Phenomenological study of heavy hadron lifetimes

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ABSTRACT: The lifetimes of bottom and charmed hadrons are analyzed within the framework of the heavy quark expansion (HQE). Lifetime differences arise from the spectator effects such as weak W-annihilation and Pauli interference. Spectator effects originating from dimension-7 four-quark operators are derived. Hadronic matrix elements of four-quark operators are parameterized in a model-independent way. Using the dimension-6 bag parameters recently determined from HQET sum rules and the vacuum insertion approximation for meson matrix elements of dimension-7 operators, the calculated B meson lifetime ratios $\tau(B^+)/\tau(B_d^0) = 1.074^{+0.017}_{-0.016}$ and $\tau(B_s^0)/\tau(B_d^0) = 0.9962 \pm 0.0024$ are in excellent agreement with experiment. Likewise, based on the quark model evaluation of baryon matrix elements, the resulting baryon lifetime ratios $\tau(\Xi_b^-)/\tau(\Lambda_b^0)$, $\tau(\Xi_b^-)/\tau(\Xi_b^0)$, $\tau(\Omega_b^-)/\tau(\Xi_b^-)$ and the $\Lambda_b - B^0$ lifetime ratio $\tau(\Lambda_b^0)/\tau(B_d^0) = 0.953$ also agree well with the data. Contrary to the bottom hadron sector where the HQE in $1/m_b$ works well, the HQE to $1/m_c^3$ fails to give a satisfactory description of the lifetimes of both charmed mesons and charmed baryons. This calls for the subleading $1/m_Q$ corrections to spectator effects. The relevant dimension-7 spectator effects are in the right direction for explaining the large lifetime ratio of $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$. However, the destructive $1/m_c$ corrections to $\Gamma(\Omega_c^0)$ are too large to justify the validity of the HQE, namely, the predicted Pauli interference and semileptonic rates for Ω_c^0 become negative. Demanding these rates to be positive for a sensible HQE, we find the lifetime pattern $\tau(\Xi_c^+) > \tau(\Omega_c^0) > \tau(\Lambda_c^+) > \tau(\Xi_c^0)$, contrary to the current hierarchy $\tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0)$. We conclude that the Ω_c^0, which is naively expected to be shortest-lived in the charmed baryon system owing to the large constructive Pauli interference, could live longer than the Λ_c^+ due to the suppression from $1/m_c$ corrections arising from dimension-7 four-quark operators. The new charmed baryon lifetime pattern needs to be tested in forthcoming experiments.

KEYWORDS: Heavy Quark Physics, Perturbative QCD

ARXIV EPRINT: 1807.00916

Contents

1 Introduction

It was realized since the late 1970s and 1980s that the lifetime differences of singly heavy hadrons containing a heavy quark Q arise mainly from the spectator effects like W-exchange and Pauli interference due to the identical quarks produced in heavy quark decay and in the wave function of the heavy hadron [1–4]. The spectator effects were expressed in 1980s in terms of local four-quark operators by relating the total widths to the imaginary part of certain forward scattering amplitudes [5–7]. With the advent of heavy quark effective theory (HQET), it was recognized in early 1990s that nonperturbative corrections to the parton picture can be systematically expanded in powers of $1/m_Q$ [1–4]. Within the QCD-based framework, namely the heavy quark expansion (HQE), which is a generalization of the operator product expansion (OPE) in $1/m_Q$ in the Minkowski space, some phenomenological assumptions in 1980s acquired a firm theoretical footing in 1990s and nonperturbative effects can be systematically studied (for a review, see [8]).

On the basis of the OPE approach for the analysis of inclusive weak decays, the inclusive rate of the heavy hadron H_Q is schematically represented by

$$\Gamma(H_Q \to f) = \frac{G_F^2 m_Q^5}{192\pi^3} V_{\rm CKM} \left(A_0 + \frac{A_2}{m_Q^2} + \frac{A_3}{m_Q^3} + \frac{A_4}{m_Q^4} + \mathcal{O}\left(\frac{1}{m_Q^5}\right) \right), \qquad (1.1)$$

where V_{CKM} is the relevant CKM matrix element. The A_0 term comes from the decay of the heavy quark Q and is common to all heavy hadrons H_Q. There is no linear $1/m_Q$ corrections to the inclusive decay rate due to the lack of gauge-invariant dimension-four operators [1, 9, 10], a consequence known as Luke's theorem [11]. Nonperturbative corrections start at order $1/m_Q^2$ and are model independent. Spectator effects in inclusive decays due to the Pauli interference and W-exchange contributions account for $1/m_Q^3$ corrections. The estimate of spectator effects is model dependent; the hadronic four-quark matrix elements are usually evaluated by assuming the factorization approximation for mesons and the quark model for baryons. Moreover, there is a two-body phase-space enhancement factor of $16\pi^2$ for spectator effects relative to the three-body phase space for heavy quark decay. This means that $1/m_Q^3$ corrections can be quite significant. Moreover, spectator effects in charm hadron decays, being of order $1/m_c^3$, can be comparable to and even exceed the A_0 term.

Based on the HQE approach for the analysis of inclusive weak decays, the first correction to bottom hadron lifetimes is of order $1/m_b^2$ and it is model independent. For example, it was found in [12] that $\tau(B^-)/\tau(B_d) \approx \tau(B_s)/\tau(B_d) = 1 + \mathcal{O}(1/m_b^3)$ and $\tau(\Lambda_b)/\tau(B_d) = 0.98 + \mathcal{O}(1/m_b^3)$. The $1/m_b^3$ corrections can be expressed in a model-independent manner [12]

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} \simeq 0.98 - 0.17\varepsilon_1 + 0.20\varepsilon_2 - (0.012 + 0.021\tilde{B})r, \tag{1.2}$$

where ε_i, B_i, $\tilde{B}$, r are the hadronic parameters to be introduced below in section 3.1. Experimentally, the Λ_b^0 lifetime was significantly shorter than the B meson one in the early measurements. Taking the current B^0 meson lifetime $\tau(B^0) = (1.520 \pm 0.004)$ ps [13] as a benchmark, $\tau(\Lambda_b)$ was found to be (1.14 ± 0.08) ps in 1996 [14]. The world-averaged lifetime ratio then was

$$\frac{\tau(\Lambda_b^0)}{\tau(B^0)} = 0.79 \pm 0.06 \qquad (1996), \tag{1.3}$$

dominated by CERN e^+e^- collider LEP experiments [15]. This lifetime ratio remained essentially unchanged even in 2004 [16]

$$\frac{\tau(\Lambda_b^0)}{\tau(B^0)} = 0.803 \pm 0.047, \qquad \text{HFAG 2004}. \tag{1.4}$$

Since the two parameters ε_1 and ε_2 obey the constraint $\varepsilon_1 \approx 0.3\varepsilon_2$ [12] and they vanish under vacuum insertion approximation, it is very difficult to employ the HQE prediction (1.2) to accommodate the experimental value of $\tau(\Lambda_b)/\tau(B_d)$ without invoking too large a value of r and/or $\tilde{B}$. It is thus reasonable to conclude that the $1/m_b^3$ corrections in the HQE do not suffice to describe the observed lifetime difference between Λ_b and B_d.

Motivated by the conflict between theory and experiment for the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$, it was suggested in [17] that the assumption of local duality is not correct for nonleptonic inclusive width and that the presence of linear $1/m_b$ corrections prohibited in the HQE is strongly implied by the data. As shown in [18] and [19], the simple ansatz of replacing Γ_{NL} by $\Gamma_{\text{NL}}(m_{\Lambda_b}/m_b)^5$ not only solves the lifetime ratio problem but also provides the correct absolute decay widths for the Λ_b baryon and the B meson. However,

there exist several insurmountable difficulties with this scenario and some of them were already discussed in [18].

Nowadays we know that the issue with the low value of $\Lambda_b - B^0$ lifetime ratio or the short Λ_b lifetime was on the experimental side. The first direct measurement of the lifetime ratio $\tau(\Lambda_b^0)/\tau(B^0)$ was carried out by the D0 Collaboration in 2005 with the result $0.87^{+0.17}_{-0.14} \pm 0.03$ [20]. Also, the CDF experiment measured the Λ_b lifetime in exclusive decay to $J/\psi\Lambda$ [21] and showed that the Λ_b lifetime is significantly longer than either previous Λ_b lifetime measures or state-of-the-art calculation based on the HQE at the time. The world averages as of today are [13]

$$\tau(\Lambda_b^0) = (1.470 \pm 0.010) \times 10^{-12}\, s\,, \qquad \tau(\Lambda_b^0)/\tau(B_d^0) = 0.964 \pm 0.007\,. \qquad (1.5)$$

As we shall see in section 4.1 below, the current value of the $\Lambda_b - B^0$ lifetime ratio can be explained nicely in the HQE approach.

The major theoretical uncertainties of the HQE predictions for hadron lifetimes come from the hadronic matrix elements of four-quark operators. In the meson sector, the meson matrix elements can be expressed in a model-independent manner in terms of four bag parameters $B_{1,2}$ and $\epsilon_{1,2}$. These parameters have been calculated using lattice QCD and QCD sum rules (see [8] for a review). Based on HQET sum rules, they have been updated recently in [22]. For the baryon matrix elements, they can be parameterized in terms of four parameters $L_{1,\cdots,4}$, but only two of them are independent.

Although the heavy quark expansion in $1/m_b$ works well for B mesons and bottom baryons, the HQE in $1/m_c$ fails to give a satisfactory description of the lifetimes of both charmed mesons and charmed baryons. First of all, to order $1/m_c^3$, the destructive Pauli interference in D^+ decay overcomes the c quark decay rate so that the inclusive rate and the lifetime of D^+ become negative. Hence, it is not meaningful to discuss the lifetime ratio $\tau(D^+)/\tau(D^0)$ at this level. Second, the lifetime pattern of charmed baryons is understandable only qualitatively, but not quantitatively. The quantitative estimates of charmed baryon lifetimes and their ratios are still rather poor [18]. For example, $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$ is calculated to be 1.03 (see section 4.2 below), while experimentally it is measured to be 2.21 ± 0.15 [13]. Therefore, it is natural to consider the effects stemming from the next-order $1/m_c$ expansion. Specifically, we shall consider the subleading $1/m_c$ corrections to the spectator effects.

The $1/m_Q$ corrections to the spectator effects are computed by expanding the forward scattering amplitude in the light-quark momentum and matching the result onto the operators containing derivative insertions. Dimension-7 terms are either the four-quark operators times the spectator quark mass or the four-quark operators with an additional derivative [23, 24]. Dimension-7 operators were first studied in [25] for the width difference in the $B_s - \bar{B}_s$ system, in [23, 24] for the lifetime differences of heavy hadrons and in [26] for D-meson lifetimes.

In this work we will study spectator effects in inclusive nonleptonic and semileptonic decays, analyze the lifetime pattern of heavy hadrons, and pay attention to the effects of dimension-7 operators on the heavy hadron lifetimes, especially for the charmed mesons and baryons. Our goal is to see if the aforementioned problems such as the negative lifetime

of the D^+ meson and the closeness of Ξ_c^+ and Λ_c^+ lifetimes can be resolved by the inclusion of subleading $1/m_Q$ corrections to the spectator effects.

This work is organized as follows. In section 2 we give general heavy quark expansion expressions for inclusive nonleptonic and semileptonic widths. We then study lifetimes of B and D mesons in section 3 and bottom and charmed baryons in section 4 with the evaluation of hadronic four-quark matrix elements. Discussions and conclusions are given in section 5. In appendix A we sketch the derivation of dimension-7 four-quark operators relevant for the spectator effects in heavy baryon decays. Appendix B is devoted to the evaluation of baryon matrix elements in the quark model.

2 Framework

In this section we write down the general expressions for the inclusive decay widths of heavy hadrons and leave the evaluation of the relevant hadronic matrix elements to the next section. It is known that the inclusive decay rate is governed by the imaginary part of an effective nonlocal forward transition operator T. When the energy released in the decay is large enough, the nonlocal effective action can be recast as an infinite series of local operators with coefficients containing inverse powers of the heavy quark mass m_Q. Under this heavy quark expansion, the inclusive nonleptonic decay rate of a singly heavy hadron H_Q containing a heavy quark Q is given by [1–4]

$$\Gamma(H_Q) = \frac{1}{2m_{H_Q}} \text{Im} \, \langle H_Q | T | H_Q \rangle = \frac{1}{2m_{H_Q}} \langle H_Q | \int d^4x \, T[\mathcal{L}_W^\dagger(x)\mathcal{L}_W(0)] | H_Q \rangle, \qquad (2.1)$$

where the second T appearing in the integral is a time-ordering symbol. Under the operator product expansion, the transition operator T can be expressed in terms of local quark operators

$$\text{Im} \, T = \frac{G_F^2 m_Q^5}{192\pi^3} \xi \left(c_{3,Q} \bar{Q}Q + \frac{c_{5,Q}}{m_Q^2} \bar{Q}\sigma \cdot GQ + \frac{c_{6,Q}}{m_Q^3} T_6 + \frac{c_{7,Q}}{m_Q^4} T_7 + \cdots \right), \qquad (2.2)$$

where ξ is the relevant CKM matrix element, the dimension-6 T_6 consists of the four-quark operators $(\bar{Q}\Gamma q)(\bar{q}\Gamma Q)$ with Γ representing a combination of the Lorentz and color matrices, while a subset of dimension-7 T_7 is governed by the four-quark operators containing derivative insertions (see section 2.2 below). Since $\sigma \cdot G = -2\vec{\sigma} \cdot \vec{B}$, the $\bar{Q}\sigma \cdot GQ$ term describes the interaction of the heavy Q quark spin with the gluon field. Explicitly,

$$\Gamma_{\text{NL}}(H_Q) = \frac{G_F^2 m_Q^5}{192\pi^3} \xi \left\{ c_{3,Q}^{\text{NL}} \left[1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_Q^2} \right] + 2c_{5,Q}^{\text{NL}} \frac{\mu_G^2}{m_Q^2} \right. $$
$$\left. + \frac{c_{6,Q}^{\text{NL}}}{m_Q^3} \frac{\langle H_Q | T_6 | H_Q \rangle}{2m_{H_Q}} + \frac{c_{7,Q}^{\text{NL}}}{m_Q^4} \frac{\langle H_Q | T_7 | H_Q \rangle}{2m_{H_Q}} + \cdots \right\}, \qquad (2.3)$$

where use of

$$\frac{\langle H_Q | \bar{Q}Q | H_Q \rangle}{2m_{H_Q}} = 1 - \frac{\mu_\pi^2}{2m_Q^2} + \frac{\mu_G^2}{2m_Q^2} \qquad (2.4)$$

has been made with

$$\mu_\pi^2 \equiv \frac{1}{2m_{H_Q}} \langle H_Q | \bar{Q}(i\vec{D})^2 Q | H_Q \rangle = -\frac{1}{2m_{H_Q}} \langle H_Q | \bar{Q}(iD_\perp)^2 Q | H_Q \rangle = -\lambda_1,$$

$$\mu_G^2 \equiv \frac{1}{2m_{H_Q}} \langle H_Q | \bar{Q} \frac{1}{2} \sigma \cdot G Q | H_Q \rangle = d_H \lambda_2. \tag{2.5}$$

In heavy quark effective theory, the mass of the heavy hadron H_Q is of the form

$$m_{H_Q} = m_Q + \bar{\Lambda}_{H_Q} - \frac{\lambda_1}{2m_Q} - \frac{d_H \lambda_2}{2m_Q}, \tag{2.6}$$

where the three nonperturbative HQET parameters $\bar{\Lambda}_{H_Q}$, λ_1 and λ_2 are independent of the heavy quark mass and $\bar{\Lambda}_{H_Q}$ can be regarded as the binding energy of the heavy hadron in the infinite mass limit.[1] Since the chromomagnetic field is produced by the light cloud inside the heavy hadron, it is clear that $\sigma \cdot G$ is proportional to $\vec{S}_Q \cdot \vec{S}_\ell$, where $\vec{S}_Q$ ($\vec{S}_\ell$) is the spin operator of the heavy quark (light cloud). The parameter d_H is given by

$$\begin{aligned} d_H &= -\langle H_Q | 4\vec{S}_Q \cdot \vec{S}_\ell | H_Q \rangle \\ &= -2[S_{\text{tot}}(S_{\text{tot}} + 1) - S_Q(S_Q + 1) - S_\ell(S_\ell + 1)]. \end{aligned} \tag{2.7}$$

Therefore, $d_H = 3$ for B and D mesons, $d_H = -1$ for B^* and D^* mesons, $d_H = 0$ for the antitriplet baryon T_Q, $d_H = 4$ for the spin-$\frac{1}{2}$ sextet baryon S_Q and $d_H = -2$ for the spin-$\frac{3}{2}$ sextet baryon S_Q^*. It follows from eq. (2.6) that

$$\lambda_2^{\text{meson}} = \frac{1}{4}(m_{P^*}^2 - m_P^2) = \begin{cases} 0.12\,\text{GeV}^2 & for \text{ B } meson, \\ 0.14\,\text{GeV}^2 & for \text{ D } meson, \end{cases}$$

$$\lambda_2^{\text{baryon}} = \frac{1}{6}(m_{S_Q^*}^2 - m_{S_Q}^2). \tag{2.8}$$

Numerically (in units of GeV^2),

$$\lambda_2^{\Sigma_c} = 0.054, \qquad \lambda_2^{\Xi_c'} = 0.061, \qquad \lambda_2^{\Omega_c} = 0.064,$$

$$\lambda_2^{\Sigma_b} = 0.040, \qquad \lambda_2^{\Xi_b'} = 0.040, \qquad \lambda_2^{\Omega_b} = 0.041. \tag{2.9}$$

It is interesting to note that the large-N_c relation [28, 29]

$$\lambda_2^{\text{meson}} \sim N_c \lambda_2^{\text{baryon}} \tag{2.10}$$

is fairly satisfied especially for bottom hadrons. As for the kinetic energy parameter λ_1, we shall use [30]

$$\lambda_1^{\text{meson}} \sim \lambda_1^{\text{baryon}} = -(0.432 \pm 0.068)\,\text{GeV}^2. \tag{2.11}$$

Summing over the contributions from $b \to c\bar{c}s$, $b \to c\bar{c}d$, $b \to c\bar{u}d$ and $b \to c\bar{u}s$ processes, we have [1–4]

$$\begin{aligned} c_{3,b}^{\text{NL}} &= \left(N_c c_1^2 + N_c c_2^2 + 2c_1 c_2\right)(I_0(x, 0, 0) + I_0(x, x, 0)), \\ c_{5,b}^{\text{NL}} &= -\left(N_c c_1^2 + N_c c_2^2 + 2c_1 c_2\right)(I_1(x, 0, 0) + I_1(x, x, 0)) \\ &\quad - 8c_1 c_2(I_2(x, 0, 0) + I_2(x, x, 0)), \end{aligned} \tag{2.12}$$

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[1]For a review of the nonperturbative HQET parameters see ref. [27].

where $x = (m_c/m_b)^2$ and the good approximations $|V_{ud}|^2 + |V_{us}|^2 \approx 1$, $|V_{cs}|^2 + |V_{cd}|^2 \approx 1$ have been made. In the above equation, c_1, c_2 are Wilson coefficient functions, $N_c = 3$ is the number of color, I_0, I_1 and I_2 are the phase-space factors:

$$
\begin{aligned}
I_0(x,0,0) &= (1-x^2)(1-8x+x^2) - 12x^2 \ln x, \\
I_1(x,0,0) &= \frac{1}{2}\left(2 - x\frac{d}{dx}\right) I_0(x,0,0) = (1-x)^4, \\
I_2(x,0,0) &= (1-x)^3,
\end{aligned}
\tag{2.13}
$$

for $b \to c\bar{u}d$ and $b \to c\bar{u}s$ ($x = m_c^2/m_b^2$) or $c \to su\bar{d}$ ($x = m_s^2/m_c^2$) transition and

$$
\begin{aligned}
I_0(x,x,0) &= v(1 - 14x - 2x^2 - 12x^3) + 24x^2(1-x^2)\ln\frac{1+v}{1-v}, \\
I_1(x,x,0) &= \frac{1}{2}\left(2 - x\frac{d}{dx}\right) I_0(x,x,0) = v(1-2x)(1-4x-6x^2) + 24x^4\ln\frac{1+v}{1-v}, \\
I_2(x,x,0) &= v\left(1 + \frac{x}{2} + 3x^2\right) - 3x(1-2x^2)\ln\frac{1+v}{1-v},
\end{aligned}
\tag{2.14}
$$

for $b \to c\bar{c}s$ or $c \to s\bar{s}u$ transition with $v \equiv \sqrt{1-4x}$. For the c quark decay, contributions from $c \to s\bar{d}u$ and $c \to s\bar{s}u$ yield

$$
\begin{aligned}
c_{3,c}^{\mathrm{NL}} &= \left(N_c c_1^2 + N_c c_2^2 + 2c_1 c_2\right)\left(I_0(x,0,0)|V_{ud}|^2 + I_0(x,x,0)|V_{us}|^2\right), \\
c_{5,c}^{\mathrm{NL}} &= -\left(N_c c_1^2 + N_c c_2^2 + 2c_1 c_2\right)\left(I_1(x,0,0)|V_{ud}|^2 + I_1(x,x,0)|V_{us}|^2\right) \\
&\quad - 8c_1 c_2 \left(I_2(x,0,0)|V_{ud}|^2 + I_2(x,x,0)|V_{us}|^2\right),
\end{aligned}
\tag{2.15}
$$

with $x = (m_s/m_c)^2$.

It is now ready to deduce the inclusive semileptonic widths from eq. (2.3) by putting $c_1 = 1$, $c_2 = 0$ and $N_c = 1$:

$$
\Gamma_{\mathrm{SL}}(H_Q) = \frac{G_F^2 m_Q^5}{192\pi^3}\,\xi\left\{c_{3,Q}^{\mathrm{SL}}\left[1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_Q^2}\right] + 2c_{5,Q}^{\mathrm{SL}}\frac{\mu_G^2}{m_Q^2}\right\},
\tag{2.16}
$$

where

$$
\begin{aligned}
c_{3,b}^{\mathrm{SL}}(x,x_\tau) &= 2I_0(x,0,0) + I_0(x,x_\tau,0), \\
c_{5,b}^{\mathrm{SL}}(x,x_\tau) &= -\left(2I_1(x,0,0) + I_1(x,x_\tau,0)\right),
\end{aligned}
\tag{2.17}
$$

and

$$
\begin{aligned}
c_{3,c}^{\mathrm{SL}}(x,x_\mu) &= I_0(x,0,0) + I_0(x,x_\mu,0), \\
c_{5,c}^{\mathrm{SL}}(x,x_\mu) &= -\left(I_1(x,0,0) + I_1(x,x_\mu,0)\right),
\end{aligned}
\tag{2.18}
$$

with $x_\ell = (m_\ell/m_Q)^2$. For the expression of $I_{0,1}(x,y,0)$ with $y \neq x$, see [31] or the appendix of [32] with $C_0 = I_0(x,y,0)$ and $C_{\mu_G^2} = I_1(x,y,0)$.

