$$\Delta_{\text{sct}}^2 = s_d S_{\text{sct}}^2 = \frac{-\hbar^2 e^4}{256\pi^4} \frac{\text{Tr}(\mathcal{Y}_R^4)}{6} \left( \frac{1}{\epsilon^2} - \frac{17}{12\epsilon} \right) \int d^d x (\overline{\partial}_\mu c) (\widehat{\partial}^2 \overline{A}^\mu) \\ - \frac{\hbar^2 e^5}{256\pi^4} \frac{1}{3\epsilon} \sum_j (\mathcal{Y}_R^j)^3 \left( \frac{5}{2} (\mathcal{Y}_R^j)^2 - \frac{2}{3} \text{Tr}(\mathcal{Y}_R^2) \right) \int d^d x c \overline{\partial}_\mu (\overline{\psi} \overline{\gamma}^\mu \mathbb{P}_R \psi), \quad (7.9)$$

<sup>8</sup>Note that the superscript in the notation  $\delta e^2$  refers to the 2-loop contribution to  $\delta e$ , not to a square!

where we have used the BRST invariance of  $\overline{S_{AA}}$  and  $S_{AA}$ . Again we note that, in contrast to the one-loop case, this BRST breaking contains a non-evanescent contribution (the second term).

## 8 BRST symmetry breaking and its restoration at two-loop; evaluation of the two-loop finite counterterm action $S_{\text{fct}}^{(2)}$

This section is devoted to restoring BRST symmetry at the two-loop (or  $\hbar^2$ ) order. We again follow the general procedure outlined in section 4 and proceed along the same lines as at the one-loop level in section 6.

At the two-loop level the structure of renormalization can be written as

$$\Gamma_{\text{DReg}}^{(2)} = \Gamma^{(2)} + S_{\text{sct}}^2 + S_{\text{fct}}^2, \tag{8.1a}$$

$$\Delta_{\text{ct}}^2 = \mathcal{S}_d(S_0 + S_{\text{ct}})^2. \tag{8.1b}$$

The conditions (4.3), (4.9) on the counterterms specialize to

$$S_{\text{sct}}^2 + \Gamma_{\text{div}}^2 = 0, \tag{8.2a}$$

$$(\widehat{\Delta} \cdot \Gamma^2 + \Delta_{\text{ct}}^1 \cdot \Gamma^1 + \Delta_{\text{ct}}^2)_{\text{div}} = 0, \tag{8.2b}$$

$$\text{LIM}_{d \rightarrow 4}(\widehat{\Delta} \cdot \Gamma^2 + \Delta_{\text{ct}}^1 \cdot \Gamma^1 + \Delta_{\text{ct}}^2)_{\text{fin}} = 0. \tag{8.2c}$$

Like at the one-loop level, the second of these equations must hold automatically and provides a consistency check; the third equation determines the finite symmetry-restoring counterterms. We again rewrite it as

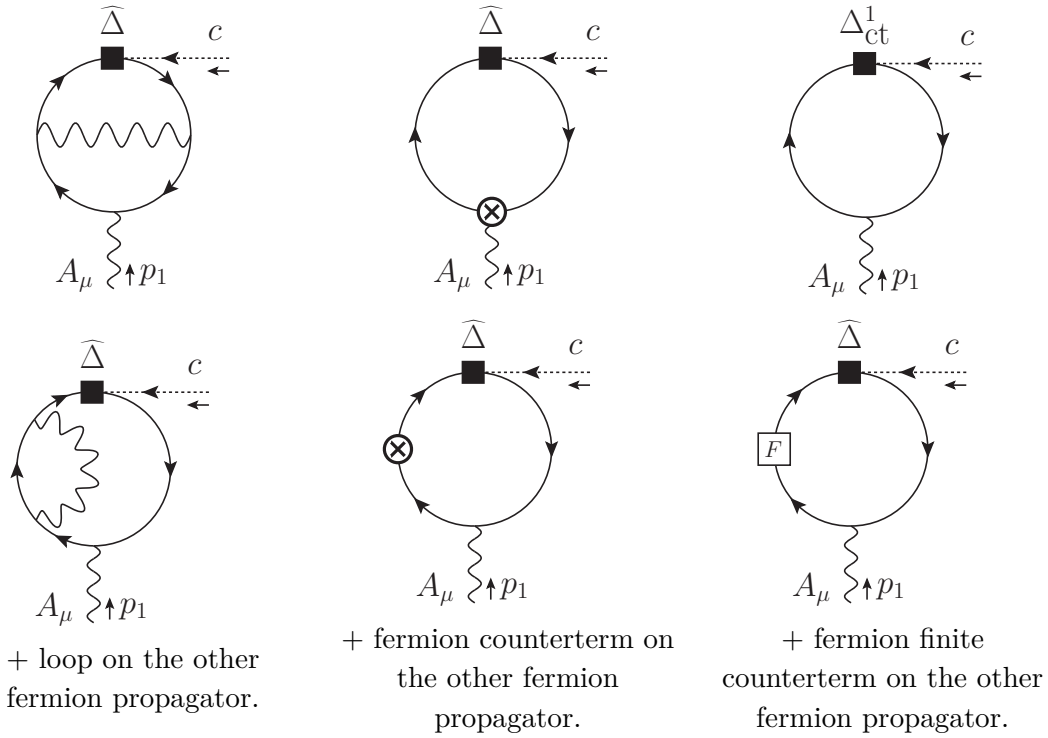
$$N[\widehat{\Delta} \cdot \Gamma^2 + \Delta_{\text{ct}}^1 \cdot \Gamma^1] + \Delta_{\text{fct}}^2 = 0, \tag{8.3}$$

