Published for SISSA by Springer

Received: September 23, 2013
Revised: October 31, 2013
Accepted: November 15, 2013
Published: November 25, 2013

Planar AdS black holes in Lovelock gravity with a nonminimal scalar field

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Abstract: In arbitrary dimension D, we consider a self-interacting scalar field nonminimally coupled with a gravity theory given by a particular Lovelock action indexed by an integer k. To be more precise, the coefficients appearing in the Lovelock expansion are fixed by requiring the theory to have a unique AdS vacuum with a fixed value of the cosmological constant. This yields to $k = 1, 2, \cdots, \left[\frac{D-1}{2}\right]$ inequivalent possible gravity theories; here the case $k = 1$ corresponds to the standard Einstein-Hilbert Lagrangian. For each par (D, k), we derive two classes of AdS black hole solutions with planar event horizon topology for particular values of the nonminimal coupling parameter. The first family of solutions depends on a unique constant and is valid only for $k \geq 2$. In fact, its GR counterpart $k = 1$ reduces to the pure AdS metric with a vanishing scalar field. The second family of solutions involves two independent constants and corresponds to a stealth black hole configuration; that is a nontrivial scalar field together with a black hole metric such that both side of the Einstein equations (gravity and matter parts) vanishes identically. In this case, the standard GR case $k = 1$ reduces to the Schwarzschild-AdS-Tangherlini black hole metric with a trivial scalar field. We show that the two-parametric stealth solution defined in D dimension can be promoted to the uniparametric black hole solution in $(D+1)$ dimension by fixing one of the two constants in term of the other and by adding a transversal coordinate. In both cases, the existence of these solutions is strongly inherent of the presence of the higher order curvature terms $k \geq 2$ of the Lovelock gravity. We also establish that these solutions emerge from a stealth configuration defined on the pure AdS metric through a Kerr-Schild transformation. Finally, in the last part, we include multiple exact $(D-1)$−forms homogenously distributed and coupled to the scalar field. For a specific coupling, we obtain black hole solutions for arbitrary value of the nonminimal coupling parameter generalizing those obtained in the pure scalar field case.

Keywords: Classical Theories of Gravity, Black Holes

ArXiv ePrint: 1309.3338

doi:10.1007/JHEP11(2013)177

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Contents

1 Introduction

Since the advent of string theory, the interests on higher-dimensional physics have grown up in the last decades, and particulary concerning the higher-dimensional General Relativity (GR). String corrections to the standard higher-dimensional Einstein-Hilbert action arise as a low energy expansion in powers in α' and involve higher powers of the curvatures. As shown in [1] and [2], in order to the graviton amplitude to be ghost-free a special combination of quadratic corrections which is nothing but the Gauss-Bonnet expression is required. One of the interesting features of the Gauss-Bonnet Lagrangian lies in the fact that its variation yields second-order field equations for the metric in spite of the presence of quadratic terms in the curvature. The Einstein-Hilbert-Gauss-Bonnet gravity piece is part of a more general gravity theory build out of the same principles as GR. Indeed, two of the main fundamental assumptions in GR are the requirements of general covariance and the fact that the field equations for the metric to be at most of second order. In view of this, it is natural to describe the spacetime geometry in three or four dimensions by the standard Einstein-Hilbert action (with eventually a cosmological constant term) while for dimensions greater than four a more general theory can be used. This fact has been first noticed by Lanczos in five dimensions [3] and then generalized in higher dimension D by Lovelock [4]. The resulting action is the so-called Lovelock gravity action which is a $D-$form constructed out of the vielbein, the spin connection and their exterior derivative. By construction, the Lovelock Lagrangian which contains higher powers of the curvatures remains invariant under local Lorentz transformations. In odd dimension, this gauge symmetry can be extended to a local anti de Sitter (AdS) or Poincaré symmetry through a particular choice of the coefficients appearing in the Lovelock expansion. In both cases, the resulting Lagrangian is a Chern-Simons form since its exterior derivative is an invariant homogeneous polynomial in the AdS or Poincaré curvatures, and their supersymmetric extensions are also known; see [5] for a good review on Chern-Simons (super)gravity. The Lovelock gravity or its Chern-Simons particular case have been shown to possess (topological) AdS black hole solutions with interesting thermodynamical properties [6–9] generalizing those obtained in the Einstein-Gauss-Bonnet case [10, 11]; for good reviews on Einstein-Gauss-Bonnet black holes, see e.g. [12–14].

