Time-reversal asymmetries and angular distributions in $\Lambda_b \to \Lambda V$

Chao-Qiang Geng[a,b,c] **and Chia-Wei Liu**[a,b]

[a]*School of Fundamental Physics and Mathematical Sciences,*
Hangzhou Institute for Advanced Study, UCAS,
Hangzhou 310024, China

[b]*International Centre for Theoretical Physics Asia-Pacific,*
Beijing/Hangzhou, China

[c]*Chongqing University of Posts & Telecommunications,*
Chongqing 400065, China

E-mail: cqgeng@ucas.ac.cn, ron1a2l3d4@gmail.com

ABSTRACT: We study the spin correlations to probe time-reversal (T) asymmetries in the decays of $\Lambda_b \to \Lambda V$ ($V = \phi, \rho^0, \omega, K^{*0}$). The eigenstates of the T-odd operators are obtained along with definite angular momenta. We obtain the T-odd spin correlations from the complex phases among the helicity amplitudes. We give the angular distributions of $\Lambda_b \to \Lambda(\to p\pi^-)V(\to PP')$ and show the corresponding spin correlations, where $P^{(\prime)}$ are the pseudoscalar mesons. Due to the helicity conservation of the s quark in Λ, we deduce that the polarization asymmetries of Λ are close to -1. Since the decay of $\Lambda_b \to \Lambda\phi$ in the standard model (SM) is dictated by the single weak phase from the product of CKM elements, $V_{tb}V_{ts}^*$, the true T and CP asymmetries are suppressed, providing a clean background to test the SM and search for new physics. In the factorization approach, as the helicity amplitudes in the SM share the same complex phase, T-violating effects are absent. Nonetheless, the experimental branching ratio of $Br(\Lambda_b \to \Lambda\phi) = (5.18 \pm 1.29) \times 10^{-6}$ suggests that the nonfactorizable effects or some new physics play an important role. By parametrizing the nonfactorizable contributions with the effective color number, we calculate the branching ratios and direct CP asymmetries. We also explore the possible T-violating effects from new physics.

KEYWORDS: CP violation, Discrete Symmetries, Heavy Quark Physics

ARXIV EPRINT: 2109.09524

Contents

1   Introduction

The time-reversal (T) symmetry demands that physics shall be invariant under reversing the motions, which corresponds to flipping time t to $-t$ and taking the complex conjugate of the quantum states. Its anti-linear property makes T-violation and CP- violation become synonyms, according to the CPT symmetry. In two or three-body decays, T-odd observables can be constructed in the spin correlations among the particles [1–10]. Since the particle spins could not be directly measured in the current high energy physics experiments, we have to extract their effects through the angular distributions in the cascade decays [11–24].

There are more than forty naive T-odd observables, which have been measured in the B meson decays [25], indicating the angular analyses have been well developed in the experiments. Particularly, the longitudinally polarized fraction, $f_L(B^0 \to \phi K^{*0}) \approx 0.5$, measured by BarBar [26] and Belle [27], shows that the nonfactorizable (NF) effects play an important role in the B decays with vector mesons in the final states. On the other hand, the angular analyses in $\Lambda_b \to J/\psi \Lambda$ [28–30] from LHCb show that the polarization fractions are consistent with zero. Furthermore, the $\Lambda_b - \overline{\Lambda}_b$ production asymmetry has also been studied [31]. Recently, the branching ratios of $\Lambda_b \to \Lambda \phi$ and $\Lambda_b \to \Lambda \gamma$ have been measured to be $(5.18 \pm 1.29) \times 10^{-6}$ [32] and $(7.1 \pm 1.7) \times 10^{-6}$ [33], respectively. However, there have been no complete angular analyses for $\Lambda_b \to \Lambda V (V = \phi, \rho^0, \omega, K^{*0})$.

On the theoretical side, to deal with Λ_b decays, various approaches have been made [34–38]. The most simple one is the naive factorization [18, 19, 39, 40], in which the mesons are produced from weak vertices directly. In particular, the experimental branching ratio of $\Lambda_b \to p\pi^-$ can be explained by light-cone QCD sum rules [39], and the experimental

results in $\Lambda_b \to J/\psi\Lambda$ are well compatible with those in the covariant confined quark model [18, 30]. In refs. [41–44], the generalized factorization approach has been considered, in which the renormalization dependence of the Wilson coefficients has been absorbed into the effective ones in the next to leading log precision [45]. However, the effective color number, found as $N_c \approx 2$ in the decays of B mesons, has to be included to parametrize the NF effects in the branching ratios. On the other hand, the NF amplitudes can be calculated systematically based on the QCD factorization [46] and perturbative QCD [47] approaches, in the heavy quark limit. Nevertheless, they suffer large uncertainties in the penguin dominated decays, due to the unknown baryon wave functions. In this work, we adopt the generalized factorization approach to estimate the results in the standard model (SM).

We will study T-violation systematically in $\Lambda_b \to \Lambda V$ based on the group theory approach. The T-violating observables are often given by the triple vector products asymmetries, read as

$$\mathcal{A}_T = \left[\Gamma(\hat{T} > 0) - \Gamma(\hat{T} < 0)\right]/\Gamma, \tag{1.1}$$

where Γ corresponds to the decay width, $\hat{T} = (\vec{v}_a \times \vec{v}_b) \cdot \vec{v}_c$, and $\vec{v}_i$ represents either the spin ($\vec{s}$) or 3-momentum ($\vec{p}$) of the particle labeled by i with $i = a$ or b or c.[1] Under the T transformation, $\hat{T}$ flips its sign, and therefore $\mathcal{A}_T$ are naively T-odd observables. However, cares must be taken when one applies it on the vector products involving spins. For instance, $(\vec{s} \times \vec{p}) \cdot \vec{J}$ is Hermitian, but $(\vec{s} \times \vec{J}) \cdot \vec{p}$ is *not*, with $\vec{J}$ the angular momentum operator. The Hermiticity can be understood by the commutation relation,

$$[(\vec{A} \times \vec{B})_i, J_i] = 0, \tag{1.2}$$

where $\vec{A}$ and $\vec{B}$ are arbitrary vectors. In contrast, the equality,

$$(\vec{A} \times \vec{J}) \cdot \vec{B} = -(\vec{A} \times \vec{B}) \cdot \vec{J} + 2i\vec{A} \cdot \vec{B}, \tag{1.3}$$

indicates that $(\vec{s} \times \vec{J}) \cdot \vec{p}$ is *not* Hermitian. Hence, it clearly makes no sense to discuss the case with $\vec{v}_b = \vec{J}$. Another issue arises from an incompatible set of observables, causing inconsistent in the analyses, which will be discussed carefully.

This paper is organized as follows. In section 2, we study the angular distributions of $\Lambda_b \to \Lambda(\to p\pi^-)V(\to PP')$, where $P^{(\prime)}$ are the pseudoscalar mesons in the cascade decays. In section 3, we define the T-odd observables and identify their effects in terms of the angular distributions. In section 4, we examine the decays in the SM based on the generalized factorization approach. We also discuss possible right-handed current contributions to T-violating observables from new physics. Finally, we give conclusions in section 5.

2 Angular distributions

The decay distributions in two body decays are often described by the helicity amplitudes [48]. In general, the decays amplitudes of $\Lambda_b \to \Lambda V$ take the form,

$$\langle \Lambda V; \{\lambda_f\}|\mathcal{H}_{\text{eff}}|\Lambda_b\rangle, \tag{2.1}$$

---

[1]Since angular momentum is always conserved, the spin of Λ_b can be identified as the angular momentum in the final state.

where $\mathcal{H}_{\text{eff}}$ is the effective Hamiltonian of weak transitions, and λ_f correspond to physical quantities to characterize the states properly. Often, λ_f are chosen as the momenta and spins of the particles, resulting in expanding the amplitudes with the Dirac spinors and polarization vector, given as

$$\xi^{\mu*}\overline{u}_\Lambda \left[A_1 \gamma_\mu + \frac{A_2 P_\mu}{m_{\Lambda_b}} - \left(B_1 \gamma_\mu + \frac{B_2 P_\mu}{m_{\Lambda_b}} \right) \gamma_5 \right] u_{\Lambda_b} , \qquad (2.2)$$

where A_1, A_2, B_1 and B_2 are the effective couplings, ξ^μ represents the polarization vector of V, $P^\mu = p^\mu_{\Lambda_b} + p^\mu_\Lambda$, and $p^\mu_{\Lambda_{(b)}}$ $(u_{\Lambda_{(b)}})$ are the four momenta (Dirac spinors) of $\Lambda_{(b)}$. Expanding the amplitudes in momenta and spins has the advantage in the dynamical aspect, for that the amplitudes are easier to be parametrized in the calculations, such as the framework of the factorization approach.

On the other hand, from the kinematical consideration, it is more preferable to describe the states in terms of J^2 and J_z. Since the values of J^2 and J_z are always constrained by the parent particle, one may eliminate the redundancies caused by the SO(3) rotation group (SO(3)$_R$), which are independent of the dynamical details. Nonetheless, J^2 and J_z do not specify the states unambiguously. We can choose a set of commuting SO(3)$_R$ scalars to identify the states further. Concerning the angular distributions in the sequential decays, the helicites can be a good choice. In the rest frames of Λ and V, the spins can be read directly.

In the center of the momentum frame of the ΛV system, the helicity and 3-momentum states are related as [48, 49]

$$|\lambda_1, \lambda_2; J = 1/2, J_z = M\rangle = \frac{1}{2\pi} \int d\Omega |\vec{p}_1, \lambda_1, \lambda_2\rangle e^{iM\phi} d^{\frac{1}{2}}(\theta)^M{}_{\lambda_1 - \lambda_2} , \qquad (2.3)$$

where $d^{\frac{1}{2}}$ stands for the Wigner d-matrix for $J = 1/2$, $\lambda_{1(2)}$ represents the helicity of Λ (V), and $\vec{p}_1$ corresponds to the 3-momentum of Λ in the Λ_b rest frame. In eq. (2.3), the left-hand side is the so-called helicitiy state, while the right-hand one is made of the linear superposition of the 3-momenta. To have a nonzero value in $d^{\frac{1}{2}}$, one must have $1/2 \geq |\lambda_1 - \lambda_2|$. The helicity states are given as

$$|a_\pm\rangle = |\pm 1/2, 0\rangle , \quad |b_\pm\rangle = |\mp 1/2, \mp 1\rangle , \qquad (2.4)$$

where the first and second entries correspond to λ_1 and λ_2, respectively, with $J = J_z = 1/2$. In this work, unless explicitly stated, J and J_z will not be written out explicitly. Here, a (b) indicates the vector meson is longitudinally (transversely) polarized, while the subscripts denote the angular momenta in the $\hat{p}_1$ direction, read as $\vec{J} \cdot \hat{p}_1 = \lambda_1 - \lambda_2 = \pm 1/2$. Under the parity transformation, the helicities flip signs, given as

$$I_s |a(b)_\pm\rangle = |a(b)_\mp\rangle , \qquad (2.5)$$

where I_s is the parity transformation operator.

