Standard Model fragmentation functions at very high energies

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Abstract: We compute the leading-order evolution of parton fragmentation functions for all the Standard Model fermions and bosons up to energies far above the electroweak scale, where electroweak symmetry is restored. We discuss the difference between double-logarithmic and leading-logarithmic resummation, and show how the latter can be implemented through a scale choice in the SU(2) coupling. We present results for a wide range of partonic center-of-mass energies, including the polarization of fermion and vector boson fragmentation functions induced by electroweak evolution.

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Contents

1 Introduction

It is a well known fact that electroweak corrections to hard processes at proton or electron colliders contain logarithmically enhanced contributions of the form $\alpha^n L^{2n}$, where $L = \ln(q/m_V)$, q being the hard process scale and $m_V \sim m_{W/Z}$. This is the case even for observables that are completely inclusive over the final state, and can be traced back to the fact that the initial state protons are not singlets under the SU(2) gauge group. Due to this double-logarithmic scaling, the convergence of electroweak perturbation theory becomes worse as the center-of-mass energy increases, and ultimately breaks down completely, namely when $\alpha L^2 \sim 1$. To obtain reliable predictions at these energy scales requires a reorganization of the perturbative expansion such that these large logarithms are resummed to all orders in perturbation theory.

Most of the studies of electroweak logarithms have considered completely exclusive observables, such that the final state is fixed. In this case the only contributions to logarithmically enhanced electroweak corrections arise from the virtual exchange of massive

gauge bosons. The mass of the gauge boson regulates the IR divergences present in massless gauge theories, giving rise to the logarithmic sensitivity on m_V. These electroweak Sudakov logarithms have been studied for a long time [1–17], and a systematic way to resum them using soft-collinear effective theory (SCET) [18–21] was developed in [15, 16]. Just as for massless theories, the real radiation of gauge bosons leads to infrared sensitivity, and therefore logarithmic sensitivity to m_V is present in such real emission contributions as well. An analogy with parton showers allowed the resummation of the enhanced corrections to leading logarithmic (LL) accuracy [22].

As already discussed, even fully inclusive observables contain double logarithmic sensitivity to the ratio q/m_V, due to the fact that the initial state is not an SU(2) singlet. For an observable that is completely inclusive over the final state, all logarithmically enhanced terms arise from initial-state radiation of W bosons. To LL accuracy, the large logarithms arise from emissions of heavy gauge bosons that are both collinear and soft, and are described by the DGLAP evolution of parton distribution functions [23–37], where one needs the full set of particles in the Standard Model. These DGLAP equations were first derived in [31], and the phenomenology of this DGLAP evolution in the complete SM was studied in [38, 39]. As will be shown in this paper, while the DGLAP evolution presented in [38, 39] was only accurate to double-logarithmic level, the full LL structure can be obtained for such completely inclusive observables through an appropriate scale choice in the SU(2) coupling constant.

Most realistic observables, however, contain a final state which is neither fully inclusive nor fully exclusive. The results of [40] allow one to obtain NLL predictions where a subset of the final state particles is fixed, while being inclusive over the emission of additional particles. So for example, they allow one to compute the cross section of the process $pp \to e^+e^- + X$, where X denotes additional particles in the final state. For the most general case, where one wants to include additional final state particles only partially (for example only in a given kinematic range, or only those that decay in a particular way) one needs to use an electroweak parton shower, which generates an arbitrary final state. If formulated correctly, such a parton shower will resum all LL electroweak Sudakov logarithms, and furthermore include many (but not all) of the NLL logarithms. A final-state parton shower including emissions from all interactions in the Standard Model was developed [41], which also paid special attention to important threshold effects for longitudinal gauge bosons.

To obtain the full NLL accuracy of [40] requires four types of input: the hard cross sections evaluated at the partonic center-of-mass energy in the unbroken SU(3)⊗SU(2)⊗U(1) Standard Model, the parton distributions functions (PDFs) describing the collinear evolution of the initial-state particles, the fragmentation functions (FFs) describing the collinear evolution of the final-state particles, and a soft function describing the wide-angle soft radiation. The collinear evolution needs to be performed with the full gauge structure SU(3) ⊗ SU(2) ⊗ U(1) and was discussed for the PDFs in detail in [38, 39]. In this paper we will perform a similar analysis for the FFs, including numerical results showing the impact of the logarithmic terms. Our results can be used as one of the inputs to [40], which allows full NLL accuracy. When used on their own, one relies on the scaling $\alpha L \ll 1$ being sufficient to assume that LL accuracy, matched with fixed-order electroweak corrections as

discussed in [39], will be sufficient. Under this assumption, one can omit the soft functions and use the hard cross sections only in combination with the collinear evolution of PDFs and FFs, but one has to keep in mind that large factors in the NLL terms could invalidate this naive logarithmic counting.[1]

This paper is organized as follows: in section 2 we discuss the form of the fragmentation function and their DGLAP evolution with q. This discussion is correct to double logarithmic accuracy, and we discuss in section 3 how the results can be modified to achieve full leading-logarithmic accuracy through an appropriate scale choice of the SU(2) coupling α_2. After a brief discussion of some implementation details in section 4, we present the results for the fragmentation functions in section 5. Our conclusions are given in section 6, and in appendix A and B we give details of an isospin and CP basis that decouples parts of the DGLAP evolution and the equations used in the forward evolution.

2 Resummation of collinear final-state logarithms

Electroweak logarithms arise from the exchange of massive gauge bosons in loops, or from the real radiation of massive gauge bosons. To LL accuracy, the only contributions are from gauge bosons that are collinear to one of the initial- or final-state particles. These are precisely the contributions that are contained in the DGLAP evolution of PDFs (for emissions collinear to initial-state particles) and FFs (for emissions collinear to final-state particles).

In the strong sector, the DGLAP equations only give rise to single logarithmic terms. This is because the limits where emissions are simultaneously soft and collinear cancel between virtual and real contributions to the DGLAP equations. This fact is easy to understand, since an arbitrarily soft emission of a gluon cannot be observed experimentally, so the divergence associated with this emission needs to cancel against the virtual contribution. This is different from the case of the soft emission of a W boson, which can always be observed through the change of flavor (or SU(2) quantum numbers) of the emitting particle. Thus, as long as a process is sensitive to the SU(2) quantum numbers of the external particles, soft radiation of W bosons from these particles gives rise to double logarithms.

Any observable at hadron or lepton colliders is sensitive to the SU(2) quantum numbers of the initial state, since the particles being collided are not SU(2) singlets. This leads to the important prediction that electroweak double logarithms are present for any observable, even if they are completely inclusive over the final state. For observables where one identifies the SU(2) properties of the final state (for example by demanding to find two leptons of given flavors), additional double logarithms arise from the collinear radiation off final state particles (even if one is completely inclusive over the momenta of said particles, and also over extra particles being radiated). These collinear logarithms can be resummed by solving the DGLAP evolution of FFs, as we will now discuss.

DGLAP equations for FFs are very similar to those for PDFs, and the discussion of them closely follows [38, 39]. We will therefore be relatively brief in this work, and refer the reader to the previous papers for more discussion.

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[1]An analysis of the size of various contributions to the full NLL resummation in exclusive processes was performed in [42].

Our solutions to the SM evolution equations are obtained in the approximation of exact SU(3)×SU(2)×U(1) symmetry. That is, we neglect fermion and Higgs masses and the Higgs vacuum expectation value, the effects of these being power-suppressed at high scales. We impose an infra-red cutoff m_V on interactions that involve the emission of an electroweak vector boson, $V = W^i$ for SU(2) or B for U(1).[2] Leading-order evolution kernels and one-loop running couplings are used. All the electroweak FFs are generated dynamically by evolving upwards from a scale $q_0 \sim m_V$. In practice we take $q_0 = m_V = 100\,\text{GeV}$. More details of the input FFs will be given in section 4.

2.1 Definition of the fragmentation functions

The fragmentation function $D_i^k(x, q)$ gives the distribution of the momentum fraction x for particle species k in a jet initiated by a parton of type i produced in a hard process at momentum scale q. As in the case of PDFs, it is convenient to define the momentum-weighted FFs,

$$d_i^k(x, q) = x\, D_i^k(x, q). \qquad (2.1)$$

Note that when we omit one of the labels i or k, our expressions apply independent of its value. One important thing to realize is that only particles in the broken-symmetry phase (or the products of their decay or hadronization) can be observed with a given momentum in the detector, and the index k therefore only runs over the particles in the broken basis, that is, the fermions, photon, gluon, Higgs, $W^\pm$ and Z^0 bosons. Furthermore, one typically does not distinguish between left- and right-handed particles, or the different polarizations of the vector bosons, in a detector. Thus, the total number of fermions is 6 quarks and anti-quarks, and 6 leptons and anti-leptons, giving 24 fermions. There are a total of 5 vector bosons and one Higgs, giving a total of 30 particles we need to consider for k.

Since the index i denotes the object produced at a high scale that initiates the jet, we define it in the unbroken-symmetry phase. When i is a fermion, one needs to separate left- and right-handed chirality states, which evolve differently as they belong to different representations of the SU(2) ⊗ U(1) symmetry. This gives a total of 12 quarks and anti-quarks, and 9 leptons and anti-leptons, making 42 fermions.

For each transversely-polarized SM vector boson, we need separate positive and negative helicity FFs, $d_{V_\pm}^k$, since boson polarization is generated during evolution and transmitted to the fermions.[3] Interference between different helicity states cancels upon azimuthal integration of transverse momenta in successive parton splittings, so there are no mixed-helicity boson FFs.

Since SU(3) is unbroken, we need only a single gluon FF of each helicity for each fragmentation product, $d_{g_+}^k$ and $d_{g_-}^k$. For the SU(2) ⊗ U(1) symmetry, there are 8 transversely-polarized gauge bosons ($W_\pm^+$, $W_\pm^-$, $W_\pm^3$ and $B_\pm$), For the neutral bosons B and W^3, one also needs to take into account the two mixed $BW_\pm$ FFs, representing the interference

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[2]The cutoff is not strictly necessary for B emission, but we keep it because the B and W^3 are mixed in the physical Z^0 boson.

[3]The original version of [38] did not discuss the effects of polarized vector bosons, the importance of which for electroweak evolution was first pointed out in [40, 43].

k \ i	f_{light}	V	H	sum
f	42×24	42×5	42	42×30
$g_\pm$	2×24	2×5	2	2×30
$W_\pm^\pm$	4×24	4×5	4	4×30
$V_\pm^0$	6×24	6×5	6	6×30
$H^\pm$	2×24	2×5	2	2×30
H^0	2×24	2×5	2	2×30
$H^0/\overline{H}^0$	2×24	2×5	2	2×30
sum	60×24	60×5	60	60×30

Table 1. Total number of fragmentation functions required. For a given final-state particle k, one requires a total of 60 FFs, which is the same as the number of PDFs needed for the initial state. Each object i can fragment into 30 particles k (the total number of particles and antiparticles in the Standard Model). Thus, in general $60 \times 30 = 1800$ FFs are required.

contribution when i could have been either of them. Such mixed FFs arise from the fact that the left-handed fermions and Higgs carry both isospin and hypercharge, such that an interference between two amplitudes, one with a $B_\pm$ and one with a $W_\pm^3$ boson in the final state, can arise. This is the very similar to the case of mixed PDFs when such an interference arises in the initial state [31, 38]. Thus there are a total of 12 gauge boson labels required. There are 4 unmixed Higgs bosons $H^\pm$, H^0 and $\overline{H}^0$ and 2 mixed $H^0/\overline{H}^0$ FFs in the unbroken phase. This brings the total to 60, the same number as for PDFs, as summarized in table 1.