2.1 Dimension-6 operators

Defining

$$\mathcal{T}_6 = \frac{G_F^2 m_Q^2}{192\pi^3} \xi \, c_{6,Q}^{\mathrm{NL}} \, T_6, \tag{2.19}$$

the dimension-six four-quark operators in eq. (2.3) responsible for spectator effects in inclusive decays of heavy baryons (denoted by $\mathcal{B}_Q$) are given by [5–7]

$$\mathcal{T}_{6,ann}^{\mathcal{B}_Q,q_1} = \frac{G_F^2 m_Q^2}{2\pi} \xi (1-x)^2 \Big\{ (c_1^2 + c_2^2)(\bar{Q}Q)(\bar{q}_1 q_1) + 2c_1 c_2 (\bar{Q}q_1)(\bar{q}_1 Q) \Big\},$$

$$\mathcal{T}_{6,int-}^{\mathcal{B}_Q,q_2} = -\frac{G_F^2 m_Q^2}{6\pi} \xi (1-x)^2 \Big\{ c_1^2 \Big[\Big(1+\frac{x}{2}\Big)(\bar{Q}Q)(\bar{q}_2 q_2) - (1+2x)\bar{Q}^\alpha(1-\gamma_5)q_2^\beta \bar{q}_2^\beta(1+\gamma_5)Q^\alpha \Big]$$

$$+ (2c_1 c_2 + N_c c_2^2) \Big[\Big(1+\frac{x}{2}\Big)(\bar{Q}q_2)(\bar{q}_2 Q) - (1+2x)\bar{Q}(1-\gamma_5)q_2 \bar{q}_2(1+\gamma_5)Q \Big] \Big\},$$

$$\mathcal{T}_{6,int-}^{\mathcal{B}_Q,q_3} = -\frac{G_F^2 m_Q^2}{6\pi} \xi \sqrt{1-4x} \Big\{ c_1^2 \Big[(1-x)(\bar{Q}Q)(\bar{q}_3 q_3) - (1+2x)\bar{Q}^\alpha(1-\gamma_5)q_3^\beta \bar{q}_3^\beta(1+\gamma_5)Q^\alpha \Big]$$

$$+ (2c_1 c_2 + N_c c_2^2) \Big[(1-x)(\bar{Q}q_3)(\bar{q}_3 Q) - (1+2x)\bar{Q}(1-\gamma_5)q_3 \bar{q}_3(1+\gamma_5)Q \Big] \Big\},$$

$$\mathcal{T}_{6,int+}^{\mathcal{B}_Q,q_3} = -\frac{G_F^2 m_Q^2}{6\pi} \xi \Big\{ c_2^2 \Big[(\bar{Q}Q)(\bar{q}_3 q_3) - \bar{Q}^\alpha(1-\gamma_5)q_3^\beta \bar{q}_3^\beta(1+\gamma_5)Q^\alpha \Big]$$

$$+ (2c_1 c_2 + N_c c_1^2) \Big[(\bar{Q}q_3)(\bar{q}_3 Q) - \bar{Q}(1-\gamma_5)q_3 \bar{q}_3(1+\gamma_5)Q \Big] \Big\}, \tag{2.20}$$

where $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$, α, β are color indices and ξ is the relevant CKM matrix element for the quark-mixing-favored decay. Note that for charm decay, $Q = c$, $q_1 = d$, $q_2 = u$ and $q_3 = s$ and for bottom decay, $Q = b$, $q_1 = u$, $q_2 = d$, $q_3 = s$. In the baryon sector, the first term $\mathcal{T}_{6,ann}^{\mathcal{B}_Q,q_1}$ corresponds to a W-exchange (or generically weak annihilation) contribution (see figure 1(a)), the rest to contributions from Pauli interference. For example, $\mathcal{T}_{6,int-}^{\mathcal{B}_Q,q_2}$ arises from the destructive interference of the q_2 quark produced in the heavy quark Q decay with the q_2 quark in the wave function of the heavy baryon $\mathcal{B}_Q$ (figure 1(b)). The last term $\mathcal{T}_{6,int+}^{\mathcal{B}_Q,q_3}$ in (2.20) is due to the constructive interference of the s quark and hence it occurs only in charmed baryon decays, i.e. $Q = c$ and $q_3 = s$ (figure 1(c)). The third term $\mathcal{T}_{6,int-}^{\mathcal{B}_Q,q_3}$ comes from the destructive Pauli interference due to $b \to c\bar{c}s$ (figure 1(b)) or $c \to s\bar{s}u$. This term exists in bottom decays with $c\bar{c}$ intermediate states and in charm decays with $s\bar{s}$ intermediate states.

As we shall see in section 4, Pauli interferences described by $\mathcal{T}_{6,int-}^{\mathcal{B}_Q,q_2}$ and $\mathcal{T}_{6,int-}^{\mathcal{B}_Q,q_3}$ are destructive as the relevant Wilson coefficient of the form $(N_c c_2^2 + 2c_1 c_2 - \tilde{B} c_1^2)$ with the hadronic parameter $\tilde{B}$ defined in eq. (4.2) is negative in both charm and bottom sectors, whereas the Pauli interference from $\mathcal{T}_{6,int+}^{\mathcal{B}_Q,q_3}$ is constructive as the relevant Wilson coefficient $(N_c c_1^2 + 2c_1 c_2 - \tilde{B} c_2^2)$ is positive. This is not necessarily true for dimension-7 Pauli interference effects to be described below.
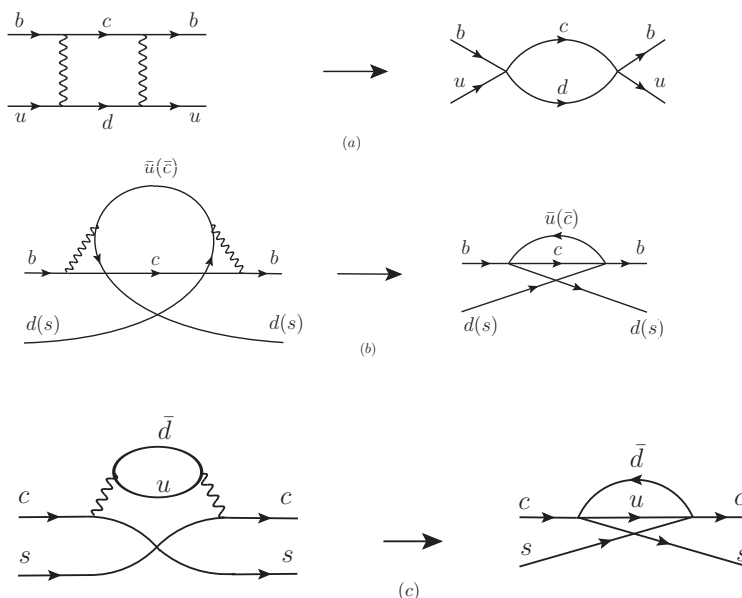
Figure 1. Spectator effects in heavy baryon decays: (a) W-exchange, (b) destructive Pauli interference for $b \to c\bar{u}d$ and $b \to c\bar{c}s$, and (c) constructive Pauli interference for $c \to u\bar{d}s$.

In the heavy meson sector, the W-exchange contribution to the heavy meson corresponds to the Pauli interference term $\mathcal{T}_{6,int-}^{\mathcal{B}_Q}$ in eq. (2.20) in heavy baryon decays, while the Pauli interference in inclusive nonleptonic decays of heavy mesons corresponds to the annihilation term $\mathcal{T}_{6,ann}^{\mathcal{B}_Q}$ in heavy baryon decays. This will be discussed in section 3.

It is clear from eqs. (2.3) and (2.20) that there is a two-body phase-space enhancement factor of $16\pi^2$ for spectator effects relative to the three-body phase space for the heavy quark decay. This implies that spectator effects, being of order $1/m_Q^3$, are comparable to and even exceed the $1/m_Q^2$ terms. Note that the Wilson coefficients and four-quark operators in eq. (2.20) are renormalized at the heavy quark mass scale. Sometimes the so-called hybrid renormalization [6, 33] is performed to evolve the four-quark operators (not the Wilson coefficients!) from m_Q down to a low-energy scale, say, a typical hadronic scale $\mu_{\rm had}$. The evolution from m_Q down to $\mu_{\rm had}$ will in general introduce new structures such as penguin operators. Nevertheless, in the present paper we will follow [12] to employ (2.3) and (2.20) as our starting point for describing inclusive weak decays since it is equivalent to first evaluating the four-quark matrix elements renormalized at the m_Q scale and then relating them to the hadronic matrix elements renormalized at $\mu_{\rm had}$ through the renormalization group equation, provided that the effect of penguin operators is neglected.

For inclusive semileptonic decays, apart from the heavy quark decay contribution there is an additional spectator effect in charmed-baryon semileptonic decay originating from the Pauli interference of the s quark [34]; that is, the s quark produced in $c \to s\ell^+\nu_\ell$ has an interference with the s quark in the wave function of the charmed baryon (see figure 2). It is now ready to deduce this term from $\mathcal{T}_{6,int+}^{q_3}$ in eq. (2.20) by putting $c_1 = 1$, $c_2 = 0$,
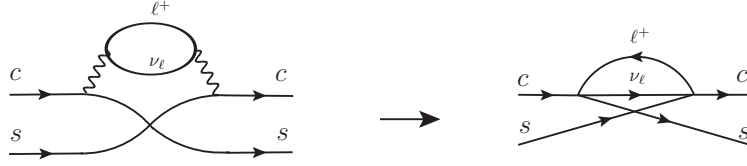
Figure 2. Spectator effect in the charmed-baryon semileptonic decay.

$N_c = 1$ and $q_3 = s$:

$$\Gamma_{6,int}^{\mathrm{SL}}(\mathcal{B}_c) = -\frac{G_F^2 m_c^2}{6\pi}|V_{cs}|^2 \frac{1}{2m_{\mathcal{B}_c}}\langle\mathcal{B}_c|(\bar{c}s)(\bar{s}c) - \bar{c}(1-\gamma_5)s\bar{s}(1+\gamma_5)c|\mathcal{B}_c\rangle. \qquad (2.21)$$

Obviously, this term occurs only in the semileptonic decays of Ξ_c and Ω_c baryons.

2.2 Dimension-7 operators

To the order of $1/m_Q^4$ in the heavy quark expansion in eq. (2.3), we need to consider dimension-7 operators. For our purposes, we shall focus on the $1/m_Q$ corrections to the spectator effects discussed in the last subsection and neglect the operators with gluon fields. As mentioned in the Introduction, the relevant dimension-7 terms are either the four-quark operators times the spectator quark mass or the four-quark operators with an additional derivative [23, 24]. We shall follow [26] to define the following dimension-7 four-quark operators:

$$P_1^q = \frac{m_q}{m_Q}\bar{Q}(1-\gamma_5)q\bar{q}(1-\gamma_5)Q, \qquad\qquad P_2^q = \frac{m_q}{m_Q}\bar{Q}(1+\gamma_5)q\bar{q}(1+\gamma_5)Q, \qquad (2.22)$$

$$P_3^q = \frac{1}{m_Q^2}\bar{Q}\overleftarrow{D}_\rho\,\gamma_\mu(1-\gamma_5)D^\rho q\bar{q}\gamma^\mu(1-\gamma_5)Q, \quad P_4^q = \frac{1}{m_Q^2}\bar{Q}\overleftarrow{D}_\rho\,(1-\gamma_5)D^\rho q\bar{q}(1+\gamma_5)Q,$$

$$P_5^q = \frac{1}{m_Q}\bar{Q}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)(i\!\not{D})Q, \qquad P_6^q = \frac{1}{m_Q}\bar{Q}(1-\gamma_5)q\bar{q}(1+\gamma_5)(i\!\not{D})Q,$$

and the color-octet operators S_i^q $(i=1,\ldots,6)$ obtained from P_i^q by inserting t^a in the two currents of the respective color singlet operators, for example, $S_1^q = \frac{m_q}{m_Q}\bar{Q}(1-\gamma_5)t^a q\bar{q}(1-\gamma_5)t^a Q$.

Following the prescription outlined in [26], one can derive the dimension-7 operators relevant to heavy baryon decays. Explicitly (see appendix A for details),

$$\mathcal{T}_{7,ann}^{\mathcal{B}_Q,q_1} = \frac{G_F^2 m_Q^2}{6\pi}\xi\,(1-x)\bigg\{\left(2N_c c_1 c_2 + c_1^2 + c_2^2\right)\left[2(1+x)P_3^{q_1} + (1-x)P_5^{q_1}\right]$$

$$+6(c_1^2 + c_2^2)\left[2(1+x)S_3^{q_1} + (1-x)S_5^{q_1}\right]\bigg\},$$

$$\mathcal{T}_{7,int}^{\mathcal{B}_Q,q_2} = \frac{G_F^2 m_Q^2}{6\pi}\xi(1-x)\bigg\{\left(\frac{1}{N_c}c_1^2 + 2c_1 c_2 + N_c c_2^2\right)\left[-(1-x)(1+2x)(P_1^{q_2} + P_2^{q_2})\right.$$

$$+2(1+x+x^2)P_3^{q_2} - 12x^2 P_4^{q_2} - (1-x)\left(1+\frac{x}{2}\right)P_5^{q_2} + (1-x)(1+2x)P_6^{q_2}\bigg]$$

$$+2c_1^2\Big[-(1-x)(1+2x)(S_1^{q_2}+S_2^{q_2})+2(1+x+x^2)S_3^{q_2}-12x^2S_4^{q_2}$$

$$-(1-x)\left(1+\frac{x}{2}\right)S_5^{q_2}+(1-x)(1+2x)S_6^{q_2}\Big]\Big\},\tag{2.23}$$

$$\mathcal{T}_{7,int}^{\mathcal{B}_Q,q_3}=\frac{G_F^2m_Q^2}{6\pi}\xi\sqrt{1-4x}\Bigg\{\left(\frac{1}{N_c}c_1^2+2c_1c_2+N_cc_2^2\right)\Big[-(1+2x)(P_1^{q_3}+P_2^{q_3})$$

$$+\frac{2}{1-4x}(1-2x-2x^2)P_3^{q_3}-\frac{24x^2}{1-4x}P_4^{q_3}-(1-x)P_5^{q_3}+(1+2x)P_6^{q_3}\Big]$$

$$+2c_1^2\Big[-(1+2x)(S_1^{q_3}+S_2^{q_3})+\frac{2}{1-4x}(1-2x-2x^2)S_3^{q_3}-\frac{24x^2}{1-4x}S_4^{q_3}$$

$$-(1-x)S_5^{q_3}+(1+2x)S_6^{q_3}\Big]\Bigg\},$$

$$\mathcal{T}_{7,int}^{\mathcal{B}_c,s}=\frac{G_F^2m_c^2}{6\pi}\xi\Bigg\{\left(\frac{1}{N_c}c_2^2+2c_1c_2+N_cc_1^2\right)\Big[-P_1^s-P_2^s+2P_3^s-P_5^s+P_6^s\Big]$$

$$+2c_2^2\Big[-S_1^s-S_2^s+2S_3^s-S_5^s+S_6^s\Big]\Bigg\}.$$

However, we shall see later that in order to evaluate the baryon matrix elements, it is more convenient to express dimension-7 operators in terms of P_i^q and $\tilde{P}_i^q$ ones, where $\tilde{P}_i$ denotes the color-rearranged operator that follows from the expression of P_i by interchanging the color indices of the q_i and $\bar{q}_j$ Dirac spinors. For example, $\tilde{P}_1^q=\frac{m_q}{m_Q}\bar{Q}_i(1-\gamma_5)q_j\bar{q}_i(1-\gamma_5)Q_j$. Using the relation

$$S_i=-\frac{1}{2N_c}P_i+\frac{1}{2}\tilde{P}_i,\tag{2.24}$$

we obtain

$$\mathcal{T}_{7,ann}^{\mathcal{B}_Q,q_1}=\frac{G_F^2m_Q^2}{2\pi}\xi\,(1-x)\Bigg\{2c_1c_2\Big[2(1+x)P_3^{q_1}+(1-x)P_5^{q_1}\Big]$$

$$+(c_1^2+c_2^2)\Big[2(1+x)\tilde{P}_3^{q_1}+(1-x)\tilde{P}_5^{q_1}\Big]\Bigg\},$$

$$\mathcal{T}_{7,int}^{\mathcal{B}_Q,q_2}=\frac{G_F^2m_Q^2}{6\pi}\xi(1-x)\Bigg\{\left(2c_1c_2+N_cc_2^2\right)\Big[-(1-x)(1+2x)(P_1^{q_2}+P_2^{q_2})$$

$$+2(1+x+x^2)P_3^{q_2}-12x^2P_4^{q_2}-(1-x)\left(1+\frac{x}{2}\right)P_5^{q_2}+(1-x)(1+2x)P_6^{q_2}\Big]$$

$$+c_1^2\Big[-(1-x)(1+2x)(\tilde{P}_1^{q_2}+\tilde{P}_2^{q_2})+2(1+x+x^2)\tilde{P}_3^{q_2}-12x^2\tilde{P}_4^{q_2}$$

$$-(1-x)\left(1+\frac{x}{2}\right)\tilde{P}_5^{q_2}+(1-x)(1+2x)\tilde{P}_6^{q_2}\Big]\Bigg\},\tag{2.25}$$

$$\mathcal{T}_{7,int}^{\mathcal{B}_Q,q_3} = \frac{G_F^2 m_Q^2}{6\pi} \xi \sqrt{1-4x} \Bigg\{ \left(2c_1c_2 + N_c c_2^2\right) \Bigg[-(1+2x)(P_1^{q_3} + P_2^{q_3})$$

$$+ \frac{2}{1-4x}(1-2x-2x^2)P_3^{q_3} - \frac{24x^2}{1-4x}P_4^{q_3} - (1-x)P_5^{q_3} + (1+2x)P_6^{q_3} \Bigg]$$

$$+ c_1^2 \Bigg[-(1+2x)(\tilde{P}_1^{q_3} + \tilde{P}_2^{q_3}) + \frac{2}{1-4x}(1-2x-2x^2)\tilde{P}_3^{q_3} - \frac{24x^2}{1-4x}\tilde{P}_4^{q_3}$$

$$- (1-x)\tilde{P}_5^{q_3} + (1+2x)\tilde{P}_6^{q_3} \Bigg] \Bigg\},$$

$$\mathcal{T}_{7,int}^{\mathcal{B}_c,s} = \frac{G_F^2 m_c^2}{6\pi} \xi \Bigg\{ \left(2c_1c_2 + N_c c_1^2\right) \Bigg[-P_1^s - P_2^s + 2P_3^s - P_5^s + P_6^s \Bigg]$$

$$+ c_2^2 \Bigg[-\tilde{P}_1^s - \tilde{P}_2^s + 2\tilde{P}_3^s - \tilde{P}_5^s + \tilde{P}_6^s \Bigg] \Bigg\}.$$

For the dimension-7 operators relevant to heavy meson decays, see the next section.

2.3 Lifetime ratio

In order to compare the HQE predictions with the experimental results, we often consider the lifetime ratio of two heavy hadrons H_1 and H_2, which reads

$$\frac{\tau(H_1)}{\tau(H_2)} = 1 + \frac{\Gamma_2 - \Gamma_1}{\Gamma_1} = 1 + \frac{\mu_\pi^2(H_1) - \mu_\pi^2(H_2)}{2m_Q^2} + \frac{c_{3,Q} + 2c_{5,Q}}{c_{3,Q}} \frac{\mu_G^2(H_1) - \mu_G^2(H_2)}{2m_Q^2}$$

$$+ \frac{c_{6,Q}}{c_{3,Q}} \frac{\langle H_2|T_6|H_2\rangle}{2m_Q^3 m_{H_2}} - \frac{c_{6,Q}}{c_{3,Q}} \frac{\langle H_1|T_6|H_1\rangle}{2m_Q^3 m_{H_1}}$$

$$+ \frac{c_{7,Q}}{c_{3,Q}} \frac{\langle H_2|T_7|H_2\rangle}{2m_Q^4 m_{H_2}} - \frac{c_{7,Q}}{c_{3,Q}} \frac{\langle H_1|T_7|H_1\rangle}{2m_Q^4 m_{H_1}} + \cdots , \quad (2.26)$$

where use of eqs. (2.3) and (2.16) has been made, $c_{3,Q} \equiv c_{3,Q}^{\rm NL} + c_{3,Q}^{\rm SL}$ and likewise for $c_{5,Q}$. Note that the lifetime ratio computed in this manner is valid for B mesons and bottom baryons where the HQE in $1/m_b$ converges nicely, but not for charmed hadrons where the inclusive rates are not dominated by the $c_{3,c}$ term.

3 Lifetimes of heavy mesons

3.1 Lifetimes of bottom mesons

We shall first fix the b quark mass from the measured inclusive semileptonic decay rate. Experimentally [13],

$$\mathcal{B}(B^+ \to X_c e^+ \nu_e) = (10.8 \pm 0.4)\%, \qquad \mathcal{B}(B^0 \to X_c e^+ \nu_e) = (10.1 \pm 0.4)\%,$$

$$\mathcal{B}(B^+/B^0 \text{ admixture} \to X_c e^+ \nu_e) = (10.65 \pm 0.16)\%. \quad (3.1)$$

Theoretically, eq. (2.16) leads to[2]

$$\Gamma(B \to X_c e^+ \nu_e) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left\{ I_0(x,0,0) \left(1 + a\frac{\alpha_s}{\pi}\right) \left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) \right.$$
$$\left. + (I_0(x,0,0) - 4I_1(x,0,0)) \left(1 + b\frac{\alpha_s}{\pi}\right) \frac{\mu_G^2}{2m_b^2} \right\}, \quad (3.2)$$

where we have included the radiative corrections to order α_s characterized by the parameters a and b. The order α_s corrections alone without μ_π^2/m_b^2 or μ_G^2/m_b^2 terms were first calculated in [36–38]. Corrections of order $\alpha_s \mu_\pi^2/m_b^2$ have been calculated in [39, 40], while the $\mathcal{O}(\alpha_s \mu_G^2/m_b^2)$ terms in [41, 42]. The analytic expression of the coefficient a can be found in [38] and the b term in [42].