which implicitly fixes the choice of finite, evanescent counterterms. The meaning of this equation is as follows: the breaking of the Slavnov-Taylor identity is given via the quantum action principle by Green functions with breaking insertions. The finite symmetry-restoring counterterms are defined such that they cancel the finite, purely 4-dimensional part of the breaking. As at the one-loop level, we have used that the BRST variation of the singular counterterms  $\Delta_{\text{sct}}^2$  contains no terms of order  $\epsilon^0$  and we could drop the index ‘DReg’.

In the following we first describe the required Feynman diagrammatic computation, then we carry out the check corresponding to eq. (8.2b) and finally we determine the finite, symmetry-restoring counterterms.

### 8.1 Computation of the 2-loop breaking of BRST symmetry

Here we provide details on the computation of the Feynman diagrams describing the two-loop symmetry breakings. As described in section 4 the quantum action principle implies that they are given by diagrams with insertions of the symmetry breaking of the tree-level and counterterm action. At the two-loop level eqs. (8.2b) and (8.3) show that the divergent and finite parts of the following kinds of diagrams are required.  $\widehat{\Delta} \cdot \Gamma^2$  contains genuine two-loop diagrams with one insertion of the tree-level breaking  $\widehat{\Delta}$ , and it contains one-loop



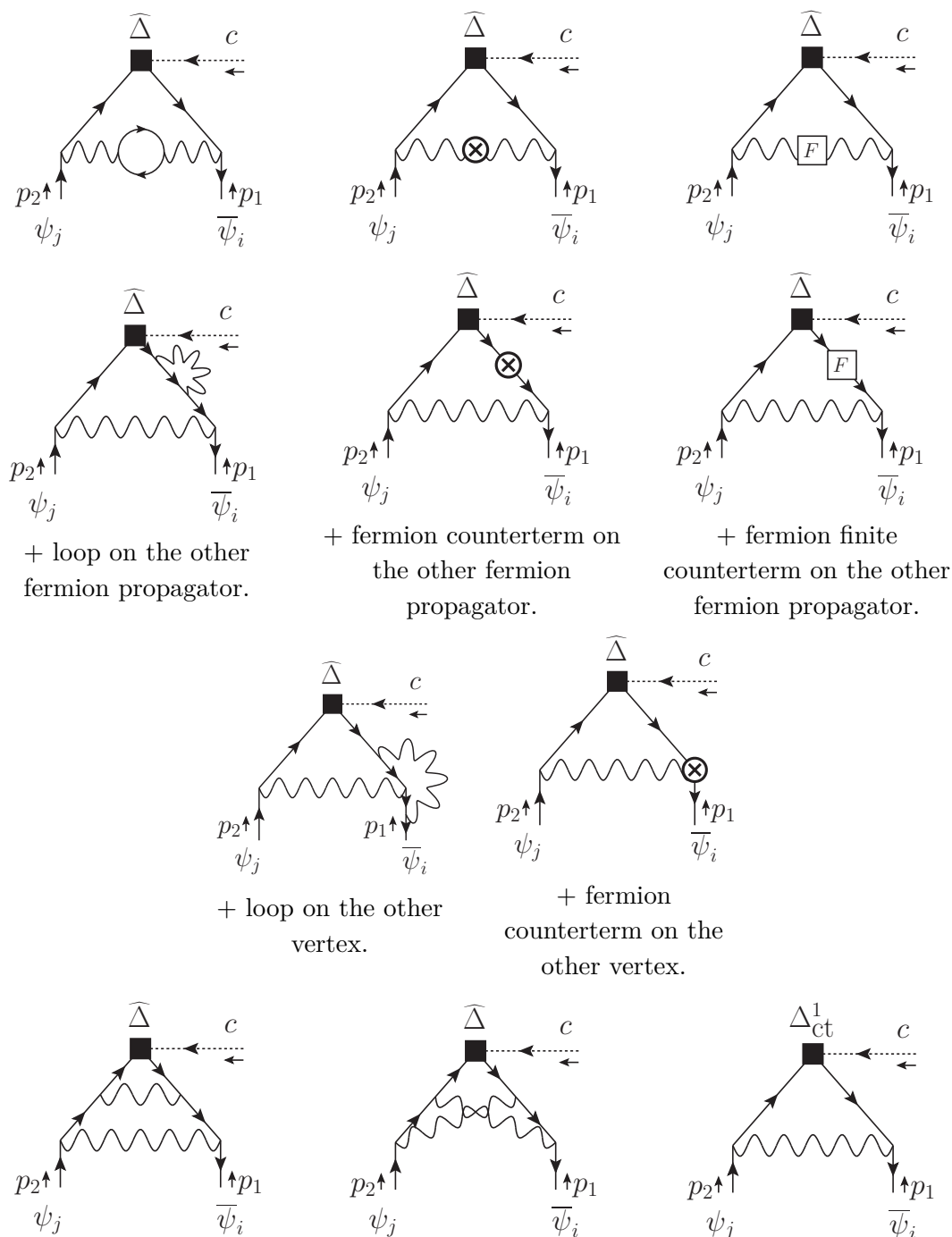
**Figure 2.** List of Feynman diagrams for the ghost-photon breaking contribution given in eq. (8.4). The diagrams in the first column are genuine two-loop diagrams with one insertion of the tree-level breaking  $\widehat{\Delta}$ . The diagrams in the second column are one-loop diagrams with one insertion of a one-loop singular counterterm, denoted as a circled cross. The third column contains a one-loop diagram with an insertion of a one-loop symmetry-restoring counterterm, denoted by a boxed  $F$ , and a one-loop diagram with an insertion of the one-loop breaking  $\Delta_{\text{ct}}^1$ .