Instead of using the unbroken basis, where all particles have definite quantum numbers under the SU(3) $\otimes$ SU(2) $\otimes$ U(1), one can also work in the broken basis, where instead of H^0 and $\overline{H}^0$ one has the Higgs boson h and the longitudinally-polarized Z^0, and instead of the neutral gauge bosons B and W_3, one has the photon and transversely-polarized Z^0. In the latter case, one can construct the FFs for the photon, the Z^0 and their mixed γZ state as transformations of the FFs for the B, the W_3 and their mixed state. This is anyway necessary at the electroweak scale, below which the symmetry is broken. Using $A = c_W B + s_W W_3$ and $Z^0 = -s_W B + c_W W_3$, the relation between FFs containing $i = \gamma, Z, \gamma Z$ and those with $i = B, W_3, BW$ is

$$\begin{pmatrix} d_\gamma \\ d_Z \\ d_{\gamma Z} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & c_W s_W \\ s_W^2 & c_W^2 & -c_W s_W \\ -2c_W s_W & 2c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} d_B \\ d_{W_3} \\ d_{BW} \end{pmatrix}, \qquad (2.2)$$

and thus

$$\begin{pmatrix} d_B \\ d_{W_3} \\ d_{BW} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -c_W s_W \\ s_W^2 & c_W^2 & c_W s_W \\ 2c_W s_W & -2c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} d_\gamma \\ d_Z \\ d_{\gamma Z} \end{pmatrix}, \qquad (2.3)$$

where the weak mixing parameters are given by

$$s_W \equiv s_W(q) = \sqrt{\frac{\alpha_1(q)}{\alpha_1(q) + \alpha_2(q)}}$$

$$c_W \equiv c_W(q) = \sqrt{\frac{\alpha_2(q)}{\alpha_1(q) + \alpha_2(q)}} \,. \tag{2.4}$$

In the neutral Higgs sector, the relation between the broken and unbroken fields is

$$H^0 = \frac{(h - iZ_L)}{\sqrt{2}} \,, \qquad \bar{H}^0 = \frac{(h + iZ_L)}{\sqrt{2}} \,, \tag{2.5}$$

where h and Z_L represent the Higgs and the longitudinal Z^0 fields, respectively. The corresponding FFs are

$$d_{H^0} = \frac{1}{2} \left[d_h + d_{Z_L} + i \left(d_{hZ_L} - d_{Z_L h} \right) \right] \,, \tag{2.6}$$

$$d_{\bar{H}^0} = \frac{1}{2} \left[d_h + d_{Z_L} - i \left(d_{hZ_L} - d_{Z_L h} \right) \right] \,. \tag{2.7}$$

There are also the mixed FFs

$$d_{H^0 \bar{H}^0} = \frac{1}{2} \left[d_h - d_{Z_L} - i \left(d_{hZ_L} + d_{Z_L h} \right) \right] \,, \tag{2.8}$$

$$d_{\bar{H}^0 H^0} = \frac{1}{2} \left[d_h - d_{Z_L} + i \left(d_{hZ_L} + d_{Z_L h} \right) \right] \,, \tag{2.9}$$

which describe the difference between the Higgs and longitudinal Z^0 FFs:

$$d_h = \frac{1}{2} \left(d_{H^0} + d_{\bar{H}^0} + d_{H^0 \bar{H}^0} + d_{\bar{H}^0 H^0} \right) , \tag{2.10}$$

$$d_{Z_L} = \frac{1}{2} \left(d_{H^0} + d_{\bar{H}^0} - d_{H^0 \bar{H}^0} - d_{\bar{H}^0 H^0} \right) . \tag{2.11}$$

Although the flavor basis chosen above is the most intuitive, the fact that many of the 60 FFs are coupled to one another makes it quite difficult to solve the evolution equations. To decouple some of the equations, it helps to change the basis such that the ingredients have definite total isospin **T** and CP quantum numbers, which (neglecting the tiny CP violation) are conserved in the Standard Model. Then only FFs with the same quantum numbers can mix. The FFs for each set of quantum numbers required are shown in table 2. In the case of the vector bosons, the unpolarized FFs $d_{V_+}^k + d_{V_-}^k$ can have $\{\mathbf{T}, \mathrm{CP}\} = \{0, +\}, \{1, -\}$ or $\{2, +\}$, while the helicity asymmetries $d_{V_+}^k - d_{V_-}^k$ have $\{0, -\}, \{1, +\}$ or $\{2, -\}$. The mixed Higgs FFs have unit isospin and we can form the combinations with definite CP:

$$d_{HH}^{1\pm} = \frac{1}{2} \left(d_{H^0 \bar{H}^0} \pm d_{\bar{H}^0 H^0} \right) , \tag{2.12}$$

where only the $\{1, +\}$ state contributes to the physical Higgs and Z_L^0 FFs. Further details of the isospin and CP basis are given in appendix A.

$\{\mathbf{T}, \mathrm{CP}\}$	fields
$\{0, \pm\}$	$2n_g \times q_R , n_g \times \ell_R , n_g \times q_L , n_g \times \ell_L , g , W , B , H$
$\{1, \pm\}$	$n_g \times q_L , n_g \times \ell_L , W , BW, H, HH$
$\{2, \pm\}$	W

Table 2. The 60 FFs required for the SM evolution can written in a basis with definite conserved quantum numbers. $2(5n_g + 4)$ FFs contribute to the $\{0, \pm\}$ states, $2(2n_g + 4)$ to each to the $\{1, \pm\}$ and 2 to the $\{2, \pm\}$, where $n_g = 3$ stands for the number of generations.

Note that in general there can be additional mixed FFs, which however are zero in our matching conditions at scale q_0 and are not generated in the evolution. In particular, there can be states mixing left-and right-handed fermions, but they are not present when we consider only the FFs d_i^k for unpolarized particles k.

2.2 General evolution equations

We consider the x-weighted FFs of parton species i at momentum fraction x and scale q, $d_i(x, q)$. In leading order they satisfy evolution equations of the following approximate form:[4]

$$q\frac{\partial}{\partial q}d_i^k(x,q) = \sum_I \frac{\alpha_I(q)}{\pi} \left[P_{i,I}^V(q)\, d_i^k(x,q) + \sum_j C_{ji,I} \int_x^{z_{\max}^{ji,I}(q)} \mathrm{d}z\, P_{ji,I}^R(z) d_j^k(x/z,q) \right]$$
$$\equiv \sum_I \left[q\frac{\partial}{\partial q}d_i^k(x,q) \right]_I . \tag{2.13}$$

Here, the sum over I goes over the different interactions in the Standard Model and the notation $\left[q\,\partial/\partial q\, d_i^k(x,q) \right]_I$ implies that we only keep the terms proportional to the coupling α_I when taking the derivative.[5] We denote by $I = 1, 2, 3$ the pure U(1), SU(2) and SU(3) gauge interactions, by $I = Y$ the Yukawa interactions, and by $I = M$ the mixed interaction proportional to

$$\alpha_M(q) = \sqrt{\alpha_1(q)\,\alpha_2(q)} . \tag{2.14}$$

The first contribution in eq. (2.13), proportional to $P_{i,I}^V$, denotes the virtual contribution to the FF evolution, while the second contribution is the real contribution from the splitting of parton i into parton j. Notice that i and j are interchanged here relative to the PDF evolution equations, because d_j^k represents the fragmentation of the outgoing parton from the splitting, rather than the distribution of the incoming one. The maximum value of z in the integration of the real contribution depends on the type of splitting and interaction,

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[4]In section 3 we present a modification of the evolution equations to achieve full leading-logarithmic accuracy.

[5]Note that $[\ldots]_I$ is only introduced for notational convenience and should not be interpreted as setting all other couplings to zero. In particular, the FFs appearing on the right-hand side of eq. (2.13) still depend on the values of all coupling constants.

and we choose

$$z_{\max}^{ji,I}(q) = \begin{cases} 1 - \dfrac{m_V}{q} & \text{for } I = 1, 2, \text{ and } i, j \notin V \text{ or } i, j \in V \\ 1 & \text{otherwise} \end{cases}, \qquad (2.15)$$

that is, we apply an infrared cutoff m_V, of the order of the electroweak scale, when a B or W boson is emitted. This regulates the divergence of the splitting function for those emissions as $z \to 1$. Such a cutoff is mandatory for $I = 2$ because there are FF contributions that are SU(2) non-singlets. The evolution equations for SU(3) are regular in the absence of a cutoff, as hadron FFs are color singlets. Similarly for U(1), the unpolarized FFs have zero hypercharge,[6] but we include the same cutoff for $I = 1$, since the B and W_3 are mixed in the physical Z and γ states.

It was shown in [38] that the virtual corrections for the fermion, unmixed scalar and unmixed, unpolarized vector boson PDFs, which are the same for the corresponding FFs, are given by

$$P_{i,I}^V(q) = -\sum_j C_{ji,I} \int_0^{z_{\max}^{ji,I}(q)} z \, \mathrm{d}z \, P_{ji,I}^R(z) \text{ for } i \neq \mathrm{BW, HH}. \qquad (2.16)$$

The virtual corrections for the individual vector boson helicity states are the same as the unpolarized ones. For the mixed vector boson FFs one has

$$P_{BW,1}^V(q) = \frac{1}{2}P_{B,1}^V(q), \qquad P_{BW,2}^V(q) = \frac{1}{2}P_{W,2}^V(q), \qquad (2.17)$$

while the virtual contribution for $i = BW$ is zero for the other interactions. The virtual contributions for the mixed Higgs FFs are the same as those for the unmixed states:

$$P_{HH,I}^V(q) = P_{H,I}^V(q). \qquad (2.18)$$

Thus for the unmixed FFs we have simply

$$\left[q\frac{\partial}{\partial q}d_i^k(x,q)\right]_I = \frac{\alpha_I(q)}{\pi}\sum_j C_{ji,I} \int_0^{z_{\max}^{ji,I}(q)} \mathrm{d}z \, P_{ji,I}^R(z) \left[d_j^k(x/z,q) - z\,d_i^k(x,q)\right]. \qquad (2.19)$$

This implies that the DGLAP equations are defined by the splitting functions $P_{ji,I}^R(z)$ and the coefficients $C_{ji,I}$.

Most of the splitting functions can be obtained from the seminal paper of Altarelli and Parisi [44]. For the gauge interactions of the Standard Model, $I = 1, 2, 3$ and the mixed

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[6]Although there can be contributions with non-zero hypercharge for transversely polarized fermions [31].

interaction M, which we denoted collectively by $I = G$, one finds

$$P^R_{f_L f_L, G}(z) = P^R_{f_R f_R, G}(z) = \frac{2}{1-z} - (1+z)\,, \tag{2.20}$$

$$P^R_{V_+ f_L, G}(z) = P^R_{V_- f_R, G}(z) = \frac{(1-z)^2}{z}\,, \tag{2.21}$$

$$P^R_{V_- f_L, G}(z) = P^R_{V_+ f_R, G}(z) = \frac{1}{z}\,, \tag{2.22}$$

$$P^R_{f_L V_+, G}(z) = P^R_{f_R V_-, G}(z) = \frac{1}{2}(1-z)^2\,, \tag{2.23}$$

$$P^R_{f_L V_-, G}(z) = P^R_{f_R V_+, G}(z) = \frac{1}{2}z^2\,, \tag{2.24}$$

$$P^R_{V_+ V_+, G}(z) = P^R_{V_- V_-, G}(z) = \frac{2}{1-z} + \frac{1}{z} - 1 - z(1+z)\,, \tag{2.25}$$

$$P^R_{V_+ V_-, G}(z) = P^R_{V_- V_+, G}(z) = \frac{(1-z)^3}{z}\,, \tag{2.26}$$

$$P^R_{HH, G}(z) = \frac{2}{1-z} - 2\,, \tag{2.27}$$

$$P^R_{V_\pm H, G}(z) = \frac{1}{z} - 1\,, \tag{2.28}$$

$$P^R_{HV_\pm, G}(z) = \frac{1}{2}z(1-z)\,. \tag{2.29}$$

The factor of $1/2$ in P^R_{fV} has to be included since we are considering fermions with definite chirality. For splitting to and from antifermions we have, from CP invariance,

$$P^R_{\bar{f}_L V_+, G}(z) = P^R_{f_L V_-, G}(z)\,, \quad P^R_{\bar{f}_R V_+, G}(z) = P^R_{f_R V_-, G}(z)\,, \tag{2.30}$$

$$P^R_{V_+ \bar{f}_L, G}(z) = P^R_{V_- f_L, G}(z)\,, \quad P^R_{V_+ \bar{f}_R, G}(z) = P^R_{V_- f_R, G}(z)\,. \tag{2.31}$$

Finally for the Yukawa interaction (Y), one has

$$P^R_{ff, Y}(z) = \frac{1-z}{2}\,, \tag{2.32}$$

$$P^R_{Hf, Y}(z) = P^R_{ff, Y}(1-z)\,, \tag{2.33}$$

$$P^R_{fH, Y}(z) = \frac{1}{2}\,. \tag{2.34}$$

We now give the necessary coefficients $C_{ij,I}$ for the five interactions.

I=3: SU(3) interaction. We start by considering the well known case of SU(3) interactions. The relevant degrees of freedom are the gluon, as well as left and right-handed quarks. The coupling coefficients are

$$C_{qq,3} = C_{gq,3} = C_F\,, \qquad C_{qg,3} = T_R\,, \qquad C_{gg,3} = C_A\,, \tag{2.35}$$

where $C_F = 4/3$, $C_A = 3$, $T_R = 1/2$. Note that since SU(3) has the same coupling to left- and right-handed quarks, it does not produce a polarization asymmetry on its own, unless an initial asymmetry is present due to polarized initial states. However, due to the different electroweak evolution of the left- and right-handed fermions, even the gluon FFs develop a polarization asymmetry above the electroweak scale.