Consequently, the decays amplitudes are given by

$$a_\pm = H_{\pm\frac{1}{2},0} = \langle a_\pm; \text{``out''}|\mathcal{H}_{\text{eff}}|\Lambda_b\rangle ,$$
$$b_\pm = H_{\mp\frac{1}{2},\mp 1} = \langle b_\pm; \text{``out''}|\mathcal{H}_{\text{eff}}|\Lambda_b\rangle , \qquad (2.6)$$

where "out" denotes $t \to \infty$. If $\mathcal{H}_{\text{eff}}$ respects the T symmetry, one has that[2] [49]

$$a(b)_\pm = \langle \Lambda_b | \mathcal{H}_{\text{eff}} | a(b)_\pm; \text{``in''} \rangle \,, \qquad (2.7)$$

where "in" denotes $t \to -\infty$. Furthermore, if the final state interactions (FSIs) are absent, we can interchange "in" and "out" freely, resulting in

$$a(b)_\pm = \langle \Lambda_b | \mathcal{H}_{\text{eff}} | a(b)_\pm; \text{``in''} \rangle = a^*(b^*)_\pm \,. \qquad (2.8)$$

As a result, one concludes that $a(b)_\pm$ are real. This is a common approach in analyzing the T symmetry with complex phases.

The angular distributions are parametrized with five angles, $\theta, \theta_{1,2}$ and $\phi_{1,2}$, shown in figure 1. In the following, we take $\Lambda_b \to \Lambda(\to p\pi^-)\rho^0(\to \pi^+\pi^-)$ as a concrete example. In this case, θ, θ_1, and θ_2 are defined as the angles between $(\hat{n}_{\Lambda_b}, \vec{p}_\Lambda)$, $(\vec{p}_\Lambda, \vec{p}_p)$, and $(\vec{p}_{\rho^0}, \vec{p}_{\pi^+})$, respectively, where $\vec{n}_{\Lambda_b}$ is the unit vector pointing toward the polarization of Λ_b, $\vec{p}_{\Lambda,\rho^0}$ is the 3-momentum defined in the rest frame of Λ_b, and $\vec{p}_p$ $(\vec{p}_{\pi^+})$ is defined in the helicity frame of Λ (ρ^0). On the other hand, ϕ_1 (ϕ_2) is the azimuthal angle between the decay planes of $\hat{n}_{\Lambda_b} \times \vec{p}_\Lambda$ and $\vec{p}_\Lambda \times \vec{p}_p$ $(\vec{p}_{\rho^0} \times \vec{p}_{\pi^+})$. The angular distributions are then given by

$$\mathcal{D}(\vec{\Omega}) \equiv \frac{1}{\Gamma_c} \frac{\partial^5 \Gamma_c}{\partial \cos\theta \, \partial \cos\theta_1 \, \partial \cos\theta_2 \, \partial\phi_1 \, \partial\phi_2} = \frac{1}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2} \frac{3}{8\pi^2} \times$$

$$\sum_{\lambda_1', M=\pm 1/2} \rho_{M,M} \left| \sum_{\lambda_1, \lambda_2} H_{\lambda_1, \lambda_2} A_{\lambda_1'} d^{\frac{1}{2}}(\theta)^M{}_{\lambda_1-\lambda_2} d^{\frac{1}{2}}(\theta_1)^{\lambda_1}{}_{\lambda_1'} d^1(\theta_2)^{\lambda_2}{}_0 e^{i(\lambda_1\phi_1+\lambda_2\phi_2)} \right|^2 \qquad (2.9)$$

where Γ_c is the decay width of $\Lambda_b \to \Lambda(\to p\pi^-)\rho^0(\to \pi^+\pi^-)$, $\rho_{M,M}$ are the density matrix elements of Λ_b in the polarized direction, $\rho_{\pm 1/2, \pm 1/2} = (1 \pm P_b)/2$, with P_b the polarized fractions, and $A_\pm$ are given by

$$|A_+|^2 = \frac{1+\alpha}{2}, \quad |A_-|^2 = \frac{1-\alpha}{2}\,, \qquad (2.10)$$

with α the up-down asymmetry parameter for $\Lambda \to p\pi^-$ [50]. For the charge conjugate decays, one has the same formula with $\bar{\alpha} = -\alpha$ by assuming CP conserved in $\Lambda \to p\pi^-$. From the formalism, it is obvious that the complex phases of $A_\pm$ would not affect the results. Here, we see the merit of the helicity formalism in which the angles are untangled in the amplitudes, i.e., the helicities are independent of the Λ_b polarization and each other. In the next section, we will find that it is not the case, when the states are expanded with the triple vector products.

For the practical purpose, $\mathcal{D}(\vec{\Omega})$ is further written as

$$\mathcal{D}(\vec{\Omega}) = \frac{1}{32\pi^2} \frac{1}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2} \sum_{i=1}^{20} f_i(a_\pm, b_\pm) D_i(\alpha_\Lambda, P_b, \theta, \theta_1, \phi_1, \theta_2, \phi_2), \quad (2.11)$$

---

[2]In practice, J_z flip sign under the TR transformation. Nonetheless, since the angular momentum is conserved, we can rotate both sides back without affecting the amplitudes.
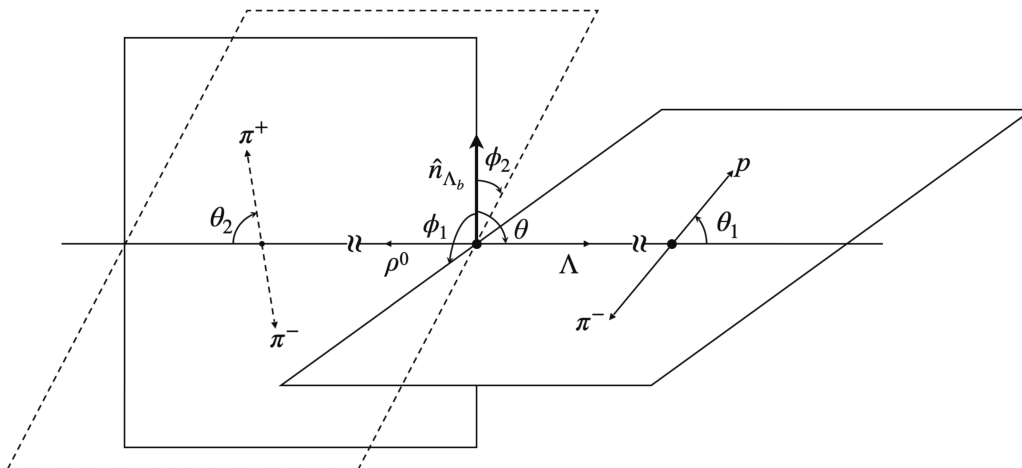
Figure 1. The angles given in eq. (2.9), where $\hat{n}_{\Lambda_b}$ is the unit vector in the Λ_b polarized direction.

where f_i are made of the amplitudes related to $\Lambda_b \to \Lambda \rho^0$, and D_i depend on the angles, P_b and α_Λ. The explicit forms are given in table 1, where we have used the Legendre polynomial of $P_2 = (3\cos^2\theta_2 - 1)/2$ for the later convenience. In general, $\mathcal{D}(\vec{\Omega})$ can be applied to any sequential decay in the form of $1/2 \to 1/2(\to 1/2, 0)1(\to 0, 0)$ with a straightforward replacement. For instance, the distribution of $\Xi_c^0 \to \Lambda(\to p\pi^-)\overline{K}^{*0}(\to \pi^+ K^-)$ can be given by substituting Ξ_c^0 and $\overline{K}^{*0}$ for Λ_b and ρ^0, respectively [52].

In $\mathcal{D}(\vec{\Omega})$, f_i depend on 4 complex amplitudes, in which a real parameter and a complex phase could be diminished by the normalization, resulting in 3 real parameters and 3 relative complex phases left. Although the explicit forms are complicated, there have some rules which $\mathcal{D}(\vec{\Omega})$ follows:

- In the unpolarized case, $P_b = 0$, $\mathcal{D}(\vec{\Omega})$ shall only depend on θ_1, θ_2 and $\phi_1 + \phi_2$ (see figure 1).

- Since ρ^0 decays strongly, $\mathcal{D}(\vec{\Omega})$ is invariant under interchanging $\pi^\pm$ in the cascade decay of ρ^0. Therefore, it does not matter which pseudoscalar meson we choose to define θ_2. Formally, it means that D_i is invariant under the transformation of $(\phi_2, \theta_2) \to (\phi_2 + \pi, \pi - \theta_2)$.

- On the other hand, Λ decays weakly, and hence it is important that θ_1 is chosen as the polar angle between $\hat{n}_{\Lambda_b}$ and $\vec{p}_\Lambda$. Formally, D_i is unaltered under the transformation of $(\phi_1, \theta_1, \alpha) \to (\phi_1 + \pi, \pi - \theta_1, -\alpha)$.

Even when Λ_b is unpolarized, it is still possible to determine $|a_\pm|^2$ and $|b_\pm|^2$ with $f_{2,3,4}$ and the decay widths. Furthermore, the relative phase between b_+ (b_-) and a_+ (a_-) can be extracted from $f_{5,6}$.

To reduce the uncertainties caused by P_b, one can sum over the degree of freedoms in the Λ_b polarization. To this end, one defines that

$$\begin{aligned} \chi_1 &= \phi_1 + \phi_2\,, & 0 < \chi_1 < 2\pi\,, \\ \chi_2 &= \frac{1}{2}(\phi_1 - \phi_2)\,, & -\pi < \chi_2 < \pi\,. \end{aligned} \tag{2.12}$$