The inclusive rate is very sensitive to the quark mass m_b. The reliability of the calculation depends on the ability to control the higher order contributions in the double series expansion in α_s and $\Lambda_{\rm QCD}/m_b$. The pole mass definition for heavy quark masses does not converge very well and moreover it is plagued by the renormalon ambiguity [43, 44]. For the short-distance $\overline{\rm MS}$ mass $\bar{m}_b(\bar{m}_b)$, it is not under good control for the smaller scale $\mu \sim 1\,\text{GeV}$. Two different schemes commonly used to define the short-distance b-quark mass are the kinetic [45, 46] and the 1S [47] schemes. We follow [30] for a recent global fit of inclusive semileptonic B decays in the kinetic scheme. This analysis includes higher power corrections $\mathcal{O}(1/m_b^4)$ and $\mathcal{O}(m_b^5)$ and next-to-leading order QCD (NLO-QCD) corrections $\mathcal{O}(\alpha_s^2)$. In this scheme, it is conventional to constrain the charm quark mass to be the $\overline{\rm MS}$ one $\bar{m}_c(3\,\text{GeV}) = 0.987 \pm 0.013\,\text{GeV}$ which yields a better convergence of the perturbative series. The results of the fit are [30]:

$$m_b^{\rm kin}(1\,\text{GeV}) = 4.546 \pm 0.021\,\text{GeV}, \quad \mu_\pi^2 = 0.432 \pm 0.068\,\text{GeV}^2, \quad \mu_G^2 = 0.355 \pm 0.060\,\text{GeV}^2. \quad (3.3)$$

The corresponding $\overline{\rm MS}$ mass $\bar{m}_b(\bar{m}_b)$ for $m_b^{\rm kin}(1\,\text{GeV})$ is close to the usual one.

To the leading-order QCD (LO-QCD), the definition of the quark mass is very arbitrary. If everything is calculated consistently to NLO-QCD, the dependence of the final result on the quark mass definition will be considerably weak when the relations between different quark mass schemes are used consistently at the NLO accuracy.[3] Although dimension 3, 4 and 6 Wilson coefficients up to NLO-QCD are available for heavy B and D mesons, they are still absent for heavy baryons. Dimension-7 Wilson coefficients are known only to the LO level for both heavy mesons and baryons. For this reason, in this work we shall focus on the LO-QCD study. In the bottom hadron sector, we use the quark masses $m_b = 4.546\,\text{GeV}$ and $m_c = 0.987\,\text{GeV}$ obtained in eq. (3.3). The reason is that the calculated inclusive semileptonic rate to LO, $\Gamma(B \to X_c e^+ \nu_e) = 4.59 \times 10^{-14}\,\text{GeV}$ using the kinetic b quark mass is very close to the experimental measurement:

$$\Gamma(B^+/B^0\,\text{admixture} \to X_c e^+ \nu_e) = (4.476 \pm 0.067) \times 10^{-14}\text{GeV}, \quad (3.4)$$

---

[2]Corrections to inclusive semileptonic B decays have been calculated to order $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(1/m_b^5)$ [35].

[3]Besides the above-mentioned inclusive semileptonic B decays, another example is given in [48] for inclusive nonleptonic decay rates to NLO which are calculated in various quark mass schemes. The numerical results are similar for different short-distance quark masses.

where the average lifetime $\tau(B^+/B^0/B_s/b-\text{baryon admixture}) = (1.566\pm0.003)$ ps [13] and branching fraction (3.1) have been made. If the running quark masses $\bar{m}_b(\bar{m}_b) = 4.248$ GeV and $\bar{m}_c(\bar{m}_c) = 1.277$ GeV are employed, the obtained $\Gamma(B \to X_c e^+ \nu_e)$ to LO will be too small by 47% compared to experiment. It should be stressed that to the NLO-QCD, the dependence of the inclusive semileptonic rate on the quark mass definition is considerably weak.

We next turn to the spectator effects of order $1/m_b^3$. The W-exchange contributions to B_d and B_s correspond to the Pauli interference terms $\mathcal{T}_{6,int}^{\mathcal{B}_Q,q_2}$ and $\mathcal{T}_{6,int}^{\mathcal{B}_Q,q_3}$, respectively, in eq. (2.20) for heavy baryon decays, while the Pauli interference in inclusive nonleptonic B^- decay corresponds to the annihilation term $\mathcal{T}_{6,ann}^{\mathcal{B}_Q,q_1}$ in heavy baryon decays:

$$
\mathcal{T}_{6,ann}^{B_d} = -\frac{G_F^2 m_b^2}{6\pi} \xi (1-x)^2 \Bigg\{ \left(\frac{1}{N_c}c_1^2 + 2c_1 c_2 + N_c c_2^2 \right) \left[\left(1 + \frac{x}{2} \right) (\bar{b}d)(\bar{d}b) \right.
$$
$$
\left. - (1+2x)\bar{b}(1-\gamma_5)d\bar{d}(1+\gamma_5)b \right] + 2c_1^2 \left[\left(1 + \frac{x}{2} \right) (\bar{b}t^a b)(\bar{d}t^a d) \right.
$$
$$
\left. - (1+2x)\bar{b}(1-\gamma_5)t^a d\bar{d}(1+\gamma_5)t^a b \right] \Bigg\},
$$

$$
\mathcal{T}_{6,ann}^{B_s} = -\frac{G_F^2 m_b^2}{6\pi} \xi \sqrt{1-4x} \Bigg\{ \left(\frac{1}{N_c}c_1^2 + 2c_1 c_2 + N_c c_2^2 \right) \left[(1-x)(\bar{b}s)(\bar{s}b) \right.
$$
$$
\left. - (1+2x)\bar{b}(1-\gamma_5)s\bar{s}(1+\gamma_5)b \right] + 2c_1^2 \left[(1-x)(\bar{b}t^a b)(\bar{s}t^a s) \right.
$$
$$
\left. - (1+2x)\bar{b}(1-\gamma_5)t^a s\bar{s}(1+\gamma_5)t^a b \right] \Bigg\}, \tag{3.5}
$$

$$
\mathcal{T}_{6,int}^{B_u} = \frac{G_F^2 m_b^2}{6\pi} \xi (1-x)^2 \Bigg\{ \left(2N_c c_1 c_2 + c_1^2 + c_2^2 \right) (\bar{b}u)(\bar{u}b)
$$
$$
+ 2N_c(c_1^2 + c_2^2)(\bar{b}t^a u)(\bar{u}t^a b) \Bigg\},
$$

where $(\bar{q}_1 t^a q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) t^a q_2$ with $t^a = \lambda^a/2$ and we have applied the relation

$$
(\bar{Q}_\alpha t_{\alpha\beta}^a q_\beta)(\bar{q}_\rho t_{\rho\sigma}^a Q_\sigma) = \frac{1}{2}(\bar{Q}Q)(\bar{q}q) - \frac{1}{2N_c}(\bar{Q}q)(\bar{q}Q) \tag{3.6}
$$

to the transition operators so that they are more suitable for the matrix element evaluation in the meson case. Likewise, dimension-7 operators relevant for heavy meson decays can be read from eq. (2.23):

$$
\mathcal{T}_{7,ann}^{B_d} = \frac{G_F^2 m_b^2}{6\pi} \xi (1-x) \Bigg\{ \left(\frac{1}{N_c}c_1^2 + 2c_1 c_2 + N_c c_2^2 \right) \left[-(1-x)(1+2x)(P_1^d + P_2^d) \right.
$$
$$
\left. + 2(1+x+x^2)P_3^d - 12x^2 P_4^d - (1-x)\left(1 + \frac{x}{2}\right)P_5^d + (1-x)(1+2x)P_6^d \right]
$$

$$+2c_1^2\left[-(1-x)(1+2x)(S_1^d+S_2^d)+2(1+x+x^2)S_3^d-12x^2S_4^d\right.$$

$$\left.-(1-x)\left(1+\frac{x}{2}\right)S_5^d+(1-x)(1+2x)S_6^d\right]\Bigg\}, \tag{3.7}$$

$$\mathcal{T}_{7,ann}^{B_s}=\frac{G_F^2m_Q^2}{6\pi}\,\xi\sqrt{1-4x}\Bigg\{\left(\frac{1}{N_c}c_1^2+2c_1c_2+N_cc_2^2\right)\left[-(1+2x)(P_1^s+P_2^s)\right.$$

$$\left.+\frac{2}{1-4x}(1-2x-2x^2)P_3^s-\frac{24x^2}{1-4x}P_4^s-(1-x)P_5^s+(1+2x)P_6^s\right]$$

$$+2c_1^2\left[-(1+2x)(S_1^s+S_2^s)+\frac{2}{1-4x}(1-2x-2x^2)S_3^s-\frac{24x^2}{1-4x}S_4^s\right.$$

$$\left.-(1-x)S_5^s+(1+2x)S_6^s\right]\Bigg\},$$

$$\mathcal{T}_{7,int}^{B_u}=\frac{G_F^2m_b^2}{6\pi}\,\xi\,(1-x)\Bigg\{\left(2N_cc_1c_2+c_1^2+c_2^2\right)\left[2(1+x)P_3^u+(1-x)P_5^u\right]$$

$$+6(c_1^2+c_2^2)\left[2(1+x)S_3^u+(1-x)S_5^u\right]\Bigg\}.$$

For the meson matrix elements of four-quark operators, we follow [12] to define the bag parameters B_i and ε_i to parametrize the hadronic matrix elements in a model-independent way:

$$\langle B_q|(\bar{b}q)(\bar{q}b)|B_q\rangle = f_{B_q}^2m_{B_q}^2B_1,$$
$$\langle B_q|\bar{b}(1-\gamma_5)q\bar{q}(1+\gamma_5)b|B_q\rangle = f_{B_q}^2m_{B_q}^2B_2,$$
$$\langle B_q|(\bar{b}\,t^aq)(\bar{q}\,t^ab)|B_q\rangle = f_{B_q}^2m_{B_q}^2\varepsilon_1, \tag{3.8}$$
$$\langle B_q|b\,t^a(1-\gamma_5)q\bar{q}\,t^a(1+\gamma_5)b|B_q\rangle = f_{B_q}^2m_{B_q}^2\varepsilon_2.$$

Under the vacuum-insertion approximation, bag parameters are given by $B_i=1$ and $\varepsilon_i=0$, but they will be treated as free parameters here. In the large-N_c limit, it is expected that $B_i\sim\mathcal{O}(1)$ and $\varepsilon_i\sim\mathcal{O}(1/N_c)$. Likewise, the matrix elements of dimension-7 four-quark operators read [26]

$$\langle B_q|P_i^q|B_q\rangle = -\frac{m_q}{m_b}f_B^2m_B^2\rho_i^q, \qquad\qquad i=1,2,$$

$$\langle B_q|P_i^q|B_q\rangle = (-1)^if_B^2m_B^2\frac{1}{2}\left(\frac{m_B^2}{m_b^2}-1\right)\rho_i^q, \qquad i=3,4, \tag{3.9}$$

$$\langle B_q|P_i^q|B_q\rangle = (-1)^if_B^2m_B^2\frac{1}{2}\left(\frac{m_B^2}{m_b^2}-1\right)\rho_i^q, \qquad i=5,6,$$

and similar parametrization for the color-octet operators with the replacement of $P\to S$ and $\rho_i\to\sigma_i$. Under the vacuum-insertion approximation, $\rho_i^q=1$ and all σ's vanish.

Applying eqs. (3.8) and (3.9) to evaluate the B-meson matrix elements of dimension-6 and dimension-7 four-quark operators, (3.5) and (3.7), respectively, the spectator effects

$$\Gamma^{\rm spec}(B_q)=\frac{\langle B_q|\mathcal{T}_6+\mathcal{T}_7|B_q\rangle}{2m_{B_q}} \tag{3.10}$$

have the expressions

$$\Gamma^{\text{ann}}(B_d) = -\frac{G_F^2 m_b^2}{\pi}|V_{cb}V_{ud}|^2|\psi_{b\bar{q}}^B(0)|^2(1-x)^2\Bigg\{\left(\frac{1}{N_c}c_1^2+2c_1c_2+N_cc_2^2\right)\bigg[\left(1+\frac{x}{2}\right)B_1$$

$$-(1+2x)B_2+\left(\frac{1+x+x^2}{1-x}\rho_3^d+\frac{6x^2}{1-x}\rho_4^d-\frac{1}{2}\left(1+\frac{x}{2}\right)\rho_5^d-\frac{1}{2}(1+2x)\rho_6^d\right)$$

$$\times\left(\frac{m_B^2}{m_b^2}-1\right)\bigg]+2c_1^2\bigg[\left(1+\frac{x}{2}\right)\varepsilon_1-(1+2x)\varepsilon_2+\left(\frac{1+x+x^2}{1-x}\sigma_3^d\right.$$

$$+\frac{6x^2}{1-x}\sigma_4^d-\frac{1}{2}\left(1+\frac{x}{2}\right)\sigma_5^d-\frac{1}{2}(1+2x)\sigma_6^d\bigg)\left(\frac{m_B^2}{m_b^2}-1\right)\bigg]\Bigg\},$$

$$\Gamma^{\text{ann}}(B_s) = -\frac{G_F^2 m_b^2}{\pi}|V_{cb}V_{cs}|^2|\psi_{b\bar{s}}^{B_s}(0)|^2\sqrt{1-4x}\Bigg\{\left(\frac{1}{N_c}c_1^2+2c_1c_2+N_cc_2^2\right)\bigg[(1-x)B_1 \quad (3.11)$$

$$-(1+2x)B_2+\left(\frac{1-2x-2x^2}{1-4x}\rho_3^s+\frac{12x^2}{1-4x}\rho_4^s-\frac{1-x}{2}\rho_5^s-\frac{1+2x}{2}\rho_6^s\right)$$

$$\times\left(\frac{m_{B_s}^2}{m_b^2}-1\right)\bigg]+2c_1^2\bigg[(1-x)\varepsilon_1-(1+2x)\varepsilon_2+\left(\frac{1-2x-2x^2}{1-4x}\sigma_3^s\right.$$

$$+\frac{12x^2}{1-4x}\sigma_4^s-\frac{1-x}{2}\sigma_5^s-\frac{1+2x}{2}\sigma_6^s\bigg)\left(\frac{m_{B_s}^2}{m_b^2}-1\right)\bigg]\Bigg\},$$

$$\Gamma^{\text{int}}(B_u) = \frac{G_F^2 m_b^2}{\pi}|V_{cb}V_{ud}|^2|\psi_{b\bar{q}}^B(0)|^2(1-x)^2\Bigg\{\left(2N_cc_1c_2+c_1^2+c_2^2\right)\bigg[B_1-\left(\frac{1+x}{1-x}\rho_3^u+\frac{1}{2}\rho_5^u\right)$$

$$\times\left(\frac{m_B^2}{m_b^2}-1\right)\bigg]+6(c_1^2+c_2^2)\bigg[\varepsilon_1-\left(\frac{1+x}{1-x}\sigma_3^u+\frac{1}{2}\sigma_5^u\right)\left(\frac{m_B^2}{m_b^2}-1\right)\bigg]\Bigg\},$$

where $|\psi_{b\bar{q}}^B(0)|^2 = \frac{1}{12}f_B^2 m_B$ is the B meson wave function at the origin squared. Since $(m_B^2/m_b^2 - 1) \sim \mathcal{O}(1/m_b)$, it is evident that contributions from dimension-7 operators are suppressed by Λ/m_b relative to the dimension-6 ones. As stressed in [12], the coefficients of B_i in $\Gamma^{\text{ann}}(B_d)$ are one to two orders of magnitude smaller than that of ε_i. Therefore, contributions of B_i can be safely neglected at least in $\Gamma^{\text{ann}}(B_d)$. There exist several estimates of the bag parameters B_i and ε_i based on sum rules [49–52] and lattice QCD [53, 54]. On the basis of HQET sum rules,[4] these parameters have been updated recently to be [22]

$$B_1 = 1.028^{+0.064}_{-0.056}, \quad B_2 = 0.988^{+0.087}_{-0.079}, \quad \varepsilon_1 = -0.107^{+0.028}_{-0.029}, \quad \varepsilon_2 = -0.033^{+0.021}_{-0.021}, \quad (3.12)$$

evaluated at the $\mu = \bar{m}_b(\bar{m}_b)$ scale. For the parameters ρ_i^q and σ_i^q we shall use the vacuum-insertion estimates, namely $\rho_i^q = 1$ and $\sigma_i^q = 0$.

To compute the nonleptonic decay rate we apply the Wilson coefficient functions

$$c_1(\mu) = 1.14, \qquad c_2(\mu) = -0.31, \tag{3.13}$$

---

[4]The HQET sum rule calculation in the literature relies on the work of [55] where the necessary three-loop HQET diagrams have been computed and on the work of [56] where these results have been first used for an estimate of the bag parameter.

	Γ^{dec}	Γ^{ann}	Γ_-^{int}	Γ^{semi}	Γ^{tot}	$\tau(10^{-12}s)$	$\tau_{\text{expt}}(10^{-12}s)$
B^+	3.102	0	-0.267	1.000	3.834	1.717	1.638 ± 0.004
B_d^0	3.102	0.039	0	1.000	4.141	1.590	1.520 ± 0.004
B_s^0	3.102	0.053	0	1.000	4.155	1.584	1.510 ± 0.005

Table 1. Various contributions to the decay rates (in units of $10^{-13}\,\text{GeV}$) of B mesons. Experimental values are taken from [13].

which are evaluated at $\mu = 4.4\,\text{GeV}$ to the leading logarithmic approximation (see table XIII of [57]). The total rate reads

$$\Gamma = \Gamma^{\text{dec}} + \Gamma^{\text{ann}} + \Gamma^{\text{int}} + \Gamma^{\text{semi}}, \tag{3.14}$$

where the decay rate of the heavy quark b of the B meson is given by

$$\Gamma^{\text{dec}}(B) = \frac{G_F^2 m_b^5}{192\pi^3} \xi \left\{ c_{3,b}^{\text{NL}} \left[1 - \frac{\lambda_1}{2m_b^2} + \frac{d_B \lambda_2}{2m_b^2} \right] + 2c_{5,b}^{\text{NL}} \frac{d_B \lambda_2}{m_b^2} \right\} \tag{3.15}$$

with $d_B = 3$. Now we make a comparison with [8] on the b quark lifetime. From eqs. (2.12) and (2.17) we obtain $c_{3,b} = c_{3,b}^{\text{NL}} + c_{3,b}^{\text{SL}} = 5.61$ to LO-QCD level for $m_b = 4.546\,\text{GeV}$ and $m_c = 0.987\,\text{GeV}$, while Lenz got $c_{3,b} = 5.29 \pm 0.35$ to LO-QCD for $\bar{m}_b(\bar{m}_b) = 4.248\,\text{GeV}$, $\bar{m}_c(\bar{m}_b) = 0.997\,\text{GeV}$ and 6.88 ± 0.74 with NLO-QCD corrections. We have the lifetime of a free b quark $\tau_b = 1.51$ ps, while Lenz obtained $\tau_b = (1.65 \pm 0.24)$ ps [8], where

$$\Gamma_b = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 c_{3,b}. \tag{3.16}$$

The calculated lifetimes of B mesons shown in table 1 are longer than the free b quark lifetime for two reasons: (i) $1/m_b^2$ effects characterized by λ_1 and λ_2 will suppress the nonleptonic rate slightly, and (ii) inclusive semileptonic rate is slightly suppressed by QCD corrections, the a and b terms in eq. (3.2).

Eq. (3.11) implies a constructive W-exchange to B_d and B_s and a destructive Pauli interference to B_u. From eq. (2.26) we obtain model-independent expressions for the lifetime ratios

$$\frac{\tau(B^+)}{\tau(B_d^0)} = 1 + (0.037B_1 + 0.0008B_2 - 0.57\epsilon_1 + 0.15\epsilon_2)_{\text{dim}-6} + (-0.015\rho_3 - 0.0064\rho_5$$
$$- 0.00014\rho_6 + 0.11\sigma_3 - 0.0007\sigma_4 + 0.099\sigma_5 + 0.026\sigma_6)_{\text{dim}-7}, \tag{3.17}$$
$$\frac{\tau(B_s^0)}{\tau(B_d^0)} = 1 + (0.0003B_1 - 0.0005B_2 + 0.060\epsilon_1 - 0.079\epsilon_2)_{\text{dim}-6} + (0.0002\rho_3 - 0.0001\rho_5$$
$$- 0.0001\rho_6 + 0.039\sigma_3 + 0.002\sigma_4 - 0.015\sigma_5 - 0.019\sigma_6)_{\text{dim}-7},$$

where we have decomposed the lifetime ratios in terms of dimension-6 and dimension-7 contributions and dropped the superscripts of ρ_i^q and σ_i^q by assuming their flavor independence

for simplicity. Using eq. (3.12) for dimension-6 bag parameters and the vacuum-insertion approximation for dimension-7 ρ_i^q and σ_i^q,[5] we obtain

$$\left.\frac{\tau(B^+)}{\tau(B_d^0)}\right|_{\text{theo}} = 1.074^{+0.017}_{-0.016}, \qquad \left.\frac{\tau(B^+)}{\tau(B_d^0)}\right|_{\text{expt}} = 1.076 \pm 0.004,$$

$$\left.\frac{\tau(B_s^0)}{\tau(B_d^0)}\right|_{\text{theo}} = 0.9964 \pm 0.0024, \qquad \left.\frac{\tau(B_s^0)}{\tau(B_d^0)}\right|_{\text{expt}} = 0.994 \pm 0.004, \qquad (3.18)$$

to be compared with $\left.\frac{\tau(B^+)}{\tau(B_d)}\right|_{\text{theo}} = 1.082^{+0.022}_{-0.026}$ and $\left.\frac{\tau(B_s)}{\tau(B_d)}\right|_{\text{theo}} = 0.9994 \pm 0.0025$ found in [22]. The theoretical uncertainties in (3.18) arise mainly from the bag parameters given in eq. (3.12). Our results are in excellent agreement with experiment. If we apply naive vacuum-insertion approximation also to dimension-6 bag parameters, we will have

$$\left.\frac{\tau(B^+)}{\tau(B_d^0)}\right|_{\text{VIA}} = 1.016, \qquad \left.\frac{\tau(B_s^0)}{\tau(B_d^0)}\right|_{\text{VIA}} = 1.000. \qquad (3.19)$$

This implies that the main contribution to the $B_u - B_d$ lifetime ratio arises from the color-octet terms $-0.57\epsilon_1 + 0.15\epsilon_2 + \cdots$ in eq. (3.17). The predicted ϵ_1 and ϵ_2 from HQET sum rules given in eq. (3.12) yield an excellent description of lifetime ratios. Note that $\epsilon_1 \approx 3\epsilon_2$ here rather than $\epsilon_1 \approx 0.3\epsilon_2$ as originally argued in [12].

Several remarks are in order. (i) Weak annihilation contributions to B_d and B_s are suppressed relative to the Pauli interference due to a large cancelation between the bag parameters B_1 and B_2 and the partial cancelation between ϵ_1 and ϵ_2, see eq. (3.11) and table 1. (ii) The annihilation contribution to B_s is larger than that of B_d owing to SU(3) breaking in the decay constants and masses. This explains why the lifetime B_s is slightly shorter than B_d. (iii) To order $1/m_c^3$, we obtain $\tau(B^+)/\tau(B_d^0) = 1.0945$, which can be checked from eq. (3.17). Hence, it is necessary to introduce dimension-7 operators in order to improve the agreement with experiment.