I=1: U(1) interaction. For U(1) the relevant degrees of freedom are left- and right-handed fermions (denoted by the subscript f), as well as the U(1) gauge boson B. The coefficients involving fermions and gauge bosons are

$$C_{ff,1} = C_{Bf,1} = Y_f^2, \qquad C_{fB,1} = N_f Y_f^2, \qquad C_{BB,1} = 0, \tag{2.36}$$

where the hypercharges of the different fermions are given by $Y_{q_L} = 1/6$, $Y_{u_R} = 2/3$, $Y_{d_R} = -1/3$, $Y_{\ell_L} = -1/2$ and $Y_{e_R} = -1$. The color factor N_f is equal to $N_c = 3$ for quarks and 1 for leptons. The coefficients involving the Higgs bosons are

$$C_{hh,1} = C_{Bh,1} = C_{hB,1} = \frac{1}{4}, \tag{2.37}$$

where h here stands for any of the four Higgs boson FFs.

I=2: SU(2) interaction. Denoting by u_L and d_L any up- and down-type left-handed fermion, one finds

$$C_{u_L d_L,2} = C_{d_L u_L,2} = C_{W^+ u_L,2} = C_{W^- d_L,2} = \frac{1}{2}, \tag{2.38}$$

$$C_{u_L u_L,2} = C_{W_3 u_L,2} = C_{d_L d_L,2} = C_{W_3 d_L,2} = \frac{1}{4}, \tag{2.39}$$

$$C_{u_L W^+,2} = C_{d_L W^-,2} = N_f \frac{1}{2}, \tag{2.40}$$

$$C_{u_L W_3,2} = C_{d_L W_3,2} = N_f \frac{1}{4}, \tag{2.41}$$

$$C_{W^\pm W^\pm,2} = C_{W^\pm W_3,2} = C_{W_3 W^\pm,2} = 1, \tag{2.42}$$

where as before the color factor $N_f = 3$ for quarks, 1 for leptons. The coupling coefficients of the Higgs bosons are given by

$$C_{H^+ H^+,2} = C_{H^0 H^0,2} = C_{W_3 H^+,2} = C_{W_3 H^0,2},$$
$$= C_{H^+ W_3,2} = C_{H^0 W_3,2} = \frac{1}{4}, \tag{2.43}$$

$$C_{H^+ H^0,2} = C_{H^0 H^+,2} = C_{H^+ W^+,2} = C_{W^+ H^+,2},$$
$$= C_{H^0 W^-,2} = C_{W^- H^0,2} = \frac{1}{2}. \tag{2.44}$$

The couplings for the charge-conjugate states are the same.

I=Y: Yukawa interaction. In this work we only keep the top Yukawa coupling, setting all others to zero. This gives the following coefficients:

$$C_{q_L^3 t_R,Y} = C_{H^0 t_R,Y} = C_{H^+ t_R,Y} = C_{t_R q_L^3,Y} = C_{\bar{H}^0 t_L,Y} = C_{H^- b_L,Y} = 1, \tag{2.45}$$

where q_L^3 denotes either the left-handed top or bottom quark. We furthermore need

$$C_{t_R H^0,Y} = C_{t_R H^+,Y} = C_{t_L \bar{H}^0,Y} = C_{b_L H^-,Y} = N_c. \tag{2.46}$$

I=M: mixed $B - W_3$ interaction. Finally, we need to consider the evolution involving the mixed BW boson FF. The non-vanishing couplings are

$$C_{BW f_u,M} = -C_{BW f_d,M} = 2\frac{Y_f}{2}\,, \tag{2.47}$$

$$C_{f_u BW,M} = -C_{f_d BW,M} = N_f \frac{Y_f}{2}\,, \tag{2.48}$$

where f_u and f_d represent the up- and down-type left-handed fermions and anti-fermions of all generations. Since $Y_{\bar{f}} = -Y_f$ and $T_{3\bar{f}} = -T_{3f}$, the couplings for fermions and anti-fermions are identical. The factor of 2 in the first line comes from our definition of f_{BW} as the sum of BW and WB contributions. The diagonal coefficients $C_{f_u f_u,M}$ and $C_{f_d f_d,M}$ are zero because there is no vector boson with both U(1) and SU(2) interactions. The couplings involving the Higgs bosons are

$$C_{BW H^+,M} = -C_{BW H^0,M} = \frac{1}{2}\,, \tag{2.49}$$

$$C_{H^+ BW,M} = -C_{H^0 BW,M} = \frac{1}{4}\,, \tag{2.50}$$

where, as for the fermions, the same relations hold for the charge-conjugate states.

The resulting evolution equations in the $\{\mathbf{T}, \mathrm{CP}\}$ basis are given in full in appendix B.

2.3 Double logarithmic evolution

Any combination of FFs that is not SU(2)-symmetric has a component that evolves double-logarithmically. For example, from eqs. (2.19) and (2.38)–(2.42), the combination of left-handed quark FFs that has $\{\mathbf{T}, \mathrm{CP}\} = \{1, -\}$,

$$d_q^{1-} = \frac{1}{4}\left(d_{u_L} - d_{d_L} - d_{\bar{u}_L} + d_{\bar{d}_L}\right)\,, \tag{2.51}$$

satisfies the evolution equation

$$\left[q\frac{\partial}{\partial q}d_q^{1-}(x,q)\right]_2 = \frac{\alpha_2(q)}{\pi}\left\{\int_0^{1-m_V/q}\mathrm{d}z\, P_{ff,G}^R(z)\left[-\frac{1}{4}d_q^{1-}(x/z,q) - \frac{3}{4}z\, d_q^{1-}(x,q)\right]\right.$$
$$\left. + \int_0^1 \mathrm{d}z\, P_{Vf,G}^R(z)\left[\frac{1}{2}d_W^{1-}(x/z,q) - \frac{3}{4}z\, d_q^{1-}(x,q)\right]\right\}\,, \tag{2.52}$$

where

$$d_W^{1-} = \frac{1}{2}\left(d_{W_+^+} - d_{W_+^-} + d_{W_-^+} - d_{W_-^-}\right)\,. \tag{2.53}$$

The mismatch between the coefficients of $d_q^{1-}(x/z,q)$ and $d_q^{1-}(x,q)$ on the right-hand side of eq. (2.52) induces a logarithmic sensitivity to m_V and hence a double-logarithmic term in the evolution. In fact, noting that the SU(2) contribution to the fermion Sudakov factor is

$$\Delta_{f,2}(q) = \exp\left\{-\frac{3}{4}\int_{m_V}^q \frac{\mathrm{d}q'}{q'}\frac{\alpha_2(q')}{\pi}\left[\int_0^{1-m_V/q'}\mathrm{d}z\, z\, P_{ff,G}^R(z) + \int_0^1 \mathrm{d}z\, z\, P_{Vf,G}^R(z)\right]\right\}$$
$$= \exp\left\{-\frac{3}{4}\int_{m_V}^q \frac{\mathrm{d}q'}{q'}\frac{\alpha_2(q')}{\pi}\left[\int_0^{1-m_V/q'}\mathrm{d}z\, P_{ff,G}^R(z) + \mathcal{O}(m_V/q')\right]\right\}\,, \tag{2.54}$$

we have

$$\left[q\frac{\partial}{\partial q}d_q^{1-}(x,q)\right]_2 = \frac{\alpha_2(q)}{\pi}\left\{-\frac{1}{4}\int_0^1 \mathrm{d}z\, P_{ff,G}^R(z)\left[d_q^{1-}(x/z,q)-d_q^{1-}(x,q)\right]\right.$$

$$\left.+\frac{1}{2}\int_0^1 \mathrm{d}z\, P_{Vf,G}^R(z)d_W^{1-}(x/z,q)+\frac{4}{3}d_q^{1-}(x,q)\,q\frac{\mathrm{d}}{\mathrm{d}q}\ln\Delta_{f,2}(q)+\mathcal{O}(m_V/q)\right\}. \tag{2.55}$$

The integrals are now independent of m_V and therefore only produce single-logarithmic evolution. All the double-logarithmic dependence comes from the Sudakov factor and we can write

$$d_q^{1-}(x,q) = \tilde{d}_q^{1-}(x,q)\left[\Delta_{f,2}(q)\right]^{4/3} \tag{2.56}$$

where $\tilde{d}_q^{1-}$ has only single-logarithmic evolution. Similarly, all other FF combinations that are not SU(2)-symmetric are suppressed at high energy by powers of the corresponding SU(2) Sudakov factor [31].

While for fermions there are only isospin 0 and 1 combinations possible, for vector bosons one can also form combinations with $\mathbf{T}=2$:

$$d_W^{2\pm} = \frac{1}{6}\left[\left(d_{W_+^+}+d_{W_-^+}-2d_{W_+^3}\right)\pm\left(d_{W_+^-}+d_{W_-^-}-2d_{W_-^3}\right)\right]. \tag{2.57}$$

The double-logarithmic dependence in fact only depends on the value of the isospin, and in general one finds

$$d_i^{T\pm}(x,q) = \tilde{d}_i^{T\pm}(x,q)\,\Delta_i^{(T)}(q) \tag{2.58}$$

where in double-logarithmic approximation

$$\Delta_i^{(T)}(q) \simeq \exp\left[-T(T+1)\int_{m_V}^q \frac{\mathrm{d}q'}{q'}\frac{\alpha_2}{\pi}\int_0^{1-m_V/q'}\frac{\mathrm{d}z}{1-z}\right] = \exp\left[-T(T+1)\frac{\alpha_2}{2\pi}\ln^2\left(\frac{q}{m_V}\right)\right]. \tag{2.59}$$

2.4 Momentum conservation

The total momentum fraction carried by particle species k in a jet initiated by a parton of type i at scale q is given by

$$\langle d_i^k(q)\rangle \equiv \int_0^1 \mathrm{d}x\, d_i^k(x,q). \tag{2.60}$$

Noting that

$$\int_0^1 \mathrm{d}x\, P_{ji,I}^R \otimes d_j^k = Q_{ji,I}(q)\,\langle d_j^k(q)\rangle, \tag{2.61}$$

where

$$Q_{ji,I}(q) = \int_0^{z_{\max}^{ji,I}(q)}\mathrm{d}z\, z\, P_{ji,I}^R(z), \tag{2.62}$$

we have from the evolution equation (2.19) for unmixed FFs

$$\left[q\frac{\mathrm{d}}{\mathrm{d}q}\langle d_i^k(q)\rangle\right]_I = \frac{\alpha_I(q)}{\pi}\sum_j C_{ji,I}Q_{ji,I}(q)\left[\langle d_j^k(q)\rangle - \langle d_i^k(q)\rangle\right]. \tag{2.63}$$

Writing

$$F_{ij}(q) = \sum_I \frac{\alpha_I(q)}{\pi} \left[C_{ji,I} Q_{ji,I}(q) - \delta_{ij} \sum_l C_{li,I} Q_{li,I}(q) \right] \tag{2.64}$$

this gives

$$q \frac{\mathrm{d}}{\mathrm{d}q} \langle d_i^k(q) \rangle = \sum_j F_{ij}(q) \langle d_j^k(q) \rangle . \tag{2.65}$$

This is a set of ordinary differential equations that can be solved by finding the eigenvalues and eigenvectors of the matrix $F_{ij}(q)$. One of the eigenvalues, corresponding to the eigenvector $(1, 1, \ldots, 1)$, is zero, so for every species k and unmixed interaction I there is a linear combination of the momentum fractions $\langle d_i^k \rangle$ that is scale-independent. Furthermore, since the sum of momenta of all species k in the jet must equal that of the initial parton i, for the unmixed FFs we have

$$\sum_k \langle d_i^k(q) \rangle = 1 \tag{2.66}$$

for every value of i, and thus

$$\left[q \frac{\mathrm{d}}{\mathrm{d}q} \sum_k \langle d_i^k(q) \rangle \right]_I = 0 , \tag{2.67}$$

so the momentum sum is conserved by each interaction separately.

For the mixed vector boson FF, $i = BW$, of either helicity, the real emission term involves the difference between the momentum sums for up- and down-type fermions and scalars, which vanishes, so that, from eq. (2.17),

$$q \frac{\mathrm{d}}{\mathrm{d}q} \sum_k \langle d_{BW}^k \rangle = \frac{1}{2\pi} \left[\alpha_1(q) P_{B,1}^V(q) + \alpha_2(q) P_{W,2}^V(q) \right] \sum_k \langle d_{BW}^k \rangle \tag{2.68}$$

and hence

$$\sum_k \langle d_{BW}^k(q) \rangle = \exp \left\{ \int_{q_0}^q \frac{1}{2\pi} \frac{\mathrm{d}q'}{q'} \left[\alpha_1(q') P_{B,1}^V(q') + \alpha_2(q') P_{W,2}^V(q') \right] \right\} \sum_k \langle d_{BW}^k(q_0) \rangle$$
$$= \Delta_{BW}(q) \sum_k \langle d_{BW}^k(q_0) \rangle , \tag{2.69}$$

where $\Delta_{BW}(q)$ is the BW Sudakov factor. Now from eq. (2.3) we have

$$\sum_k \langle d_{BW}^k \rangle = \sum_k \left[2 c_W s_W \left(\langle d_\gamma^k \rangle - \langle d_Z^k \rangle \right) + \left(c_W^2 - s_W^2 \right) \langle d_{\gamma Z}^k \rangle \right]$$
$$= \left(c_W^2 - s_W^2 \right) \sum_k \langle d_{\gamma Z}^k \rangle , \tag{2.70}$$

and, as will be discussed in section 4, the mixed γZ FFs $d_{\gamma Z}^k$ all vanish at the electroweak scale $q = q_0$. Hence the momentum sum for the mixed FF, of either helicity, vanishes at all scales:

$$\sum_k \langle d_{BW}^k(q) \rangle \equiv 0 . \tag{2.71}$$

Similarly the mixed Higgs FFs (2.12) make equal and opposite contributions to the Higgs and Z_L^0, and do not mix with other FFs, so they also do not contribute to the momentum sum.