i	f_i	D_i
1	$\lvert a_+\rvert^2 + \lvert a_-\rvert^2 + \lvert b_+\rvert^2 + \lvert b_-\rvert^2$	1
2	$2\lvert a_+\rvert^2 + 2\lvert a_-\rvert^2 - \lvert b_+\rvert^2 - \lvert b_-\rvert^2$	P_2
3	$\lvert a_+\rvert^2 - \lvert a_-\rvert^2 - \lvert b_+\rvert^2 + \lvert b_-\rvert^2$	$\alpha\cos\theta_1$
4	$2\lvert a_+\rvert^2 - 2\lvert a_-\rvert^2 + \lvert b_+\rvert^2 - \lvert b_-\rvert^2$	$\alpha\cos\theta_1 P_2$
5	$\frac{3}{\sqrt{2}}\Re(b_+ a_+^* - b_- a_-^*)$	$\alpha\sin\theta_1\sin(2\theta_2)\cos(\phi_1+\phi_2)$
6	$\frac{3}{\sqrt{2}}\Im(b_+ a_+^* + b_- a_-^*)$	$\alpha\sin\theta_1\sin(2\theta_2)\sin(\phi_1+\phi_2)$
7	$\lvert a_+\rvert^2 - \lvert a_-\rvert^2 + \lvert b_+\rvert^2 - \lvert b_-\rvert^2$	$P_b\cos\theta$
8	$2\lvert a_+\rvert^2 - 2\lvert a_-\rvert^2 - \lvert b_+\rvert^2 + \lvert b_-\rvert^2$	$P_b\cos\theta P_2$
9	$\frac{3}{\sqrt{2}}\Re(a_+ b_-^* - a_- b_+^*)$	$P_b\sin\theta\sin(2\theta_2)\cos\phi_2$
10	$\frac{3}{\sqrt{2}}\Im(a_+ b_-^* + a_- b_+^*)$	$P_b\sin\theta\sin(2\theta_2)\sin\phi_2$
11	$\lvert a_+\rvert^2 + \lvert a_-\rvert^2 - \lvert b_+\rvert^2 - \lvert b_-\rvert^2$	$P_b\alpha\cos\theta\cos\theta_1$
12	$2\lvert a_+\rvert^2 + 2\lvert a_-\rvert^2 + \lvert b_+\rvert^2 + \lvert b_-\rvert^2$	$P_b\alpha\cos\theta\cos\theta_1 P_2$
13	$\frac{3}{\sqrt{2}}\Re(b_+ a_+^* + b_- a_-^*)$	$P_b\alpha\cos\theta\sin\theta_1\sin(2\theta_2)\cos(\phi_1+\phi_2)$
14	$\frac{3}{\sqrt{2}}\Im(b_+ a_+^* - b_- a_-^*)$	$P_b\alpha\cos\theta\sin\theta_1\sin(2\theta_2)\sin(\phi_1+\phi_2)$
15	$\frac{3}{\sqrt{2}}\Re(a_+ b_-^* + a_- b_+^*)$	$P_b\alpha\sin\theta\cos\theta_1\sin(2\theta_2)\cos\phi_2$
16	$\frac{3}{\sqrt{2}}\Im(a_+ b_-^* - a_- b_+^*)$	$P_b\alpha\sin\theta\cos\theta_1\sin(2\theta_2)\sin\phi_2$
17	$-2\Re(a_- a_+^*)$	$P_b\alpha\sin\theta\sin\theta_1(1+2P_2)\cos\phi_1$
18	$-2\Im(a_- a_+^*)$	$P_b\alpha\sin\theta\sin\theta_1(1+2P_2)\sin\phi_1$
19	$2\Re(b_+ b_-^*)$	$P_b\alpha\sin\theta\sin\theta_1(1-P_2)\cos(\phi_1+2\phi_2)$
20	$2\Im(b_+ b_-^*)$	$P_b\alpha\sin\theta\sin\theta_1(1-P_2)\sin(\phi_1+2\phi_2)$

Table 1. The parametrized angular distributions with the angles shown in figure 1, where P_2 is the Legendre polynomial with $P_2 = (3\cos^2\theta_2 - 1)/2$.

The unpolarized angular distribution is then given as

$$\mathcal{D}^0(\vec{\Omega}^0) = \int_{-\pi}^{\pi}\int_{-1}^{1}\mathcal{D}^\chi(\vec{\Omega}^\chi)d\cos\theta d\chi_2\,, \qquad (2.13)$$

where only D_i with $i \leq 6$ survive from the selection rule. In analogy to eq. (2.9), we have

$$\mathcal{D}^0(\vec{\Omega}^0) = \frac{1}{\Gamma_c}\frac{\partial^3\Gamma_c}{\partial\cos\theta_1\partial\cos\theta_2\partial\chi_1}$$

$$= \frac{1}{8\pi}\frac{1}{\lvert a_+\rvert^2 + \lvert a_-\rvert^2 + \lvert b_+\rvert^2 + \lvert b_-\rvert^2}\sum_{i=1}^{6}f_i(a_\pm, b_\pm)D_i^0(\alpha_\Lambda, \theta_1, \theta_2, \chi_1)\,, \qquad (2.14)$$

which is clearly independent of P_b. Here, $\mathcal{D}^\chi(\vec{\Omega}^\chi)$ and D_i^0 are obtained by changing the parametrization of the azimuthal angles from (ϕ_1, ϕ_2) to (χ_1, χ_2) in $\mathcal{D}(\vec{\Omega})$ and D_i, respectively.

By integrating θ_2 and χ_1 in $\mathcal{D}^0(\vec{\Omega}^0)$, we obtain that

$$\frac{1}{\Gamma_c}\frac{d\Gamma_c}{d\cos\theta_1} = (1 + \alpha_{\lambda_1}\alpha\cos\theta_1)/2\,, \tag{2.15}$$

where

$$\alpha_{\lambda_1} \equiv \frac{\Gamma(\lambda_1 = 1/2) - \Gamma(\lambda_1 = -1/2)}{\Gamma(\lambda_1 = 1/2) + \Gamma(\lambda_1 = -1/2)} = \frac{|a_+|^2 + |b_-|^2 - |a_-|^2 - |b_+|^2}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2}\,, \tag{2.16}$$

describing the polarization asymmetry of Λ. Likewise, we can integrate θ_1 and χ_1, given by

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_2} = \frac{1}{2} + \frac{P_2}{4} + \frac{3}{4}P_2\alpha_{\lambda_2}\,, \tag{2.17}$$

where

$$\alpha_{\lambda_2} \equiv \frac{|a_+|^2 + |a_-|^2 - |b_+|^2 - |b_-|^2}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2}\,, \tag{2.18}$$

representing the asymmetry between the longitudinal and transverse polarizations of ρ^0. With eq. (2.5), it is straightforward to see that α_{λ_1} and α_{λ_2} are P-odd and P-even, respectively. In addition, $\alpha_{\lambda_{1,2}}$ are independent of P_b, which is a reasonable result since $\lambda_{1,2}$ are $SO(3)_R$ scalars and shall not be affected by the spin direction of Λ_b alone.

The structures of the angular distributions can be traced back to the spin correlations in $\Lambda_b \to \Lambda V$. In the next section, we will study the T-odd spin correlations and identify their effects on $\mathcal{D}(\vec{\Omega})$.

In the near future, it is not likely that the experiments have enough data points to reconstruct the full angular distributions in eq. (2.9). Nonetheless, the experiments at LHCb are able to carry out the partial angular distribution analyses, which require a handful of parameters to be fitted, such as the ones in eqs. (2.14), (2.15) and (2.17) (see eq. (3.19) also). In particular, aiming on probing T-violation, the partial angular analysis of $\Lambda_b \to \Lambda\phi$ has been studied at LHCb. [32], although the effects of the FSIs have not been considered.

3 T-odd observables

In general, to observe T-violating effects, one should compare $\Lambda_b \to \Lambda V$ to the time-reversed processes, $\Lambda V \to \Lambda_b$, which are difficult to prepare in the experiments. However, in the first order of the weak interaction, it is possible to constrain the amplitudes with the T symmetry as demonstrated in the helicity formalism (see eq. (2.8) and appendix A also). Our goal in this section is to obtain naive T-odd observables with eq. (A.4) and subtract the effects of the FSIs by comparing them with the charge conjugate ones. To this end, we ought to find the eigenstates of the T-odd operators, $\hat{T}_i$, in the ΛV systems, which satisfy

$$I_t|\Lambda V; \lambda_{t_i}, \{\lambda_f\}\rangle = |\Lambda V; -\lambda_{t_i}, \{\lambda_f^T\}\rangle\,, \tag{3.1}$$

where I_t is the T operator, λ_{t_i} are the eigenvalues of $\hat{T}_i$, $\{\lambda_f\}$ corresponds to a set of compatible physical quantities to specify the states, and λ_f^T represent the values after the T transformation.

From eq. (A.2), if $\mathcal{H}_{\text{eff}}$ respects the T symmetry, we have

$$|\langle \Lambda V; \lambda_{t_i}, \text{"out"}, \{\lambda_f\}|\mathcal{H}_{\text{eff}}|B\rangle|^2 - |\langle \Lambda V; -\lambda_{t_i}, \text{"in"}, \{\lambda_f^T\}|\mathcal{H}_{\text{eff}}|B\rangle|^2 = 0. \tag{3.2}$$

In our study, except for J_z which can always be flipped back by $\mathrm{SO}(3)_R$,[3] λ_f are chosen as T-even. The naive T-violating observables are then given by

$$\Delta_{t_i} = \frac{\Gamma(\lambda_{t_i}) - \Gamma(-\lambda_{t_i})}{\Gamma(\lambda_{t_i}) + \Gamma(-\lambda_{t_i})}, \tag{3.3}$$

where we always choose $\lambda_{t_i} > 0$ to fix the ambiguity. Notice that if "in" and "out" can be interchanged freely, the left-handed side of eq. (3.2) would be proportional to the numerator of Δ_{t_i}, resulting in that Δ_{t_i} are T-odd observables. Furthermore, since $\hat{T}_i$ involve more than two spins in two-body decays, Δ_{t_i} are also referred to as the T-odd spin correlations.

Nonetheless, $|\pm\lambda_{t_i}\rangle$ could potentially oscillate, leading to

$$\langle -\lambda_{t_i}; \text{"out"}|\lambda_{t_i}; \text{"in"}\rangle \neq 0, \tag{3.4}$$

for the rescattering from $\lambda_{t_i} \to -\lambda_{t_i}$, due to the FSIs. To subtract the rescattering effects, we have to compare them with their charge conjugates, as long as the FSIs are not tractable. Accordingly, the true T-violating quantities can be given as

$$\mathcal{T}_{t_i} \equiv (\Delta_{t_i} - (\pm)\overline{\Delta}_{t_i})/2, \tag{3.5}$$

where the overline denotes the charge conjugate, and the signs correspond to the parities of $\hat{T}_i$. For example, if $\hat{T}_j$ are the vector products of two spins and one 3-momentum, it is P-odd with the minus sign for $\mathcal{T}_{t_j}$. Here, as we have used the CPT symmetry to relate the charge conjugate processes, $\mathcal{T}_{t_i}$ are not only T-violating but also CP-violating observables.

To define the T-odd operators, we start with the definition of the spin, given by [51]

$$m\vec{s} = P^0\vec{J} - \vec{p} \times \vec{K} - \frac{1}{P^0 + m}\vec{p}(\vec{p} \cdot \vec{J}), \tag{3.6}$$

where m is the particle's mass, and $\vec{K}$ is the generator of the Lorentz boost. Despite that the definition seems complicated, each term can be understood separately. On the right-hand side of eq. (3.6), the first term describes that $\vec{s}$ shall be reduced to $\vec{J}$ at the rest frame, the second one ensures $[s_i, p_j] = 0$, and the last one guarantees that the algebra is closed, i.e., $[s_i, s_j] = i\epsilon_{ijk}s_k$. As a result, we have [51]

$$s_i L|\vec{p} = 0, J_z = M\rangle = L J_i|\vec{p} = 0, J_z = M\rangle, \tag{3.7}$$

where L is an arbitrary Lorentz boost, indicating that $\vec{s}$ is treated as a 3-momentum state of $\vec{J}$ in its rest frame.