3.2 Lifetimes of charmed mesons

The semileptonic inclusive decay $D \to X_s e^+ \nu_e$, the analog of $B \to X_c e^+ \nu_e$, has not been measured. Instead, what we have are [13]

$$\mathcal{B}(D^+ \to X e^+ \nu_e) = (16.07 \pm 0.30)\%, \qquad \mathcal{B}(D^0 \to X e^+ \nu_e) = (6.49 \pm 0.11)\%,$$
$$\mathcal{B}(D_s^0 \to X e^+ \nu_e) = (6.5 \pm 0.4)\%. \qquad (3.20)$$

We begin with the inclusive semileptonic decay rate of the D meson given by eq. (2.16)

$$\Gamma(D \to X e^+ \nu_e) = \frac{G_F^2 m_c^5}{192\pi^3} \left(|V_{cs}|^2 \eta(x,0,0) \left\{ c_{3,c}^{\text{SL}}(x,0) \left[1 + \frac{\lambda_1 + d_D \lambda_2}{2m_c^2} \right] + 2c_{5,c}^{\text{SL}}(x,0) \frac{d_D \lambda_2}{m_c^2} \right\} \right.$$

$$\left. + |V_{cd}|^2 \eta(0,0,0) \left\{ c_{3,c}^{\text{SL}}(0,0) \left[1 + \frac{\lambda_1 + d_D \lambda_2}{2m_c^2} \right] + 2c_{5,c}^{\text{SL}}(0,0) \frac{d_D \lambda_2}{m_c^2} \right\} \right), \quad (3.21)$$

---

[5]We have followed [22] to assign fixed uncertainties to both ρ_i and σ_i, namely $\rho_i = 1 \pm 1/2$ and $\sigma_i = 0 \pm 1/6$.

where $x = (m_s/m_c)^2$, $d_D = 3$ and the Wilson coefficients $c_{3,c}^{\text{SL}}$ and $c_{5,c}^{\text{SL}}$ are given by eq. (2.18). In the above equation we have included the radiative correction $\eta(x, 0, 0)$ given by the $(1 + a\alpha_s/\pi)$ term in eq. (3.2). We find that the experimental values for D^+ and D^0 semileptonic widths can be fitted by the charm quark mass $m_c = 1.56\,\text{GeV}$.[6] For the Wilson coefficients, we shall use the lowest order values

$$c_1(\mu) = 1.346, \qquad c_2(\mu) = -0.636 \tag{3.22}$$

evaluated at the scale $\mu = 1.25\,\text{GeV}$ with $\Lambda_{\overline{\text{MS}}}^{(4)} = 325\,\text{MeV}$ (see tables VI and VII of [57]).

Just as the B meson case, the spectator effects in the D meson sector read

$$
\begin{aligned}
\Gamma^{\text{ann}}(D^0) = {} & -\frac{G_F^2 m_c^2}{12\pi}|V_{cs}V_{ud}|^2 f_D^2 m_D (1-x)^2 \Bigg\{ \left(\frac{1}{N_c}c_1^2 + 2c_1 c_2 + N_c c_2^2\right)\left[\left(1 + \frac{x}{2}\right)B_1 \right. \\
& -(1+2x)B_2 + \left(\frac{1+x+x^2}{1-x}\rho_3^u + \frac{6x^2}{1-x}\rho_4^u - \frac{1}{2}\left(1 + \frac{x}{2}\right)\rho_5^u - \frac{1}{2}(1+2x)\rho_6^u\right) \\
& \times \left.\left(\frac{m_D^2}{m_c^2} - 1\right)\right] + 2c_1^2\left[\left(1 + \frac{x}{2}\right)\varepsilon_1 - (1+2x)\varepsilon_2 + \left(\frac{1+x+x^2}{1-x}\sigma_3^u\right.\right. \\
& + \left.\left.\frac{6x^2}{1-x}\sigma_4^u - \frac{1}{2}\left(1 + \frac{x}{2}\right)\sigma_5^u - \frac{1}{2}(1+2x)\sigma_6^u\right)\left(\frac{m_D^2}{m_c^2} - 1\right)\right]\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^{\text{int}}(D^+) = {} & \frac{G_F^2 m_c^2}{12\pi}|V_{cs}V_{ud}|^2 f_D^2 m_D (1-x)^2 \Bigg\{ (2N_c c_1 c_2 + c_1^2 + c_2^2)\left[B_1 - \left(\frac{1+x}{1-x}\rho_3^d + \frac{1}{2}\rho_5^d\right)\right.\\
& \times \left.\left(\frac{m_D^2}{m_c^2} - 1\right)\right] + 6(c_1^2 + c_2^2)\left[\varepsilon_1 - \left(\frac{1+x}{1-x}\sigma_3^d + \frac{1}{2}\sigma_5^d\right)\left(\frac{m_D^2}{m_c^2} - 1\right)\right]\Bigg\}, \tag{3.23}
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^{\text{ann}}(D_s^+) = {} & -\frac{G_F^2 m_c^2}{12\pi}|V_{cs}|^2 f_{D_s}^2 m_{D_s} \Bigg\{ \left(\frac{1}{N_c}c_2^2 + 2c_1 c_2 + N_c c_1^2\right)|V_{ud}|^2\left[B_1 - B_2 - \frac{1}{2}(\rho_1^s + \rho_2^s)\frac{m_s}{m_c}\right.\\
& + \left.\left(\rho_3^s - \frac{1}{2}(\rho_5^s + \rho_6^s)\right)\left(\frac{m_{D_s}^2}{m_c^2} - 1\right)\right] + 2c_2^2|V_{ud}|^2\left[\varepsilon_1 - \varepsilon_2 - \frac{1}{2}(\sigma_1^s + \sigma_2^s)\frac{m_s}{m_c}\right.\\
& + \left.\left(\sigma_3^s - \frac{1}{2}(\sigma_5^s + \sigma_6^s)\right)\left(\frac{m_{D_s}^2}{m_c^2} - 1\right)\right] + \left[1 + (1-z)^2\left(1 + \frac{z}{2}\right)\right]B_1\\
& - \left[1 + (1-z)^2(1+2z)\right]B_2 - \left[1 + (1-z)^2(1+2z)\right](\rho_1^s + \rho_2^s)\frac{m_s}{m_c}\\
& + \left([1 + (1-z)(1+z+z^2)]\rho_3^s + 6z^2(1-z)\rho_4^s\right)\left(\frac{m_{D_s}^2}{m_c^2} - 1\right)\Bigg\},
\end{aligned}
$$

where $z = m_\mu^2/m_c^2$. We have followed [26] to derive the expression for the inclusive rate of D_s^+. Note that the contributions involving the z terms arise from the leptonic intermediate states.

---

[6]The semileptonic widths of D^+ and D^0 are very similar, while the D_s^+ one is smaller by 15%.

It is well known that D^+ has a longer lifetime than D^0 because of destructive Pauli interference [5, 59, 60]. To a good approximation to $1/m_c^3$ expansion, we have

$$\Gamma(D^+) \approx \frac{G_F^2 m_c^5}{192\pi^3}\left[3(c_1^2 + c_2^2) + 2c_1 c_2\right] + \frac{G_F^2 m_c^2}{12\pi}(c_1^2 + c_2^2 + 6c_1 c_2)f_D^2 m_D + \Gamma^{\text{semi}},$$

$$\Gamma(D^0) \approx \frac{G_F^2 m_c^5}{192\pi^3}\left[3(c_1^2 + c_2^2) + 2c_1 c_2\right] + \Gamma^{\text{semi}}, \tag{3.24}$$

where $\Gamma^{\text{semi}} \approx G_F^2 m_c^5/(96\pi^3)$. For the decay constant f_D of order $205\,\text{MeV}$ (see [58] for a review), it is easily seen that the Pauli interference $\Gamma^{\text{int}}(D^+)$ to order $1/m_c^3$ overcomes the c quark decay rate so that $\Gamma(D^+)$ becomes negative no matter which charmed quark mass is employed, the $\overline{\text{MS}}$ mass $\bar{m}_c(\bar{m}_c) = 1.279\,\text{GeV}$ or the fit mass $m_c = 1.56\,\text{GeV}$. This remains to be true even if other sets of the bag parameters are used so long as they are not far from the vacuum insertion expectation.

In the literature, the lifetime ratio is often computed using the relation[7]

$$R_D \equiv \frac{\tau(D^+)}{\tau(D^0)} = 1 + [\Gamma(D^0) - \Gamma(D^+)]\tau(D^+)_{\text{expt}}, \tag{3.25}$$

where the experimental value of $\tau(D^+)$ is utilized on the r.h.s. of the above equation. However, it is important to keep in mind that the calculated D^+ lifetime is negative to order $1/m_c^3$. Hence, it does not make sense to apply the HQE to $\mathcal{O}(1/m_c^3)$ to predict a "positive" lifetime ratio $\tau(D^+)/\tau(D^0)$ in spite of a negative D^+ lifetime predicted by the HQE at this level. Therefore, we should also consider the ratio

$$R_D' \equiv \frac{\tau(D^0)}{\tau(D^+)} = 1 + [\Gamma(D^+) - \Gamma(D^0)]\tau(D^0)_{\text{expt}} \tag{3.26}$$

to ensure that $R_D' = 1/R_D$. Their experimental values are given by [13]

$$\frac{\tau(D^+)}{\tau(D^0)}\bigg|_{\text{expt}} = 2.536 \pm 0.019, \qquad \frac{\tau(D^0)}{\tau(D^+)}\bigg|_{\text{expt}} = 0.394 \pm 0.003\,. \tag{3.27}$$

It follows from eqs. (3.23), (3.25) and (3.26) that

$$R_D = 1 + (2.88B_1 + 0.11B_2 - 17.25\epsilon_1 + 3.71\epsilon_2)_{\text{dim}-6}$$
$$+ (-1.34\rho_3 - 0.62\rho_5 + 0.024\rho_6 + 4.25\sigma_3 + 3.70\sigma_5 + 0.80\sigma_6)_{\text{dim}-7}, \tag{3.28}$$
$$R_D' = 1 + (-1.13B_1 - 0.043B_2 + 6.80\epsilon_1 - 1.46\epsilon_2)_{\text{dim}-6}$$
$$+ (0.53\rho_3 + 0.24\rho_5 - 0.0093\rho_6 - 1.68\sigma_3 + 1.46\sigma_5 - 0.31\sigma_6)_{\text{dim}-7}.$$

We can use the experimental values of R_D and R_D' to constrain the bag parameters. To order $1/m_c^3$, we find $R_D = 3.98$ and $R_D' = -0.18$ for $B_i = 1$ and $\epsilon_i = 0$. This implies a negative D^+ lifetime. Indeed, the calculated $\tau(D^+)$ is $-8.4 \times 10^{-13}s$. The subleading $1/m_c$

---

[7]It is not meaningful to apply eq. (2.26) to compute the lifetime ratio R_D because (i) the $\Gamma(D^+)$ rate is not dominated by the c_3 term, and (ii) when eq. (2.26) is applied to the ratio R_D', it will lead to $R_D' = 2 - R_D$ which is negative.

corrections to the Pauli interference term, namely, Γ_7^{int} obtained from dimension-7 four-quark operators at $1/m_c^4$ level, contributes constructively to the D^+ width (see eq. (3.23)). Hence, it is conceivable that the $1/m_c$ corrections to the Pauli interference will be able to render $\Gamma(D^+)$ positive in certain rages of the bag parameters. With $m_c = 1.56\,\text{GeV}$, we find $R_D = 2.06$ and $R'_D = 0.58$ in the presence of $1/m_c^4$ corrections with $\rho_i = 1$ and $\sigma_i = 0$.

For the D_s^+ meson, we follow [26] to define a substracted D_s^+ lifetime by

$$\bar{\tau}(D_s^+) = \frac{\tau(D_s^+)}{1 - \mathcal{B}(D_s^+ \to \tau^+ \nu_\tau)} = (0.533 \pm 0.004) \times 10^{-12} s \tag{3.29}$$

as the decay $D_s^+ \to \tau^+ \nu_\tau$ cannot be properly described by HQE due to its small energy release. In analogue to the D^+ meson, we consider the lifetime ratios

$$R_{D_s} \equiv \frac{\bar{\tau}(D_s^+)}{\tau(D^0)} = 1 + [\Gamma(D^0) - \bar{\Gamma}(D_s^+)]\bar{\tau}(D_s^+)_{\text{expt}},$$

$$R'_{D_s} \equiv \frac{\tau(D^0)}{\bar{\tau}(D_s^+)} = 1 + [\bar{\Gamma}(D_s^+) - \Gamma(D^0)]\tau(D^0)_{\text{expt}}, \tag{3.30}$$

with the experimental values [13]

$$R_{D_s} = 1.30 \pm 0.01, \qquad R'_{D_s} = 0.77 \pm 0.01. \tag{3.31}$$

We obtain

$$\begin{aligned}
R_{D_s} &= 1 + (4.81 B_1 - 4.82 B_2 - 1.21 \epsilon_1 + 1.22 \epsilon_2)_{\text{dim}-6} + (-0.17 \rho_1 - 0.17 \rho_2 + 2.86 \rho_3 \\
&\quad - 0.94 \rho_5 - 0.94 \rho_6 - 0.017 \sigma_1 - 0.017 \sigma_2 - 0.41 \sigma_3 + 0.21 \sigma_5 + 0.21 \sigma_6)_{\text{dim}-7}, \\
R'_{D_s} &= 1 + (-3.70 B_1 + 3.71 B_2 + 0.93 \epsilon_1 - 0.94 \epsilon_2)_{\text{dim}-6} + (0.13 \rho_1 + 0.13 \rho_2 - 2.20 \rho_3 \\
&\quad + 0.72 \rho_5 + 0.72 \rho_6 + 0.013 \sigma_1 + 0.013 \sigma_2 + 0.32 \sigma_3 - 0.16 \sigma_5 - 0.16 \sigma_6)_{\text{dim}-7}. \tag{3.32}
\end{aligned}$$

In principle, if the vacuum-insertion expectation for ρ's and σ's is assumed, the four unknown parameters $B_{1,2}$ and $\epsilon_{1,2}$ can be obtained by solving the four equations for R_D, R'_D, R_{D_s} and R'_{D_s} given in eqs. (3.28) and (3.32). In practice, the solutions are very sensitive to the experimental values within errors. We pick up the solutions not far from the vacuum-insertion expectation.[8] For example, one of the solutions is

$$B_1 = 0.840, \quad B_2 = 0.919, \quad \epsilon_1 = -0.060, \quad \epsilon_2 = -0.025, \tag{3.33}$$

at the scale $\mu = m_c$. They reproduce the experimental values of R_D, R'_D, R_{D_s} and R'_{D_s}. Note that in order to accommodate the experimental values of R_{D_s} and R'_{D_s}, it is necessary to have $B_2 > B_1$. To see this, we take vacuum-insertion values for ρ's and σ's and find from eq. (3.32) that

$$\begin{aligned}
R_{D_s} &= 1.65 + (4.81 B_1 - 4.82 B_2 - 1.21 \epsilon_1 + 1.22 \epsilon_2)_{\text{dim}-6}, \\
R'_{D_s} &= 0.50 + (-3.70 B_1 + 3.71 B_2 + 0.93 \epsilon_1 - 0.94 \epsilon_2)_{\text{dim}-6}. \tag{3.34}
\end{aligned}$$

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[8]Some solutions, for example, $B_1 = 0.138, B_2 = 0.144, \epsilon_1 = -0.296$ and $\epsilon_2 = -0.552$ can also reproduce the data, but they are ruled out as $B_{1,2}$ are too small, whereas $\epsilon_{1,2}$ are too large.

It is clear that if $B_2 \approx B_1$ and $\epsilon_2 \approx \epsilon_1$, the value of R_{Ds} (R'_{D_s}) will be too large (small) compared to the data. Hence, one needs $B_2 > B_1$ and $|\epsilon_1| > |\epsilon_2|$ in order to suppress R_{D_s} and enhance R'_{D_s} simultaneously.

Our results are to be compared with the recent estimates based on HQET sum rules by Kirk, Lenz and Rauh (KLR) [22]

$$B_1 = 0.902^{+0.077}_{-0.051}, \quad B_2 = 0.739^{+0.124}_{-0.073}, \quad \epsilon_1 = -0.132^{+0.041}_{-0.046}, \quad \epsilon_2 = -0.005^{+0.032}_{-0.032}, \quad (3.35)$$

evaluated at the scale $\mu = 3\,\text{GeV}$. While KLR have updated the prediction of $\tau(D^+)/\tau(D^0)$, they did not perform the similar update for the lifetime ratio of D_s^+ and D^0 mesons. As discussed above, the explanation of the R_{D_s} data requires that $B_1 < B_2$.

Finally, we notice that the size of the subleading $1/m_c$ corrections is $[(\Gamma(D^0) - \Gamma(D^+)]_{\text{dim-7}}/[(\Gamma(D^0)-\Gamma(D^+)]_{\text{dim-6}} \approx -56\%$, which is compatible with a convergent series.

4   Lifetimes of heavy baryons

4.1   Lifetimes of bottom baryons

The spectator effects in inclusive heavy bottom baryon decays arising from dimension-6 and dimension-7 operators are given by eqs. (2.20) and (2.25), respectively. We shall rely on the quark model to evaluate the baryon matrix elements of four-quark operators. In [18] we have studied the matrix elements in the MIT bag model [61–64] and the nonrelativistic quark model (NQM). In analogue to eq. (3.8), we parameterize the four baryon matrix elements in a model-independent manner:[9]

$$\begin{aligned}
\langle \mathcal{B}_b | (\bar{b}q)(\bar{q}b) | \mathcal{B}_b \rangle &= f_{B_q}^2 m_{B_q} m_{\mathcal{B}_b} L_1^{\mathcal{B}_b}, \\
\langle \mathcal{B}_b | \bar{b}(1-\gamma_5)q\bar{q}(1+\gamma_5)b | \mathcal{B}_b \rangle &= f_{B_q}^2 m_{B_q} m_{\mathcal{B}_b} L_2^{\mathcal{B}_b}, \\
\langle \mathcal{B}_b | (\bar{b}b)(\bar{q}q) | \mathcal{B}_b \rangle &= f_{B_q}^2 m_{B_q} m_{\mathcal{B}_b} L_3^{\mathcal{B}_b}, \\
\langle \mathcal{B}_b | \bar{b}^\alpha(1-\gamma_5)q^\beta\bar{q}^\beta(1+\gamma_5)b^\alpha | \mathcal{B}_b \rangle &= f_{B_q}^2 m_{B_q} m_{\mathcal{B}_b} L_4^{\mathcal{B}_b},
\end{aligned} \qquad (4.1)$$

where $\mathcal{B}_b$ stands for the antitriplet bottom baryon T_b (Λ_b or Ξ_b) or the sextet bottom baryon Ω_b, f_{B_q} and m_{B_q} are the decay constant and the mass of the heavy meson B_q, respectively. The four hadronic parameters $L_1, \cdots, L_4$ are not all independent.

First, since the color wavefunction for a baryon is totally antisymmetric, the matrix element of $(\bar{b}b)(\bar{q}q)$ is the same as that of $(\bar{b}q)(\bar{q}b)$ except for a sign difference. Thus we follow [12] to define a parameter $\tilde{B}$

$$L_3^{\mathcal{B}_b} = -\tilde{B}L_1^{\mathcal{B}_b}, \qquad L_4^{\mathcal{B}_b} = -\tilde{B}L_2^{\mathcal{B}_b}, \qquad \text{for } \mathcal{B}_b = T_b, \Omega_b, \qquad (4.2)$$

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[9]This is similar to the hadronic parameters defined in eq. (28) of [65].

so that $\tilde{B} = 1$ in the valence-quark approximation. Second, in the quark model evaluation we obtain (see appendix B for derivation) [18]

$$\langle T_b|(\bar{b}q)(\bar{q}b)|T_b\rangle/(2m_{T_b}) = \begin{cases} -\left|\psi_{bq}^{T_b}(0)\right|^2, & NQM \\ -(a_q + b_q), & MIT \end{cases}$$

$$\langle \Omega_b|(\bar{b}s)(\bar{s}b)|\Omega_b\rangle/(2m_{\Omega_b}) = \begin{cases} -6\left|\psi_{bs}^{\Omega_b}(0)\right|^2, & NQM \\ -\frac{1}{3}(18a_s + 2b_s + 32c_s), & MIT \end{cases} \tag{4.3}$$

and

$$\langle T_b|\bar{b}(1-\gamma_5)q\bar{q}(1+\gamma_5)b|T_b\rangle/(2m_{T_b}) = \begin{cases} \frac{1}{2}\left|\psi_{bq}^{T_b}(0)\right|^2, & NQM \\ \frac{1}{2}(a_q + b_q), & MIT \end{cases}$$

$$\langle \Omega_b|\bar{b}(1-\gamma_5)s\bar{s}(1+\gamma_5)b|\Omega_b\rangle/(2m_{\Omega_b}) = \begin{cases} -\left|\psi_{bs}^{\Omega_b}(0)\right|^2, & NQM \\ -(a_s - \frac{5}{3}b_s - \frac{16}{3}c_s), & MIT \end{cases} \tag{4.4}$$

where a_q, b_q and c_q are the four-quark overlap integrals used in the MIT bag model:

$$a_q = \int d^3r \left[u_q^2(r)u_b^2(r) + v_q^2(r)v_b^2(r) \right],$$

$$b_q = \int d^3r \left[u_q^2(r)v_b^2(r) + v_q^2(r)u_b^2(r) \right],$$

$$c_q = \int d^3r \, u_q(r)v_q(r)u_b(r)v_b(r), \tag{4.5}$$

which are expressed in terms of the large and small components $u(r)$ and $v(r)$, respectively, of the quark wavefunction (see e.g., ref. [66] for the technical detail of the bag model evaluation). In deriving eq. (4.4), use of

$$\langle T_b|\bar{b}^\alpha\gamma_\mu\gamma_5 b^\beta \bar{q}^\beta\gamma^\mu(1-\gamma_5)q^\alpha|T_b\rangle/(2m_{T_b}) = 0,$$

$$\langle \Omega_b|\bar{b}^\alpha\gamma_\mu\gamma_5 b^\beta \bar{s}^\beta\gamma^\mu(1-\gamma_5)s^\alpha|\Omega_b\rangle/(2m_{\Omega_b}) = \begin{cases} 4\left|\psi_{bs}^{\Omega_b}(0)\right|^2 & NQM \\ 4\left(a_s - \frac{b_s}{3}\right) & MIT \end{cases} \tag{4.6}$$

and

$$\bar{b}^\alpha\gamma_\mu\gamma_5 b^\beta \bar{q}^\beta\gamma^\mu(1-\gamma_5)q^\alpha = -\bar{b}(1-\gamma_5)q\bar{q}(1+\gamma_5)b - \frac{1}{2}(\bar{b}q)(\bar{q}b), \tag{4.7}$$

has been made. The first relation in eq. (4.6) is a model-independent consequence of heavy quark spin symmetry [12]. It follows from eqs. (4.2), (4.3) and (4.4) that

$$L_2^{T_b} = -\frac{1}{2}L_1^{T_b}, \qquad L_2^{\Omega_b} = \frac{1}{6}L_1^{\Omega_b}, \tag{4.8}$$

where the second relation for the Ω_b is exact in the NQM but only an approximation in the MIT bag model.