3 Achieving full (next-to-) leading-logarithmic accuracy

We have seen in section 2.3 that fragmentation functions that are not iso-singlets experience double-logarithmic evolution. This is due to the fact that the soft singularity as $z \to 1$ in the splitting functions $P^R_{ii,G}(z)$ do not cancel between the virtual and real contributions. This is the origin of the SU(2) Sudakov factor, which according to eq. (2.16) is given by

$$
\Delta_{i,2}(q) = \exp \left[\int_{m_V}^{q} \frac{\mathrm{d}q'}{q'} \frac{\alpha_2(q')}{\pi} P^V_{i,2}(q') \right]
$$
$$
= \exp \left[- \int_{m_V}^{q} \frac{\mathrm{d}q'}{q'} \frac{\alpha_2(q')}{\pi} \sum_j C_{ji,I} \int_0^{z^{ji,I}_{\max}(q')} z \, \mathrm{d}z \, P^R_{ji,I}(z) \right] . \tag{3.1}
$$

The leading logarithmic contribution arises from the term in the splitting function that is divergent as $z \to 1$ and one can write

$$
C_{ji,I} \int_0^{z^{ji,I}_{\max}(q')} z \, \mathrm{d}z \, P^R_{ji,I}(z) \sim 2 \, C_{i,2} \int_0^{1 - m_V/q'} \frac{\mathrm{d}z}{1-z} = 2 \, C_{i,2} \ln \left(\frac{q'}{m_V} \right), \tag{3.2}
$$

where $C_{f,2} = C_{H,2} = 3/4$ and $C_{W,2} = 2$. For a fixed coupling α_2 we then obtain the double-logarithmic (DL) approximation to the Sudakov factor,

$$
\Delta^{DL}_{i,2}(q) = \exp \left[-C_{i,2} \frac{\alpha_2}{\pi} \ln^2 \left(\frac{q}{m_V} \right) \right] \equiv \exp \left[-\frac{C_{i,2}}{\pi} \alpha_2 L^2 \right]. \tag{3.3}
$$

However, it is well known that in general, Sudakov factors take the form

$$
\Delta_{i,2}(q) = \exp \left[L \, g_1(\alpha L) + g_2(\alpha L) + \alpha \, g_3(\alpha L) + \ldots \right] \tag{3.4}
$$

where in the case at hand $\alpha \equiv \alpha_2(q)$. The functions $g_i(x)$ are polynomial functions satisfying $g_i(0) = 0$ and determine the logarithmic terms necessary in the expansion when the size of the log is such that $\alpha L \sim 1$. The DL approximation amounts to only keeping the $\mathcal{O}(\alpha)$ term of the g_1 function. It is only sufficient if the size of the log satisfies $\alpha L^2 \sim 1$ but $\alpha L \ll 1$. The leading logarithmic (LL) approximation amounts to keeping the whole function g_1, which sums all terms of order $\alpha^n L^{n+1}$ while the next-to-leading logarithmic (NLL) approximation requires keeping in addition the function g_2, which sums logs of order $\alpha^n L^n$ and so on.

At the highest energies reachable at the LHC and a future 100 TeV collider the logarithm can be as large 5 and 7, respectively. Given that $\alpha_2 \sim 0.03$, this means that $\alpha_2 L^2 \sim 1$, but one still has $\alpha_2 L \ll 1$. In [38, 39] and so far in this paper we have given DL accurate results that only reproduce the term of order αL^2 in the exponent, even though they also produce an incomplete set of subleading terms. In the absence of large numerical factors, which can spoil the naive scaling of the logarithmic terms, DL accurate results would be sufficient to describe the physics at these energies. However, it is known [11] that for fermion scattering processes the single logarithmic coefficient is about a factor of 3 larger that the double logarithmic coefficient, such that these simple scaling rules might not provide reliable answers. In the following we will therefore only specify the dominant

	DL		LL		NLL	
	no match	match	no match	match	no match	match
missed term	$\alpha_2 L$	$\alpha_2^2 L^3$	$\alpha_2 L$	$\alpha_2^2 L^3$	α_2	$\alpha_2^2 L^2$

Table 3. Scaling of the dominant missed term in the perturbative expansion, for the double log expansion, where only the leading αL^2 term in the exponent is kept, the LL expansion, where the whole function $g_1(\alpha L)$ is kept, and the NLL expansion, where the functions $g_1(\alpha L)$ and $g_2(\alpha L)$ are kept. For each of these, we show the scaling of the first missed term if just the logarithmic resummation is used, and also the scaling if the resummed result is matched with the fixed order NLO calculation (such that the full α dependence is reproduced).

term missed in a given logarithmic expansion (assuming $\alpha L \ll 1$), without discussing the actual size of the effect.

In table 3 we show the dominant term missed when using DL, LL and NLL resummation. For each we also give the first missed term if the resummed results are matched to the full $\mathcal{O}(\alpha)$ calculation. One can see that using the full LL resummation vs DL accurate results [using the complete function $g_1(\alpha L)$, rather than just its $\mathcal{O}(\alpha)$ term] does not improve the situation, since one is still missing a term of order αL or $\alpha^2 L^3$ if the result is matched with the fixed order calculation at NLO, as described in [39]. The $\alpha^2 L^3$ in the matched calculation comes from the missed αL term of the function $g_2(\alpha L)$ multiplying the αL^2 term of $Lg_1(\alpha L)$. This term is only reproduced once the complete NLL resummation is taken into account. This of course makes sense, since the full LL resummation is only formally improving the accuracy of the DL resummation when counting $\alpha L \sim 1$. In that limit, however, the NLL resummation provides an $\mathcal{O}(1)$ effect, which needs to be included as well. Note that the two different choices for the scaling of the logarithm were already discussed in some detail in [16]. From this pure logarithmic counting, one expects DL accuracy to be the same as LL accuracy, and NLL effects to be subdominant as long as $\alpha L \ll 1$. But as mentioned before, large numerical coefficients can spoil this behavior, with the details depending on the process being studied.

Even though the full LL resummation does not improve the situation over matched DL resummation for feasible collider energies, we will show how it can be obtained in the DGLAP formalism by choosing the scale of the running SU(2) coupling appropriately. It is well known in standard QCD resummation and parton shower algorithms, that for double logarithmically sensitive observables the evolution should be angular-ordered and the running coupling should be evaluated at the transverse momentum of gauge boson emission [45, 46]. This means that instead of using $\alpha_2(q)$ as we have been doing in the DGLAP evolution, one should use $\alpha_2(q(1-z))$. Then since

$$\alpha_2(q') = \frac{\alpha_2(q)}{1 + \beta_0^{(2)} \frac{\alpha_2(q)}{\pi} \ln \frac{q'}{q}}, \tag{3.5}$$

with $\beta_0^{(2)} = 19/12$, the ratio of these two scale choices is given by the expansion

$$\frac{\alpha_2(q(1-z))}{\alpha_2(q)} = 1 - \frac{\alpha_2(q)}{\pi} \beta_0^{(2)} \ln(1-z) + \left[\frac{\alpha_2(q)}{\pi} \beta_0^{(2)} \ln(1-z) \right]^2 + \dots . \tag{3.6}$$

Note that these logarithmic terms in $1 - z$ only give rise to large logarithms if integrated against a singular function $f(z) \sim 1/(1-z)$. Thus, in standard DGLAP evolution in QCD, where the soft divergence as $z \to 1$ cancels between the virtual and real contributions, the difference between these two scales do not lead to logarithmic terms that need to be resummed. For the case of SU(2) DGLAP evolution of PDFs or FFs that are not iso-singlets, however, this cancelation does not happen, and one finds

$$\int_0^{1-\frac{m}{q}} \mathrm{d}z \frac{\alpha_2(q(1-z))}{\pi} \frac{1}{1-z} = \frac{\alpha_2(q)}{\pi} L + \frac{\alpha_2^2(q)}{\pi^2} \frac{\beta_0^{(2)}}{2} L^2 + \dots, \tag{3.7}$$

which generates the LL function $g_1(\alpha_2 L)$. The full LL resummation is therefore obtained by changing the SU(2) splitting functions that are singular as $z \to 1$ as

$$P_{ff,2}^R(z) \to P_{ff,2}^R(z,q) = \frac{\alpha_2[q(1-z)]}{\alpha_2(q)} \frac{2}{1-z} - (1+z), \tag{3.8}$$

$$P_{V_+ V_+,2}^R(z) \to P_{VV,2}^R(z,q) = \frac{\alpha_2[q(1-z)]}{\alpha_2(q)} \frac{2}{1-z} + \frac{1}{z} - 1 - z(1+z), \tag{3.9}$$

$$P_{HH,G}^R(z) \to P_{HH,G}^R(z,q) = \frac{\alpha_2[q(1-z)]}{\alpha_2(q)} \frac{2}{1-z} - 2. \tag{3.10}$$

By making one more change one can in fact also reproduce the full NLL resummation of the collinear evolution. The only missing term is the 2-loop cusp anomalous dimension, which can be included using the CMW prescription [47] for the coupling constant. This amounts to changing

$$\alpha_2[q(1-z)] \to \alpha_2^{\mathrm{CMW}}[q(1-z)] \tag{3.11}$$

in eqs. (3.8)–(3.10), where

$$\alpha_2^{\mathrm{CMW}}[q(1-z)] \equiv \alpha_2[q(1-z)] \left[1 + \frac{\Gamma_{\mathrm{cusp,f}}^{(2)}}{\Gamma_{\mathrm{cusp,f}}^{(1)}} \frac{\alpha_2[q(1-z)]}{\pi} \right] \simeq \alpha_2[k_{\mathrm{CMW}} q(1-z)], \quad (3.12)$$

$$k_{\mathrm{CMW}} = \exp\left(-\frac{1}{\beta_0^{(2)}} \frac{\Gamma_{\mathrm{cusp,f}}^{(2)}}{\Gamma_{\mathrm{cusp,f}}^{(1)}} \right), \tag{3.13}$$

and $\Gamma_{\mathrm{cusp,f}}^{(n)}$ and $\Gamma_{\mathrm{cusp,a}}^{(n)}$ denote the cusp anomalous dimension in the fundamental and adjoint representations at n-loop order. For n_g fermion generations and n_H Higgs doublets [48]

$$\frac{\Gamma_{\mathrm{cusp,f}}^{(2)}}{\Gamma_{\mathrm{cusp,f}}^{(1)}} = \frac{\Gamma_{\mathrm{cusp,a}}^{(2)}}{\Gamma_{\mathrm{cusp,a}}^{(1)}} = \frac{67}{18} - \frac{\pi^2}{6} - \frac{5}{9} n_g - \frac{1}{9} n_H = \frac{35}{18} - \frac{\pi^2}{6}, \tag{3.14}$$

which gives

$$k_{\mathrm{CMW}} = \exp\left(\frac{6\pi^2 - 70}{57} \right) = 0.828. \tag{3.15}$$

One can verify that this reproduces the complete NLL resummation in the collinear sector by comparing directly against the results of [40]. For observables that are completely

inclusive over the final state, where no soft function is required, this therefore reproduces the full NLL resummation. For less inclusive observables, it misses the logarithmic resummation coming from the evolution of the soft function, which was discussed in [40] and is not included here.

As we have explained, including the full LL resummation, compared with only the DL resummation, does not improve the formal accuracy of the calculation, unless the full NLL effects are included at the same time. Since the NLL contributions that were obtained above by including the 2-loop cusp anomalous dimension only include the NLL effects from the collinear evolution, but miss the NLL contributions from the soft evolution from [40], their inclusion does not raise the formal accuracy either. Nevertheless, we compare the results from the collinear NLL resummation discussed above with DL accurate results obtained in previous work when presenting results in section 5.