In the present work, we will consider a gravity action given by a particular Lovelock Lagrangian with fixed coefficients such that the resulting theory has a unique anti-de Sitter vacuum with a fixed cosmological constant while our matter action will be concerned with a nonminimal self-interacting scalar field. For this Lovelock gravity model with nonminimal scalar field source, we will look for topological black hole solutions with planar event horizon topology. Note that the first examples of topological black holes in GR without source were discussed in [15, 16]. The reasons of considering such a matter source are multiples. Firstly, the ideas behind the anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [17] have been recently extended to non-relativistic physics particularly to gain a better understanding of some unconventional superconductors [18–20]. In this context, black holes with scalar hair at low temperature which disappears at low temperature play an important role since they will reproduce the correct behavior of the superconductor phase diagram. It is well-known now from the BBMB solution (solution of the Einstein equations with a conformal source given by a scalar field) in four dimensions [21, 22] that scalar fields nonminimally coupled can be useful to escape standard no-hair theorems [23]. Note that the BBMB solution has been extended in presence of a cosmological constant with a potential term in four dimensions and for the conformal nonminimal coupling parameter $\xi = 1/6$, [24–27]. In fact, scalar fields nonminimally coupled to curved spacetimes play an important role in different branches of physics and are also of interest for pure mathematical proposals (as for example for the Yamabe problem). The introduction of nonminimal couplings in spite of complicating the calculations may be of extreme relevance for many problems. For example, the nonminimal couplings are generated by quantum corrections even if they are absent in the classical action [28], and they are required in order to renormalize the theory or at least to enhance their renormalizability properties [29–31]. In cosmological context, it has been argued that in most of the inflationary scenarios with scalar fields, the presence of the nonminimal coupling is unavoidable and its correct value depends on the gravity and the scalar field models adopted [32, 33]. Secondly, we have already considered such matter source in the case of a particular combination of the Einstein-Hilbert-Gauss-Bonnet gravity action and establish the existence of some black hole configurations for particular values of the nonminimal coupling parameter [34]. The present work is then the natural extension of the work done in [34] in order to reinforce our conviction that these black hole solutions are strongly inherent to the presence of the higher-order curvature terms.

The plan of the paper is organized as follows. In the next section, we present the model of a scalar field nonminimally coupled with a gravity action given by a particular

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Lovelock Lagrangian. After deriving the field equations, we will present two classes of topological black hole solutions with planar base manifold. In section 3, we will add to the starting action exact $(D-1)-$forms and obtain a more general class of black hole solutions generalizing those obtained in the pure scalar field case. Finally, the last section is devoted to our conclusions, comments and further works. Two appendices are also added. In the first one, we show that these black hole solutions can be constructed from a stealth configuration on the pure AdS metric through a Kerr-Schild transformation. In the second appendix, we establish that the existence of these solutions is inherent to the higher-order curvature terms $k \geq 2$ of the Lovelock Lagrangian and they can not be promoted to black hole solutions in the standard GR case $k = 1$.

2 Planar AdS black holes for a particular Lovelock gravity with a non-minimal scalar field

We start with a generalization of the Einstein-Hilbert gravity action in arbitrary dimension D yielding at most to second-order field equations for the metric and known as the Lovelock Lagrangian. This latter is a $D-$form constructed with the vielbein e^a, the spin connection ω^{ab}, and their exterior derivatives without using the Hodge dual. The Lovelock action is a polynomial of degree $[D/2]$ (where $[x]$ denotes the integer part of x) in the curvature two-form, $R^{ab} = d\,\omega^{ab} + \omega^a_{\ c} \wedge \omega^{cb}$ as

$$\int \sum_{p=0}^{[D/2]} \alpha_p \; L^{(p)}, \tag{2.1a}$$

$$L^{(p)} = \epsilon_{a_1 \cdots a_d} R^{a_1 a_2} \cdots R^{a_{2p-1} a_{2p}} e^{a_{2p+1}} \cdots e^{a_d}, \tag{2.1b}$$

where the α_p are arbitrary dimensionful coupling constants and where wedge products between forms are understood. Here $L^{(0)}$ and $L^{(1)}$ are proportional respectively to the cosmological term and the Einstein-Hilbert Lagrangian. Now, as shown in ref. [8], requiring the Lovelock action to have a unique AdS vacuum with a unique cosmological constant, fixes the α_p yielding to a series of actions indexed by an integer k, and given by