In quantum mechanics, one should find a set of commuting operators to characterize the states properly, which has often been mishandled in the studies concerning T-violation. For instance, let us consider the most simple T-odd operator, given by

$$\hat{T}_1 = (\vec{s}_1 \times \vec{s}_2) \cdot \hat{p}_1, \tag{3.8}$$

---

[3]See the footnote followed by eq. (2.8).

where $\vec{s}_1$ and $\vec{s}_2$ are the spins of Λ and V, respectively, and $\hat{p}_1$ is the normalized unit vector of $\vec{p}_1$. In the literature, it is often examined in terms of the Dirac spinors and polarization vector as eq. (2.2), which essentially expand the final states by $(\vec{p}_1, \vec{s}_1 \cdot \hat{n}_1)$ and $(\vec{p}_2, \vec{s}_2 \cdot \hat{n}_2)$, where $\hat{n}_1$ and $\hat{n}_2$ are arbitrary unit vectors in the three-dimensional space. However, the approach is questionable since $\vec{s}_1 \cdot \hat{n}_1$ does not commute with $\hat{T}_1$. It is odd (wrong) to study any physical quantity with incompatible observables.

For each T-odd operator, one should find a set of compatible observables, in which J^2 and J_z are always included, resulting in that only T-odd $SO(3)_R$ scalars are studied. These T-odd scalars are classified into 4 categories, depending on the spins. First, we study $\hat{T}_1$ defined in eq. (3.8), which can be sizable even when $P_b = 0$, since it does not contain $\vec{J}$. Second, we work out the case in which $\vec{s}_1$ is not involved, given as

$$\hat{T}_2 = (\vec{s}_2 \times \hat{p}_1) \cdot \vec{J}, \tag{3.9}$$

which can be large even when $\alpha = 0$. Third, we discuss the case with $\vec{s}_1$, given as

$$\hat{T}_3 \equiv (\vec{s}_1 \times \hat{p}_1) \cdot \vec{J}, \tag{3.10}$$

which requires both $P_b \neq 0$ and $\alpha \neq 0$. Note that if we set $\alpha = 0$, D_i will be independent of θ_1 and ϕ_1 as in the cascade decays of $1/2 \rightarrow 1/2, 0$ shown in eq. (2.15). Finally, we examine the cases in which all the spins are involved.

3.1 T-odd observables with $\vec{s}_1$ and $\vec{s}_2$

In this subsection, we consider the T-odd scalar operators made from $\vec{s}_1$, $\vec{s}_2$, $\hat{p}_1$ and $\vec{J} \cdot \hat{p}_1$. Since $\hat{p}_1$ commutes with each of them, the T-odd scalar operators must also commute with $\vec{J} \cdot \hat{p}_1$. The most simple case is $\hat{T}_1$, given in eq. (3.8). By direct computation with eqs. (2.3) and (3.7), the eigenstates are given by

$$|\lambda_{t_1} = \pm\frac{1}{\sqrt{2}}, \lambda = \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left(|a_+\rangle \mp i|b_+\rangle\right),$$
$$|\lambda_{t_1} = \pm\frac{1}{\sqrt{2}}, \lambda = -\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left(|a_-\rangle \pm i|b_-\rangle\right), \tag{3.11}$$

where λ_{t_1} and $\lambda = \lambda_1 - \lambda_2$ are the eigenvalues of $\hat{T}_1$ and $\vec{J} \cdot \hat{p}_1$, respectively. Due to the anti-linear property of I_t, one can easily check that the eigenstates in eq. (3.11) satisfy the criterion in eq. (3.1).

In addition, it is straightforward to see the helicities are entangled in $|\lambda_{t_1}\rangle$. Nonetheless, they are still independent of $\hat{n}_{\Lambda_b}$. Since $\hat{T}_1$ commutes with $\vec{J} \cdot \hat{p}_1$, $|a_+\rangle$ and $|a_-\rangle$ share the same θ dependence with $|b_+\rangle$ and $|b_-\rangle$, respectively. For instance, from eq. (2.3), we have

$$|\lambda_{t_1} = \pm\frac{1}{\sqrt{2}}, \lambda = \frac{1}{2}\rangle = \frac{1}{2\sqrt{2}\pi} \int d\Omega \left(|\vec{p}_1, 1/2, 0\rangle \mp i|\vec{p}_1, -1/2, \mp 1\rangle\right) e^{-iM\phi} d^{\frac{1}{2}}(\theta)^M{}_{\frac{1}{2}}. \tag{3.12}$$

Clearly, the helicities do not depend on θ and hence $\hat{n}_{\Lambda_b}$.

Notice the close relations between the opposite λ. The eigenstates in eq. (3.11) are related by the parity, which can be seen from the following identity, read as[4]

$$I_t I_s |\lambda_{t_1}, \lambda\rangle = I_t| - \lambda_{t_1}, -\lambda\rangle = |\lambda_{t_1}, -\lambda\rangle \,. \tag{3.13}$$

Here, the first and second equalities are due to that $\lambda_{(t_1)}$ and λ are P-odd and T-even, respectively. In general, λ_{t_1} is degenerated as long as $\lambda \neq 0$.

The naive T-odd observable, defined in eq. (3.3), is then given as

$$\Delta_{t_1} \equiv \frac{\Gamma(\lambda_{t_1}) - \Gamma(-\lambda_{t_1})}{\Gamma(\lambda_{t_1}) + \Gamma(-\lambda_{t_1})} = \frac{2\Im(a_+ b_+^* - a_- b_-^*)}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2} \,, \tag{3.14}$$

which is also P-odd. Consequently, the true T-odd quantity is given by

$$\mathcal{T}_{t_1} = (\Delta_{t_1} + \overline{\Delta}_{t_1})/2 \,. \tag{3.15}$$

From eq. (3.14), it is obvious that Δ_{t_1} vanishes if $a_\pm$ and $b_\pm$ are real. This is consistent with eq. (2.8).

To construct a P-even and T-odd observable, we define

$$\hat{T}_1^{\ p} = \vec{J} \cdot \hat{p}_1 \hat{T}_1 \,. \tag{3.16}$$

By comparing eqs. (3.16) and (3.12), it is easy to see that the eigenstates of $\hat{T}_1^p$ are identical to those of $\hat{T}_1$ with the eigenvalues of $\lambda_{t_1}^p = \lambda_{t_1}\lambda$. Accordingly, the naive T-odd observable is then given as

$$\Delta_{t_1}^p = \frac{2\Im(a_+ b_+^* + a_- b_-^*)}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2} \,, \tag{3.17}$$

and the true one is

$$\mathcal{T}_{t_1}^p = (\Delta_{t_1}^p - \overline{\Delta}_{t_1}^p)/2 \,. \tag{3.18}$$

With the unpolarized Λ_b, there are only 3 independent observables in the decay distributions, $\vec{p}_a$, $\vec{p}_b$ and $\vec{p}_c$, where a, b and c correspond to the particles after the cascade decays. As a result, to manifest the T-odd quantity, it can only take the P-odd form, $(\vec{p}_a \times \vec{p}_b) \cdot \vec{p}_c$. Hence, one naively expects that Δ_{t_1} (P-odd) shows up in $\mathcal{D}^0(\vec{\Omega}^0)$, whereas $\Delta_{t_1}^p$ (P-even) can not be observed. It is indeed the case in our previous work [10], in which we consider $B \to VV'$. Interestingly, things go the other way round in Λ_b. We see that Δ_{t_1} appears in f_{14}, which requires $P_b \neq 0$, whereas $\Delta_{t_1}^p$ is found in f_6, which is independent of P_b. The reason for this opposite behavior is that to manifest the helicities of Λ, α is always needed (see eq. (2.15)), which is P-odd, and therefore inverts the argument.

Since the observation of $\Delta_{t_1}^p$ does not demand Λ_b to be polarized, the uncertainties caused by P_b can be eliminated. From $\mathcal{D}^0(\vec{\Omega}^0)$, we find that

$$\Delta_{t_1}^p = -\frac{128\sqrt{2}}{3\pi^2\alpha} \int_0^{2\pi} \int_{-1}^1 \int_{-1}^1 \mathcal{D}^0(\vec{\Omega}^0) d\cos\theta_1 \, (\cos\theta_2 d\cos\theta_2) \, (\sin\chi_1 d\chi_1) \,. \tag{3.19}$$

---

[4]In practice, J_z flips the sign under I_t. However, we can always rotate it back with $R_2(-\pi)$ without affecting λ_{t_i} and λ, and therefore, the conclusion still holds.

Note that in contrast to the ordinary integral, we have added the weight factors, $\cos\theta_2$ and $\sin\chi_1$. Here, $\cos\theta_2$ is designed to diminish the uncertainties, satisfying the orthogonal relation, given as

$$\int_{-1}^{1} D_i \cos\theta_2 d\cos\theta_2 = 0 \quad \text{for} \quad i = 1, 2, 3, 4\,, \tag{3.20}$$

while $\sin\chi_1$ corresponds to the familiar Fourier transformation.

3.2 T-odd observables with $\vec{J}$ and $\vec{s}_2$

Similar to the previous subsection, we consider the T-odd scalar operators made of $\vec{s}_2$, $\hat{p}_1$, $\vec{J}$ and $\vec{s}_1 \cdot \hat{p}_1$. Since $\vec{s}_1 \cdot \hat{p}_1$ commutes with each of them, the T-odd operators commute with $\vec{s}_1 \cdot \hat{p}_1$. From eq. (3.13) with substituting λ_1 for λ, we anticipate that the eigenvalues are degenerated as $\lambda_1 \neq 0$. For the most simple case of $\hat{T}_2$ in eq. (3.9), we have that

$$|\lambda_{t_2} = \pm 1/\sqrt{2}\,, \lambda_1 = 1/2\rangle = \frac{1}{\sqrt{2}} \left(|a_+\rangle \pm i|b_-\rangle\right),$$

$$|\lambda_{t_2} = \pm 1/\sqrt{2}\,, \lambda_1 = -1/2\rangle = \frac{1}{\sqrt{2}} \left(|a_-\rangle \mp i|b_+\rangle\right), \tag{3.21}$$

where λ_{t_2} and λ_1 are the eigenvalues of $\hat{T}_2$ and $\vec{s}_1 \cdot \hat{p}_1$, respectively, while the naive T-odd observable is

$$\Delta_{t_2} = \frac{2\Im(b_- a_+^* - b_+ a_-^*)}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2}\,, \tag{3.22}$$

where the spins correlations are handed down to f_{10}. With the charge conjugate, the true T-odd observable is then given as

$$\mathcal{T}_{t_2} = (\Delta_{t_2} + \overline{\Delta}_{t_2})/2\,. \tag{3.23}$$

In analogy to the previous subsection, we define $\hat{T}_2^p = \vec{s}_1 \cdot \hat{p}_1 \hat{T}_2$, resulting in the naive T-odd observable as

$$\Delta_{t_2}^P = \frac{2\Im(b_- a_+^* + b_+ a_-^*)}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2}\,, \tag{3.24}$$

which can be found in f_{16}, and the true T-odd observable is

$$\mathcal{T}_{t_2}^p = (\Delta_{t_2}^p - \overline{\Delta}_{t_2}^p)/2\,. \tag{3.25}$$

In section 4, we will see that $\Lambda_b \to \Lambda V$ is predominant by a_- and b_+, making $\Delta_{t_2}^{(p)}$ a good observable to test the SM.