4 Implementation details and input FFs

For simplicity we start the evolution of all FFs at the electroweak breaking scale $q_0 \sim m_V$, which in practice we take to be $100\,\mathrm{GeV}$. Each value of the fragmentation product k requires a separate run of the evolution code. For a quark or charged lepton, $k = f$, assuming that the helicity of the fragmentation product is not detected, we take as input

$$d_{f_L}^f(x, q_0) = d_{f_R}^f(x, q_0) = \delta(1-x)\,, \tag{4.1}$$

setting all other initial FFs to zero. Then the FFs for all 60 SM states i fragmenting into f are generated by evolving these input FFs to higher scales using the SM DGLAP equations given in section 3. To obtain FFs at scales below q_0, the resulting FFs $d_i^f(x, q > q_0)$ should be convoluted with the $\mathrm{SU}(3) \otimes \mathrm{U}(1)_{\mathrm{em}}$-evolved and hadronized FF of a jet of flavor f produced at scale q_0. The neutrinos $k = \nu$ have no right-handed states, so the initial condition becomes

$$d_{\nu_L}^\nu(x, q_0) = \delta(1-x)\,, \quad d_i^\nu(x, q_0) = 0 \text{ otherwise}\,, \tag{4.2}$$

for evolution from scale q_0. The resulting FFs can be interpreted directly as neutrino momentum fraction distributions, since the neutrinos do not evolve below the electroweak scale.

For fragmentation into a gauge boson V we again assume the helicity is not detected, so the input is

$$d_{V_+}^V(x, q_0) = d_{V_-}^V(x, q_0) = \delta(1-x)\,, \quad d_i^V(x, q_0) = 0 \text{ otherwise}\,. \tag{4.3}$$

For the gluon, the SM-evolved FFs at higher scales then need to be convoluted with the FFs of a gluon jet produced at scale q_0. For the $W^\pm$, on the other hand, the boson can simply be allowed to decay at scale q_0. For the neutral gauge bosons $V = \gamma$, Z^0 we resolve them into the unbroken B, W^3 and BW states according to eq. (2.2) at scale q_0 and evolve these upwards. Again, the heavy bosons can then decay directly at scale q_0, while the photon can either be treated as a stable particle or fragmented by $\mathrm{U}(1)_{\mathrm{em}}$ evolution at lower scales. Similarly the Higgs and longitudinal gauge boson FFs are resolved as in equations (2.10) and (2.11), and the unbroken FFs are evolved to higher scales using the unbroken SM.

The DGLAP evolution equations were solved directly in x-space using a two-stage Runge-Kutta method (Heun's method). Evaluations were on a grid of (501,71) points in (x,q) for each FF. The x-grid was uniform in $\log x$ from 10^{-7} to 0.5 and uniform in $\log(1-x)$ above 0.5:

$$
\begin{aligned}
x_i &= 0.5 \left(2 \times 10^{-7}\right)^{1-\frac{i}{250}} && \text{for} && 0 \leq i \leq 250 \\
&= 1 - 0.5 \left(2 \times 10^{-7}\right)^{\frac{i}{250}-1} && \text{for} && 251 \leq i \leq 500.
\end{aligned} \tag{4.4}
$$

The q-grid was uniform in $\log q$:

$$
q_i = 10^{1+\frac{i}{10}} \text{ GeV for } 0 \leq i \leq 70. \tag{4.5}
$$

The input FFs at the matching scale are proportional to $\delta(1-x)$, which was approximated by the following smooth function of x:

$$
f_\delta(x) = \begin{cases} \dfrac{1}{4a^4}(a+1-x)(2a-1+x)^2 & \text{for } x > 1-2a \\ 0 & \text{otherwise,} \end{cases} , \tag{4.6}
$$

with $a = 10^{-3}$. Results were confirmed to be stable for this value of a. Integrals were evaluated to relative precision 10^{-6} by adaptive gaussian integration with five-point polynomial interpolation in the x-grid.

Notice that the momentum conservation relations (2.66) and (2.71) involve sums over independent runs of the evolution code for the 30 possible fragmentation products k, and must hold for each one of the 60 fragmenting objects i, which provides a valuable check on the correctness and precision of the code.

5 Results

As already mentioned, there are a total of $60 \times 30 = 1800$ distinct FFs, and we can clearly only show a small subset of all possible results. We therefore choose a few illustrative choices of i (left-and right-handed down quarks, the left- and right-handed electron, the SU(2) bosons W^+ and W_3, the U(1) boson B and the gluon), and for each i group the 30 possible values of k into a few representative sets. Readers interested in more details can request all data as LHAPDF files from the authors. The main results use the full NLL accuracy of the DGLAP evolution that was discussed in section 3. Note that to obtain full NLL accuracy for a cross section prediction requires the inclusion of single logarithms arising from the evolution of the soft function that were computed in [40].

We begin by showing in figure 1 the results for the momentum fractions $\langle d_i^k(q) \rangle$ defined in eq. (2.60). For each species i, we show how the total momentum is shared between fragmentation particles k at scales q ranging from 100 to 10^6 GeV. We stack the various sets for k on top of each other, such that momentum conservation implies that each plot sums to unity for all values of q once all particles are included. To show the size of the difference between DL and NLL accuracy, we show in dashed lines also the results obtained using DL accuracy. The reason that for several curves the DL result is not visible is because
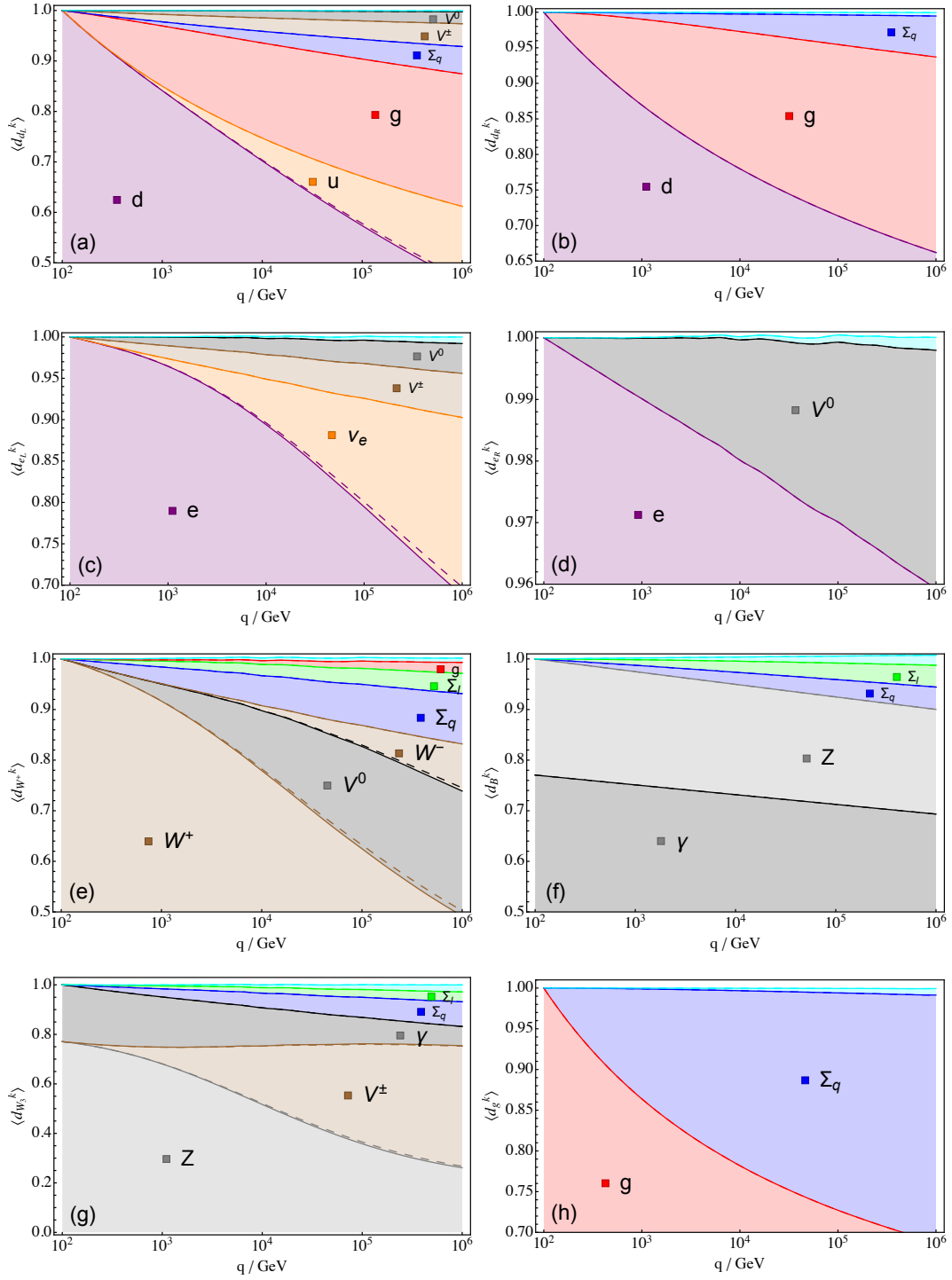
Figure 1. The momentum averaged fragmentation functions $\langle d_i^k \rangle$ for (a,b) $i = d_L, d_R$, (c,d) e_L, e_R, (e,f) W^+, B, (g,h) W_3, g. The different values of k are stacked on top of each other, such that the total equals one, as demanded by the sum rule. Dashed/solid lines show DL/NLL resummed results.

it is indistinguishable from the NLL result.[7] One can also clearly see that at $q = 100\,\text{GeV}$, the only contribution is for $i = k$. Since i and k are chosen in the unbroken and broken basis, respectively, for the W_3 and B the relative probability of Z^0 and γ are given by the weak mixing angle. As we evolve to larger values of q, other flavors k are generated.

In the first row we show the fragmentation of left- and right-handed down quarks, $i = d_L, d_R$. In the left-handed case (a) one can see that the SU(2) interaction has a significant effect. Left-handed up quarks are generated with double logarithmic probability, such that at large enough values of q the amount of u_L and d_L become of the same order of magnitude, and SU(2) bosons are produced at an appreciable rate. Gluons are produced at a larger rate, which is obviously due to the relative strength of the SU(3) and SU(2) interactions. For the right handed down quark (b), the fragmentation is completely dominated by QCD evolution, such that a large fraction of gluons and a smaller fraction of quarks other than d_R get generated. Other particles, shown by the remaining contribution in cyan, only make up a tiny fraction, even at $q = 10^6\,\text{GeV}$.

The fragmentation of left- and right-handed electrons is shown in the second row of figure 1. In the left-handed case (c) one can again see the importance of the SU(2) interactions at large values of q, and for $q \sim 10^6\,\text{GeV}$ the relative fraction of electrons and neutrinos becomes comparable, with the momentum fraction contained in gauge bosons at the 10% level. For the right-handed electron (d), the evolution is only given by the U(1) interaction, such that one generates only a small fraction of U(1) bosons, and an even smaller fraction of other particles.

Gauge boson fragmentation is shown in the third and fourth rows of figure 1. For the W^+ boson (e), one sees that the other SU(2) gauge bosons are generated quite rapidly as q becomes larger than $100\,\text{GeV}$. Asymptotically, for $q \to \infty$, the three SU(2) gauge bosons will evolve to have equal momentum fractions, and while one can see the trend for them to become equal, one needs to go to much higher values than are shown here. Quarks and leptons are also produced at an appreciable rate, with more quarks owing to the colour factor. For the U(1) boson (f), the only non-vanishing fragmentation at $q = 100\,\text{GeV}$ is into Z bosons and photons, with relative fraction $\tan^{-2}\theta_W$. Since the coupling constant α_1 is smaller than α_2, quarks and leptons are produced at a lower rate than for the W^+ boson. However, the quark and lepton rates are more equal, because the colour factor of the quarks is largely compensated by the higher hypercharges of the leptons. For an initial W_3 boson, shown in (g), one again starts off with only Z bosons and photons, with relative fraction $\tan^2\theta_W$. Quickly the neutral SU(2) boson evolves into charged Ws, and also into quarks and leptons. Finally, we show in (h) the evolution of the gluon. As expected, it is completely dominated by the strong interaction, such that it mostly evolves into quarks.