$$I_k = -\frac{1}{2k(D-3)!} \int \sum_{p=0}^{k} \frac{C_p^k}{(D-2p)} \; L^{(p)}, \qquad 1 \leq k \leq \left[\frac{D-1}{2}\right], \tag{2.2}$$

where C_p^k corresponds to the combinatorial factor. The global factor in front of the integral is chosen such that the gravity action (2.2) can be re-written in the standard fashion as

$$I_k = \frac{1}{2} \int d^D x \sqrt{-g} \Big[\; R + \frac{(D-1)(D-2)}{k} + \frac{(k-1)}{2(D-3)(D-4)} \mathcal{L}_{\rm GB} + \frac{(k-1)(k-2)}{3!(D-3)(D-4)(D-5)(D-6)} \mathcal{L}_{(3)} + \cdots \Big], \tag{2.3}$$

where $\mathcal{L}_{GB} = R^2 - 4\, R_{\mu\nu} R^{\mu\nu} + R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu}$ stands for the Gauss-Bonnet Lagrangian, and $\mathcal{L}_{(3)}$ is given by

$$\begin{aligned}\mathcal{L}_{(3)} \,=\, & R^3 - 12 R R_{\mu\nu} R^{\mu\nu} + 16\, R_{\mu\nu} R^{\mu}{}_{\rho} R^{\nu\rho} + 24 R_{\mu\nu} R_{\rho\sigma} R^{\mu\rho\nu\sigma} + 3 R R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ & - 24 R_{\mu\nu} R^{\mu}{}_{\rho\sigma\kappa} R^{\nu\rho\sigma\kappa} + 4 R_{\mu\nu\rho\sigma} R^{\mu\nu\eta\zeta} R^{\rho\sigma}{}_{\eta\zeta} - 8 R_{\mu\rho\nu\sigma} R^{\mu}{}_{\eta}{}^{\nu}{}_{\zeta} R^{\rho\eta\sigma\zeta}.\end{aligned}$$

Note that in odd dimension $D = 2n - 1$ and for $k = n - 1$, the corresponding action I_{n-1} is a Chern-Simons action, that is a $(2n-1)-$form whose exterior derivative can be written as the contraction of an invariant tensor with the wedge product of n curvatures two-forms. In even dimension $D = 2n$, the maximal value of k is $n - 1$, and in this case the resulting gravity action has a Born-Infeld like structure since it can be written as the Pfaffian of the $2-$form $\bar{R}^{ab} = R^{ab} + e^a e^b$. The gravity theories I_k have been shown to possess black hole solutions with interesting features, in particular concerning their thermodynamics properties, see [8] and [9]. In what follows, we will consider a scalar field nonminimally coupled together with the gravity actions given by I_k, (2.2). More precisely, we will consider the following action for any integer $k \geq 2$,[1]

$$S_k = I_k - \int d^D x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{\xi}{2} R \Phi^2 + U(\Phi) \right], \tag{2.4}$$

The field equations read

$$\mathcal{G}^{(k)}_{\mu\nu} = T_{\mu\nu}, \tag{2.5a}$$

$$\Box \Phi = \xi R \Phi + \frac{dU}{d\Phi}, \tag{2.5b}$$

where $\mathcal{G}^{(k)}_{\mu\nu}$ is the gravity tensor associated to the variation of the action I_k (2.2),

$$\begin{aligned}\mathcal{G}^{(k)}_{\mu\nu} \,=\, & G_{\mu\nu} - \frac{(D-1)(D-2)}{2k} g_{\mu\nu} + \frac{(k-1)}{2(D-3)(D-4)} K_{\mu\nu} + \\ & \frac{(k-1)(k-2)}{3!(D-3)(D-4)(D-5)(D-6)} S_{\mu\nu} + \cdots\end{aligned}$$

where $K_{\mu\nu}$ is the Gauss-Bonnet tensor

$$K_{\mu\nu} = 2\big(R R_{\mu\nu} - 2 R_{\mu\rho} R^{\rho}{}_{\nu} - 2 R^{\rho\sigma} R_{\mu\rho\nu\sigma} + R_{\mu}{}^{\rho\sigma\gamma} R_{\nu\rho\sigma\gamma}\big) - \frac{1}{2}\, g_{\mu\nu} \mathcal{L}_{GB}$$