diagrams with one insertion of a one-loop counterterm and one insertion of  $\widehat{\Delta}$ . The object  $\Delta_{\text{ct}}^1 \cdot \Gamma^1$  consists of one-loop diagrams with tree-level vertices and with one insertion of the breaking of the one-loop counterterm action  $\Delta_{\text{ct}}^1$ . Note that in our case for the U(1) model we have  $(\Delta_{\text{sct}}^1 \cdot \Gamma^1)^2 = 0$ , implying  $(\Delta_{\text{ct}}^1 \cdot \Gamma^1)^2 = (\Delta_{\text{fct}}^1 \cdot \Gamma^1)^2$ , due to the fact that there are no ghost loop corrections.

Like at the one-loop level, the only relevant results are the ones which are either divergent or finite but not evanescent. Since the breaking insertions  $\widehat{\Delta}$  are themselves evanescent, such results can only arise from power-counting divergent Feynman diagrams. For this reason only a finite number of Feynman diagrams with a specific set of external fields need to be computed. The relevant diagrams with non-vanishing contributions are shown in figures 2, 3 and 4, and their results are described in the following.

The ghost-gauge boson contribution from the diagrams with external fields  $cA$  shown in figure 2 is

$$i \left( [\widehat{\Delta} + \Delta_{\text{ct}}^1] \cdot \widetilde{\Gamma} \right)_{A_\mu c}^2 = \frac{1}{256\pi^4} \frac{e^4 \text{Tr}(\mathcal{Y}_R^4)}{6} \left[ \left( \frac{1}{\epsilon^2} - \frac{17}{12\epsilon} \right) \hat{p}_1^2 \bar{p}_1^\mu - \frac{11}{4} \bar{p}_1^2 \bar{p}_1^\mu + \mathcal{O}(\cdot) \right]. \quad (8.4)$$



**Figure 3.** List of Feynman diagrams for the Ghost-fermion-fermion breaking contribution. The symbols are as in figure 2 and the results are given in eq. (8.5).

The result contains  $1/\epsilon^2$  poles and  $1/\epsilon$  poles with local, evanescent coefficients and finite, non-evanescent terms. Finite but evanescent terms are not relevant for the present context and are suppressed here and in the following.

The ghost-fermion-fermion contribution from the diagrams with external fields  $c\bar{\psi}\psi$  shown in figure 3 is

$$i \left( [\widehat{\Delta} + \Delta_{\text{ct}}^1] \cdot \widetilde{\Gamma} \right)_{\psi\bar{\psi}c}^2 = \frac{1}{256\pi^4} \frac{e^5 (\mathcal{Y}_R^j)^3}{3} (\bar{p}_1 + \bar{p}_2) \mathbb{P}_R \times \left[ \frac{1}{\epsilon} \left( \frac{5}{2} (\mathcal{Y}_R^j)^2 - \frac{2}{3} \text{Tr}(\mathcal{Y}_R^2) \right) + \frac{127}{12} (\mathcal{Y}_R^j)^2 - \frac{1}{9} \text{Tr}(\mathcal{Y}_R^2) \right]. \quad (8.5)$$

The result contains no  $1/\epsilon^2$  poles but only  $1/\epsilon$  poles with local, evanescent coefficients and finite, non-evanescent terms.