While figure 1 illustrates the evolution of the total momentum fractions carried by various particles in a given species of jet, it does not show how the evolution looks for fixed values of x. This is shown in figures 2 and 3 for the same set of particles as before, and

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[7]However, given that the results with DL accuracy include an incomplete set of higher logarithmic terms, these results do not allow the conclusion that NLL terms are numerically subdominant to the DL terms. Rather, one can only conclude that the NLL terms that arise from the improved scale setting procedure in the SU(2) running coupling are subdominant.
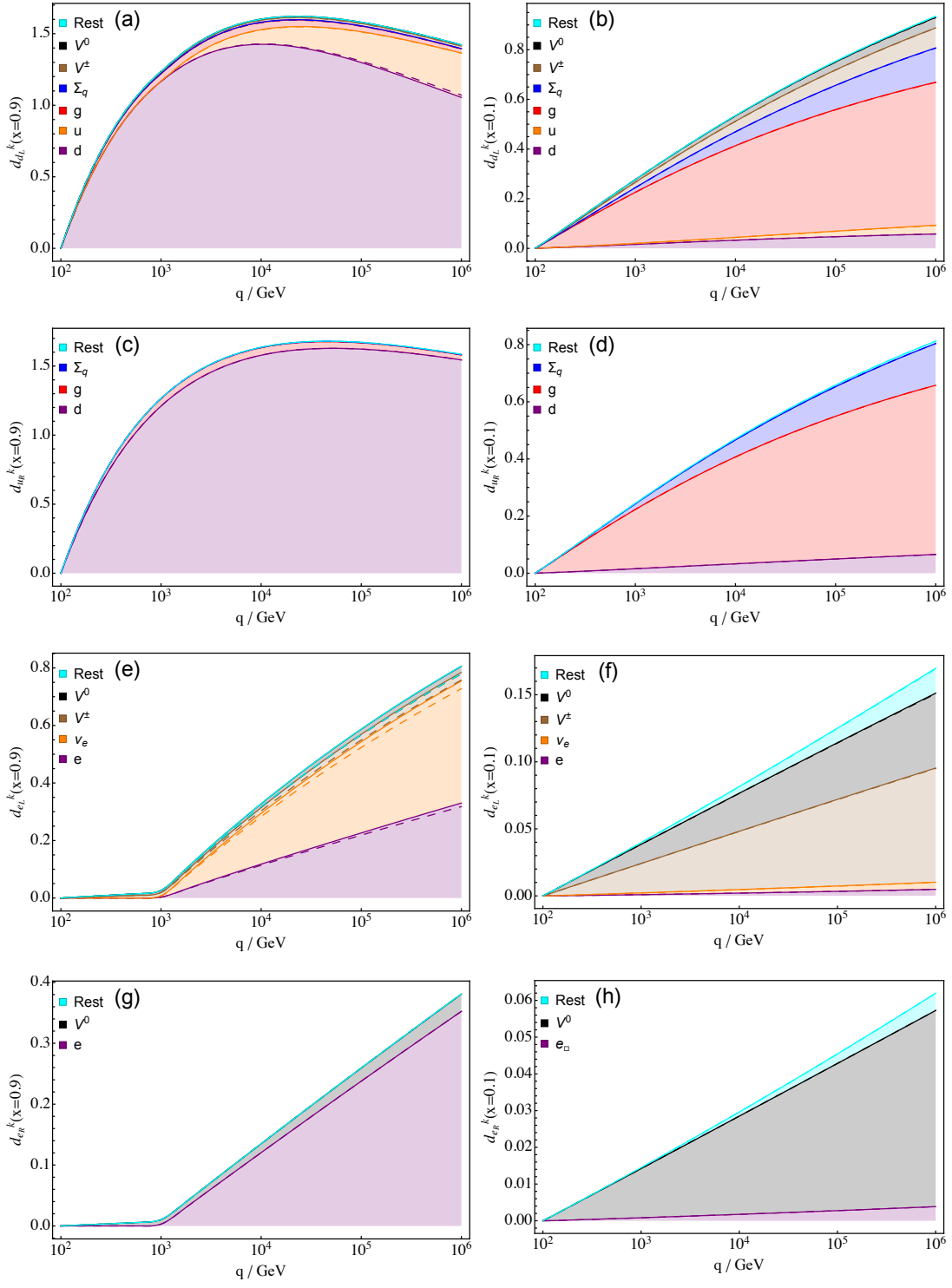
Figure 2. The fragmentation functions at (left) $x = 0.9$ and (right) $x = 0.1$ for (a,b) $i = d_L$, (c,d) d_R, (e,f) e_L, (g,h) e_R. The different values of k are stacked on top of each other. Dashed/solid lines show DL/NLL resummed results.
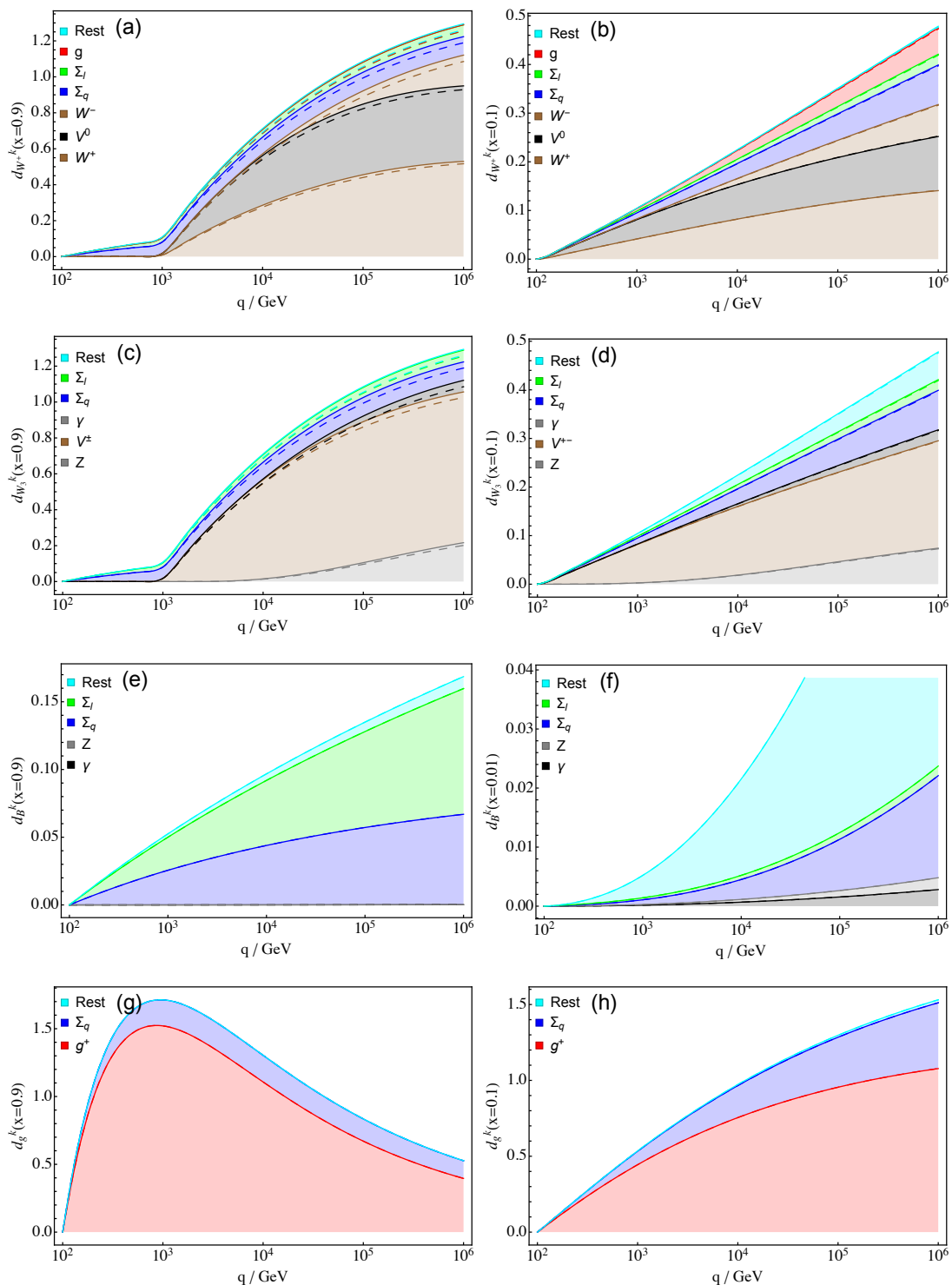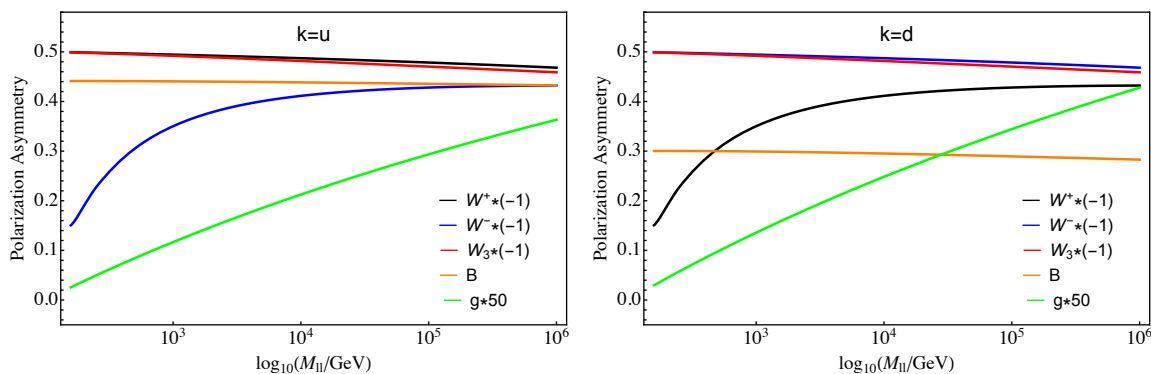
Figure 3. The fragmentation functions for (left) $x = 0.9$ and (right) $x = 0.1$ for (a,b) $i = W^+$, (c,d) W^3, (e,f) B, (g,h) g. The different values of k are stacked on top of each other. Dashed/solid lines show DL/NLL resummed results.

Figure 4. The absolute value of the polarization asymmetry, defined as $A(V) = (\langle d^k_{V_+} \rangle - \langle d^k_{V_-} \rangle)/(\langle d^k_{V_+} \rangle + \langle d^k_{V_-} \rangle)$, for fragmentation into (left) u and (right) d quarks, for the vector bosons $W^{\pm}$, W_3, B and the gluon. Note that the gluon asymmetry is scaled by a factor of 50, and that for the SU(2) bosons the negative of the asymmetry is shown. The results use the NLL accuracy as discussed in section 3.

using the values $x = 0.9$ (shown on the left) and $x = 0.1$ (on the right). As in figure 1, solid (dashed) lines correspond to NLL (DL) evolution. As explained in section 4, the initial condition at $q = 100\,\text{GeV}$ is a δ-function at $x = 1$ for $i = k$, such that the fragmentation at $x = 0.9$ is overall much more dominated by $k = i$ than at lower values of x. Notice that the constraint $x < 1 - m_V/q$ for emission of a heavy vector boson means that at $x = 0.9$ no such emission can occur below $q = m_V/(1 - x) \sim 1\,\text{TeV}$, depressing the evolution of leptons and heavy bosons below that scale. At $x = 0.1$, fragmentation into vector bosons is dominant at all scales, because of the low-z enhancement of the corresponding splitting functions.

In figure 4 we show some results on the polarization asymmetry of vector bosons fragmenting into up and down quarks. For $W^+, W_3 \to u$ and $W^-, W_3 \to d$ the asymmetry is large and negative, due to the dominance of the $V_- \to f_L$ splitting function. The $W^- \to u$ and $W^+ \to d$ asymmetries are also negative but increase from zero as they start at higher order. The B asymmetries are positive because of the dominance of fragmentation into the right-handed quarks in that case. The gluon asymmetry is a secondary effect of the different evolution of left- and right-handed quarks, the latter evolving more slowly and so remaining at higher x. Notice that there is even a slight difference between the gluon asymmetries for fragmentation into up and down quarks, due to their different electroweak evolution.

Although substantial vector bosons polarizations are generated by electroweak evolution, their effects on fragmentation into fermions and unpolarized bosons are negligible. The boson helicity asymmetries $d^k_{V_+} - d^k_{V_-}$ start from zero at the electroweak scale and cannot affect the unpolarized bosons at all, as they have opposite CP quantum numbers. They can indirectly affect only the $\{\mathbf{T}, \text{CP}\} = \{0, -\}$ and $\{1, +\}$ fermion FFs, generally producing effects at the 10^{-4} level or less in the momentum-averaged FFs of individual fermions.

6 Conclusions

In this paper we have discussed the evolution of fragmentation functions in the full Standard Model, which requires resummation of leading logarithms arising from final-state radiation and the associated virtual corrections.

At energy scales far above the electroweak symmetry breaking scale, short distance processes can be described in terms of 60 objects in the unbroken Standard Model: 12 left-handed quarks, 12 right-handed quarks, 12 left-handed leptons, 6 right-handed leptons, 2 transversely polarized gluons, 2 transversely polarized U(1) gauge bosons, 6 transversely polarized SU(2) bosons, 4 Higgs fields, 2 mixed neutral Higgs states and 2 transversely polarized states that mix the U(1) and neutral SU(2) boson. In hard interactions at such energies, any subsequent radiation is dominated by emissions that are either soft or collinear to the colliding or produced particles.

Processes that only depend on the flavor of one particle in each of these "jets" of radiation can be described solely in terms of parton distributions and fragmentation functions, which have to be evaluated at the short-distance scale of the hard interaction. The DGLAP evolution of the PDFs and FFs from the electroweak symmetry breaking scale to the hard scale q resums the logarithmic dependence on the ratio m_V/q. If the observed particles are not SU(2) singlets, one encounters double logarithms in the evolution.

We have presented the evolution of FFs in the complete Standard Model, where all three gauge interactions and the Yukawa interaction of the third generation contribute significantly to the DGLAP evolution. Together with the evolution of the PDFs, which was presented in [22], this provides all details necessary to resum the dominant logarithms for all cases where one is inclusive over the kinematics of the final state particles. Combining this with the running of soft functions presented in [40], full NLL accuracy of the electroweak evolution can be obtained.