Since the small component $v(r)$ is negligible in the NQM, baryon matrix elements of four-quark operators in the NQM and MIT models are the same except for the replacement:

$$a_q \to \int d^3r \, u_b^2(r) u_q^2(r), \qquad b_q \to 0, \qquad c_q \to 0. \tag{4.9}$$

Hence, in the NQM a_q is nothing but the baryon wave function at the origin squared $|\psi_{bq}(0)|^2$. In general, the strength of destructive Pauli interference and W-exchange is governed by $a_q + b_q$ in the bag model and $|\psi(0)|^2$ in the NQM. However, the bag model calculation of $a_q + b_q$ generally gives a much smaller value than the nonrelativistic estimate of $|\psi(0)|^2$. As argued in [18], the difference between $a_q + b_q$ and $|\psi(0)|^2$ is not simply attributed to relativistic corrections; it arises essentially from the distinction in the spatial scale of the wavefunction especially at the origin. As a consequence, both models give a quite different quantitative description for processes sensitive to $|\psi(0)|^2$. It turns out that the NQM works better for heavy baryon decays. Hence, we will follow [18] to consider the NQM estimate of baryon matrix elements.

To estimate the bottom baryon wave function in the center, consider $|\psi_{bq}^{\Lambda_b}(0)|^2$ as an example. A calculation of hyperfine splittings between Σ_b and Λ_b as well as between B^* and B based on the mass formula given in [67] yields [68]

$$|\psi_{bq}^{\Lambda_b}(0)|^2 = \frac{2m_q}{m_b - m_q} \frac{m_{\Sigma_b} - m_{\Lambda_b}}{m_{B^*} - m_B} |\psi_{b\bar{q}}^B(0)|^2, \tag{4.10}$$

where the equality $|\psi_{bq}^{\Sigma_b}(0)|^2 = |\psi_{bq}^{\Lambda_b}(0)|^2$ has been assumed. As a consequence, the wave function of a bottom baryon at the origin can be related to that of a B meson. Another approach proposed by Rosner [69] is to consider the hyperfine splittings of Σ_b and B separately so that

$$|\psi_{bq}^{\Lambda_b}(0)|^2 = |\psi_{bq}^{\Sigma_b}(0)|^2 = \frac{4}{3} \frac{m_{\Sigma_b^*} - m_{\Sigma_b}}{m_{B^*} - m_B} |\psi_{b\bar{q}}^B(0)|^2. \tag{4.11}$$

This method is presumably more reliable as $|\psi_{bq}(0)|^2$ thus determined does not depend on m_b and the constituent quark mass m_q directly. Defining the wave function ratio

$$r_{\Lambda_b} = \left| \frac{\psi_{bq}^{\Lambda_b}(0)}{\psi_{b\bar{q}}^B(0)} \right|^2, \tag{4.12}$$

and noting that $|\psi_{b\bar{q}}^B(0)|^2 = \frac{1}{12} f_B^2 m_B$, we see from eqs. (4.3), (4.4) and (4.6) that the parameters $L_1, \cdots, L_4$ for bottom baryons now read

$$L_1^{T_b} = -\frac{1}{6} r_{T_b}, \qquad L_2^{T_b} = \frac{1}{12} r_{T_b}, \qquad L_3^{T_b} = \frac{1}{6} \tilde{B} r_{T_b}, \qquad L_4^{T_b} = -\frac{1}{12} \tilde{B} r_{T_b},$$
$$L_1^{\Omega_b} = -r_{\Omega_b}, \qquad L_2^{\Omega_b} = -\frac{1}{6} r_{\Omega_b}, \qquad L_3^{\Omega_b} = \tilde{B} r_{\Omega_b}, \qquad L_4^{\Omega_b} = \frac{1}{6} \tilde{B} r_{\Omega_b}. \tag{4.13}$$

Hence, the baryon matrix elements are expressed in terms of two independent parameters $r_{\mathcal{B}_c}$ and $\tilde{B}$.

For dimension-7 four-quark operators, the baryon matrix elements are given by

$$\langle T_b|P_1^q|T_b\rangle = \langle T_b|P_2^q|T_b\rangle = \frac{1}{48}f_{B_q}^2 m_{B_q} m_{T_b} r_{T_b}\left(\frac{m_{T_b}^2 - m_{\rm diq}^2}{m_b^2} - 1\right)\eta_{1,2}^q,$$

$$\langle T_b|P_3^q|T_b\rangle = -2\langle T_b|P_4^q|T_b\rangle = -\frac{1}{24}f_{B_q}^2 m_{B_q} m_{T_b} r_{T_b}\left(\frac{m_{T_b}^2 - m_{\rm diq}^2}{m_b^2} - 1\right)\eta_{3,4}^q, \quad (4.14)$$

$$\langle \Omega_b|P_1^s|\Omega_b\rangle = \langle \Omega_b|P_2^s|\Omega_b\rangle = \frac{1}{8}f_{B_s}^2 m_{B_s} m_{\Omega_b} r_{\Omega_b}\left(\frac{m_{\Omega_b}^2 - m_{\{ss\}}^2}{m_b^2} - 1\right)\eta_{1,2}^s,$$

$$\langle \Omega_b|P_3^s|\Omega_b\rangle = 6\langle \Omega_b|P_4^s|\Omega_b\rangle = -\frac{1}{4}f_{B_s}^2 m_{B_s} m_{\Omega_b} r_{\Omega_b}\left(\frac{m_{\Omega_b}^2 - m_{\{ss\}}^2}{m_b^2} - 1\right)\eta_{3,4}^s,$$

where $m_{\rm diq}$ is the mass of the scalar diquark of T_b and the parameters η_i^q are expected to be of order unity. We shall follow [70] to use $m_{[ud]} = 710\,\text{MeV}$ for Λ_b, $m_{[us]} = 948\,\text{MeV}$ for Ξ_b^0, $m_{[ds]} = 948\,\text{MeV}$ for Ξ_b^- and $m_{\{ss\}} = 1203\,\text{MeV}$ for Ω_b with $[qq']$ antisymmetric in flavor and $\{ss\}$ symmetric in flavor denoting scalar and axial-vector diquarks, respectively. For the matrix elements of the operators $\tilde{P}_i^q$, we follow eq. (4.2) to introduce a parameter $\tilde{\beta}_i^q$

$$\langle \mathcal{B}_b|\tilde{P}_i^q|\mathcal{B}_b\rangle = -\tilde{\beta}_i^q\langle \mathcal{B}_b|P_i^q|\mathcal{B}_b\rangle, \quad (4.15)$$

so that $\tilde{\beta}_i^q = 1$ under the valence quark approximation.

Two remarks are in order. First, unlike the meson matrix elements $\langle B_q|P_{1,2}^q|B_q\rangle$ in eq. (3.9) which are explicitly of order m_q/m_b because of the definition of the operators $P_{1,2}^q$, the baryon matrix elements $\langle \mathcal{B}_b|P_{1,2}^q|B_b\rangle$ in eq. (4.14) are not explicitly proportional to m_q/m_b. Nevertheless, it is easily seen that $(m_{T_b}^2 - m_{\rm diq}^2)/m_b^2 - 1$, for example, is indeed of order m_q/m_b. Second, unlike the meson case we do not know how to evaluate the baryon matrix elements of $P_{5,6}^q$. Since the operators $P_{5,6}^q$ arise from by expressing the QCD four-quark operators in terms of HQET operators

$$\bar{b}\Gamma q\bar{q}\Gamma b = \bar{h}_v\Gamma q\bar{q}\Gamma h_v + \frac{1}{2m_b}\left[\bar{h}_v(-i\overleftarrow{\slashed{D}})\Gamma q\bar{q}\Gamma h_v + \bar{h}_v\Gamma q\bar{q}\Gamma(i\slashed{D})h_v\right], \quad (4.16)$$

we shall follow [23, 24] to assume that dimension-7 operators contain full QCD b quark fields. Therefore, to evaluate the baryon matrix elements of dimension-7 operators given in eq. (2.25), we will drop the operators $P_{5,6}^q$ and $\tilde{P}_{5,6}^q$.

To estimate the hadronic parameter $r_{\mathcal{B}_Q}$ in the NQM, we find from eq. (4.11) that

$$r_{\Lambda_b} = \frac{4}{3}\frac{m_{\Sigma_b^*} - m_{\Sigma_b}}{m_{B^*} - m_B}, \qquad r_{\Xi_b} = \frac{4}{3}\frac{m_{\Xi_b^*} - m_{\Xi_b'}}{m_{B^*} - m_B}, \qquad r_{\Omega_b} = \frac{4}{3}\frac{m_{\Omega_b^*} - m_{\Omega_b}}{m_{B^*} - m_B}, \quad (4.17)$$

and likewise for r_{Λ_c}, r_{Ξ_c} and r_{Ω_c}. Notice that the heavy-quark spin-violating mass relation [28]

$$(m_{\Sigma_Q^*} - m_{\Sigma_Q}) + (m_{\Omega_Q^*} - m_{\Omega_Q}) = 2(m_{\Xi_Q^*} - m_{\Xi_Q'}) \quad (4.18)$$

holds very accurately for $Q = b, c$. Numerically, we obtain[10]

$$
\begin{array}{lll}
r_{\Lambda_c} = 0.610, & r_{\Xi_c} = 0.656, & r_{\Omega_c} = 0.664, \\
r_{\Lambda_b} = 0.607, & r_{\Xi_b} = 0.601, & r_{\Omega_b} = 0.601,
\end{array} \quad (4.19)
$$

----

[10]For a summary of the earlier estimates of $r_{\mathcal{B}_Q}$, see [8].

and

$$|\psi_{bq}^{\Lambda_b}(0)|^2 = 0.92 \times 10^{-2}\text{GeV}^3, \qquad |\psi_{bq}^{\Xi_b}(0)|^2 = 0.91 \times 10^{-2}\text{GeV}^3,$$
$$|\psi_{bs}^{\Omega_b}(0)|^2 = 1.42 \times 10^{-2}\text{GeV}^3, \tag{4.20}$$

for $f_{B_q} = 186\,\text{MeV}$ and $f_{B_s} = 230\,\text{MeV}$ [58]. Therefore, the NQM estimate of $|\psi_{bq}^{\mathcal{B}_b}(0)|^2$ is indeed larger than the analogous bag model quantity: $a_q + b_q \sim 3 \times 10^{-3}\text{GeV}^3$.

Except for the weak annihilation term, the expression of Pauli interference will be very lengthy if the hadronic parameters η_i^q and $\tilde{\beta}_i^q$ are all treated to be different from each other. Since in realistic calculations we will set $\tilde{\beta}_i^q(\mu_h) = 1$ under valence quark approximation and put η_i^q to unity, we shall assume for simplicity that $\eta_i^q = \eta$ and $\tilde{\beta}_i^q = \tilde{\beta}$. The spectator effects in nonleptonic decays of bottom baryons are now readily obtained from eqs. (2.20) and (2.25):

$$\Gamma^{\text{ann}}(\Lambda_b^0) = \frac{G_F^2 m_b^2}{2\pi}|V_{cb}|^2 r_{\Lambda_b}\left|\psi_{b\bar{q}}^B(0)\right|^2\left\{\left(\tilde{B}(c_1^2+c_2^2)-2c_1c_2\right)(1-x)^2\right.$$
$$\left. +\frac{1}{2}\left(\tilde{\beta}(c_1^2+c_2^2)-2c_1c_2\right)\eta(1-x^2)\left(\frac{m_{\Lambda_b}^2-m_{[ud]}^2}{m_b^2}-1\right)\right\},$$

$$\Gamma^{\text{ann}}(\Xi_b^0) = \frac{G_F^2 m_b^2}{2\pi}|V_{cb}|^2 r_{\Xi_b}\left|\psi_{b\bar{q}}^B(0)\right|^2\left\{\left(\tilde{B}(c_1^2+c_2^2)-2c_1c_2\right)(1-x)^2\right.$$
$$\left. +\frac{1}{2}\left(\tilde{\beta}(c_1^2+c_2^2)-2c_1c_2\right)\eta(1-x^2)\left(\frac{m_{\Xi_b}^2-m_{[us]}^2}{m_b^2}-1\right)\right\},$$

$$\Gamma_-^{\text{int}}(\Lambda_b^0) = -\frac{G_F^2 m_b^2}{4\pi}|V_{cb}V_{ud}|^2 r_{\Lambda_b}\left|\psi_{b\bar{q}}^B(0)\right|^2\left\{\left(\tilde{B}c_1^2-2c_1c_2-N_cc_2^2\right)\left((1-x)^2(1+x)\right.\right.$$
$$\left. +\left|\frac{V_{cd}}{V_{ud}}\right|^2\sqrt{1-4x}\right)-\frac{1}{2}\left(\tilde{\beta}c_1^2-2c_1c_2-N_cc_2^2\right)\eta(1-x)(1+x+2x^2)\left(\frac{m_{\Lambda_b}^2-m_{[ud]}^2}{m_b^2}-1\right)\right\},$$

$$\Gamma_-^{\text{int}}(\Xi_b^0) = -\frac{G_F^2 m_b^2}{4\pi}|V_{cb}V_{cs}|^2\sqrt{1-4x}\, r_{\Xi_b}\left|\psi_{b\bar{q}}^B(0)\right|^2\left\{\left(\tilde{B}c_1^2-2c_1c_2-N_cc_2^2\right)\right. \tag{4.21}$$
$$\left. +\frac{1}{6}\left(\tilde{\beta}c_1^2-2c_1c_2-N_cc_2^2\right)\eta\left(1+2x+\frac{2(1-2x+10x^2)}{1-4x}\right)\left(\frac{m_{\Xi_b}^2-m_{[us]}^2}{m_b^2}-1\right)\right\},$$

$$\Gamma_-^{\text{int}}(\Xi_b^-) = -\frac{G_F^2 m_b^2}{4\pi}|V_{cb}|^2 r_{\Xi_b}\left|\psi_{b\bar{q}}^B(0)\right|^2\left\{\left(\tilde{B}c_1^2-2c_1c_2-N_cc_2^2\right)\left(|V_{ud}|^2(1-x)^2(1+x)\right.\right.$$
$$\left. +|V_{cs}|^2\sqrt{1-4x}\right)+\left(\tilde{\beta}c_1^2-2c_1c_2-N_cc_2^2\right)\eta\left[-\frac{1}{2}|V_{ud}|^2(1-x)(1+x+2x^2)\right.$$
$$\left.\left. +\frac{1}{6}|V_{cs}|^2\sqrt{1-4x}\left(1+2x+\frac{2(1-2x+10x^2)}{1-4x}\right)\right]\left(\frac{m_{\Xi_b}^2-m_{[ds]}^2}{m_b^2}-1\right)\right\},$$

	Γ^{dec}	Γ^{ann}	Γ^{int}_-	Γ^{semi}	Γ^{tot}	$\tau(10^{-12}s)$	$\tau_{\text{expt}}(10^{-12}s)$
Λ_b^0	3.108	0.228	-0.053	1.055	4.338	1.517	1.470 ± 0.010
Ξ_b^0	3.108	0.232	-0.084	1.055	4.310	1.527	1.479 ± 0.031
Ξ_b^-	3.108		-0.130	1.055	4.032	1.633	1.571 ± 0.040
Ω_b^-	3.105		-0.341	1.039	3.803	1.730	$1.64^{+0.18}_{-0.17}$

Table 2. Various contributions to the decay rates (in units of $10^{-13}\,\text{GeV}$) of bottom baryons with the hadronic scale $\mu_{\text{had}} = 0.825\,\text{GeV}$.

$$\Gamma^{\text{int}}_-(\Omega_b^-) = -\frac{G_F^2 m_b^2}{6\pi} |V_{cb}V_{cs}|^2 \sqrt{1-4x}\; r_{\Omega_b} \left| \psi_{b\bar{s}}^{B_s}(0) \right|^2 \left\{ \left(\tilde{B}c_1^2 - 2c_1 c_2 - N_c c_2^2 \right)(5-8x) \right.$$

$$\left. + \frac{3}{2}\left(\tilde{\beta}c_1^2 - 2c_1 c_2 - N_c c_2^2 \right)\eta \left(1 + 2x + \frac{2(1-2x-8x^2)}{1-4x} \right) \left(\frac{m_{\Omega_b}^2 - m_{\{ss\}}^2}{m_b^2} - 1 \right) \right\},$$

where use has been made of eqs. (4.13) and (4.14). Note that there is no weak annihilation contribution to the Ξ_b^- and Ω_b and that there are two Cabibbo-allowed Pauli interference terms in Ξ_b^- decay, and one Cabibbo-allowed as well as one Cabibbo-suppressed interferences in Λ_b decay.

To compute the decay widths of bottom baryons, we have to specify the values of $\tilde{B}$ and r. Since $\tilde{B} = 1$ in the valence-quark approximation and since the wavefunction squared ratio r is evaluated using the quark model, it is reasonable to assume that the NQM and the valence-quark approximation are most reliable when the baryon matrix elements are evaluated at a typical hadronic scale μ_{had}. As shown in [12], the parameters $\tilde{B}$ and r renormalized at two different scales are related via the renormalization group equation to be

$$\tilde{B}(\mu)r(\mu) = \tilde{B}(\mu_{\text{had}})r(\mu_{\text{had}}), \qquad \tilde{B}(\mu) = \frac{\tilde{B}(\mu_{\text{had}})}{\kappa + \frac{1}{N_c}(\kappa - 1)\tilde{B}(\mu_{\text{had}})}, \tag{4.22}$$

with

$$\kappa = \left(\frac{\alpha_s(\mu_{\text{had}})}{\alpha_s(\mu)} \right)^{3N_c/2\beta_0} = \sqrt{\frac{\alpha_s(\mu_{\text{had}})}{\alpha_s(\mu)}} \tag{4.23}$$

and $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$. We consider the hadronic scale in the range of $\mu_{\text{had}} \sim 0.65 - 1\,\text{GeV}$. Taking the medium scale $\mu_{\text{had}} = 0.825\,\text{GeV}$ as an illustration, we obtain $\alpha_s(\mu_{\text{had}}) = 0.59$, $\tilde{B}(\mu) = 0.54\tilde{B}(\mu_{\text{had}}) \simeq 0.54$ and $r(\mu) \simeq 1.86\, r(\mu_{\text{had}})$. The parameter $\tilde{\beta}$ is treated in a similar way. Using the values of $r(\mu_{\text{had}})$ given in eq. (4.19), the calculated inclusive decay rates of bottom baryons are summarized in table 2. We find that the lifetimes of bottom baryons stay almost constant with variation of μ_{had}.

We see from table 2 that the bottom baryon lifetimes follow the pattern

$$\tau(\Omega_b^-) > \tau(\Xi_b^-) > \tau(\Xi_b^0) \simeq \tau(\Lambda_b^0). \tag{4.24}$$

Theoretically, this pattern originates from the fact that while Λ_b, Ξ_b^0, Ξ_b^-, Ω_b all receive contributions from the destructive Pauli interference, only Λ_b and Ξ_b^0 have weak annihilation

effects and that the destructive Pauli interference $\Gamma_-^{\rm int}$ in Ω_b is the largest due to the presence of two valence s quarks in its quark content. The Ξ_b^- has the second largest $\Gamma_-^{\rm int}$ due to the Pauli interference of identical s quarks and the interference of identical d quarks.

Several remarks are in order. (i) There is a tiny difference between the semileptonic decays of the antitriplet Λ_b or Ξ_b and the sextet Ω_b. It comes from the fact that the chromomagnetic operator contributes to the matrix element of Ω_b but not to Λ_b or Ξ_b as the light degrees of freedom in the latter are spinless. (ii) It is evident from tables 1 and 2 that W-annihilation contribution in B decays is much smaller than that in bottom baryon decays. The W-exchange in B decays is helicity suppressed, while it is neither helicity nor color suppressed in the heavy baryon case. (iii) As pointed out in [65], b-flavor-conserving decays such as $\Xi_b^- \to \Lambda_b^0 \pi^-, \Lambda_b^0 e^- \bar{\nu}_e$ and $\Xi_b^0 \to \Lambda_b^0 \pi^0$ could affect the total rates of the Ξ_b. These heavy-flavor-conserving weak decays were studied more than two decades ago within the framework that incorporates both heavy-quark and chiral symmetries [71, 72]. The branching fraction of $\Xi_b \to \Lambda_b \pi$ is found to be of order $(0.1 \sim 1)\%$, consistent with the recent LHCb measurement which lies in the range from $(0.57 \pm 0.21)\%$ to $(0.19 \pm 0.07)\%$ [73]. Hence, contributions from b-flavor-conserving decays can be safely neglected for our present purpose.