3.3 T-odd observables with $\vec{J}$ and $\vec{s}_1$

We now expand the states in terms of $\vec{s}_2 \cdot \hat{p}_1$ and $\hat{T}_3$. The eigenstates with nonzero eigenvalues are

$$|\lambda_{t_3} = \pm\frac{1}{2}\rangle = \frac{1}{\sqrt{2}} \left(|a_+\rangle \pm i|a_-\rangle\right), \tag{3.26}$$

where λ_{t_3} are the eigenvalues of $\hat{T}_3$. On the other hand, similar to eq. (3.13), we have

$$\hat{T}_3|b_\pm\rangle = 0\,. \tag{3.27}$$

The naive T- odd observable is

$$\Delta_{t_3} \equiv \frac{2\Im(a_-a_+^*)}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2}, \tag{3.28}$$

which manifests itself in f_{18}. The true T-odd observable is

$$\mathcal{T}_{t_3} = (\Delta_{t_3} - \overline{\Delta}_{t_3})/2, \tag{3.29}$$

which is also a CP-violating observable due to the CPT symmetry.

3.4 T-odd observables with triple spin correlations

Among the T-odd parameters in $D(\vec{\Omega})$, only f_{20} has not been discussed yet. The responsible T-odd operator is quite complicated, read as

$$\hat{T}_4 = \left[\hat{T}_1 \left(\vec{s}_2 \cdot \vec{J} - \frac{1}{2}(\vec{s}_2 \cdot \hat{p}_1)^2 \right) + \frac{1}{\sqrt{2}} \hat{T}_3 \right] + h.c., \tag{3.30}$$

where $h.c.$ stands for the Hermitian conjugate. The eigenstates are

$$|\lambda_{t_4} = \pm\sqrt{2}\rangle = \frac{1}{\sqrt{2}} \left(|b_+\rangle \pm i|b_-\rangle \right), \tag{3.31}$$

where $\hat{T}_4|a_\pm\rangle = 0$. The corresponding naive T-odd observable is then given as

$$\Delta_{t_4} \equiv \frac{2\Im(b_-b_+^*)}{|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2}, \tag{3.32}$$

which is proportional to the relative phase between $b_\pm$, while the true T-odd observable is

$$\mathcal{T}_{t_4} = (\Delta_{t_4} + \overline{\Delta_{t_4}})/2. \tag{3.33}$$

Here, the nonzero value of Δ_{t_4} could be caused by the FSIs. However, to oscillate between $|b_\pm\rangle$, the helicities of V must alter twice ($\lambda_2 : 1 \to 0 \to -1$), and therefore, the oscillations are expected to be suppressed, as the case in B mesons decays [10]. Clearly, it is interesting to see whether the suppression holds in the baryon systems or not.

Before ending this section, let us collect the results. We have found the T-odd spin correlations in $\Lambda_b \to \Lambda V$, with their effects on the sequential decays identified. In particular, we have shown that $\Delta_{t_1}, \Delta_{t_1}^p, \Delta_{t_2}, \Delta_{t_2}^p, \Delta_{t_3}$ and Δ_{t_4} correspond to $f_{14}, f_6, f_{10}, f_{16}, f_{18}$ and f_{20}, respectively. Notably, all the relative phases among $a_\pm$ and $b_\pm$ can be described by $\Delta_{t_{1,2,3,4}}^{(p)}$, which complete our study on all possible T-odd observables.

4 Numerical results

In the SM, the effective Hamiltonian responsible for $b \to d/s$ transitions, obtained from the operator product expansions, is given by [53]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{f=,d,s} \left[V_{ub}V_{uf}^* \left(C_1 O_1^f + C_2 O_2^f \right) - V_{tb}V_{tq}^* \sum_{i=3}^{10} C_i O_i^f \right], \tag{4.1}$$

where G_F stands for the Fermi constant, C_i represent the Wilson coefficients, $V_{qq'}$ correspond to the CKM matrix elements, and O_{1-10}^f are the operator products, given by [53]

$$O_1^f = (\overline{u}_\alpha b_\alpha)_L (\overline{f}_\beta u_\beta)_L \,, \qquad\qquad O_2^f = (\overline{u}_\alpha b_\beta)_L (\overline{f}_\beta u_\alpha)_L \,,$$

$$O_3^f = (\overline{f}_\alpha b_\alpha)_L \sum_q (\overline{q}_\beta q_\beta)_L \,, \qquad O_4^f = (\overline{f}_\alpha b_\beta)_L \sum_q (\overline{q}_\beta q_\alpha)_L \,,$$

$$O_5^f = (\overline{f}_\alpha b_\alpha)_L \sum_q (\overline{q}_\beta q_\beta)_R \,, \qquad O_6^f = (\overline{f}_\alpha b_\beta)_L \sum_q (\overline{q}_\beta q_\alpha)_R \,,$$

$$O_7^f = \frac{3}{2}(\overline{f}_\alpha b_\alpha)_L \sum_q e_q(\overline{q}_\beta q_\beta)_R \,, \qquad O_8^f = \frac{3}{2}(\overline{f}_\alpha b_\beta)_L \sum_q e_q(\overline{q}_\beta q_\alpha)_R \,,$$

$$O_9^f = \frac{3}{2}(\overline{f}_\alpha b_\alpha)_L \sum_q e_q(\overline{q}_\beta q_\beta)_L \,, \qquad O_{10}^f = \frac{3}{2}(\overline{f}_\alpha b_\beta)_L \sum_q e_q(\overline{q}_\beta q_\alpha)_L \,, \qquad (4.2)$$

with L and R in the subscripts denoting the left and right-handed currents, respectively. The amplitudes are given by sandwiching $\mathcal{H}_{\text{eff}}$ between the initial and final states. Note that both C_i and O_i depend on the renormalization schemes and energy scales.

In the following, we adopt the generalized factorization approach [45], in which the quarks and antiquarks of vector mesons are created by weak vertices. The amplitudes are simplified as [18, 46]

$$\frac{G_F}{\sqrt{2}} \mathcal{C}_V \langle V|V^\mu|0\rangle\langle\Lambda|(\overline{s}b)_L|\Lambda_b\rangle = \frac{G_F}{\sqrt{2}} \mathcal{C}_V f_V M_V \xi^{\mu*}\langle\Lambda|(\overline{s}b)_L\gamma_\mu b|\Lambda_b\rangle \,, \qquad (4.3)$$

where V^μ, f_V, M_V, and ξ^μ are the currents, decay constants, masses, and polarizations of the vector mesons (V), and $\mathcal{C}_V$ are given as

$$\mathcal{C}_\phi = -V_{ts}^* V_{tb} \left(a_3 + a_4 + a_5 - \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) \,,$$

$$\mathcal{C}_{\rho^0} = \frac{1}{\sqrt{2}} \left[V_{us}^* V_{ub} a_2 - \frac{3}{2} V_{ts}^* V_{tb} (a_7 + a_9) \right] \,,$$

$$\mathcal{C}_\omega = \frac{1}{\sqrt{2}} \left\{ V_{us}^* V_{ub} a_2 - V_{ts}^* V_{tb} \left[2a_3 + 2a_5 + \frac{1}{2}(a_7 + a_9) \right] \right\} \,,$$

$$\mathcal{C}_{K^{*0}} = -V_{td}^* V_{tb} \left(a_4 - \frac{1}{2}a_{10} \right) \,, \qquad (4.4)$$

respectively, with $a_i = c_i^{\text{eff}} + c_{i+(-1)}^{\text{eff}}/N_c$ for $i =$odd (even). Here, one has $N_c = 3$ in the absence of the NF contributions. In the numerical calculations, the Wolfenstein parametrization is used for the CKM matrix elements in the SM, taken to be [25]

$$\lambda = 0.22650 \pm 0.00048 \,, \quad A = 0.790_{-0.012}^{+0.017} \,, \quad \rho = 0.141_{-0.017}^{+0.016} \,, \quad \eta = 0.357 \pm 0.011 \,, \quad (4.5)$$

and the effective Wilson coefficients are given in table 2 [43, 45].

The baryon transitions in eq. (4.3) can be parametrized by the form factors, defined by

$$\langle\Lambda|\overline{s}\gamma^\mu b|\Lambda_b\rangle = \overline{u}_\Lambda \left(f_1(q^2)\gamma^\mu - f_2(q^2)i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Lambda_b}} + f_3(q^2)\frac{q^\mu}{M_{\Lambda_b}} \right) u_{\Lambda_b} \,,$$

$$\langle\Lambda|\overline{s}\gamma^\mu\gamma^5 b|\Lambda_b\rangle = \overline{u}_\Lambda \left(g_1(q^2)\gamma^\mu - g_2(q^2)i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Lambda_b}} + g_3(q^2)\frac{q^\mu}{M_{\Lambda_b}} \right) \gamma^5 u_{\Lambda_b} \,, \qquad (4.6)$$

	$b \to d$	$\bar{b} \to \bar{d}$	$b \to s$	$\bar{b} \to \bar{s}$
c_1^{eff}	1.168	1.168	1.168	1.168
c_2^{eff}	-0.365	-0.365	-0.365	-0.365
c_3^{eff}	$238 + 14i$	$254 + 43i$	$243 + 31i$	$241 + 32i$
c_4^{eff}	$-497 - 42i$	$-545 - 130i$	$-512 - 94i$	$-506 - 97i$
c_5^{eff}	$145 + 14i$	$162 + 43i$	$150 + 31i$	$148 + 32i$
c_6^{eff}	$-633 - 42i$	$-682 - 130i$	$-649 - 94i$	$-643 - 97i$
c_7^{eff}	$-1.0 - 1.0i$	$-1.4 - 1.8i$	$-1.1 - 2.2i$	$-1.1 - 1.3i$
c_8^{eff}	5.0	5.0	5.0	5.0
c_9^{eff}	$-112 - 1i$	$-112 - 3i$	$-112 - 2i$	$-112 - 2i$
c_{10}^{eff}	20	20	20	20

Table 2. The effective Wilson coefficients with the NDR and $\overline{\text{MS}}$ schemes at the energy scale of $\mu = 2.5\,\text{GeV}$, where $c_{1,2}^{\text{eff}}$ and c_{3-10}^{eff} are in the units of 10^0 and 10^{-4}, respectively.

	f_1	f_2	f_3	g_1	g_2	g_3
$q^2 = m_\rho^2$	1.60 ± 0.09	0.02 ± 0.00	-0.18 ± 0.00	1.60 ± 0.09	0.05 ± 0.00	-0.21 ± 0.00
$q^2 = m_{K^*}^2$	1.62 ± 0.09	0.02 ± 0.00	-0.19 ± 0.01	1.62 ± 0.09	0.05 ± 0.00	-0.22 ± 0.01
$q^2 = m_\phi^2$	1.63 ± 0.07	0.04 ± 0.01	-0.19 ± 0.01	1.64 ± 0.08	0.05 ± 0.01	-0.22 ± 0.00
$q^2 = (M_{\Lambda_b} - M_\Lambda)^2$	11.0 ± 0.00	4.04 ± 0.02	0.00 ± 0.13	11.5 ± 0.0	0.66 ± 0.07	-3.05 ± 0.11

Table 3. Form factors with the unit 10^{-1}, where the uncertainties come from the bag radius.

where $q^\mu = p_{\Lambda_b}^\mu - p_\Lambda^\mu$, and $f_{1,2,3}$ and $g_{1,2,3}$ are the form factors. In this study, we calculate them with the modified MIT bag model [44], in which the center motions of the baryon waves are removed. The hadrons parameters are given as [44, 54]

$$m_b = 4.8 \text{ GeV}, \quad m_s = 0.28 \text{ GeV}, \quad m_u = m_d = 0.005 \text{ GeV}, \quad R^{-1} = 0.21 \pm 0.01 \text{GeV}, \tag{4.7}$$

where R is the bag radius, and $(f_\phi, f_\omega, f_\rho) = (215 \pm 5, 187 \pm 5, 216 \pm 3)\,\text{MeV}$ [55]. The numerical results of the form factors with the q^2 dependencies are given in table 3. The uncertainties come mainly from the bag radius. We see that our results are consistent with those having $f_1 = g_1$ demanded by the heavy quark symmetry.