While the dominant terms are of double logarithmic origin (scaling as $\alpha^n L^{2n}$ in a cross section), we also showed how the complete LL resummation (terms scaling as $\alpha^n L^{n+1}$ in the logarithm of a cross section) may be achieved by an appropriate choice for the scale of the running SU(2) coupling in the singular terms of the evolution. While this does not improve the accuracy in the relevant limit $\alpha_2 L^2 \sim 1$, and has a small numerical effect on the resulting FFs, it is necessary when the results from the DGLAP evolution are combined with the soft function evolution to obtain full NLL accuracy.

Numerically, the electroweak logarithms lead to appreciable effects at the highest energy scales that can be reached at the LHC and a future $100\,\mathrm{TeV}$ pp collider, but they still tend to be slightly smaller than what might be expected from the naive scaling of $\alpha_2 L^2$. For example, a left handed lepton produced at $3\,\mathrm{TeV}$ ($30\,\mathrm{TeV}$) has a 6% (15%) probability to fragment into a different particle defined at the electroweak scale $q_0 \sim 100\,\mathrm{GeV}$. The effect is larger for SU(2) bosons produced at the high scale. A charged W boson produced at $3\,\mathrm{TeV}$ ($30\,\mathrm{TeV}$) has a 14% (30%) probability to fragment into a different particle defined at $100\,\mathrm{GeV}$.

We have also studied for the first time the phenomenology of electroweak boson polarization in the FFs. Although large polarizations are generated, they have minimal effects as long as the polarization of fragmentation products is not detected.

Acknowledgments

A Isospin and CP basis

As already explained in section 1, the set of 60 evolution equations can be decoupled to some degree by switching to a basis of well-defined isospin $\mathbf{T}$ and CP. Writing a fermion FF with given $\{\mathbf{T}, \mathrm{CP}\}$ as $d_i^{\mathbf{T}\mathrm{CP}}$, the left-handed fermion FFs are

$$d_{f_L}^{0\pm} = \frac{1}{4}\left[(d_{u_L} + d_{d_L}) \pm (d_{\bar{u}_L} + d_{\bar{d}_L})\right], \tag{A.1}$$

$$d_{f_L}^{1\pm} = \frac{1}{4}\left[(d_{u_L} - d_{d_L}) \pm (d_{\bar{u}_L} - d_{\bar{d}_L})\right], \tag{A.2}$$

where u_L and d_L refer to left-handed up- and down-type fermions. Right-handed fermion FFs are given by

$$d_{f_R}^{0\pm} = \frac{1}{2}\left(d_{f_R} \pm d_{\bar{f}_R}\right). \tag{A.3}$$

The SU(3) and U(1) boson FFs have $\mathbf{T} = 0$, with the unpolarized and helicity asymmetry combinations having CP $= +$ and $-$, respectively:

$$d_g^{0\pm} = d_{g_+} \pm d_{g_-}, \qquad d_B^{0\pm} = d_{B_+} \pm d_{B_-}. \tag{A.4}$$

The SU(2) bosons can have $\{\mathbf{T}, \mathrm{CP}\} = \{0, +\}, \{1, -\}, \{2, +\}$ for the unpolarized FFs and $\{0, -\}, \{1, +\}, \{2, -\}$ for the asymmetries:

$$d_W^{0\pm} = \frac{1}{3}\left[\left(d_{W_+^+} + d_{W_+^-} + d_{W_+^3}\right) \pm \left(d_{W_-^+} + d_{W_-^-} + d_{W_-^3}\right)\right], \tag{A.5}$$

$$d_W^{1\pm} = \frac{1}{2}\left[\left(d_{W_+^+} - d_{W_+^-}\right) \mp \left(d_{W_-^+} - d_{W_-^-}\right)\right], \tag{A.6}$$

$$d_W^{2\pm} = \frac{1}{6}\left[\left(d_{W_+^+} + d_{W_+^-} - 2d_{W_+^3}\right) \pm \left(d_{W_-^+} + d_{W_-^-} - 2d_{W_-^3}\right)\right]. \tag{A.7}$$

The mixed BW boson FFs are a combination of 0^- and 1^- states, and therefore they have the opposite CP to the corresponding W boson FFs:

$$d_{BW}^{1\pm} = d_{BW_+} \pm d_{BW_-}. \tag{A.8}$$

For the Higgs boson, one writes similarly to the fermions

$$d_H^{0\pm} = \frac{1}{4}\left[(d_{H^+} + d_{H^0}) \pm (d_{H^-} + d_{\bar{H}^0})\right], \tag{A.9}$$

$$d_H^{1\pm} = \frac{1}{4}\left[(d_{H^+} - d_{H^0}) \pm (d_{H^-} - d_{\bar{H}^0})\right]. \tag{A.10}$$

There are also the mixed $H^0/\bar{H}^0$ FFs, which carry non-zero hypercharge and have unit isospin:

$$d_{HH}^{1\pm} = \frac{1}{2}\left(d_{H^0\bar{H}^0} \pm d_{\bar{H}^0 H^0}\right). \tag{A.11}$$

The even-CP mixed FF d_{HH}^{1+} represents the difference between the Higgs and longitudinal Z^0 FFs. The longitudinal vector boson and Higgs FFs are given by

$$d_{W_L^+} = d_H^{0+} + d_H^{1+} + d_H^{0-} + d_H^{1-}, \tag{A.12}$$

$$d_{W_L^-} = d_H^{0+} + d_H^{1+} - d_H^{0-} - d_H^{1-}, \tag{A.13}$$

$$d_{Z_L} = d_H^{0+} - d_H^{1+} - d_{HH}^{1+}, \tag{A.14}$$

$$d_h = d_H^{0+} - d_H^{1+} + d_{HH}^{1+}. \tag{A.15}$$

The odd-CP mixed FF d_{HH}^{1-} does not contribute to any physical states and does not mix with any other FFs, so it remains zero and we do not consider it further.

The resulting evolution equations are collected in appendix B.

B Equations used in the forward evolution

As in [38], we define

$$P_{ji,I}^R \otimes d_j^k = \int_x^{z_{\max}^{ji,I}(q)} \mathrm{d}z\, P_{ji,I}^R(z) d_j^k(x/z, q). \tag{B.1}$$

For splittings involving gauge bosons, we define

$$P_{VV,I}^R \otimes d_i \equiv \left(P_{V_+V_+,I}^R + P_{V_+V_-,I}^R\right) \otimes d_i, \tag{B.2}$$

$$P_{Vf,I}^R \otimes d_i \equiv \left(P_{V_+f_L,I}^R + P_{V_-f_L,I}^R\right) \otimes d_i, \tag{B.3}$$

$$P_{fV,I}^R \otimes d_i \equiv \left(P_{f_L V_+,I}^R + P_{f_L V_-,I}^R\right) \otimes d_i. \tag{B.4}$$

The '+'-prescription is

$$P_{ii,I}^+ \otimes d_i \equiv P_{ii,I}^R \otimes d_i + \frac{P_{i,I}^V}{C_{i,I}} d_i, \tag{B.5}$$

where $C_{i,I}$ is the coefficient in the corresponding Sudakov factor:

$$\Delta_{i,I}(q) = \exp\left[\int_{q_0}^q \frac{\mathrm{d}q'}{q'} \frac{\alpha_I(q')}{\pi} P_{i,I}^V(q')\right]$$

$$= \exp\left[-C_{i,I} \int_{q_0}^q \frac{\mathrm{d}q'}{q'} \frac{\alpha_I(q')}{\pi} \int_0^{z_{\max}^{ii,I}(q)} z\, \mathrm{d}z\, P_{ii,I}^R(z) + \ldots\right], \tag{B.6}$$

and $\ldots$ represents less divergent terms. For convenience we also define the isospin suppression factors

$$\Delta_i^{(T)}(q) = [\Delta_{i,2}(q)]^{T(T+1)/(2C_{i,2})}. \tag{B.7}$$

For gauge bosons we also need the helicity asymmetry splitting functions:

$$P_{VV,I}^A \otimes d_i \equiv \left(P_{V_+V_+,I}^R - P_{V_+V_-,I}^R \right) \otimes d_i + \frac{P_{V,I}^V}{C_{V,I}} d_i \,, \tag{B.8}$$

$$P_{Vf,I}^A \otimes d_i \equiv \left(P_{V_+f_L,I}^R - P_{V_-f_L,I}^R \right) \otimes d_i \,, \tag{B.9}$$

$$P_{fV,I}^A \otimes d_i \equiv \left(P_{f_LV_+,I}^R - P_{f_LV_-,I}^R \right) \otimes d_i \,. \tag{B.10}$$

B.1 SU(3) interaction

- $\mathbf{T} = 0$ and CP $= \pm$:

$$\left[q \frac{\partial}{\partial q} d_{q_{L,R}}^{0+} \right]_3 = \frac{\alpha_3}{\pi} C_F \left[P_{ff,G}^+ \otimes d_{q_{L,R}}^{0+} + P_{Vf,G}^R \otimes d_g^{0+} \right] \,, \tag{B.11}$$

$$\left[q \frac{\partial}{\partial q} d_g^{0+} \right]_3 = \frac{\alpha_3}{\pi} \left[C_A P_{VV,G}^+ \otimes d_g^{0+} + T_R P_{fV,G}^R \otimes d_{\sum_g}^{0+} \right] \,, \tag{B.12}$$

$$\left[q \frac{\partial}{\partial q} d_{q_{L,R}}^{0-} \right]_3 = \frac{\alpha_3}{\pi} C_F \left[P_{ff,G}^+ \otimes d_{q_{L,R}}^{0-} \pm P_{Vf,G}^A \otimes d_g^{0-} \right] \,, \tag{B.13}$$

$$\left[q \frac{\partial}{\partial q} d_g^{0-} \right]_3 = \frac{\alpha_3}{\pi} \left[C_A P_{VV,G}^A \otimes d_g^{0-} + T_R P_{fV,G}^A \otimes d_{\sum_g}^{0-} \right] \,. \tag{B.14}$$

Here

$$d_{\sum_g}^{0\pm} = 4 \sum_{q_L} d_{q_L}^{0\pm} \pm 2 \sum_{q_R} d_{q_R}^{0\pm} \,, \tag{B.15}$$

where the sums run over all left-handed quark doublets and all right-handed quarks. The factors of 4 and 2 are due to the different normalizations in eqs. (A.1) and (A.3).

- All other states:

$$\left[q \frac{\partial}{\partial q} d_q \right]_3 = \frac{\alpha_3}{\pi} C_F P_{ff,G}^+ \otimes d_q \,. \tag{B.16}$$

B.2 U(1) interaction

- $\mathbf{T} = 0$ and CP $= +$:

$$\left[q \frac{\partial}{\partial q} d_f^{0+} \right]_1 = \frac{\alpha_1}{\pi} Y_f^2 \left[P_{ff,G}^+ \otimes d_f^{0+} + P_{Vf,G}^R \otimes d_B^{0+} \right] \,, \tag{B.17}$$

$$\left[q \frac{\partial}{\partial q} d_B^{0+} \right]_1 = \frac{\alpha_1}{\pi} \left[P_{B,1}^V d_B^{0+} + P_{fV,G}^R \otimes d_{\sum_B f}^{0+} + P_{HV,G}^R \otimes d_H^{0+} \right] \,, \tag{B.18}$$

$$\left[q \frac{\partial}{\partial q} d_H^{0+} \right]_1 = \frac{\alpha_1}{\pi} \frac{1}{4} \left[P_{HH,G}^+ \otimes d_H^{0+} + P_{VH,G}^R \otimes d_B^{0+} \right] \,, \tag{B.19}$$

where

$$d_{\sum_B f}^{0\pm} = 4 \sum_{f_L} N_f Y_{f_L}^2 d_{f_L}^{0\pm} \pm 2 \sum_{f_R} N_f Y_{f_R}^2 d_{f_R}^{0\pm} \,. \tag{B.20}$$

- **T** $= 0$ and CP $= -$:

$$\left[q\frac{\partial}{\partial q}d_{f_{L,R}}^{0-}\right]_1 = \frac{\alpha_1}{\pi}Y_f^2\left[P_{ff,G}^+ \otimes d_{f_{L,R}}^{0-} \pm P_{Vf,G}^A \otimes d_B^{0-}\right], \tag{B.21}$$

$$\left[q\frac{\partial}{\partial q}d_B^{0-}\right]_1 = \frac{\alpha_1}{\pi}\left[P_{B,1}^V d_B^{0-} + P_{fV,G}^A \otimes d_{\sum_B f}^{0-}\right], \tag{B.22}$$

$$\left[q\frac{\partial}{\partial q}d_H^{0-}\right]_1 = \frac{\alpha_1}{\pi}\frac{1}{4}P_{HH,G}^+ \otimes d_H^{0-}. \tag{B.23}$$