The ghost-two gauge bosons contribution from diagrams with external fields  $cAA$  turns out to vanish. Hence

$$i \left( [\widehat{\Delta} + \Delta_{\text{ct}}^1] \cdot \widetilde{\Gamma} \right)_{AAc}^2 = 0. \quad (8.6)$$

The ghost-three gauge bosons contribution from the diagrams with external fields  $cAAA$  shown in figure 4 is

$$i \left( [\widehat{\Delta} + \Delta_{\text{ct}}^1] \cdot \widetilde{\Gamma} \right)_{A_\rho A_\nu A_\mu c}^2 = \frac{1}{256\pi^4} 3e^6 \text{Tr}(\mathcal{Y}_R^6) (\bar{p}_1 + \bar{p}_2 + \bar{p}_3)_\sigma \times (\bar{g}^{\mu\nu} \bar{g}^{\rho\sigma} + \bar{g}^{\mu\rho} \bar{g}^{\nu\sigma} + \bar{g}^{\mu\sigma} \bar{g}^{\nu\rho}). \quad (8.7)$$

Notice that this result contains no UV divergence but only finite terms.

Collecting the results of eqs. (8.4), (8.5), (8.6) and (8.7), one obtains the following result for the two-loop breaking of the Slavnov-Taylor identity of two-loop subrenormalized Green functions:

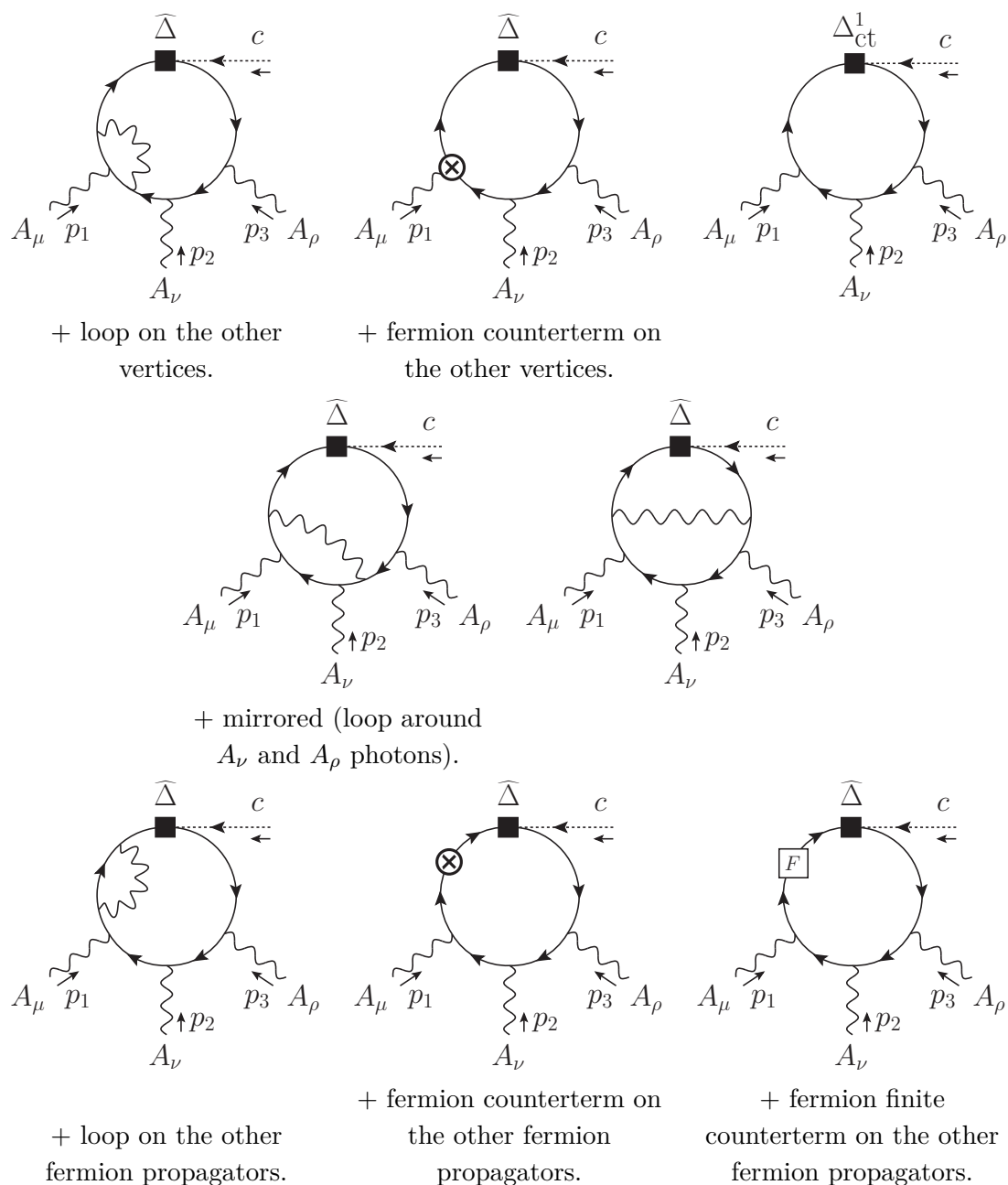
$$\begin{aligned} \left( [\widehat{\Delta} + \Delta_{\text{ct}}^1] \cdot \Gamma \right)^2 &= \frac{\hbar^2 e^4}{256\pi^4} \int d^d x \\ &\times \left\{ -\frac{\text{Tr}(\mathcal{Y}_R^4)}{6} \left[ \left( \frac{1}{\epsilon^2} - \frac{17}{12\epsilon} \right) c \bar{\partial}_\mu \widehat{\partial}^2 \bar{A}^\mu - \frac{11}{4} c \bar{\partial}_\mu \bar{\partial}^2 \bar{A}^\mu \right] \right. \\ &+ e \sum_j \frac{(\mathcal{Y}_R^j)^3}{3} \left[ \frac{1}{\epsilon} \left( \frac{5}{2} (\mathcal{Y}_R^j)^2 - \frac{2}{3} \text{Tr}(\mathcal{Y}_R^2) \right) \right. \\ &\quad \left. \left. + \frac{127}{12} (\mathcal{Y}_R^j)^2 - \frac{1}{9} \text{Tr}(\mathcal{Y}_R^2) \right] c \bar{\partial}_\mu (\bar{\psi}_j \bar{\gamma}^\mu \mathbb{P}_R \psi_j) \right. \\ &\left. + \frac{3e^2 \text{Tr}(\mathcal{Y}_R^6)}{2} c \bar{\partial}_\mu (\bar{A}^\mu \bar{A}_\nu \bar{A}^\nu) \right\} + \mathcal{O}(\cdot). \end{aligned} \quad (8.8)$$