Finally, the helicity amplitudes are related to the form factors as [18]

$$a_\pm = \frac{G_F}{\sqrt{2}} \mathcal{C}_V f_V M_V \left[\sqrt{Q_-} \left(f_1 \frac{M_+}{M_V} + f_2 \frac{M_V}{M_{\Lambda_b}} \right) \mp \sqrt{Q_+} \left(g_1 \frac{M_-}{M_V} - g_2 \frac{M_V}{M_{\Lambda_b}} \right) \right],$$

$$b_\pm = \frac{G_F}{\sqrt{2}} \mathcal{C}_V f_V M_V \left[\sqrt{2Q_-} \left(-f_1 - f_2 \frac{M_+}{M_{\Lambda_b}} \right) \mp \sqrt{2Q_+} \left(-g_1 + g_2 \frac{M_-}{M_{\Lambda_b}} \right) \right], \tag{4.8}$$

where $M_\pm = M_{\Lambda_b} \pm M_\Lambda$ and $Q_\pm = M_\pm^2 - M_V^2$. The decay widths and direct CP asymmetries

are given by

$$\Gamma_{\Lambda_b \to \Lambda V} = \frac{1}{16\pi} \frac{|\vec{p}_\Lambda|}{M_{\Lambda_b}^2} \left(|a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2 \right) ,$$

$$A_{CP} = \frac{\Gamma(\Lambda_b \to \Lambda\,V) - \Gamma(\overline{\Lambda}_b \to \overline{\Lambda}\,\overline{V})}{\Gamma(\Lambda_b \to \Lambda\,V) + \Gamma(\overline{\Lambda}_b \to \overline{\Lambda}\,\overline{V})} . \tag{4.9}$$

The numerical results are shown in table 4, where the first and second uncertainties arise from the bag radius and CKM matrix elements, respectively. We see that with $N_c = 3$, $\Lambda_b \to \Lambda\omega$ is suppressed at the level of 10^{-8} due to the cancellation of the effective Wilson coefficients, which is also found in the framework of the QCD factorization [46]. To take account the NF effects, we treat N_c as a parameter in the decay branching ratios, which are in general related to the decay processes. Nonetheless, in this work, we will assume that N_c is independent of the vector mesons. With the experimental branching ratio in $\Lambda_b \to \Lambda\phi$, we find that $N_c = 2.0 \pm 0.3$, which is consistent with the B meson decays.

In the factorization approach, α_{λ_1} and α_{λ_2} are independent of $\mathcal{C}_V$ as can be seen from eqs. (2.16), (2.18) and (4.8). In addition, we find that in the bag model, they also depend little on the bag radius and the vector mesons. Explicitly, we have that

$$\alpha_{\lambda_1} = -0.99 \approx -1 , \qquad \alpha_{\lambda_2} = 0.86 , \tag{4.10}$$

for $\Lambda_b \to \Lambda V$ with $V = \{\phi, \rho^0, \omega, K^{*0}\}$. The values can be understood in the heavy quark limit, in which the light quark chiralities are related to the helicities. In eq. (4.3), the s quark in Λ is left-handed, and the quark and anti-quark in V have the same chiralities, resulting in $\lambda_1 = -1/2$ and $\lambda_2 = 0$. We conclude that the amplitudes are dominated by a_- in the framework of the factorization in the SM.

On the other hand, once the NF contributions are considered, the arguments would not hold. In the B meson decays with $b \to s$ transitions, we know that the NF contributions are mainly found in the negative helicities [56–61], which lead to the so-called polarization puzzles in $B^0 \to K^{*0}\phi$ [26, 27, 62, 63]. In analogy, we assume that the NF effects attribute solely to b_+ in the $b \to s$ transitions of Λ_b decays. As a result, since $b_+ \approx 0$ in the factorizable amplitudes, the branching ratios increase along with the NF contributions, which meet well with the results in table 4.

In the $b \to s$ transitions, we assume that a_- are totally factorizable, calculated with $N_c = 3$, and $|b_+|^2$ can be obtained by subtracting the $|a_-|^2$ contributions in the branching ratios. The numerical values of α_{λ_1} remain unaltered, since $|a_-|^2$ and $|b_+|^2$ share the same sign in eq. (2.16). In contrast, α_{λ_2} decrease along with N_c. The results are given in table 4. We see that α_{λ_2} are sensitive to N_c. With $N_c = 2.0 \pm 0.3$, we get that

$$\alpha_{\lambda_2}^N = 0.0 \pm 0.3 , \ -1 , \ 0.6 \pm 0.2 \tag{4.11}$$

for $\Lambda_b \to (\Lambda\phi, \Lambda\omega, \Lambda\rho^0)$, where the superscript of "N" denotes the NF contribution in the scenario of the effective color number. Here, we find that $\Lambda_b \to \Lambda\omega$ is dictated by the NF contribution, which is consistent with those in the literature [42, 46].

In the factorization approach, the relative phases of $a_\pm$ and $b_\pm$ would vanish (see eq. (4.8)), and therefore, one predicts that $\Delta_{t_i}^{(p)} = 0$ for $i = 1, 2, 3, 4$. Although it only holds

V		N_c=1.7	N_c=2.0	N_c=2.3	N_c=3	Exp
ϕ	Br	$6.90 \pm 0.65 \pm 0.30$	$5.22 \pm 0.49 \pm 0.24$	$4.12 \pm 0.39 \pm 0.18$	$2.67 \pm 0.25 \pm 0.12$	5.18 ± 1.29
	A_{CP}	$1.14 \pm 0.00 \pm 0.00$	$1.16 \pm 0.00 \pm 0.00$	$1.18 \pm 0.00 \pm 0.00$	$1.21 \pm 0.00 \pm 0.00$	
	$\alpha^N_{\lambda_2}$	-0.28	-0.05	0.21	0.86	
ρ^0	Br	$0.30 \pm 0.03 \pm 0.02$	$0.27 \pm 0.03 \pm 0.02$	$0.25 \pm 0.03 \pm 0.02$	$0.24 \pm 0.03 \pm 0.01$	
	A_{CP}	$-2.15 \pm 0.00 \pm 0.04$	$-1.61 \pm 0.00 \pm 0.01$	$-1.11 \pm 0.00 \pm 0.01$	$-0.18 \pm 0.00 \pm 0.00$	
	$\alpha^N_{\lambda_2}$	0.43	0.59	0.71	0.86	
ω	Br	$2.98 \pm 0.35 \pm 0.15$	$1.36 \pm 0.16 \pm 0.07$	$0.57 \pm 0.06 \pm 0.03$	$0.01 \pm 0.00 \pm 0.00$	
	A_{CP}	$-1.88 \pm 0.00 \pm 0.07$	$-1.87 \pm 0.00 \pm 0.07$	$-1.86 \pm 0.00 \pm 0.07$	$-1.55 \pm 0.00 \pm 0.04$	
	$\alpha^N_{\lambda_2}$	-1	-0.99	-0.98	0.86	
K^{*0}	Br	$0.10 \pm 0.01 \pm 0.00$	$0.11 \pm 0.01 \pm 0.00$	$0.12 \pm 0.02 \pm 0.01$	$0.14 \pm 0.02 \pm 0.01$	
	A_{CP}	$-14.1 \pm 0.0 \pm 0.4$	$-13.5 \pm 0.0 \pm 0.3$	$-13.1 \pm 0.0 \pm 0.3$	$-12.6 \pm 0.0 \pm 0.5$	

Table 4. Branching ratios and direct CP asymmetries in units of 10^{-6} and % for $\Lambda_b \to \Lambda V$, where the first and second uncertainties come from the bag radius and the CKM matrix elements in the SM, respectively, while the experimental branching ratio of $\Lambda_b \to \Lambda\phi$ is taken from ref. [32].

in the framework of the factorization, with the scenario that the NF effects attribute to b_+ solely, Δ_{t_1} would remain suppressed. Furthermore, $\Lambda_b \to \Lambda\phi$ is dominated by one weak phase, $V_{ts}^* V_{tb}$, so that the effects of CP- and T-violation are highly suppressed. Explicitly, we have

$$\mathcal{T}^{(p)}_{t_i}(\Lambda_b \to \Lambda\phi) \approx 0, \qquad \text{for} \ \ i = 1, 2, 3, 4, \tag{4.12}$$

which are independent of the factorization ansatz, providing a clean background to test the SM and search for new physics.

We now explore the possible contributions from new physics to the T-violating observables. Note that in $\Lambda_b \to \Lambda\omega/\rho^0$, the s quark is essentially left-handed in the SM, and thus the experimental results with $\alpha_{\lambda_1} \geq 0$ can be a smoking gun for new physics. Particularly, $\Lambda_b \to \Lambda\omega$ acquires large contributions from right-handed penguin operators. To illustrate the effects, we concentrate on the possible effective Lagarangian from new physics, given by [5, 64, 65]

$$\mathcal{L}^{\mathcal{N}}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \tilde{c}_5 (\bar{s}_\alpha b_\alpha)_R \sum_q \left(\bar{q}_\beta q_\beta \right)_L, \tag{4.13}$$

which contributes mainly to a_+ in the factorization approach, given as

$$a_+^{\mathcal{N}} = G_F \tilde{c}_5 f_\omega M_\omega \left[\sqrt{Q_-} \left(f_1 \frac{M_+}{M_\omega} + f_2 \frac{M_\omega}{M_{\Lambda_b}} \right) + \sqrt{Q_+} \left(g_1 \frac{M_-}{M_\omega} - g_2 \frac{M_\omega}{M_{\Lambda_b}} \right) \right], \tag{4.14}$$

where $\tilde{c}_5$ is the effective coefficient and the superscript of $\mathcal{N}$ denotes new physics. With $\tilde{c}_5 > 10^{-3}$, new physics could potentially flip the sign of α_{λ_1}.