- **T** $= 1$ and CP $= +$:

$$\left[q\frac{\partial}{\partial q}d_{HH}^{1+}\right]_1 = \frac{\alpha_1}{\pi}\frac{1}{4}P_{HH,G}^+ \otimes d_{HH}^{1+}, \tag{B.24}$$

$$\left[q\frac{\partial}{\partial q}d_{BW}^{1+}\right]_1 = \frac{\alpha_1}{\pi}\frac{1}{2}P_{B,1}^V d_{BW}^{1+}. \tag{B.25}$$

- **T** $= 1$ and CP $= -$:

$$\left[q\frac{\partial}{\partial q}d_{BW}^{1-}\right]_1 = \frac{\alpha_1}{\pi}\frac{1}{2}P_{B,1}^V d_{BW}^{1-}. \tag{B.26}$$

- All other states:

$$\left[q\frac{\partial}{\partial q}d_f\right]_1 = \frac{\alpha_1}{\pi}Y_f^2 P_{ff,G}^+ \otimes d_f, \tag{B.27}$$

$$\left[q\frac{\partial}{\partial q}d_H\right]_1 = \frac{\alpha_1}{\pi}\frac{1}{4}P_{HH,G}^+ \otimes d_H. \tag{B.28}$$

B.3   SU(2) interaction

- **T** $= 0$ and CP $= +$:

$$\left[q\frac{\partial}{\partial q}d_{f_L}^{0+}\right]_2 = \frac{\alpha_2}{\pi}\frac{3}{4}\left[P_{ff,G}^+ \otimes d_{f_L}^{0+} + P_{Vf,G}^R \otimes d_W^{0+}\right], \tag{B.29}$$

$$\left[q\frac{\partial}{\partial q}d_W^{0+}\right]_2 = \frac{\alpha_2}{\pi}\left[2P_{VV,G}^+ \otimes d_W^{0+} + \sum_{f_L} N_f P_{fV,G}^R \otimes d_{f_L}^{0+} + P_{HV,G}^R \otimes d_H^{0+}\right], \tag{B.30}$$

$$\left[q\frac{\partial}{\partial q}d_H^{0+}\right]_2 = \frac{\alpha_2}{\pi}\frac{3}{4}\left[P_{HH,G}^+ \otimes d_H^{0+} + P_{VH,G}^R \otimes d_W^{0+}\right]. \tag{B.31}$$

- **T** $= 0$ and CP $= -$:

$$\left[q\frac{\partial}{\partial q}d_{f_L}^{0-}\right]_2 = \frac{\alpha_2}{\pi}\frac{3}{4}\left[P_{ff,G}^+ \otimes d_{f_L}^{0-} + P_{Vf,G}^A \otimes d_W^{0-}\right], \tag{B.32}$$

$$\left[q\frac{\partial}{\partial q}d_W^{0-}\right]_2 = \frac{\alpha_2}{\pi}\left[2P_{VV,G}^A \otimes d_W^{0-} + \sum_{f_L} N_f P_{fV,G}^A \otimes d_{f_L}^{0-}\right], \tag{B.33}$$

$$\left[q\frac{\partial}{\partial q}d_H^{0-}\right]_2 = \frac{\alpha_2}{\pi}\frac{3}{4}P_{HH,G}^+ \otimes d_H^{0-}. \tag{B.34}$$

- **T = 1 and CP = +:**

$$\left[\Delta_f^{(1)} q \frac{\partial}{\partial q} \frac{d_{f_L}^{1+}}{\Delta_f^{(1)}} \right]_2 = \frac{\alpha_2}{\pi} \left[-\frac{1}{4} P_{ff,G}^+ \otimes d_{f_L}^{1+} + \frac{1}{2} P_{Vf,G}^A \otimes d_W^{1+} \right], \tag{B.35}$$

$$\left[\Delta_V^{(1)} q \frac{\partial}{\partial q} \frac{d_W^{1+}}{\Delta_V^{(1)}} \right]_2 = \frac{\alpha_2}{\pi} \left[P_{VV,G}^A \otimes d_W^{1+} + \sum_{f_L} N_f P_{fV,G}^A \otimes d_{f_L}^{1+} \right], \tag{B.36}$$

$$\left[\Delta_H^{(1)} q \frac{\partial}{\partial q} \frac{d_H^{1+}}{\Delta_H^{(1)}} \right]_2 = \frac{\alpha_2}{\pi} \left[-\frac{1}{4} P_{HH,G}^+ \otimes d_H^{1+} \right], \tag{B.37}$$

$$\left[\Delta_H^{(1)} q \frac{\partial}{\partial q} \frac{d_{HH}^{1+}}{\Delta_H^{(1)}} \right]_2 = \frac{\alpha_2}{\pi} \left[-\frac{1}{4} P_{HH,G}^+ \otimes d_{HH}^{1+} \right], \tag{B.38}$$

$$\left[\Delta_V^{(1)} q \frac{\partial}{\partial q} \frac{d_{BW}^{1+}}{\Delta_V^{(1)}} \right]_2 = 0. \tag{B.39}$$

- **T = 1 and CP = −:**

$$\left[\Delta_f^{(1)} q \frac{\partial}{\partial q} \frac{d_{f_L}^{1-}}{\Delta_f^{(1)}} \right]_2 = \frac{\alpha_2}{\pi} \left[-\frac{1}{4} P_{ff,G}^+ \otimes d_{f_L}^{1-} + \frac{1}{2} P_{Vf,G}^R \otimes d_W^{1-} \right], \tag{B.40}$$

$$\left[\Delta_V^{(1)} q \frac{\partial}{\partial q} \frac{d_W^{1-}}{\Delta_V^{(1)}} \right]_2 = \frac{\alpha_2}{\pi} \left[P_{VV,G}^+ \otimes d_W^{1-} + \sum_{f_L} N_f P_{fV,G}^R \otimes d_{f_L}^{1-} + P_{HV,G}^R \otimes d_H^{1-} \right], \tag{B.41}$$

$$\left[\Delta_H^{(1)} q \frac{\partial}{\partial q} \frac{d_H^{1-}}{\Delta_H^{(1)}} \right]_2 = \frac{\alpha_2}{\pi} \left[-\frac{1}{4} P_{HH,G}^+ \otimes d_H^{1-} + \frac{1}{2} P_{VH,G}^R \otimes d_W^{1-} \right], \tag{B.42}$$

$$\left[\Delta_V^{(1)} q \frac{\partial}{\partial q} \frac{d_{BW}^{1-}}{\Delta_V^{(1)}} \right]_2 = 0. \tag{B.43}$$

- **T = 2 and CP = +:**

$$\left[\Delta_V^{(2)} q \frac{\partial}{\partial q} \frac{d_W^{2+}}{\Delta_V^{(2)}} \right]_2 = -\frac{\alpha_2}{\pi} P_{VV,G}^+ \otimes d_W^{2+}. \tag{B.44}$$

- **T = 2 and CP = −:**

$$\left[\Delta_V^{(2)} q \frac{\partial}{\partial q} \frac{d_W^{2-}}{\Delta_V^{(2)}} \right]_2 = -\frac{\alpha_2}{\pi} P_{VV,G}^A \otimes d_W^{2-}. \tag{B.45}$$

B.4 Yukawa interaction

- **T** = 0 and CP = +:

$$\left[q \frac{\partial}{\partial q} d_{q_L^3}^{0+} \right]_Y = \frac{\alpha_Y}{\pi} \left[P_{q_L^3,Y}^V d_{q_L^3}^{0+} + P_{ff,Y}^R \otimes d_{t_R}^{0+} + P_{Hf,Y}^R \otimes d_H^{0+} \right], \tag{B.46}$$

$$\left[q \frac{\partial}{\partial q} d_{t_R}^{0+} \right]_Y = \frac{\alpha_Y}{\pi} 2 \left[P_{t_R,Y}^V d_{t_R}^{0+} + P_{ff,Y}^R \otimes d_{q_L^3}^{0+} + P_{Hf,Y}^R \otimes d_H^{0+} \right], \tag{B.47}$$

$$\left[q \frac{\partial}{\partial q} d_H^{0+} \right]_Y = \frac{\alpha_Y}{\pi} \left[P_{H,Y}^V d_H^{0+} + N_c P_{fH,Y}^R \otimes d_{\sum_H f}^{0+} \right], \tag{B.48}$$

where

$$d_{\sum_H f}^{0+} = d_{t_R}^{0+} + d_{q_L^3}^{0+} . \tag{B.49}$$

- **T** = 0 and CP = −:

$$\left[q \frac{\partial}{\partial q} d_{q_L^3}^{0-} \right]_Y = \frac{\alpha_Y}{\pi} \left[P_{q_L^3,Y}^V d_{q_L^3}^{0-} + P_{ff,Y}^R \otimes d_{t_R}^{0-} - P_{Hf,Y}^R \otimes d_H^{0-} \right], \tag{B.50}$$

$$\left[q \frac{\partial}{\partial q} d_{t_R}^{0-} \right]_Y = \frac{\alpha_Y}{\pi} 2 \left[P_{t_R,Y}^V d_{t_R}^{0-} + P_{ff,Y}^R \otimes d_{q^3}^{0-} + P_{Hf,Y}^R \otimes d_H^{0-} \right], \tag{B.51}$$

$$\left[q \frac{\partial}{\partial q} d_H^{0-} \right]_Y = \frac{\alpha_Y}{\pi} \left[P_{H,Y}^V d_H^{0-} + N_c P_{fH,Y}^R \otimes d_{\sum_H f}^{0-} \right], \tag{B.52}$$

where

$$d_{\sum_H f}^{0-} = d_{t_R}^{0-} - d_{q_L^3}^{0-} . \tag{B.53}$$

- **T** = 1 and CP = +:

$$\left[q \frac{\partial}{\partial q} d_{q_L^3}^{1+} \right]_Y = \frac{\alpha_Y}{\pi} \left[P_{q_L^3,Y}^V d_{q_L^3}^{1+} - P_{Hf,Y} \otimes d_H^{1+} \right], \tag{B.54}$$

$$\left[q \frac{\partial}{\partial q} d_H^{1+} \right]_Y = \frac{\alpha_Y}{\pi} \left[P_{H,Y}^V d_H^{1+} - N_c P_{fH}^R \otimes d_{q_L^3}^{1+} \right], \tag{B.55}$$

$$\left[q \frac{\partial}{\partial q} d_{HH}^{1+} \right]_Y = \frac{\alpha_Y}{\pi} P_{H,Y}^V d_{HH}^{1+} . \tag{B.56}$$

- **T** = 1 and CP = −:

$$\left[q \frac{\partial}{\partial q} d_{t_L}^{1-} \right]_Y = \frac{\alpha_Y}{\pi} \left[P_{t_L,Y}^V d_{t_L}^{1-} + P_{Hf,Y} \otimes d_H^{1-} \right], \tag{B.57}$$

$$\left[q \frac{\partial}{\partial q} d_H^{1-} \right]_Y = \frac{\alpha_Y}{\pi} \left[P_{H,Y}^V d_H^{1-} + N_c P_{fH,Y}^R \otimes d_{q_L^3}^{1-} \right]. \tag{B.58}$$

B.5 Mixed interaction

- **T** = 1 and CP = +:

$$\left[q\frac{\partial}{\partial q}d_f^{1+}\right]_M = \frac{\alpha_M}{\pi}\frac{Y_f}{2}P_{Vf,G}^R \otimes d_{BW}^{1+}\,,\tag{B.59}$$

$$\left[q\frac{\partial}{\partial q}d_{BW}^{1+}\right]_M = \frac{\alpha_M}{\pi}\left[4\sum_{f_L}Y_fN_fP_{fV,G}^R \otimes d_f^{1+} + 2P_{HV,G}^R \otimes d_H^{1+}\right]\,,\tag{B.60}$$

$$\left[q\frac{\partial}{\partial q}d_H^{1+}\right]_M = \frac{\alpha_M}{\pi}\frac{1}{4}P_{VH,G}^R \otimes d_{BW}^{1+}\,.\tag{B.61}$$

- **T** = 1 and CP = −:

$$\left[q\frac{\partial}{\partial q}d_{f_L}^{1-}\right]_M = \frac{\alpha_M}{\pi}\frac{Y_f}{2}P_{Vf,G}^A \otimes d_{BW}^{1-}\,,\tag{B.62}$$

$$\left[q\frac{\partial}{\partial q}d_{BW}^{1-}\right]_M = \frac{\alpha_M}{\pi}4\sum_{f_L}Y_fN_fP_{fV,G}^A \otimes d_f^{1-}\,,\tag{B.63}$$

$$\left[q\frac{\partial}{\partial q}d_H^{1-}\right]_M = 0\,.\tag{B.64}$$

Equation (B.60) differs slightly from ref. [31] where, taking into account the definition there of $d_{B3} = d_{BW}/2$, an 8 would appear in place of 4 in the first term on the right-hand side.

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