From eq. (2.26) we obtain the following lifetime ratios[11]

$$
\left.\frac{\tau(\Xi_b^-)}{\tau(\Lambda_b^0)}\right|_{\rm theo} = 1.073^{+0.009}_{-0.004}, \qquad\qquad \left.\frac{\tau(\Xi_b^-)}{\tau(\Lambda_b^0)}\right|_{\rm expt} = 1.089 \pm 0.028,
$$

$$
\left.\frac{\tau(\Xi_b^-)}{\tau(\Xi_b^0)}\right|_{\rm theo} = 1.066^{+0.009}_{-0.004}, \qquad\qquad \left.\frac{\tau(\Xi_b^-)}{\tau(\Xi_b^0)}\right|_{\rm expt} = 1.083 \pm 0.036, \qquad (4.25)
$$

$$
\left.\frac{\tau(\Omega_b^-)}{\tau(\Xi_b^-)}\right|_{\rm theo} = 1.054^{+0.006}_{-0.002}, \qquad\qquad \left.\frac{\tau(\Omega_b^-)}{\tau(\Xi_b^-)}\right|_{\rm expt} = 1.11 \pm 0.16,
$$

and

$$
\left.\frac{\tau(\Lambda_b^0)}{\tau(B_d^0)}\right|_{\rm theo} = 0.953^{+0.006}_{-0.008}, \qquad \left.\frac{\tau(\Lambda_b^0)}{\tau(B_d^0)}\right|_{\rm expt} = 0.964 \pm 0.007 . \qquad (4.26)
$$

They are in good agreement with experiment [13]. The theoretical uncertainties for bottom baryon lifetime ratios can arise from many different places such as the nonperturbative parameters μ_π^2 and μ_G^2, where QCD sum rule and lattice calculations are still not available, and the matrix elements of dimension-6 and -7 operators. In the quark model, the unknown matrix elements are expressed in terms of two parameters $r_{\mathcal{B}_b}$, the wave function ratio, and $\tilde{B}$, which is equal to unity under the valence quark approximation. The estimate of the former is quite uncertain in the literature (see [8] for a review). Therefore, it is far more difficult to estimate the uncertainties than the B meson case. Nevertheless, there is one uncertainty which we can estimate reliably, namely, the hadronic scale $\mu_{\rm had}$ introduced before. The baryon matrix elements need to be evaluated at a typical hadronic scale $\mu_{\rm had}$ in order to comply with the valence quark approximation. We consider the hadronic scale in the range of $0.65 - 1\,{\rm GeV}$ and use $\mu_{\rm had} = 0.825 \pm 0.175\,{\rm GeV}$ to estimate the uncertainties. The theoretical errors in eq. (4.25) we have computed arise solely from the uncertainty of

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[11]The experimental lifetime ratio $\tau(\Xi_b^0)/\tau(\Xi_b^-) = 0.929 \pm 0.028$ (or $\tau(\Xi_b^-)/\tau(\Xi_b^0) = 1.076 \pm 0.032$) is quoted in the Heavy Flavor Averaging Group [74, 75].

the hadronic scale. The uncertainty in the prediction of $\tau(\Lambda_c^+)/\tau(B_d^0)$ comes from the bag parameters in (3.12) and the hadronic scale $\mu_{\rm had}$. We see that the current world average of $\tau(\Lambda_b^0)/\tau(B_d^0)$ can be nicely explained within the framework of the HQE.

4.2 Lifetimes of charmed baryons

We first summarize the spectator effects relevant to charmed baryon decays derived from eqs. (2.20) and (2.25):

$$\Gamma^{\rm ann}(\Lambda_c^+) = \frac{G_F^2 m_c^2}{2\pi}|V_{cs}V_{ud}|^2 r_{\Lambda_c}\left|\psi_{c\bar q}^D(0)\right|^2\left\{\left(\tilde B(c_1^2+c_2^2)-2c_1 c_2\right)(1-x)^2\right.$$
$$\left.+\frac{1}{2}\left(\tilde\beta(c_1^2+c_2^2)-2c_1 c_2\right)\eta(1-x^2)\left(\frac{m_{\Lambda_c}^2-m_{[ud]}^2}{m_c^2}-1\right)\right\},$$

$$\Gamma^{\rm ann}(\Xi_c^+) = \frac{G_F^2 m_c^2}{2\pi}|V_{cs}V_{us}|^2 r_{\Xi_c}\left|\psi_{c\bar q}^D(0)\right|^2\left\{\left(\tilde B(c_1^2+c_2^2)-2c_1 c_2\right)(1-x)^2\right.$$
$$\left.+\frac{1}{2}\left(\tilde\beta(c_1^2+c_2^2)-2c_1 c_2\right)\eta(1-x^2)\left(\frac{m_{\Xi_c}^2-m_{[us]}^2}{m_c^2}-1\right)\right\},$$

$$\Gamma^{\rm ann}(\Xi_c^0) = \frac{G_F^2 m_c^2}{2\pi}|V_{cs}V_{ud}|^2 r_{\Xi_c}\left|\psi_{c\bar q}^D(0)\right|^2\left\{\left(\tilde B(c_1^2+c_2^2)-2c_1 c_2\right)(1-x)^2\right.$$
$$\left.+\frac{1}{2}\left(\tilde\beta(c_1^2+c_2^2)-2c_1 c_2\right)\eta(1-x^2)\left(\frac{m_{\Xi_c}^2-m_{[ds]}^2}{m_c^2}-1\right)\right\},$$

$$\Gamma^{\rm ann}(\Omega_c^0) = 3\frac{G_F^2 m_c^2}{\pi}|V_{cs}V_{us}|^2 r_{\Omega_c}\left|\psi_{c\bar s}^{D_s}(0)\right|^2\left\{\left(\tilde B(c_1^2+c_2^2)-2c_1 c_2\right)(1-x)^2\right.$$
$$\left.+\frac{1}{2}\left(\tilde\beta(c_1^2+c_2^2)-2c_1 c_2\right)\eta(1-x^2)\left(\frac{m_{\Omega_c}^2-m_{\{ss\}}^2}{m_c^2}-1\right)\right\},$$

$$\Gamma_-^{\rm int}(\Lambda_c^+) = -\frac{G_F^2 m_c^2}{4\pi}|V_{cs}V_{ud}|^2 r_{\Lambda_c}\left|\psi_{c\bar q}^D(0)\right|^2\left\{\left(\tilde B c_1^2-2c_1 c_2-N_c c_2^2\right)\left((1-x)^2(1+x)\right.\right.$$
$$\left.+\left|\frac{V_{cd}}{V_{ud}}\right|^2\sqrt{1-4x}\right)-\frac{1}{2}\left(\tilde\beta c_1^2-2c_1 c_2-N_c c_2^2\right)\eta(1-x)(1+x+2x^2)\left(\frac{m_{\Lambda_c}^2-m_{[ud]}^2}{m_c^2}-1\right)\right\},$$

$$\Gamma_-^{\rm int}(\Xi_c^+) = -\frac{G_F^2 m_c^2}{4\pi}|V_{cs}V_{ud}|^2\sqrt{1-4x}\, r_{\Xi_c}\left|\psi_{c\bar q}^D(0)\right|^2\left\{\left(\tilde B c_1^2-2c_1 c_2-N_c c_2^2\right)\right. \qquad (4.27)$$
$$\left.+\frac{1}{6}\left(\tilde\beta c_1^2-2c_1 c_2-N_c c_2^2\right)\eta\left(1+2x+\frac{2(1-2x+10x^2)}{1-4x}\right)\left(\frac{m_{\Xi_c}^2-m_{[us]}^2}{m_c^2}-1\right)\right\},$$

$$\Gamma_+^{\rm int}(\Lambda_c^+) = \frac{G_F^2 m_c^2}{4\pi}|V_{cd}V_{ud}|^2 r_{\Lambda_c}\left|\psi_{c\bar q}^D(0)\right|^2\left\{\left(2c_1 c_2+N_c c_1^2-\tilde B c_2^2\right)\right.$$
$$\left.-\frac{1}{2}\left(2c_1 c_2+N_c c_1^2-\tilde\beta c_2^2\right)\eta\left(\frac{m_{\Lambda_c}^2-m_{[ud]}^2}{m_c^2}-1\right)\right\},$$

$$\Gamma_+^{\mathrm{int}}(\Xi_c^+) = \frac{G_F^2 m_c^2}{4\pi}|V_{cs}V_{ud}|^2 \, r_{\Xi_c} \left|\psi_{c\bar{q}}^D(0)\right|^2 \bigg\{ \left(2c_1c_2 + N_c c_1^2 - \tilde{B}c_2^2\right)\left(1 + \left|\frac{V_{us}}{V_{ud}}\right|^2 (1-x)^2(1+x)\right)$$

$$-\frac{1}{2}\left(2c_1c_2 + N_c c_1^2 - \tilde{\beta}c_2^2\right)\eta\left(1 + \left|\frac{V_{us}}{V_{ud}}\right|^2 (1-x)^2(1+x)\right)\left(\frac{m_{\Xi_c}^2 - m_{[us]}^2}{m_c^2} - 1\right)\bigg\},$$

$$\Gamma_+^{\mathrm{int}}(\Xi_c^0) = \frac{G_F^2 m_c^2}{4\pi}|V_{cs}V_{ud}|^2 \, r_{\Xi_c} \left|\psi_{c\bar{q}}^D(0)\right|^2 \bigg\{ \left(2c_1c_2 + N_c c_1^2 - \tilde{B}c_2^2\right)\left(1 + \left|\frac{V_{us}}{V_{ud}}\right|^2 (1-x)^2(1+x)\right)$$

$$-\frac{1}{2}\left(2c_1c_2 + N_c c_1^2 - \tilde{\beta}c_2^2\right)\eta\left(1 + \left|\frac{V_{us}}{V_{ud}}\right|^2 (1-x)^2(1+x)\right)\left(\frac{m_{\Xi_c}^2 - m_{[ds]}^2}{m_c^2} - 1\right)\bigg\},$$

$$\Gamma_+^{\mathrm{int}}(\Omega_c^0) = \frac{G_F^2 m_c^2}{6\pi}|V_{cs}V_{ud}|^2 \, r_{\Omega_c} \left|\psi_{c\bar{s}}^{D_s}(0)\right|^2 \bigg\{ \left(2c_1c_2 + N_c c_1^2 - \tilde{B}c_2^2\right)\left(5 + \left|\frac{V_{us}}{V_{ud}}\right|^2 (1-x)^2(5+x)\right)$$

$$-\frac{9}{2}\left(2c_1c_2 + N_c c_1^2 - \tilde{\beta}c_2^2\right)\eta\left(\frac{m_{\Omega_c}^2 - m_{\{ss\}}^2}{m_c^2} - 1\right)\bigg\},$$

where $\tilde{B}$, η and $\tilde{\beta}$ are the hadronic parameters defined in eqs. (4.2), (4.14) and (4.15), respectively, and the wavefunction ratio $r_{\mathcal{B}_c}$ defined in analog to eq. (4.12) with values given in eq. (4.19). As stated before, we follow [70] to use $m_{[ud]} = 710\,\mathrm{MeV}$, $m_{[us]} = m_{[ds]} = 948\,\mathrm{MeV}$ and $m_{\{ss\}} = 1203\,\mathrm{MeV}$ for the diquark masses.

Unlike bottom baryon decays, there exist constructive Pauli interference terms Γ_+^{int} in charmed baryon decays in addition to the destructive Pauli interference Γ_-^{int}. Cabibbo-allowed (Cabibbo-suppressed) Γ_+^{int} arises from the constructive interference between the s (d) quark produced in the c quark decay and the spectator s (d) quark in the charmed baryon (see figure 1.(c)).

For the semileptonic inclusive decay of the charmed baryons meson, the semileptonic decay rate has the same expression as eq. (3.21) except that the parameter d_D is replaced by $d_{\mathcal{B}_c}$, which is equal to 0 for the antitriplet charmed baryons Λ_c, Ξ_c and 4 for the Ω_c. For charmed baryons Ξ_c and Ω_c, there is an additional contribution to the semileptonic width coming from the Pauli interference of the s quark [34] (figure 2). The dimension-6 contribution $\Gamma_{6,int}^{\mathrm{SL}}(\mathcal{B}_c)$ is given before by eq. (2.21). As for the dimension-7 four-quark operator for semileptonic decays, it can be written as

$$\mathcal{T}_7^{\mathrm{SL}}(\mathcal{B}_c) = \frac{G_F^2 m_c^2}{6\pi}|V_{cs}|^2 \sum_{\ell = e,\mu}\left(\tilde{g}_i^{\nu\ell}P_i^s + \tilde{h}_i^{\nu\ell}S_i^s\right), \tag{4.28}$$

with the coefficients given by [26]

$$\tilde{g}_1^{\nu\mu} = -(1-z)^2(1+2z), \qquad\qquad \tilde{g}_2^{\nu\mu} = -(1-z)^2(1+2z),$$
$$\tilde{g}_3^{\nu\mu} = 2(1-z)(1+z+z^2), \qquad\qquad \tilde{g}_4^{\nu\mu} = -12z^2(1-z), \tag{4.29}$$

and $\tilde{h}_i^{\nu\mu} = 0$, where $z = (m_\mu/m_c)^2$. The coefficients $\tilde{g}_i^{\nu e}$ are given by setting $z = 0$. Noting $\Gamma_{7,int}^{\mathrm{SL}}(\mathcal{B}_c) = \langle \mathcal{B}_c|\mathcal{T}_7^{\mathrm{SL}}|\mathcal{B}_c\rangle/(2m_{\mathcal{B}_c})$ and using eqs. (4.13) and (4.14) to evaluate the baryon

matrix elements, we obtain

$$\begin{aligned}
\Gamma^{\mathrm{SL}}_{\mathrm{int}}(\Xi_c) &= \Gamma^{\mathrm{SL}}_{6,int}(\Xi_c) + \Gamma^{\mathrm{SL}}_{7,int}(\Xi_c) \\
&= \frac{G_F^2 m_c^2}{4\pi}|V_{cs}|^2 r_{\Xi_c}|\psi^{D_s}_{c\bar{s}}(0)|^2\left[1 - (1 + \frac{1}{2}z^2 - z^3)\left(\frac{m_{\Xi_c}^2 - m_{[sq]}^2}{m_c^2} - 1\right)\right], \quad (4.30) \\
\Gamma^{\mathrm{SL}}_{\mathrm{int}}(\Omega_c) &= \frac{G_F^2 m_c^2}{6\pi}|V_{cs}|^2 r_{\Omega_c}|\psi^{D_s}_{c\bar{s}}(0)|^2\left[5 - 9(1 - \frac{5}{6}z^2 + \frac{1}{3}z^3)\left(\frac{m_{\Omega_c}^2 - m_{\{ss\}}^2}{m_c^2} - 1\right)\right].
\end{aligned}$$

We shall see later that, depending on the parameter r, the spectator effect in semileptonic decay of Ξ_c and Ω_c can be very significant, in particular for the latter.

We now turn to the heavy baryon wavefunction at the origin. We learn from eq. (4.20) that $|\psi^{\mathcal{B}_b}_{bq}(0)|^2$ is of order $1 \times 10^{-2}\mathrm{GeV}^3$. Likewise, $|\psi(0)|^2$ for hyperons is also of the same order of magnitude as the bottom baryons (see [76] for details). However, for the charmed baryon we obtain $|\psi^{\Lambda_c}_{cq}(0)|^2 = r_{\Lambda_c}|\psi^D_{c\bar{q}}(0)|^2 = 3.8 \times 10^{-3}\mathrm{GeV}^3$ under the assumption that the D meson wavefunction in the center squared $|\psi^D_{c\bar{q}}(0)|^2$ is identified with $\frac{1}{12}f_D^2 m_D$. However, this is smaller than those in bottom or hyperon decays. This means that $|\psi^D_{c\bar{q}}(0)|^2$ is not simply equal to $\frac{y}{12}f_D^2 m_D$ with $y = 1$. We shall use $y = 1.75$.

For the numerical results, we first consider the semileptonic decays. The measured inclusive semileptonic rate of the Λ_c^+

$$\Gamma(\Lambda_c^+ \to Xe^+\nu_e)_{\mathrm{expt}} = (1.307 \pm 0.112) \times 10^{-13}\mathrm{GeV}, \qquad (4.31)$$

obtained from $\mathcal{B}(\Lambda_c^+ \to Xe^+\nu_e) = (3.97 \pm 0.34)\%$, an average of the Mark II measurement of $(4.5 \pm 1.7)\%$ [77] and the recent BESIII result of $(3.95 \pm 0.35)\%$ [78], is larger than that of D mesons:

$$\begin{aligned}
\Gamma(D^+ \to Xe^+\nu_e)_{\mathrm{expt}} &= (1.017 \pm 0.019) \times 10^{-13}\mathrm{GeV}, \\
\Gamma(D^0 \to Xe^+\nu_e)_{\mathrm{expt}} &= (1.042 \pm 0.018) \times 10^{-13}\mathrm{GeV}. \qquad (4.32)
\end{aligned}$$

Theoretically, the difference between Λ_c and D comes from the λ_2 terms in eq. (3.21) which are absent in the former. Our prediction

$$\Gamma(\Lambda_c^+ \to Xe^+\nu_e) = 1.415 \times 10^{-13}\mathrm{GeV}, \qquad (4.33)$$

is consistent with experiment (4.31). Writing $\Gamma^{\mathrm{SL}} = \Gamma^{\mathrm{SL}}_c + \Gamma^{\mathrm{SL}}_{\mathrm{int}}$, we see from table 3 that the spectator effects to $\mathcal{O}(1/m_c^3)$ in the semileptonic decays of Ξ_c and Ω_c are quite significant, in particular for the latter.

To proceed the hadronic decay rates, we employ the Wilson coefficients given in eq. (3.22). As before, we consider the hadronic scale in the range of $\mu_{\mathrm{had}} \sim 0.65 - 1\,\mathrm{GeV}$ and obtain $\tilde{B}(\mu) = 0.70\tilde{B}(\mu_{\mathrm{had}}) \simeq 0.70$ and $r(\mu) \simeq 1.42\,r(\mu_{\mathrm{had}})$ at the medium scale $\mu_{\mathrm{had}} = 0.825\,\mathrm{GeV}$. Repeating the same exercise as the bottom baryon case, the results of calculations to order $1/m_c^3$ are exhibited in table 3. Unlike the bottom baryon case where the lifetimes stay almost constant with variation of μ_{had}, the lifetimes of charmed baryons increase by around 10% when the hadronic scale varies from 0.65 to 1.0 GeV. Nevertheless, the lifetime ratios remain nearly constant.

	Γ^{dec}	Γ^{ann}	Γ^{int}_{-}	Γ^{int}_{+}	Γ^{semi}	Γ^{tot}	$\tau(10^{-13}s)$	$\tau_{\text{expt}}(10^{-13}s)$
Λ_c^+	0.886	1.479	-0.400	0.042	0.215	2.221	2.96	2.00 ± 0.06
Ξ_c^+	0.886	0.085	-0.431	0.882	0.726	2.148	3.06	4.42 ± 0.26
Ξ_c^0	0.886	1.591		0.882	0.726	4.084	1.61	$1.12^{+0.13}_{-0.10}$
Ω_c^0	1.019	0.515		2.974	1.901	6.409	1.03	0.69 ± 0.12

Table 3. Various contributions to the decay rates (in units of 10^{-12} GeV) of singly charmed baryons to order $1/m_c^3$ with the hadronic scale $\mu_{\text{had}} = 0.825$ GeV. Experimental values of charmed baryon lifetimes are taken from [13].

	Γ^{dec}	Γ^{ann}	Γ^{int}_{-}	Γ^{int}_{+}	Γ^{semi}	Γ^{tot}	$\tau(10^{-13}s)$	$\tau_{\text{expt}}(10^{-13}s)$
Λ_c^+	0.886	2.179	-0.211	0.022	0.215	3.091	2.12	2.00 ± 0.06
Ξ_c^+	0.886	0.133	-0.186	0.407	0.437	1.677	3.92	4.42 ± 0.26
Ξ_c^0	0.886	2.501		0.405	0.435	4.228	1.56	$1.12^{+0.13}_{-0.10}$
Ω_c^0	1.019	0.876		-0.559	-0.256	1.079	6.10	0.69 ± 0.12

Table 4. Various contributions to the decay rates (in units of 10^{-12} GeV) of singly charmed baryons after including subleading $1/m_c$ corrections to spectator effects. The hadronic scale is chosen to be $\mu_{\text{had}} = 0.825$ GeV.

We see from table 3 the lifetime pattern

$$\tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0) \tag{4.34}$$

is in accordance with experiment (for early studies of charmed baryon lifetimes, see [2–4, 7, 79–84]). This lifetime hierarchy is understandable qualitatively but not quantitatively. The Ξ_c^+ baryon is longest-lived among charmed baryons because of the smallness of W-exchange and partial cancellation between constructive and destructive Pauli interferences, while Ω_c is shortest-lived due to the presence of two s quarks in the Ω_c that renders the contribution of Γ^{int}_{+} largely enhanced. It is also clear from table III that, although the qualitative feature of the lifetime pattern is comprehensive, the quantitative estimates of charmed baryon lifetimes and their ratios are still rather poor. For example, $R_1 \equiv \tau(\Xi_c^+)/\tau(\Lambda_c^+)$ and $R_2 \equiv \tau(\Xi_c^+)/\tau(\Xi_c^0)$ are calculated to be 1.03 and 1.90, respectively, while experimentally $R_1 = 2.21 \pm 0.15$ and $R_2 = 3.95 \pm 0.47$.

It is evident that, contrary to B meson and bottom baryon cases where the HQE in $1/m_b$ leads to the lifetime ratios in excellent agreement with experiment, the heavy quark expansion in $1/m_c$ does not work well for describing the lifetime pattern of charmed baryons. Since the charm quark is not heavy enough, it is perhaps sensible to consider the subleading $1/m_c$ corrections to spectator effects as depicted in eq. (4.27). The numerical results are shown in table 4. By comparing table 4 with table 3, we see that $\Gamma(\Lambda_c^+)$ is enhanced while $\Gamma(\Xi_c^+)$ is suppressed so that the resulting lifetime ratio R_1 is enhanced from 1.03 to 1.84. This means that $1/m_c$ corrections to spectator effects described by dimension-7 operators are in the right direction. However, the calculated Ω_c lifetime becomes entirely
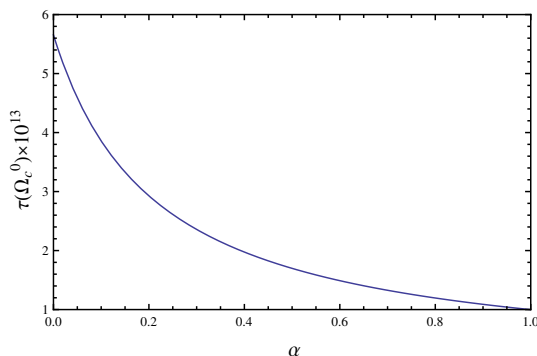
Figure 3. Lifetime of the Ω_c^0 as a function of α.

unexpected: the shortest-lived Ω_c turns out to be the longest-lived one to $\mathcal{O}(1/m_c^4)$. This is because the dimension-7 contributions $\Gamma_{+,7}^{\rm int}(\Omega_c)$ and $\Gamma_7^{\rm SL}(\Omega_c)$ are destructive and their size are so large that they overcome the dimension-6 ones and flip the sign. Of course, a negative $\Gamma^{\rm SL}(\Omega_c)$ does not make sense as the subleading corrections are too large to justify the validity of the HQE.