On the other hand, due to the helicity conservation of the s quark, (a_+, b_-) and (a_-, b_+) receive contributions from $\mathcal{L}^{\mathcal{N}}_{\text{eff}}$ and the SM, respectively, and the CP-violating effects are suppressed due to the lack of the interferences. In contrast, $\mathcal{T}^p_{t_1}$, depending on the complex phases between different helicities, can be sizable. With the assumption of

$|a_-| \approx |b_+|$ and $|a_+^{\mathcal{N}}| \approx |b_-^{\mathcal{N}}|$, we find that

$$\mathcal{T}_{t_1}^p \sim \sqrt{1 - \alpha_{\lambda_1}^2} \sin \phi_{\mathcal{N}} \,, \tag{4.15}$$

which can be large when the phase of $\phi_{\mathcal{N}}$ from new physics is sizable. As a result, we conclude that the T-violating observables are useful in testing the complex phases for new physics.

5 Conclusions

We have parametrized the helicity amplitudes in terms of the angular distributions and systematically studied the T-violating observables in $\Lambda_b \to \Lambda(\to p\pi^-)V(\to PP')$. We have shown that all the relative complex phases among $a_\pm$ and $b_\pm$ can be interpreted as the T-odd correlations. By subtracting the effects from the FSIs, we have defined the true T-violating observables, which could be measured in the experiments. In particular, we recommend the experiments on $\Delta_{t_1}^p$ and $\mathcal{T}_1^p$, which do not require Λ_b to be polarized. In addition, the polarization asymmetries of Λ and V have been defined and their effects on the cascade decays have been given.

The decays of $\Lambda_b \to \Lambda V$ in the SM have been examined with the generalized factorization approach, which leads to the domination of a_-, resulting in that $\alpha_{\lambda_1} \approx -1$. Since the complex phases among $a_\pm$ and $b_\pm$ are identical, the factorization approach suggests that T-violating observables in the decays can not be observed. Nonetheless, the measured branching ratio of $\Lambda_b \to \Lambda\phi$ indicates that the NF effects play an important role, resulting in that the nonzero values of $\Delta_{t_i}^{(p)}$ do not necessary contradict to the SM. However, as $\Lambda_b \to \Lambda\phi$ is dominated by a single weak phase, the true T-violating effects are not expected to be observed. In table 4, we have given the branching ratios and direct CP asymmetries for the different values of the effective color number N_c. We have found that α_{λ_2} depend heavily on the NF contributions. Furthermore, the absolute value of $A_{CP}(\Lambda_b \to \Lambda K^{*0})$ has been expected to be larger than 10%. We have also explored the possible effects from new physics. In particular, we have illustrated that the right-handed currents from new physics can potentially flip the sign of α_{λ_1} from negative to positive, resulting in a possible large T-violating effect. Finally, we recommend the future experiments on $\mathcal{T}_1^p(\Lambda_b \to \Lambda\phi)$ to test the SM and search for new physics.

A T-transformation

If the system respects the T symmetry, one has that [49]

$$\langle f|U(\infty, -\infty)|i\rangle = \langle i^T|U(\infty, -\infty)|f^T\rangle \,, \tag{A.1}$$

for arbitrary initial and final states $|i\rangle$ and $|f\rangle$, respectively, where $U(t, t_0)$ is the time evolution operator from t_0 to t, and the superscript T denotes the time-reversed state. Hence, in general, the T-transformation relates $i \to f$ to $f^T \to i^T$ instead of $i^T \to f^T$.

In the first order of the weak interaction, it is possible to relate the amplitudes between $i \to f$ and $i^T \to f^T$. To do this, we adopt the interaction picture, in which the weak

transition is described by $\mathcal{H}_{\text{eff}}$, and the unperturbed Hamiltonian corresponds to the strong interaction. We have [49]

$$\langle f; \text{“out”}|\mathcal{H}_{\text{eff}}|i\rangle = \langle i^T|\mathcal{H}_{\text{eff}}^\dagger|f^T; \text{“in”}\rangle = \langle i^T|\mathcal{H}_{\text{eff}}|f^T; \text{“in”}\rangle \,, \tag{A.2}$$

where “in” and “out” denote $t \to \mp\infty$, respectively. Here, the states are related as

$$U_0(\infty, -\infty)|f; \text{“in”}\rangle = |f; \text{“out”}\rangle \,, \tag{A.3}$$

where U_0 represents the time evolution operator for the unperturbed Hamiltonian (strong interaction). In eq. (A.2), we have taken $|i\rangle$ as a single particle state of a stable hadron, having $|i; \text{“in”}\rangle = |i; \text{“out”}\rangle$. Furthermore, in eq. (A.2), if $|f^T\rangle$ is an eigenstate of the FSI, we would have $|f^T; \text{“in”}\rangle = e^{ic_f}|f^T; \text{“out”}\rangle$ with c_f the elastic rescattering phases, thereby leading to

$$|\langle f; \text{“out”}|\mathcal{H}_{\text{eff}}|i\rangle|^2 = |\langle i|\mathcal{H}_{\text{eff}}|f^T; \text{“out”}\rangle|^2 \,. \tag{A.4}$$

As an application, for instance, after factorizing the pion decays, $|f\rangle$ corresponds to the vacuum and eq. (A.4) demands the pion decay constants to be real.

Acknowledgments

We would like to thank Yi-Wen Lin for the assistance on the figure.

References

[1] G. Bélanger and C.Q. Geng, *T violating muon polarization in $K_{\mu3}$ decays*, *Phys. Rev. D* **44** (1991) 2789 [INSPIRE].

[2] P. Agrawal, J.N. Ng, G. Bélanger and C.Q. Geng, *CP violation in $K^+ \to \pi^+ l\bar{l}$ decays*, *Phys. Rev. Lett.* **67** (1991) 537 [INSPIRE].

[3] C.Q. Geng and J.N. Ng, *Testing discrete symmetries with the decay $\eta \to \pi^+\pi^-\gamma$*, *Phys. Scripta T* **99** (2002) 109.

[4] W. Bensalem, A. Datta and D. London, *T violating triple product correlations in charmless Λ_b decays*, *Phys. Lett. B* **538** (2002) 309 [hep-ph/0205009] [INSPIRE].

[5] W. Bensalem, A. Datta and D. London, *New physics effects on triple product correlations in Λ_b decays*, *Phys. Rev. D* **66** (2002) 094004 [hep-ph/0208054] [INSPIRE].

[6] C.Q. Geng and Y.K. Hsiao, *T violation in $\bar{B}^0 \to \Lambda\bar{p}\pi^+$*, *Phys. Rev. D* **72** (2005) 037901 [INSPIRE].

[7] C.Q. Geng and Y.K. Hsiao, *Direct CP and T Violation in Baryonic B Decays*, *Int. J. Mod. Phys. A* **23** (2008) 3290 [arXiv:0801.0022] [INSPIRE].

[8] C.-H. Chen, C.-Q. Geng and L. Li, *New CP phase of B_s-$\bar{B}_s$ mixing on T violation in $B_{d(s)} \to K^*(\phi)\ell^+\ell^-$*, *Phys. Lett. B* **670** (2009) 374 [arXiv:0808.0127] [INSPIRE].

[9] R.-H. Li, C.-D. Lu and W. Wang, *Branching ratios, forward-backward asymmetries and angular distributions of $B \to K_2^* l^+ l^-$ in the standard model and new physics scenarios*, *Phys. Rev. D* **83** (2011) 034034 [arXiv:1012.2129] [INSPIRE].

[10] C.Q. Geng and C.-W. Liu, *Searching for possible evidences of new physics in $B \to V_1 V_2$*, arXiv:2106.10628 [INSPIRE].

[11] J.G. Korner and M. Krämer, *Exclusive nonleptonic charm baryon decays*, *Z. Phys. C* **55** (1992) 659 [INSPIRE].

[12] J. Hrivnac, R. Lednicky and M. Smizanska, *Feasibility of beauty baryon polarization measurement in $\Lambda^0 J/\psi$ decay channel by ATLAS LHC*, *J. Phys. G* **21** (1995) 629 [hep-ph/9405231] [INSPIRE].

[13] C.-W. Chiang and L. Wolfenstein, *Observables in the decays of B to two vector mesons*, *Phys. Rev. D* **61** (2000) 074031 [hep-ph/9911338] [INSPIRE].

[14] Z.J. Ajaltouni, E. Conte and O. Leitner, *Λ_b decays into Λ-vector*, *Phys. Lett. B* **614** (2005) 165 [hep-ph/0412116] [INSPIRE].

[15] O. Leitner, Z.J. Ajaltouni and E. Conte, *An Angular distribution analysis of Λ_b decays*, *Nucl. Phys. A* **755** (2005) 435 [hep-ph/0412131] [INSPIRE].

[16] O. Leitner and Z.J. Ajaltouni, *Testing CP and Time Reversal Symmetries with $\Lambda_b \to \Lambda V(1-)$ Decays*, *Nucl. Phys. B Proc. Suppl.* **174** (2007) 169 [hep-ph/0610189] [INSPIRE].

[17] T. Gutsche, M.A. Ivanov, J.G. Korner, V.E. Lyubovitskij and P. Santorelli, *Rare baryon decays $\Lambda_b \to \Lambda l^+ l^- (l = e, \mu, \tau)$ and $\Lambda_b \to \Lambda\gamma$: differential and total rates, lepton- and hadron-side forward-backward asymmetries*, *Phys. Rev. D* **87** (2013) 074031 [arXiv:1301.3737] [INSPIRE].

[18] T. Gutsche, M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij and P. Santorelli, *Polarization effects in the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-) + J/\psi(\to \ell^+\ell^-)$ in the covariant confined quark model*, *Phys. Rev. D* **88** (2013) 114018 [arXiv:1309.7879] [INSPIRE].

[19] T. Gutsche, M.A. Ivanov, J.G. Körner and V.E. Lyubovitskij, *Nonleptonic two-body decays of single heavy baryons Λ_Q, Ξ_Q, and Ω_Q $(Q = b, c)$ induced by W emission in the covariant confined quark model*, *Phys. Rev. D* **98** (2018) 074011 [arXiv:1806.11549] [INSPIRE].

[20] C.Q. Geng, C.-W. Liu and T.-H. Tsai, *Asymmetries of anti-triplet charmed baryon decays*, *Phys. Lett. B* **794** (2019) 19 [arXiv:1902.06189] [INSPIRE].

[21] J.-Y. Cen, C.-Q. Geng, C.-W. Liu and T.-H. Tsai, *Up-down asymmetries of charmed baryon three-body decays*, *Eur. Phys. J. C* **79** (2019) 946 [arXiv:1906.01848] [INSPIRE].