It is particularly noteworthy that, despite significantly more complicated computations, the structure of the terms is the same as at the one-loop level.

## 8.2 Two-loop finite symmetry-restoring counterterms

Like at the one-loop level we can first use the result to check the cancellation of the UV divergences as prescribed by eq. (8.2b). Indeed, this cancellation with  $s_d S_{\text{sct}}^{(2)}$ , eq. (7.9) takes place as it should, in the explicit form

$$\Delta_{\text{sct}}^2 = s_d S_{\text{sct}}^{(2)} = - \left( [\widehat{\Delta} + \Delta_{\text{ct}}^1] \cdot \widetilde{\Gamma} \right)_{\text{div}}^2, \quad (8.9)$$



**Figure 4.** List of Feynman diagrams for the Ghost-three gauge bosons breaking contribution (additional diagrams corresponding to  $\{(p_1, \mu), (p_2, \nu), (p_3, \rho)\}$  permutations are not shown). The symbols are as in figure 2 and the results are given in eq. (8.7).



providing a confirmation of the computation. Next we can turn to the determination of the two-loop symmetry-restoring counterterms, using eq. (8.3). Given the results of the previous subsection and a simple calculation we obtain

$$\begin{aligned}
 \Delta_{\text{fct}}^2 &= -N[\widehat{\Delta} \cdot \Gamma_{\text{DRreg}}^2 + \Delta_{\text{ct}}^1 \cdot \Gamma_{\text{DRreg}}^1] \\
 &= -\text{LIM}_{d \rightarrow 4} \left\{ \left( [\widehat{\Delta} + \Delta_{\text{ct}}^{(1)}] \cdot \Gamma \right)^{(2)} + s_d S_{\text{sct}}^{(2)} \right\} \\
 &= + \frac{\hbar^2 e^4}{256\pi^4} \text{Tr}(\mathcal{Y}_R^4) s \left( \frac{11}{48} \int d^4 x \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu \right) \\
 &\quad - \frac{\hbar^2 e^4}{256\pi^4} \sum_j (\mathcal{Y}_R^j)^2 \left( \frac{127}{36} (\mathcal{Y}_R^j)^2 - \frac{1}{27} \text{Tr}(\mathcal{Y}_R^2) \right) s \int d^4 x \bar{\psi}_j i \bar{\not{\partial}} \mathbb{P}_R \psi_j \\
 &\quad + \frac{\hbar^2 e^6}{256\pi^4} \frac{3}{8} \text{Tr}(\mathcal{Y}_R^6) s \int d^4 x \bar{A}_\mu \bar{A}^\mu \bar{A}_\nu \bar{A}^\nu.
 \end{aligned} \tag{8.10}$$

Here the right-most equation has been written as an explicit 4-dimensional BRST transformation of a local action.