In order to allow a description of the $1/m_c^4$ corrections to $\Gamma(\Omega_c)$ within the realm of perturbation theory, we introduce a parameter α so that $\Gamma_{+,7}^{\rm int}(\Omega_c)$ and $\Gamma_7^{\rm SL}(\Omega_c)$ are multiplied by a factor of $(1-\alpha)$; that is, α describes the degree of suppression. The origin of this suppression is unknown, but it could be due to the next-order $1/m_c$ correction.[12] The lifetime of Ω_c^0 is plotted as a function of α in figure 3. For the two extreme cases that $\alpha = 0$ (no suppression on dimension-7 effects) and $\alpha = 1$ (no corrections to $\Gamma_+^{\rm int}(\Omega_c)$ and $\Gamma^{\rm SL}(\Omega_c)$ from dimension-7 operators), we have $\tau(\Omega_c^0) = 6.10 \times 10^{-13}s$ and $0.97 \times 10^{-13}s$, respectively (see table 5). Our guidelines for the parameter α are (i) both $\Gamma_+^{\rm int}(\Omega_c)$ and $\Gamma^{\rm SL}(\Omega_c)$ should be positive (at least, a negative $\Gamma^{\rm SL}(\Omega_c)$ does not make sense), and (ii) $\Gamma^{\rm SL}(\Omega_c)$ is comparable to that of Λ_c^+ or Ξ_c. Under these guidelines, we get $\alpha > 0.16$ to ensure positive $\Gamma^{\rm SL}(\Omega_c)$ and $\Gamma_+^{\rm int}(\Omega_c)$ and $\alpha \sim 0.22$ (0.32) for $\Gamma^{\rm SL}(\Omega_c)$ to be comparable to that of Λ_c^+ (Ξ_c).

We see from table 5 that a reasonable range of α lies in $0.16 < \alpha < 0.32$ and the corresponding Ω_c lifetimes lies in the range

$$2.3 \times 10^{-13}s < \tau(\Omega_c^0) < 3.3 \times 10^{-13}s. \tag{4.35}$$

It should be stressed that this is our conjecture as we do not have rigorous statements on the unknown parameter α. At any rate, the Ω_c^0 lifetime is very different from the current world average of $\tau(\Omega_c^0) = (0.69 \pm 0.12) \times 10^{-13}s$ [13] from fixed target experiments. We suggest the new lifetime pattern

$$\tau(\Xi_c^+) > \tau(\Omega_c^0) > \tau(\Lambda_c^+) > \tau(\Xi_c^0), \tag{4.36}$$

which can be tested in the forthcoming LHCb measurements of charmed baryon lifetimes. Very recently, LHCb has reported a new measurement of the Ω_c^0 lifetime, $\tau(\Omega_c^0) = (2.68 \pm$

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[12]Another possibility is that, as noticed in passing, it is not clear to us what are the baryon matrix elements of dimension-7 operators $P_{5,6}^q$. This may also explain the suppression needed for $\Gamma_{+,7}^{\rm int}(\Omega_c)$ and $\Gamma_7^{\rm SL}(\Omega_c)$. Note that the above-mentioned suppression effect is not needed for the antitriplet baryons Λ_c^+, Ξ_c^+ and Ξ_c^0.

α	Γ^{dec}	Γ^{ann}	Γ^{int}_{+}	Γ^{semi}	Γ^{tot}	$\tau(10^{-13}s)$
0	1.019	0.876	-0.559	-0.256	1.079	6.10
0.12	1.019	0.876	-0.135	0.003	1.762	3.73
0.16	1.019	0.876	0.006	0.089	1.990	3.31
0.22	1.019	0.876	0.218	0.219	2.331	2.82
0.32	1.019	0.876	0.571	0.435	2.900	2.27
1	1.019	0.876	2.974	1.901	6.770	0.97

Table 5. Various contributions to the decay rates (in units of 10^{-12} GeV) of the Ω_c^0 after including subleading $1/m_c$ corrections to spectator effects. However, the dimension-7 contributions $\Gamma^{\text{int}}_{+,7}(\Omega_c^0)$ and $\Gamma^{\text{SL}}_7(\Omega_c^0)$ are multiplied by a factor of $(1-\alpha)$ with α varying from 0 to 1.

$0.24 \pm 0.10 \pm 0.02) \times 10^{-13}s$ [85], using the semileptonic decay $\Omega_b^- \to \Omega_c^0 \mu^- \bar{\nu}_\mu X$ with $\Omega_c^0 \to pK^-K^-\pi^+$. This value is nearly four times larger than the current world-average value.[13]

Finally, we would like to remark on the semileptonic widths. We see from table 3 that to order $1/m_c^3$, the constructive Pauli interference is sizeable for the Ξ_c and becomes overwhelming for the Ω_c. However, this interference effect will be partially washed out by the next-order $1/m_c$ correction, in particular for the latter (see table 5). Nevertheless, the interference effect in semileptonic inclusive decays can be tested by measuring the ratio of semileptonic branching fractions $\mathcal{B}^{\text{semi}}(\Xi_c^+)/\mathcal{B}^{\text{semi}}(\Lambda_c^+)$, where $\mathcal{B}^{\text{semi}}(\mathcal{B}_c) = \mathcal{B}(\mathcal{B}_c \to Xe^+\nu_e)$. This ratio naively of order 1.8 will be enhanced to $\mathcal{O}(3.2)$ in the presence of Pauli interference.

5 Discussions and conclusions

In this work we have analyzed the lifetimes of bottom and charmed hadrons within the framework of the heavy quark expansion. It is well known that the lifetime differences stem from spectator effects such as weak annihilation and Pauli interference. We list the dimension-6 four-quark operators responsible for various spectator effects and derive the corresponding dimension-7 ones. The hadronic matrix elements of four-quark operators are parameterized in a model-independent way.

The main results of our analysis are as follows.

- Since in this work we focus on the LO-QCD study for reason of consistency, the inclusive rate to LO is sensitive to the quark mass definition. For the b quark mass, we use the kinetic mass $m_b^{\text{kin}} = 4.546$ GeV obtained from a recent global fit to the inclusive semileptonic B decay to $X_c e^+ \nu_e$ in the kinetic scheme. Using the dimension-6 bag parameters recently determined from HQET sum rules and the vacuum-insertion approximation for meson matrix elements of dimension-7 operators, the calculated B

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[13]Our early conjecture of $\tau(\Omega_c^0)$ of order $2.3 \times 10^{-13}s$ first presented in [86] is indeed consistent with the LHCb measurement.

meson lifetime ratios $\tau(B^+)/\tau(B_d^0) = 1.074^{+0.017}_{-0.016}$ and $\tau(B_s^0)/\tau(B_d^0) = 0.9962 \pm 0.0024$ are in excellent agreement with experiment.

- Baryon matrix elements of four-quark operators parametrized in a model-independent way in terms of four parameters, but only two of them are independent. They are evaluated using the NQM and the bag model. The hadronic parameter r defined in eq. (4.12) is estimated in the NQM to be in the range 0.60 to 0.66 for both bottom and charmed baryons.

- The lifetime pattern of bottom baryons is found to be $\tau(\Omega_b^-) > \tau(\Xi_b^-) > \tau(\Xi_b^0) \simeq \tau(\Lambda_b^0)$. Spectator effects due to W-exchange and destructive Pauli interference account for their lifetime differences. The calculated lifetime ratios $\tau(\Xi_b^-)/\tau(\Lambda_b^0)$, $\tau(\Xi_b^-)/\tau(\Xi_b^0)$ and $\tau(\Omega_b^-)/\tau(\Xi_b^-)$ agree well with the data. Moreover, the $\Lambda_b - B$ lifetime ratio $\tau(\Lambda_b^0)/\tau(B_d^0) = 0.953$ is in good agreement with the experimental average, indicating that the heavy quark expansion in $1/m_b$ works well for bottom hadrons.

- It is found that W-annihilation contribution in B decays is much smaller than that in bottom baryon decays (see tables 1 and 2). The W-exchange in B decays is helicity suppressed, while it is neither helicity nor color suppressed in the heavy baryon case.

- Contrary to the bottom hadron sector where the HQE in $1/m_b$ works well, the HQE to $1/m_c^3$ fails to give a satisfactory description of the lifetimes of both charmed mesons and charmed baryons. This calls for the subleading $1/m_Q$ corrections to spectator effects.

- We have employed the experimental values for D^+ and D^0 semileptonic widths to fix the charmed quark mass to be $m_c = 1.56\,\text{GeV}$. For the charmed meson decay constant of order 200 MeV, the destructive Pauli interference leads to a negative D^+ width irrespective of which charmed quark mass is employed. We showed that $1/m_c$ corrections to the Pauli interference arising from dimension-7 four-quark operators will be able to render $\Gamma(D^+)$ positive. We use the measured lifetime ratios of $\tau(D^+)/\tau(D^0)$ and $\tau(D_s^+)/\tau(D^0)$ to constrain the bag parameters and find that $B_2 > B_1$.

- The HQE to order $1/m_c^3$ implies the lifetime hierarchy $\tau(\Xi_c^+) > \tau(\Lambda_c) > \tau(\Xi_c^0) > \tau(\Omega_c)$. However, the quantitative estimates of charmed baryon lifetimes and their ratios are still rather poor. For example, the large ratios of $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$ and $\tau(\Xi_c^+)/\tau(\Xi_c^0)$ are not quantitatively understandable.

- The calculated lifetimes for heavy baryons depend on the low normalization point. We considered the hadronic scale in the range of $\mu_{\text{had}} \sim 0.65 - 1\,\text{GeV}$ and found that the lifetimes of bottom baryons stay almost constant with variation of μ_{had}, while the charmed baryon lifetimes increase by around 10% when the hadronic scale varies from 0.65 to 1.0 GeV. Nevertheless, the charmed baryon lifetime ratios remain nearly constant.

- The relevant dimension-7 spectator effects are in the right direction for explaining the large lifetime ratio of $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$, which is enhanced from 1.05 to 1.88, in better agreement with experiment. However, the destructive $1/m_c$ corrections to $\Gamma(\Omega_c^0)$ are too large to justify the use of the HQE, namely, the predicted Pauli interference and semileptonic rates for Ω_c^0 become negative. Demanding these rates to be positive for a sensible HQE, we conjecture that the Ω_c^0 lifetime lies in the range of $(2.3 \sim 3.2) \times 10^{-13}s$. This leads to the new lifetime pattern $\tau(\Xi_c^+) > \tau(\Omega_c^0) > \tau(\Lambda_c^+) > \tau(\Xi_c^0)$, contrary to the current hierarchy $\tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0)$. This new charmed baryon lifetime pattern can be tested by LHCb.

- The Ω_c^0, which is naively expected to be shortest-lived in the charmed baryon system owing to the large constructive Pauli interference, could live longer than the Λ_c^+ due to the suppression from $1/m_c$ corrections arising from dimension-7 four-quark operators.

- For charmed baryons Ξ_c and Ω_c, there is an additional contribution to the semileptonic width coming from the constructive Pauli interference of the s quark. However, this interference effect will be partially washed out by the next-order $1/m_c$ correction, in particular for the latter. Nevertheless, this interference effect can be tested by measuring the ratio of semileptonic branching fractions $\mathcal{B}^{\text{semi}}(\Xi_c^+)/\mathcal{B}^{\text{semi}}(\Lambda_c^+)$.

Finally, we would like to remark that it is straightforward to generalize the present lifetime analysis of singly heavy baryons to doubly heavy ones. Recently, LHCb has presented the first measurement of the lifetime of the doubly charmed baryon Ξ_{cc}^{++} to be $\tau(\Xi_{cc}^{++}) = (2.56^{+0.24}_{-0.22} \pm 0.14) \times 10^{-13}s$ [87].

Acknowledgments

A   Spectator effects from dimension-7 four-quark operators

In this appendix we sketch the derivation of dimension-7 operators (2.23) relevant for the spectator effects in heavy baryon decays. The term $\mathcal{T}_{7,ann}^{\mathcal{B}_Q,q_1}$ in eq. (2.23) corresponds to $\mathcal{T}_4^{\text{PI}}$ in eq. (17) of [26]. Consider the Cabibbo-allowed charmed baryon decay so that

$$\mathcal{T}_{7,ann}^{\mathcal{B}_c,d} = \frac{G_F^2 m_c^2}{6\pi}|V_{cs}V_{ud}|^2 \sum_{i=1}^{6}\left(g_i^{su}P_i^d + h_i^{su}S_i^d\right), \tag{A.1}$$

with[14]

$$
\begin{aligned}
g_i^{qq'} &= c_2^2 g_{i,11}^{qq'} + c_1 c_2 g_{i,12}^{qq'} + c_1^2 g_{i,22}^{qq'}, \\
h_i^{qq'} &= c_2^2 h_{i,11}^{qq'} + c_1 c_2 h_{i,12}^{qq'} + c_1^2 h_{i,22}^{qq'}.
\end{aligned}
\tag{A.2}
$$

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[14]Our convention for the Wilson coefficients c_1 and c_2 is opposite to that of [26], namely their C_2 is our c_1 and vice versa.

Since (see eq. (B3) of [26])

$$g^{su}_{1,ij} = g^{su}_{2,ij} = g^{su}_{4,ij} = h^{su}_{1,ij} = h^{su}_{2,ij} = h^{su}_{4,ij} = 0, \tag{A.3}$$

it follows that

$$\mathcal{T}^{\mathcal{B}_c,d}_{7,ann} = \frac{G_F^2 m_c^2}{6\pi}|V_{cs}V_{ud}|^2 \left(g^{su}_3 P^d_3 + g^{su}_5 P^d_5 + g^{su}_6 P^d_6 + h^{su}_3 S^d_3 + h^{su}_5 S^d_5 + h^{su}_6 S^d_6\right). \tag{A.4}$$

The coefficients g^{su}_3 and h^{su}_3 are given by

$$g^{su}_3 = 2(1-x^2)(c_1^2 + 6c_1 c_2 + c_2^2), \qquad h^{su}_3 = 12(1-x^2)(c_1^2 + c_2^2). \tag{A.5}$$

The remaining coefficients $g^{sq}_{5(6),ij}$ and $h^{sq}_{5(6),ij}$ in HQET are related to those in QCD via (see eq. (B5) of [26])

$$(g^{sq}_{5,ij})_{\mathrm{HQET}} = (F^{sq}_{ij})_{\mathrm{QCD}}, \qquad (g^{sq}_{6,ij})_{\mathrm{HQET}} = (F^{sq}_{S,ij})_{\mathrm{QCD}},$$
$$(h^{sq}_{5,ij})_{\mathrm{HQET}} = (G^{sq}_{ij})_{\mathrm{QCD}}, \qquad (h^{sq}_{6,ij})_{\mathrm{HQET}} = (G^{sq}_{S,ij})_{\mathrm{QCD}}, \tag{A.6}$$

with the coefficients F^{sq} and G^{sq} given in

$$\mathcal{T}^{\mathcal{B}_c,d}_{6,ann} = \frac{G_F^2 m_c^2}{6\pi}|V_{cs}V_{ud}|^2 \left(F^{su}Q^d + F^{su}_S Q^q_S + G^{su}T^d + G^{su}_S T^d_S\right), \tag{A.7}$$

where

$$Q^d = (\bar{c}d)(\bar{d}c), \qquad Q^d_S = \bar{c}(1-\gamma_5)d\bar{d}(1+\gamma_5)c,$$
$$T^d = (\bar{c}t^a d)(\bar{d}t^a c), \qquad T^d_S = \bar{c}(1-\gamma_5)t^a d\bar{d}(1+\gamma_5)t^a c. \tag{A.8}$$

Identifing eq. (A.7) with $\mathcal{T}^{\mathcal{B}_Q,q_1}_{6,ann}$ in eq. (2.20), we see that

$$F^{su} = (1-x)^2(c_1^2 + c_2^2 + 6c_1 c_2), \qquad G^{su} = 6(1-x)^2(c_1^2 + c_2^2), \qquad F^{su}_S = G^{su}_S = 0. \tag{A.9}$$

Hence,

$$g^{su}_5 = (1-x)^2(c_1^2 + c_2^2 + 6c_1 c_2), \qquad h^{su}_5 = 6(1-x)^2(c_1^2 + c_2^2), \qquad g^{su}_6 = h^{su}_6 = 0. \tag{A.10}$$

The expression of $\mathcal{T}^{\mathcal{B}_Q,q_1}_{7,ann}$ given in eq. (2.23) with $\mathcal{B}_Q = \mathcal{B}_c$ and $q_1 = d$ is thus obtained.

Likewise, $\mathcal{T}^{\mathcal{B}_c,u}_{7,int}$ given in eq. (2.23) corresponds to the transition operator $\mathcal{T}^{\mathrm{WA}_0}_4$ in [26] and it has the expression

$$\mathcal{T}^{\mathcal{B}_c,u}_{7,int} = \frac{G_F^2 m_c^2}{6\pi}|V_{cs}V_{ud}|^2 \sum_{i=1}^{6}\left(g^{sd}_i P^u_i + h^{sd}_i S^u_i\right). \tag{A.11}$$

From appendix B of [26] we obtain

$$g^{sd}_1 = g^{sd}_2 = -(1-x)^2(1+2x)\left(\frac{1}{N_c}c_1^2 + 2c_1 c_2 + N_c c_2^2\right),$$
$$g^{sd}_3 = 2(1-x)(1+x+x^2)\left(\frac{1}{N_c}c_1^2 + 2c_1 c_2 + N_c c_2^2\right),$$
$$g^{sd}_4 = -12x^2(1-x)\left(\frac{1}{N_c}c_1^2 + 2c_1 c_2 + N_c c_2^2\right), \tag{A.12}$$
$$h^{sd}_1 = h^{sd}_2 = -2(1-x)^2(1+2x)c_1^2,$$
$$h^{sd}_3 = 4(1-x)(1+x+x^2)c_1^2, \qquad h^{sd}_4 = -24x^2(1-x)c_1^2.$$

The coefficients $g_{5,6}^{sd}$ and $h_{5,6}^{sd}$ are found by comparing

$$\mathcal{T}_{6,int}^{\mathcal{B}_c,u} = \frac{G_F^2 m_c^2}{6\pi}|V_{cs}V_{ud}|^2 \left(F^{sd}Q^u + F_S^{sd}Q_S^u + G^{sd}T^u + G_S^{sd}T_S^u\right) \tag{A.13}$$

with $\mathcal{T}_{6,int-}^{\mathcal{B}_Q,q_2}$ in eq. (2.20). Hence,

$$g_5^{sd} = -(1-x)^2\left(1+\frac{x}{2}\right)\left(\frac{1}{N_c}c_1^2 + 2c_1c_2 + N_cc_2^2\right),$$

$$g_6^{sd} = (1-x)^2(1+2x)\left(\frac{1}{N_c}c_1^2 + 2c_1c_2 + N_cc_2^2\right),$$

$$h_5^{sd} = -2(1-x)^2\left(1+\frac{x}{2}\right)c_1^2,$$

$$h_6^{sd} = -2(1-x)^2(1+2x)c_1^2. \tag{A.14}$$

This completes the derivation of $\mathcal{T}_{7,int}^{\mathcal{B}_c,u}$.

The $\mathcal{T}_{7,int}^{\mathcal{B}_b,s}$ term in eq. (2.23) describes the Pauli interference in $b \to c\bar{c}s$ (see figure 1.(b)). It can be deduced from

$$\mathcal{T}_{7,int}^{\mathcal{B}_b,s} = \frac{G_F^2 m_b^2}{6\pi}|V_{cb}V_{cs}|^2 \sum_{i=1}^6 \left(g_i^{ss}P_i^s + h_i^{ss}S_i^s\right). \tag{A.15}$$

We find

$$g_1^{ss} = g_2^{ss} = -\sqrt{1-4x}\,(1+2x)\left(\frac{1}{N_c}c_1^2 + 2c_1c_2 + N_cc_2^2\right),$$

$$g_3^{ss} = \frac{2(1-2x-2x^2)}{\sqrt{1-4x}}\left(\frac{1}{N_c}c_1^2 + 2c_1c_2 + N_cc_2^2\right),$$

$$g_4^{ss} = -\frac{24x^2}{\sqrt{1-4x}}\left(\frac{1}{N_c}c_1^2 + 2c_1c_2 + N_cc_2^2\right),$$

$$h_1^{ss} = h_2^{ss} = -2\sqrt{1-4x}\,(1+2x)c_1^2,$$

$$h_3^{ss} = \frac{4(1-2x-2x^2)}{\sqrt{1-4x}}c_1^2, \qquad h_4^{ss} = -\frac{48x^2}{\sqrt{1-4x}}c_1^2, \tag{A.16}$$

and

$$g_5^{ss} = -\sqrt{1-4x}\,(1-x)\left(\frac{1}{N_c}c_1^2 + 2c_1c_2 + N_cc_2^2\right),$$

$$g_6^{ss} = \sqrt{1-4x}\,(1+2x)\left(\frac{1}{N_c}c_1^2 + 2c_1c_2 + N_cc_2^2\right),$$

$$h_5^{ss} = -2\sqrt{1-4x}\,(1-x)c_1^2, \qquad h_6^{ss} = 2\sqrt{1-4x}\,(1+2x)c_1^2. \tag{A.17}$$

Finally, the last term $\mathcal{T}_{7,int}^{\mathcal{B}_c,s}$ in eq. (2.23) corresponds to the transition operator $\mathcal{T}_4^{\mathrm{WA_+}}$ in [26]

$$\mathcal{T}_{7,int}^{\mathcal{B}_c,s} = \frac{G_F^2 m_c^2}{6\pi}|V_{cs}V_{ud}|^2 \sum_{i=1}^6 \left(\tilde{g}_i^{ud}P_i^s + \tilde{h}_i^{ud}S_i^s\right). \tag{A.18}$$

It turns out that it has the same expression as $\mathcal{T}_{7,int}^{\mathcal{B}_Q,q_3}$ except for a vanishing x and the interchange of c_1 and c_2.