and $S_{\mu\nu}$ arises from the variation of $\mathcal{L}_{(3)}$,

$$\begin{aligned}S_{\mu\nu} \,=\, & 3\Big(R^2 R_{\mu\nu} - 4 R R_{\rho\mu} R^{\rho}{}_{\nu} - 4 R^{\rho\sigma} R_{\rho\sigma} R_{\mu\nu} + 8 R^{\rho\sigma} R_{\rho\mu} R_{\sigma\nu} - 4 R R^{\rho\sigma} R_{\rho\mu\sigma\nu} \\ & + 8 R^{\rho\kappa} R^{\sigma}{}_{\kappa} R_{\rho\mu\sigma\nu} - 16 R^{\rho\sigma} R^{\kappa}{}_{(\mu} R_{|\kappa\sigma\rho|\nu)} + 2 R R^{\rho\sigma\kappa}{}_{\mu} R_{\rho\sigma\kappa\nu} + R_{\mu\nu} R^{\rho\sigma\kappa\eta} R_{\rho\sigma\kappa\eta} \\ & - 8 R^{\rho}{}_{(\mu} R^{\sigma\kappa\eta}{}_{|\rho|} R_{|\sigma\kappa\eta|\nu)} - 4 R^{\rho\sigma} R^{\kappa\eta}{}_{\rho\mu} R_{\kappa\eta\sigma\nu} + 8 R_{\rho\sigma} R^{\rho\kappa\sigma\eta} R_{\kappa\mu\eta\nu} - 8 R_{\rho\sigma} R^{\rho\kappa\eta}{}_{\mu} R^{\sigma}{}_{\kappa\eta\nu} \\ & + 4 R^{\rho\sigma\kappa\eta} R_{\rho\sigma\zeta\mu} R_{\kappa\eta}{}^{\zeta}{}_{\nu} - 8 R^{\rho\kappa\sigma\eta} R^{\zeta}{}_{\rho\sigma\mu} R_{\zeta\kappa\eta\nu} - 4 R^{\rho\sigma\kappa}{}_{\eta} R_{\rho\sigma\kappa\zeta} R^{\eta}{}_{\mu}{}^{\zeta}{}_{\nu}\Big) - \frac{1}{2} g_{\mu\nu} \mathcal{L}_{(3)}.\end{aligned}$$

[1]The standard GR case $k = 1$ will be discussed in the appendix.

In the matter part of the equations (2.5), $T_{\mu\nu}$ represents the energy-momentum tensor of the scalar field given by

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma \Phi \partial^\sigma \Phi + U(\Phi)\right) + \xi\left(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu + G_{\mu\nu}\right)\Phi^2, \qquad (2.6)$$

where the potential $U(\Phi)$ is given by a mass term

$$U(\Phi) = \frac{8\,\xi\,D\,(D-1)}{(1-4\xi)^2}(\xi - \xi_D)(\xi - \xi_{D+1})\Phi^2, \qquad (2.7)$$

where ξ_D denotes the conformal coupling in D dimensions

$$\xi_D = \frac{D-2}{4(D-1)}. \qquad (2.8)$$

The choice of such potential will be justified in the appendix. Being a mass term and because of the presence of the term $\xi R \Phi^2$ in the action, one can define an effective mass m^2_{eff} in the case of solutions of constant curvature $R = \text{constant}$ by

$$m^2_{\text{eff}} = \xi R + \frac{16\,\xi\,D\,(D-1)}{(1-4\xi)^2}(\xi - \xi_D)(\xi - \xi_{D+1}). \qquad (2.9)$$

As for the Einstein-Gauss-Bonnet case $k = 2$ [34], we will obtain the same two classes of black hole solutions for generic value of $k \geq 2$. More precisely, for each par (D, k) with $D \geq 5$ and $k \geq 2$, we will derive two classes of AdS black hole solutions with planar event horizon topology for specific values of the nonminimal coupling parameter ξ.