This implies that the following choice of finite counterterms restores the Slavnov-Taylor identity at the two-loop level,

$$\begin{aligned}
 S_{\text{fct}}^2 &= \left( \frac{\hbar}{16\pi^2} \right)^2 \int d^4 x e^4 \left\{ \text{Tr}(\mathcal{Y}_R^4) \frac{11}{48} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + 3 e^2 \frac{\text{Tr}(\mathcal{Y}_R^6)}{8} \bar{A}_\mu \bar{A}^\mu \bar{A}_\nu \bar{A}^\nu \right. \\
 &\quad \left. - \sum_j (\mathcal{Y}_R^j)^2 \left( \frac{127}{36} (\mathcal{Y}_R^j)^2 - \frac{1}{27} \text{Tr}(\mathcal{Y}_R^2) \right) (\bar{\psi}_j i \bar{\not{\partial}} \mathbb{P}_R \psi_j) \right\}.
 \end{aligned} \tag{8.11}$$

Like at the one-loop level, three kinds of terms exist. In an obvious way they correspond to the restoration of the Ward identity relations for the photon self energy, the photon 4-point function and the fermion self energy/photon interaction.

### 8.3 Tests of Ward identities

In order to check the consistency of the previously calculated divergent and finite counterterms we may make use of Ward identities which express relations of Green's functions and their properties due to gauge invariance of the theory. In section 3.3 we have seen that in our U(1) model the Slavnov-Taylor identity straightforwardly leads to Ward identities since certain functional relations trivially survive renormalization. Once the Slavnov-Taylor identity is satisfied, the Ward identities will likewise be valid, but they provide a check that is independent of breaking diagrams. Eq. (3.21) supplies us with three well-known QED Ward identities for renormalized Green functions to check our counterterm results:

1. The transversality of the photon self energy,

$$i p_\nu \frac{\delta^2 \tilde{\Gamma}_{\text{ren}}}{\delta A_\mu(p) \delta A_\nu(-p)} = 0; \tag{8.12}$$

2. The transversality of multi-photon vertices, and in particular the photon 4-point amplitude,

$$i (p_{1+2+3})_\sigma \frac{\delta^4 \tilde{\Gamma}_{\text{ren}}}{\delta A_\rho(p_3) \delta A_\nu(p_2) \delta A_\mu(p_1) \delta A_\sigma(-p_{1+2+3})} = 0 \tag{8.13}$$

(denoting  $p_{1+2+3} \equiv p_1 + p_2 + p_3$ );

3. The relation between fermion self energy and fermion-photon interaction for vanishing photon momentum  $q = 0$ ,

$$-ie\mathcal{Y}_R \frac{\partial}{\partial p_\mu} \frac{\delta^2 \tilde{\Gamma}_{\text{ren}}}{\delta \bar{\psi}(-p) \delta \psi(p)} + i \frac{\delta^3 \tilde{\Gamma}_{\text{ren}}}{\delta A_\mu(0) \delta \bar{\psi}(-p) \delta \psi(p)} = 0. \quad (8.14)$$

With these equations, we can demonstrate the consistency and correctness of our calculations by evaluating usual loop diagrams and compare them with the results for breaking insertions.

We begin with the example of the two-loop divergent part of the photon self energy. If we contract it with one momentum, what we obtain is

$$i p_\nu \tilde{\Gamma}_{A(-p)A(p)}^{\mu\nu} \Big|_{\text{div}}^2 = \frac{ie^4}{256\pi^4} \frac{\text{Tr}(\mathcal{Y}_R^4)}{6} \left( \frac{17}{12\epsilon} - \frac{1}{\epsilon^2} \right) \hat{p}^2 \bar{p}^\mu = - \left( [\hat{\Delta} + \Delta_{\text{ct}}^1] \cdot \tilde{\Gamma} \right)_{\text{div}, A_\mu(-p)c(p)}^2. \quad (8.15)$$

The first of these equations is obtained by direct computation of the appropriate two-loop diagrams. The second equation is then an observation using eq. (7.9) and eq. (8.9). These equations mean that the part of the divergent photon self energy that would violate transversality is cancelled by the divergent counterterm calculated from the breaking insertion.