B Baryon matrix elements in the quark model

We show briefly the derivation of eqs. (4.3), (4.4) and (4.6) in the MIT bag model because the expressions in the non-relativistic quark model can be obtained from the former through a simple replacement given in eq. (4.9). Consider the four-quark operator $O = (\bar{Q}Q)(\bar{q}q)$. This operator can be written as $O = 6(\bar{Q}Q)_1(\bar{q}q)_2$, where the superscript i indicates that the quark operator acts only on the ith quark in the baryon wave function. In the bag model, it has the expression (see e.g. eq. (B2) of [66])

$$(\bar{Q}Q)_1(\bar{q}q)_2 = a_q + b_q - \left(a_q - \frac{b_q}{3} + \frac{8c_q}{3} \right) \vec{\sigma}_Q \cdot \vec{\sigma}_q, \tag{B.1}$$

where a_q, b_q and c_q are the four-quark overlap integrals defined in eq. (4.5) in terms of the large and small components of the quark wave function, $u(r)$ and $v(r)$, respectively,

$$\psi = \begin{pmatrix} iu(r)\chi \\ v(r)\vec{\sigma} \cdot \hat{\mathbf{r}}\chi \end{pmatrix}. \tag{B.2}$$

Applying the relation

$$\vec{\sigma}_1 \cdot \vec{\sigma}_2 = \frac{1}{2}(\sigma_{1+}\sigma_{2-} + \sigma_{1-}\sigma_{2+}) + \sigma_{1z}\sigma_{2z}, \tag{B.3}$$

and the wave functions

$$\Lambda_b^0 = -\frac{1}{\sqrt{6}} \left[(ud - du)\chi_A + (13) + (23) \right],$$

$$\Omega_b^- = \frac{1}{\sqrt{3}} \left[ssb\chi_S + (13) + (23) \right], \tag{B.4}$$

with obvious notation for permutation of quarks, where $abc\chi_S = (2a^\uparrow b^\uparrow c^\downarrow - a^\uparrow b^\downarrow c^\uparrow - a^\downarrow b^\uparrow c^\uparrow)/\sqrt{6}$ and $abc\chi_A = (a^\uparrow b^\downarrow c^\uparrow - a^\downarrow b^\uparrow c^\uparrow)/\sqrt{2}$, it is straightforward to show that

$$\langle \Omega_b | b_{1b}^\dagger b_{1b} b_{2s}^\dagger b_{2s} | \Omega_b \rangle = \frac{1}{3}, \qquad \langle \Omega_b | b_{1b}^\dagger b_{1b} b_{2s}^\dagger b_{2s} (\vec{\sigma}_{b1} \cdot \vec{\sigma}_{s2}) | \Omega_b \rangle = -\frac{2}{3}. \tag{B.5}$$

Hence,

$$\langle \Omega_b | (\bar{b}b)(\bar{s}s) | \Omega_b \rangle = 6 \langle \Omega_b^0 | (\bar{b}b)_1(\bar{s}s)_2 | \Omega_b \rangle = \frac{1}{3}(18a_s + 2b_s + 32c_s). \tag{B.6}$$

Likewise,

$$\langle \Lambda_b | (\bar{b}b)(\bar{q}q) | \Lambda_b \rangle = a_q + b_q. \tag{B.7}$$

Next, using the expression

$$\begin{aligned} (\bar{Q}\gamma_\mu\gamma_5 Q)_1(\bar{q}\gamma^\mu(1-\gamma_5)q)_2 &= (a_q - b_q)\vec{\sigma}_Q \cdot \vec{\sigma}_q + 2b_q(\vec{\sigma}_Q \cdot \hat{\mathbf{r}})(\vec{\sigma}_q \cdot \hat{\mathbf{r}}) \\ &= \left(a_q - \frac{b_q}{3} \right) \vec{\sigma}_Q \cdot \vec{\sigma}_q. \end{aligned} \tag{B.8}$$

we obtain

$$\langle \Omega_b | \bar{b}\gamma_\mu\gamma_5 b \bar{s}\Gamma^\mu s | \Omega_b \rangle = -4 \left(a_s - \frac{b_s}{3} \right), \qquad \langle \Lambda_b | \bar{b}\gamma_\mu\gamma_5 b \bar{q}\Gamma^\mu q | \Lambda_b \rangle = 0, \tag{B.9}$$

where $\Gamma^\mu = \gamma^\mu(1 - \gamma_5)$. With eq. (4.7) and the relation

$$\bar{b}\gamma_\mu\gamma_5 b\bar{q}\gamma^\mu(1 - \gamma_5)q = -\bar{b}^\alpha(1 - \gamma_5)q^\beta\bar{q}^\beta(1 + \gamma_5)b^\alpha - \frac{1}{2}(\bar{b}q)(\bar{q}b), \qquad \text{(B.10)}$$

we arrive at the results of (4.4) and (4.6).

Since in heavy quark effective theory, the matrix element $\langle H_Q|\bar{Q}\not{v}Q|H_Q\rangle$ is normalized to $2m_{H_Q}$, we need to put back the factor of $2m_{H_Q}$ in an appropriate place, for example, $\langle \Lambda_b^0|(\bar{b}b)(\bar{q}q)|\Lambda_b\rangle$ now reads $(a_q + b_q)(2m_{\Lambda_b})$. Note that in the quark model, the hadronic parameter $\tilde{B}$ is equal to unity which is supposed to be valid at the hadronic scale.

References

[1] I.I.Y. Bigi, N.G. Uraltsev and A.I. Vainshtein, *Nonperturbative corrections to inclusive beauty and charm decays: QCD versus phenomenological models*, *Phys. Lett.* **B 293** (1992) 430 [*Erratum ibid.* **B 297** (1992) 477] [hep-ph/9207214] [INSPIRE].

[2] B. Blok and M.A. Shifman, *The rule of discarding $1/N_c$ in inclusive weak decays. 1*, *Nucl. Phys.* **B 399** (1993) 441 [hep-ph/9207236] [INSPIRE].

[3] B. Blok and M.A. Shifman, *The rule of discarding $1/N_c$ in inclusive weak decays. 2*, *Nucl. Phys.* **B 399** (1993) 459 [hep-ph/9209289] [INSPIRE].

[4] B. Blok and M.A. Shifman, *Lifetimes of charmed hadrons revisited. Facts and fancy*, in *Tau charm factory. Proceedings*, 3$^{\text{rd}}$ *Workshop*, Marbella, Spain, 1–6 June 1993, J. Kirkby and R. Kirkby eds., Editions Frontieres, France, (1994) [hep-ph/9311331] [INSPIRE].

[5] N. Bilic, B. Guberina and J. Trampetic, *Pauli interference effect in D^+ lifetime*, *Nucl. Phys.* **B 248** (1984) 261 [INSPIRE].

[6] M.A. Shifman and M.B. Voloshin, *Preasymptotic effects in inclusive weak decays of charmed particles*, *Sov. J. Nucl. Phys.* **41** (1985) 120 [*Yad. Fiz.* **41** (1985) 187] [INSPIRE].

[7] B. Guberina, R. Rückl and J. Trampetic, *Charmed baryon lifetime differences*, *Z. Phys.* **C 33** (1986) 297 [INSPIRE].

[8] A. Lenz, *Lifetimes and heavy quark expansion*, *Int. J. Mod. Phys.* **A 30** (2015) 1543005 [arXiv:1405.3601] [INSPIRE].

[9] J. Chay, H. Georgi and B. Grinstein, *Lepton energy distributions in heavy meson decays from QCD*, *Phys. Lett.* **B 247** (1990) 399 [INSPIRE].

[10] J. Chay and S.-J. Rey, *Instanton contribution to $B \to X_\mu e\bar{\nu}$ decay*, *Z. Phys.* **C 68** (1995) 431 [hep-ph/9404214] [INSPIRE].

[11] M.E. Luke, *Effects of subleading operators in the heavy quark effective theory*, *Phys. Lett.* **B 252** (1990) 447 [INSPIRE].

[12] M. Neubert and C.T. Sachrajda, *Spectator effects in inclusive decays of beauty hadrons*, *Nucl. Phys.* **B 483** (1997) 339 [hep-ph/9603202] [INSPIRE].

[13] PARTICLE DATA GROUP collaboration, M. Tanabashi et al., *Review of particle physics*, *Phys. Rev.* **D 98** (2018) 030001 [INSPIRE].

[14] PARTICLE DATA GROUP collaboration, R.M. Barnett et al., *Review of particle physics*, *Phys. Rev.* **D 54** (1996) 1 [INSPIRE].

[15] *LEP B lifetime working group webpage*, http://lepbosc.web.cern.ch/LEPBOSC/archive_LEPBOSC/lifetimes/lepblife.html.

[16] HEAVY FLAVOR AVERAGING GROUP (HFAG) collaboration, J. Alexander et al., *Averages of b-hadron properties as of summer* 2004, hep-ex/0412073 [INSPIRE].

[17] G. Altarelli, G. Martinelli, S. Petrarca and F. Rapuano, *Failure of local duality in inclusive nonleptonic heavy flavor decays*, *Phys. Lett.* **B 382** (1996) 409 [hep-ph/9604202] [INSPIRE].

[18] H.-Y. Cheng, *A phenomenological analysis of heavy hadron lifetimes*, *Phys. Rev.* **D 56** (1997) 2783 [hep-ph/9704260] [INSPIRE].

[19] T. Ito, M. Matsuda and Y. Matsui, *New possibility of solving the problem of lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$*, *Prog. Theor. Phys.* **99** (1998) 271 [hep-ph/9705402] [INSPIRE].

[20] D0 collaboration, V.M. Abazov et al., *Measurement of the Λ_b^0 lifetime in the decay $\Lambda_b^0 \to J/\psi \Lambda^0$ with the DO detector*, *Phys. Rev. Lett.* **94** (2005) 102001 [hep-ex/0410054] [INSPIRE].

[21] CDF collaboration, A. Abulencia et al., *Measurement of the Λ_b^0 lifetime in $\Lambda_b^0 \to J/\psi \Lambda^0$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV*, *Phys. Rev. Lett.* **98** (2007) 122001 [hep-ex/0609021] [INSPIRE].

[22] M. Kirk, A. Lenz and T. Rauh, *Dimension-six matrix elements for meson mixing and lifetimes from sum rules*, *JHEP* **12** (2017) 068 [arXiv:1711.02100] [INSPIRE].

[23] F. Gabbiani, A.I. Onishchenko and A.A. Petrov, *Λ_b lifetime puzzle in heavy quark expansion*, *Phys. Rev.* **D 68** (2003) 114006 [hep-ph/0303235] [INSPIRE].

[24] F. Gabbiani, A.I. Onishchenko and A.A. Petrov, *Spectator effects and lifetimes of heavy hadrons*, *Phys. Rev.* **D 70** (2004) 094031 [hep-ph/0407004] [INSPIRE].

[25] M. Beneke, G. Buchalla and I. Dunietz, *Width difference in the B_s-$\bar{B}_s$ system*, *Phys. Rev.* **D 54** (1996) 4419 [*Erratum ibid.* **D 83** (2011) 119902] [hep-ph/9605259] [INSPIRE].

[26] A. Lenz and T. Rauh, *D-meson lifetimes within the heavy quark expansion*, *Phys. Rev.* **D 88** (2013) 034004 [arXiv:1305.3588] [INSPIRE].

[27] M. Neubert, *B decays and CP-violation*, *Int. J. Mod. Phys.* **A 11** (1996) 4173 [hep-ph/9604412] [INSPIRE].

[28] E.E. Jenkins, *Heavy baryon masses in the $1/m_Q$ and $1/N_c$ expansions*, *Phys. Rev.* **D 54** (1996) 4515 [hep-ph/9603449] [INSPIRE].

[29] E.E. Jenkins, *Update of heavy baryon mass predictions*, *Phys. Rev.* **D 55** (1997) 10 [hep-ph/9609404] [INSPIRE].

[30] P. Gambino, K.J. Healey and S. Turczyk, *Taming the higher power corrections in semileptonic B decays*, *Phys. Lett.* **B 763** (2016) 60 [arXiv:1606.06174] [INSPIRE].

[31] A.F. Falk, Z. Ligeti, M. Neubert and Y. Nir, *Heavy quark expansion for the inclusive decay $\bar{B} \to \tau \bar{\nu} X$*, *Phys. Lett.* **B 326** (1994) 145 [hep-ph/9401226] [INSPIRE].

[32] T. Mannel, A.V. Rusov and F. Shahriaran, *Inclusive semitauonic B decays to order* $\mathcal{O}(\Lambda_{\rm QCD}^3/m_b^3)$, *Nucl. Phys.* **B 921** (2017) 211 [`arXiv:1702.01089`] [INSPIRE].

[33] M.A. Shifman and M.B. Voloshin, *On annihilation of mesons built from heavy and light quark and $\bar{B}^0 \leftrightarrow B^0$ oscillations*, *Sov. J. Nucl. Phys.* **45** (1987) 292 [*Yad. Fiz.* **45** (1987) 463] [INSPIRE].

[34] M.B. Voloshin, *Spectator effects in semileptonic decay of charmed baryons*, *Phys. Lett.* **B 385** (1996) 369 [`hep-ph/9604335`] [INSPIRE].

[35] T. Mannel, S. Turczyk and N. Uraltsev, *Higher order power corrections in inclusive B decays*, *JHEP* **11** (2010) 109 [`arXiv:1009.4622`] [INSPIRE].

[36] H.-K. Quang and X.-Y. Pham, *Mass effects in QCD corrections of weak decays*, *Phys. Lett.* **B 122** (1983) 297 [INSPIRE].

[37] H.-K. Quang and X.-Y. Pham, *Exact one gluon corrections for inclusive weak processes*, *Annals Phys.* **155** (1984) 202 [INSPIRE].

[38] Y. Nir, *The mass ratio m_c/m_b in semileptonic B decays*, *Phys. Lett.* **B 221** (1989) 184 [INSPIRE].

[39] T. Becher, H. Boos and E. Lunghi, *Kinetic corrections to $B \to X_c \ell \bar{\nu}$ at one loop*, *JHEP* **12** (2007) 062 [`arXiv:0708.0855`] [INSPIRE].

[40] A. Alberti, T. Ewerth, P. Gambino and S. Nandi, *Kinetic operator effects in $\bar{B} \to X_c \ell \nu$ at $O(\alpha_s)$*, *Nucl. Phys.* **B 870** (2013) 16 [`arXiv:1212.5082`] [INSPIRE].

[41] A. Alberti, P. Gambino and S. Nandi, *Perturbative corrections to power suppressed effects in semileptonic B decays*, *JHEP* **01** (2014) 147 [`arXiv:1311.7381`] [INSPIRE].

[42] T. Mannel, A.A. Pivovarov and D. Rosenthal, *Inclusive weak decays of heavy hadrons with power suppressed terms at NLO*, *Phys. Rev.* **D 92** (2015) 054025 [`arXiv:1506.08167`] [INSPIRE].

[43] M. Beneke and V.M. Braun, *Heavy quark effective theory beyond perturbation theory: renormalons, the pole mass and the residual mass term*, *Nucl. Phys.* **B 426** (1994) 301 [`hep-ph/9402364`] [INSPIRE].

[44] I.I.Y. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, *The pole mass of the heavy quark. Perturbation theory and beyond*, *Phys. Rev.* **D 50** (1994) 2234 [`hep-ph/9402360`] [INSPIRE].

[45] I.I.Y. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, *Sum rules for heavy flavor transitions in the SV limit*, *Phys. Rev.* **D 52** (1995) 196 [`hep-ph/9405410`] [INSPIRE].

[46] I.I.Y. Bigi, M.A. Shifman, N. Uraltsev and A.I. Vainshtein, *High power n of m_b in beauty widths and $n = 5 \to \infty$ limit*, *Phys. Rev.* **D 56** (1997) 4017 [`hep-ph/9704245`] [INSPIRE].

[47] A.H. Hoang, Z. Ligeti and A.V. Manohar, *B decay and the Υ mass*, *Phys. Rev. Lett.* **82** (1999) 277 [`hep-ph/9809423`] [INSPIRE].

[48] F. Krinner, A. Lenz and T. Rauh, *The inclusive decay $b \to c\bar{c}s$ revisited*, *Nucl. Phys.* **B 876** (2013) 31 [`arXiv:1305.5390`] [INSPIRE].

[49] V. Chernyak, *Calculation of the D and B meson lifetimes and the unitarity triangle parameters*, *Nucl. Phys.* **B 457** (1995) 96 [`hep-ph/9503208`] [INSPIRE].

[50] P. Colangelo and F. De Fazio, *Role of four quark operators in the inclusive Λ_b decays*, *Phys. Lett.* **B 387** (1996) 371 [`hep-ph/9604425`] [INSPIRE].

[51] M.S. Baek, J. Lee, C. Liu and H.S. Song, *Four quark operators relevant to B meson lifetimes from QCD sum rules*, *Phys. Rev.* **D 57** (1998) 4091 [`hep-ph/9709386`] [INSPIRE].

[52] H.-Y. Cheng and K.-C. Yang, *Nonspectator effects and B meson lifetimes from a field theoretic calculation*, *Phys. Rev.* **D 59** (1999) 014011 [`hep-ph/9805222`] [INSPIRE].

[53] UKQCD collaboration, M. Di Pierro and C.T. Sachrajda, *A lattice study of spectator effects in inclusive decays of B mesons*, *Nucl. Phys.* **B 534** (1998) 373 [`hep-lat/9805028`] [INSPIRE].

[54] D. Becirevic, *Theoretical progress in describing the B meson lifetimes*, `PoS(HEP2001)098`, (2001) [`hep-ph/0110124`] [INSPIRE].

[55] A.G. Grozin and R.N. Lee, *Three-loop HQET vertex diagrams for B^0-$\bar{B}^0$ mixing*, *JHEP* **02** (2009) 047 [`arXiv:0812.4522`] [INSPIRE].

[56] A.G. Grozin, R. Klein, T. Mannel and A.A. Pivovarov, *B^0-$\bar{B}^0$ mixing at next-to-leading order*, *Phys. Rev.* **D 94** (2016) 034024 [`arXiv:1606.06054`] [INSPIRE].

[57] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [`hep-ph/9512380`] [INSPIRE].

[58] S. Aoki et al., *Review of lattice results concerning low-energy particle physics*, *Eur. Phys. J.* **C 77** (2017) 112 [`arXiv:1607.00299`] [INSPIRE].

[59] B. Guberina, S. Nussinov, R.D. Peccei and R. Rückl, *D meson lifetimes and decays*, *Phys. Lett.* **B 89** (1979) 111 [INSPIRE].

[60] Y. Koide, *Inclusive studies of charmed meson decays and* SU(4) 20-*plet dominance model*, *Phys. Rev.* **D 20** (1979) 1739 [*Erratum ibid.* **D 21** (1980) 853] [INSPIRE].

[61] A. Chodos, R.L. Jaffe, K. Johnson and C.B. Thorn, *Baryon structure in the bag theory*, *Phys. Rev.* **D 10** (1974) 2599 [INSPIRE].

[62] T.A. DeGrand, R.L. Jaffe, K. Johnson and J.E. Kiskis, *Masses and other parameters of the light hadrons*, *Phys. Rev.* **D 12** (1975) 2060 [INSPIRE].

[63] T.A. DeGrand and R.L. Jaffe, *Excited states of confined quarks*, *Annals Phys.* **100** (1976) 425 [INSPIRE].

[64] T.A. DeGrand, *Excited states of confined quarks. 2*, *Annals Phys.* **101** (1976) 496 [INSPIRE].

[65] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, *The B^+-B^0_d lifetime difference beyond leading logarithms*, *Nucl. Phys.* **B 639** (2002) 389 [`hep-ph/0202106`] [INSPIRE].

[66] H.-Y. Cheng and B. Tseng, *Nonleptonic weak decays of charmed baryons*, *Phys. Rev.* **D 46** (1992) 1042 [*Erratum ibid.* **D 55** (1997) 1697] [INSPIRE].

[67] A. De Rujula, H. Georgi and S.L. Glashow, *Hadron masses in a gauge theory*, *Phys. Rev.* **D 12** (1975) 147 [INSPIRE].

[68] J.L. Cortes and J. Sanchez Guillen, *The lifetimes of charmed baryons*, *Phys. Rev.* **D 24** (1981) 2982 [INSPIRE].

[69] J.L. Rosner, *Enhancement of the Λ_b decay rate*, *Phys. Lett.* **B 379** (1996) 267 [`hep-ph/9602265`] [INSPIRE].

[70] D. Ebert, R.N. Faustov and V.O. Galkin, *Spectroscopy and Regge trajectories of heavy baryons in the relativistic quark-diquark picture*, *Phys. Rev.* **D 84** (2011) 014025 [`arXiv:1105.0583`] [INSPIRE].

[71] H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y.C. Lin, T.-M. Yan and H.-L. Yu, *Heavy flavor conserving nonleptonic weak decays of heavy baryons*, *Phys. Rev.* **D 46** (1992) 5060 [INSPIRE].

[72] H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y.-C. Lin, T.-M. Yan and H.-L. Yu, *Heavy-flavor-conserving hadronic weak decays of heavy baryons*, *JHEP* **03** (2016) 028 [arXiv:1512.01276] [INSPIRE].

[73] LHCb collaboration, *Evidence for the strangeness-changing weak decay* $\Xi_b^- \to \Lambda_b^0 \pi^-$, *Phys. Rev. Lett.* **115** (2015) 241801 [arXiv:1510.03829] [INSPIRE].

[74] HFLAV collaboration, Y. Amhis et al., *Averages of b-hadron, c-hadron and τ-lepton properties as of summer* 2016, *Eur. Phys. J.* **C 77** (2017) 895 [arXiv:1612.07233] [INSPIRE].

[75] *Heavy Flavor Averaging Group (HFLAV) online update webpage*, http://www.slac.stanford.edu/xorg/hfag.

[76] A. Le Yaouanc et al., *Hadron transitions in the quark model*, Gordon and Breach, New York, U.S.A., (1987).

[77] G.S. Abrams et al., *Observation of charmed baryon production in e^+e^- annihilation*, *Phys. Rev. Lett.* **44** (1980) 10 [INSPIRE].

[78] BESIII collaboration, M. Ablikim et al., *Measurement of the absolute branching fraction of the inclusive semileptonic Λ_c^+ decay*, arXiv:1805.09060 [INSPIRE].

[79] M.A. Shifman and M.B. Voloshin, *Hierarchy of lifetimes of charmed and beautiful hadrons*, *Sov. Phys. JETP* **64** (1986) 698 [*Zh. Eksp. Teor. Fiz.* **91** (1986) 1180] [INSPIRE].

[80] G. Bellini, I.I.Y. Bigi and P.J. Dornan, *Lifetimes of charm and beauty hadrons*, *Phys. Rept.* **289** (1997) 1 [INSPIRE].

[81] B. Guberina and B. Melić, *Inclusive charmed baryon decays and lifetimes*, *Eur. Phys. J.* **C 2** (1998) 697 [hep-ph/9704445] [INSPIRE].

[82] B. Guberina, B. Melić and H. Stefancic, *Lifetime difference pattern of heavy hadrons*, *Phys. Lett.* **B 484** (2000) 43 [hep-ph/0004264] [INSPIRE].

[83] B. Guberina and H. Stefancic, *Cabibbo suppressed decays and the Ξ_c^+ lifetime*, *Phys. Rev.* **D 65** (2002) 114004 [hep-ph/0202080] [INSPIRE].

[84] A. Babic, B. Guberina, B. Melić and H. Stefancic, *Cabibbo-suppressed decays of the Ω_c^0: feedback to the Ξ_c^+ lifetime*, *Phys. Rev.* **D 70** (2004) 117501 [hep-ph/0406183] [INSPIRE].

[85] LHCb collaboration, *Measurement of the Ω_c^0 baryon lifetime*, *Phys. Rev. Lett.* **121** (2018) 092003 [arXiv:1807.02024] [INSPIRE].

[86] H.Y. Cheng, *Charmed hadronic decay*, talk presented at the *International Workshop on Physics at Future High Intensity Collider* @2–7 *GeV*, http://cicpi.ustc.edu.cn/hiepa2018, Huairou, Beijing, China, 19–21 March 2018.

[87] LHCb collaboration, *Measurement of the lifetime of the doubly charmed baryon* Ξ_{cc}^{++}, *Phys. Rev. Lett.* **121** (2018) 052002 [arXiv:1806.02744] [INSPIRE].