[22] C.Q. Geng, C.-W. Liu and T.-H. Tsai, *Charmed Baryon Weak Decays with Vector Mesons*, *Phys. Rev. D* **101** (2020) 053002 [arXiv:2001.05079] [INSPIRE].

[23] Y.-S. Li, X. Liu and F.-S. Yu, *Revisiting semileptonic decays of $\Lambda_{b(c)}$ supported by baryon spectroscopy*, *Phys. Rev. D* **104** (2021) 013005 [arXiv:2104.04962] [INSPIRE].

[24] F. Huang and Q.-A. Zhang, *Angular Distributions for Multi-body Semileptonic Charmed Baryon Decays*, arXiv:2108.06110 [INSPIRE].

[25] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01 [INSPIRE].

[26] BABAR collaboration, *Rates, polarizations, and asymmetries in charmless vector-vector B decays*, in *17th Les Rencontres de Physique de la Vallee d'Aoste: Results and Perspectives in Particle Physics*, (2003) [hep-ex/0303020] [INSPIRE].

[27] BELLE collaboration, *Measurement of branching fractions and polarization in $B \to \phi K^{(*)}$ decays*, *Phys. Rev. Lett.* **91** (2003) 201801 [hep-ex/0307014] [INSPIRE].

[28] ATLAS collaboration, *Measurement of the parity-violating asymmetry parameter α_b and the helicity amplitudes for the decay $\Lambda_b^0 \to J/\psi + \Lambda^0$ with the ATLAS detector*, *Phys. Rev. D* **89** (2014) 092009 [arXiv:1404.1071] [INSPIRE].

[29] CMS collaboration, *Measurement of the Λ_b polarization and angular parameters in $\Lambda_b \to J/\psi \Lambda$ decays from pp collisions at $\sqrt{s} = 7$ and 8 TeV*, *Phys. Rev. D* **97** (2018) 072010 [arXiv:1802.04867] [INSPIRE].

[30] LHCb collaboration, *Measurement of the $\Lambda_b^0 \to J/\psi\Lambda$ angular distribution and the Λ_b^0 polarisation in pp collisions*, *JHEP* **06** (2020) 110 [arXiv:2004.10563] [INSPIRE].

[31] LHCb collaboration, *Observation of the decay $\Lambda_b^0 \to pK^-\mu^+\mu^-$ and a search for CP violation*, *JHEP* **06** (2017) 108 [arXiv:1703.00256] [INSPIRE].

[32] LHCb collaboration, *Observation of the $\Lambda_b^0 \to \Lambda\phi$ decay*, *Phys. Lett. B* **759** (2016) 282 [arXiv:1603.02870] [INSPIRE].

[33] LHCb collaboration, *First Observation of the Radiative Decay $\Lambda_b^0 \to \Lambda\gamma$*, *Phys. Rev. Lett.* **123** (2019) 031801 [arXiv:1904.06697] [INSPIRE].

[34] A. Dery, M. Ghosh, Y. Grossman and S. Schacht, *$SU(3)_F$ analysis for beauty baryon decays*, *JHEP* **03** (2020) 165 [arXiv:2001.05397] [INSPIRE].

[35] J. Franklin, *Quark model relations for b-baryon decay*, *J. Phys. G* **47** (2020) 085001 [arXiv:2004.01037] [INSPIRE].

[36] J.-J. Han, R.-X. Zhang, H.-Y. Jiang, Z.-J. Xiao and F.-S. Yu, *Weak decays of bottom-charm baryons: $\mathcal{B}_{bc} \to \mathcal{B}_b P$*, *Eur. Phys. J. C* **81** (2021) 539 [arXiv:2102.00961] [INSPIRE].

[37] S. Roy, R. Sinha and N.G. Deshpande, *Nonleptonic beauty baryon decays and CP asymmetries based on an SU(3)-flavor analysis*, *Phys. Rev. D* **101** (2020) 036018 [arXiv:1911.01121] [INSPIRE].

[38] S. Roy, R. Sinha and N.G. Deshpande, *Beauty baryon nonleptonic decays into decuplet baryons and CP-asymmetries based on an SU(3)-flavor analysis*, *Phys. Rev. D* **102** (2020) 053007 [arXiv:2006.16813] [INSPIRE].

[39] A. Khodjamirian, C. Klein, T. Mannel and Y.M. Wang, *Form Factors and Strong Couplings of Heavy Baryons from QCD Light-Cone Sum Rules*, *JHEP* **09** (2011) 106 [arXiv:1108.2971] [INSPIRE].

[40] Z.-X. Zhao, *Weak decays of heavy baryons in the light-front approach*, *Chin. Phys. C* **42** (2018) 093101 [arXiv:1803.02292] [INSPIRE].

[41] C.Q. Geng and Y.K. Hsiao, *Search for direct CP-violation in baryonic b-hadron decays*, *Mod. Phys. Lett. A* **31** (2016) 1630021 [INSPIRE].

[42] Y.K. Hsiao, Y. Yao and C.Q. Geng, *Charmless two-body anti-triplet b-baryon decays*, *Phys. Rev. D* **95** (2017) 093001 [arXiv:1702.05263] [INSPIRE].

[43] C.Q. Geng, C.-W. Liu and T.-H. Tsai, *Non-leptonic two-body decays of Λ_b^0 in light-front quark model*, *Phys. Lett. B* **815** (2021) 136125 [arXiv:2102.01552] [INSPIRE].

[44] C.-Q. Geng, C.-W. Liu and T.-H. Tsai, *Nonleptonic two-body weak decays of Λ_b in modified MIT bag model*, *Phys. Rev. D* **102** (2020) 034033 [arXiv:2007.09897] [INSPIRE].

[45] A. Ali, G. Kramer and C.-D. Lu, *Experimental tests of factorization in charmless nonleptonic two-body B decays*, *Phys. Rev. D* **58** (1998) 094009 [hep-ph/9804363] [INSPIRE].

[46] J. Zhu, Z.-T. Wei and H.-W. Ke, *Semileptonic and nonleptonic weak decays of Λ_b^0*, *Phys. Rev. D* **99** (2019) 054020 [arXiv:1803.01297] [INSPIRE].

[47] C.-D. Lu, Y.-M. Wang, H. Zou, A. Ali and G. Kramer, *Anatomy of the pQCD Approach to the Baryonic Decays $\Lambda_b \to p\pi, pK$*, *Phys. Rev. D* **80** (2009) 034011 [arXiv:0906.1479] [INSPIRE].

[48] M. Jacob and G.C. Wick, *On the General Theory of Collisions for Particles with Spin*, *Annals Phys.* **7** (1959) 404 [INSPIRE].

[49] W.K. Tung, *Group Theory in Physics*, World Scientific Publishing Company (1985) [INSPIRE].

[50] BESIII collaboration, *Polarization and Entanglement in Baryon-Antibaryon Pair Production in Electron-Positron Annihilation*, *Nature Phys.* **15** (2019) 631 [arXiv:1808.08917] [INSPIRE].

[51] A. McKerrell, *Relativistic two and three-particle states*, *Nuovo Cim.* **34** (1964) 1289.

[52] BELLE collaboration, *Measurements of branching fractions and asymmetry parameters of $\Xi_c^0 \to \Lambda \bar{K}^{*0}$, $\Xi_c^0 \to \Sigma^0 \bar{K}^{*0}$, and $\Xi_c^0 \to \Sigma^+ K^{*-}$ decays at Belle*, *JHEP* **06** (2021) 160 [arXiv:2104.10361] [INSPIRE].

[53] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Effective Hamiltonians for $\Delta S = 1$ and $\Delta B = 1$ nonleptonic decays beyond the leading logarithmic approximation*, *Nucl. Phys. B* **370** (1992) 69 [*Addendum ibid.* **375** (1992) 501] [INSPIRE].

[54] R. Perez-Marcial, R. Huerta, A. Garcia and M. Avila-Aoki, *Predictions for Semileptonic Decays of Charm Baryons. 2. Nonrelativistic and MIT Bag Quark Models*, *Phys. Rev. D* **40** (1989) 2955 [*Erratum ibid.* **44** (1991) 2203] [INSPIRE].

[55] P. Ball and R. Zwicky, *$B_{d,s} \to \rho, \omega, K^*, \phi$ decay form-factors from light-cone sum rules revisited*, *Phys. Rev. D* **71** (2005) 014029 [hep-ph/0412079] [INSPIRE].

[56] H.-Y. Cheng and C.-K. Chua, *Revisiting Charmless Hadronic $B(u, d)$ Decays in QCD Factorization*, *Phys. Rev. D* **80** (2009) 114008 [arXiv:0909.5229] [INSPIRE].

[57] H.-Y. Cheng and C.-K. Chua, *QCD Factorization for Charmless Hadronic B_s Decays Revisited*, *Phys. Rev. D* **80** (2009) 114026 [arXiv:0910.5237] [INSPIRE].

[58] Z.-T. Zou, A. Ali, C.-D. Lu, X. Liu and Y. Li, *Improved Estimates of The $B_{(s)} \to VV$ Decays in Perturbative QCD Approach*, *Phys. Rev. D* **91** (2015) 054033 [arXiv:1501.00784] [INSPIRE].

[59] Q. Chang, X.-N. Li, J.-F. Sun and Y.-L. Yang, *Fits of Weak Annihilation and Hard Spectator Scattering Corrections in $B_{u,d} \to VV$ Decays*, *J. Phys. G* **43** (2016) 105004 [arXiv:1610.02747] [INSPIRE].

[60] Q. Chang, X. Li, X.-Q. Li and J. Sun, *Study of the weak annihilation contributions in charmless $B_s \to VV$ decays*, *Eur. Phys. J. C* **77** (2017) 415 [arXiv:1706.06138] [INSPIRE].

[61] C. Wang, Q.-A. Zhang, Y. Li and C.-D. Lu, *Charmless $B_{(s)} \to VV$ Decays in Factorization-Assisted Topological-Amplitude Approach*, *Eur. Phys. J. C* **77** (2017) 333 [arXiv:1701.01300] [INSPIRE].

[62] M. Bauer, B. Stech and M. Wirbel, *Exclusive Nonleptonic Decays of D-, D_s-, and B-Mesons*, *Z. Phys. C* **34** (1987) 103 [INSPIRE].

[63] H.-Y. Cheng and K.-C. Yang, *Charmless $B \to VV$ decays in QCD factorization: Implications of recent $B \to \phi K^*$ measurement*, *Phys. Lett. B* **511** (2001) 40 [hep-ph/0104090] [INSPIRE].

[64] A.L. Kagan, *Right-handed currents, CP-violation, and $B \to VV$*, hep-ph/0407076 [INSPIRE].

[65] S. Alioli, V. Cirigliano, W. Dekens, J. de Vries and E. Mereghetti, *Right-handed charged currents in the era of the Large Hadron Collider*, *JHEP* **05** (2017) 086 [arXiv:1703.04751] [INSPIRE].