The finite part of photon self energy at the two loop level is given by

$$i \tilde{\Gamma}_{AA}^{\mu\nu}(p) \Big|_{\text{fin}}^2 = \frac{ie^4}{256\pi^4} \frac{\text{Tr}(\mathcal{Y}_R^4)}{3} \left[ \left( \frac{673}{23} - 6 \log(-\bar{p}^2) - 24\zeta(3) \right) (\bar{p}^\mu \bar{p}^\nu - \bar{p}^2 \bar{g}^{\mu\nu}) + \frac{11}{8} \bar{p}^\mu \bar{p}^\nu \right], \quad (8.16)$$

and after the momentum contraction we obtain

$$i p_\nu \tilde{\Gamma}_{A(-p)A(p)}^{\mu\nu} \Big|_{\text{fin}}^2 = \frac{ie^4}{256\pi^4} \frac{\text{Tr}(\mathcal{Y}_R^4)}{6} \frac{11}{4} \bar{p}^2 \bar{p}^\mu = - \left( [\hat{\Delta} + \Delta_{\text{ct}}^1] \cdot \tilde{\Gamma} \right)_{\text{fin}, A_\mu(-p)c(p)}^2. \quad (8.17)$$

The first of these equations is again obtained by direct computation of the diagrams. It illustrates that the non-local  $\log(-\bar{p}^2)$  and transcendental  $\zeta(3)$  parts are by themselves transversal. The second equation is then observed by comparison with eq. (8.10). Hence we confirm that the violation of the symmetry is restored by our finite counterterm evaluated from breaking diagrams.

The 4-photon amplitude is finite. A direct, explicit manipulation of the corresponding Feynman diagrams shows that we can relate the breaking of the Ward identity to the breaking of the Slavnov-Taylor identity as

$$\begin{aligned} -i p_\nu \tilde{\Gamma}_{A(-p_1)A(-p_2)A(-p_3)A(p)}^{\mu_1\mu_2\mu_3\nu} \Big|_{\text{fin}}^2 &= \left( [\hat{\Delta} + \Delta_{\text{ct}}^1] \cdot \tilde{\Gamma} \right)_{\text{fin}, A_{\mu_1}(-p_1)A_{\mu_2}(-p_2)A_{\mu_3}(-p_3)c(p)}^2 \\ &= \frac{ie^6}{256\pi^4} 3 \text{Tr}(\mathcal{Y}_R^6) (\bar{p}_1 + \bar{p}_2 + \bar{p}_3)_\nu (\bar{g}^{\nu\mu_1} \bar{g}^{\mu_2\mu_3} + \bar{g}^{\nu\mu_2} \bar{g}^{\mu_1\mu_3} + \bar{g}^{\nu\mu_3} \bar{g}^{\mu_1\mu_2}). \end{aligned} \quad (8.18)$$

Via eq. (8.10) this shows again that the counterterms of eq. (8.11) appropriately restore this Ward identity.

We can investigate the Ward identity between the fermion self energy and fermion-photon interaction eq. (8.14) in a similar way. The divergent two-loop violation is given by

$$\begin{aligned}
 & - i e \mathcal{Y}_R \frac{\partial}{\partial p_\mu} \tilde{\Gamma}_{\psi(p)\bar{\psi}(-p)} \Big|_{\text{div}}^2 + i \tilde{\Gamma}_{\psi(p)\bar{\psi}(-p)A(0)}^\mu \Big|_{\text{div}}^2 \\
 & = i \mathcal{Y}_R \frac{e^5}{256\pi^4 \epsilon} \bar{\gamma}^\mu \mathbb{P}_R \left( \frac{2\mathcal{Y}_R^2 \text{Tr}(\mathcal{Y}_R^2)}{9} - \frac{5\mathcal{Y}_R^4}{6} \right) \\
 & = - \frac{\partial}{\partial q_\mu} \left( [\hat{\Delta} + \Delta_{\text{ct}}^1] \cdot \tilde{\Gamma} \right)_{\text{div}, \psi(p-q)\bar{\psi}(-p)c(q)}^2 (q=0),
 \end{aligned} \tag{8.19}$$

and the finite two-loop violation by

$$\begin{aligned}
 & - i e \mathcal{Y}_R \frac{\partial}{\partial p_\mu} \tilde{\Gamma}_{\psi(p)\bar{\psi}(-p)} \Big|_{\text{fin}}^2 + i \tilde{\Gamma}_{\psi(p)\bar{\psi}(-p)A(0)}^\mu \Big|_{\text{fin}}^2 \\
 & = i \mathcal{Y}_R \frac{e^5}{256\pi^4} \bar{\gamma}^\mu \mathbb{P}_R \left( \log(-\bar{p}^2) \left( \frac{3}{2} \mathcal{Y}_R^4 - \mathcal{Y}_R^2 \text{Tr}(\mathcal{Y}_R^2) \right) + \frac{62}{27} \mathcal{Y}_R^2 \text{Tr}(\mathcal{Y}_R^2) - \frac{109}{72} \mathcal{Y}_R^4 \right. \\
 & \quad \left. - \log(-\bar{p}^2) \left( \frac{3}{2} \mathcal{Y}_R^4 - \mathcal{Y}_R^2 \text{Tr}(\mathcal{Y}_R^2) \right) - \frac{61}{27} \mathcal{Y}_R^2 \text{Tr}(\mathcal{Y}_R^2) - \frac{145}{72} \mathcal{Y}_R^4 \right) \\
 & = i \mathcal{Y}_R \frac{e^5}{256\pi^4} \bar{\gamma}^\mu \mathbb{P}_R \left( \frac{\mathcal{Y}_R^2 \text{Tr}(\mathcal{Y}_R^2)}{27} - \frac{127\mathcal{Y}_R^4}{36} \right) \\
 & = - \frac{\partial}{\partial q_\mu} \left( [\hat{\Delta} + \Delta_{\text{ct}}^1] \cdot \tilde{\Gamma} \right)_{\text{fin}, \psi(p-q)\bar{\psi}(-p)c(q)}^2 (q=0).
 \end{aligned} \tag{8.20}$$

In each case again the first equations are obtained from explicit computation of the Feynman diagrams, and the last equations are obtained by comparing with eq. (7.9), eq. (8.9) and eq. (8.10).<sup>9</sup> As a result, it is established that the counterterms in eq. (8.11) restore all Ward identities.

## 9 Conclusions

In this work we applied the BMHV scheme for non-anticommuting  $\gamma_5$  in dimensional regularization to a chiral gauge theory at the two-loop level, and we studied the BMHV-specific aspects of renormalization. Most importantly we determined the full structure of two-loop symmetry-restoring counterterms. The present work is restricted to an abelian gauge theory with right-handed fermions and establishes the methodology. The same method will be applicable to general non-abelian gauge theories with scalar and fermionic matter.

In general, the application of the BMHV scheme leads to several specific kinds of counterterms: the ultraviolet (UV) divergences cannot be cancelled by counterterms generated by field and parameter renormalization; additional, UV divergent evanescent counterterms (corresponding to operators which vanish in strictly 4 dimensions) are needed; and the breaking of BRST symmetry needs to be repaired by adding finite, symmetry-restoring

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<sup>9</sup>The divergent  $1/\epsilon^2$  poles in (8.19) are omitted since they cancel completely. The second and third rows in (8.20) represent the full results for finite (momentum-differentiated) photon self energy and vertex interaction, respectively.

counterterms. We have evaluated all these counterterms explicitly at the one-loop and two-loop level. An important aspect of our results is that the structure at the one-loop and two-loop level is essentially the same. As expected, the UV divergences arise in the fermion and the photon self energy and in the fermion-photon interaction. The triple and quartic photon self interactions are UV finite. However, there are purely evanescent divergences in the photon self energy, and at the two-loop level there is a non-evanescent divergence in the fermion self energy, both of which require an extra counterterm which cannot be obtained from field or parameter renormalization.

The required symmetry-restoring counterterms turn out to take a rather simple structure with a straightforward interpretation. Both at the one-loop and the two-loop level there are three kinds of such counterterms. A counterterm to the photon self energy restores transversality of the renormalized photon self energy. Similarly, a counterterm to the photon 4-point function restores the Ward identity for this Green function. Finally, a counterterm to the fermion self energy restores its Ward identity-like relation to the fermion-photon interaction. An important outcome is that the precise form of these counterterms is now known, and it is established that this is the complete set of symmetry-restoring counterterms for arbitrary two-loop calculations in the model.

We applied a method which was previously applied at the one-loop level in refs. [34, 35]; refs. [36–38] applied similar techniques at the multiloop level, however in cases where the symmetry is actually unbroken by the regularization. The core of the method is the evaluation of the breaking of the Slavnov-Taylor identity by employing the regularized quantum action principle [30]. Here we presented the first such computation at the two-loop level. It involves Feynman diagrams of four different kinds: genuine two-loop diagrams with an insertion of the tree-level breaking  $\hat{\Delta}$  and one-loop diagrams with insertions of the one-loop breaking  $\Delta_{\text{ct}}^1$  or of the one-loop divergent or finite counterterms.

Since the method is now established and not restricted to abelian theories, it will be possible to apply it to general non-abelian chiral gauge theories and to the Standard Model at the two-loop level. In this way, two-loop Standard Model calculations will become feasible in the BMHV scheme without worrying about symmetry violations or scheme inconsistencies. As a further outlook, it will be of interest to explore in detail the relationship between the modified counterterm structure (with additional UV divergent and non-symmetric finite terms) and the renormalization group, similar to the one-loop discussion of ref. [34].

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