2.1 Planar AdS black hole solutions

For $k \geq 2$, an AdS black hole solution is obtained provided that the nonminimal parameter ξ takes the following form

$$\xi^{\text{b.h}}_{k,D} = \frac{(D-2)(k-1)}{4\Big[(D-1)k - (D-2)\Big]}, \qquad (2.10)$$

which in turn implies that the potential (2.7) becomes

$$U^{\text{b.h}}_{k,D}(\Phi) = \frac{(k-1)(D-2)^2(D-2+k)}{8k^2\Big[(D-1)k - (D-2)\Big]}\Phi^2. \qquad (2.11)$$

In this case, the metric solution and the scalar field are given by

$$\begin{aligned}
ds^2 &= -F^{\text{b.h}}_{k,D}(r)dt^2 + \frac{dr^2}{F^{\text{b.h}}_{k,D}(r)} + r^2 d\vec{x}^2_{D-2}, \\
F^{\text{b.h}}_{k,D}(r) &= r^2 - \frac{M}{r^{\frac{D-2(k+1)}{k}}}, \\
\Phi^{\text{b.h}}_{k,D}(r) &= M^{\frac{k-1}{2}}\sqrt{\frac{4\left[(D-1)k - (D-2)\right]}{(k-1)(D-2)}}\, r^{\frac{(k-1)(2-D)}{2k}}.
\end{aligned} \qquad (2.12)$$

Many comments can be made concerning this solution. Firstly, this black hole solution depends on a unique integration constant M, and for even k the scalar field is always real provided that M is positive constant while for odd k, the constant M can be positive or negative. The scalar field is well defined at the horizon and blows-up at the singularity $r = 0$. The solution given by (2.10), (2.11), (2.12) reduces to the one derived in the Einstein-Gauss-Bonnet case for $k = 2$ [34]. It is interesting to note that the standard GR-limit $k = 1$ (which yields to Einstein gravity with a negative cosmological constant) is possible only in the limit $M = 0$ as it can be seen from the expression of the scalar field. This is not surprising since for $k = 1$, the nonminimal coupling parameter (2.10) as well as the potential (2.11) vanish, and in this case, no-hair theorems forbid the existence of black hole solutions. We will come in detail to this point in the appendix. We would like also to emphasize that the allowed value of the nonminimal coupling parameter (2.10) which depends on (k, D) is bounded as $\xi_{k,D}^{\text{b.h}} < 1/4$ and its limit as k goes to infinity yields to the conformal coupling in D dimensions, $\lim_{k\to\infty} \xi_{k,D}^{\text{b.h}} = \xi_D$. Finally, we may observe that in even dimension given by $D = 2(k + 1)$ which corresponds to the Born-Infeld case, the lapse function $F_{k,D}^{\text{b.h}}(r)$ has a BTZ-like form [35] as it occurs in the vacuum case with a base manifold chosen to be non-Einstein [36].

2.2 Planar AdS black hole stealth solutions

The black hole stealth solution for $k \geq 2$ can be obtained in analogue way that the one obtained in the Einstein-Gauss-Bonnet case $k = 2$ [34]; this means by combining the pure gravity solution with planar base manifold [8, 9] with the stealth configuration. By stealth configuration, we mean a non-trivial solution (with a non constant scalar field) of the stealth equations

$$\mathcal{G}_{\mu\nu}^{(k)} = 0 = T_{\mu\nu}, \tag{2.13}$$

where both side (gravity and matter part) vanishes.

In fact, it is not difficult to show that a self-interacting nonminimal scalar field given by

$$\Phi(r) = A\, r^{\frac{2\xi}{4\xi-1}}, \tag{2.14}$$

has a vanishing energy-momentum tensor (2.6) $T_{\mu\nu} = 0$ on the following $\xi-$dependent spacetime geometry

$$ds^2 = -\left(r^2 - \frac{M}{r^{\frac{4(D-2)\xi-(D-3)}{4\xi-1}}}\right)dt^2 + \frac{dr^2}{\left(r^2 - \frac{M}{r^{\frac{4(D-2)\xi-(D-3)}{4\xi-1}}}\right)} + r^2 d\vec{x}_{D-2}^2. \tag{2.15}$$

On the other hand, the pure gravity equations $\mathcal{G}_{\mu\nu}^{(k)} = 0$ has a black hole solution with a lapse function $F(r)$ given by [8, 9]

$$F(r) = r^2 - \frac{M}{r^{\frac{D-(2k+1)}{k}}}.$$

Now, in order for this metric function to coincide with the stealth metric (2.15), the non-minimal coupling parameter must be chosen as

$$\xi_{k,D}^{\text{stealth}} = \frac{(D-1)(k-1)}{4\left[Dk-(D-1)\right]}, \tag{2.16}$$

and hence the mass term potential (2.7) becomes

$$U_{k,D}^{\text{stealth}}(\Phi) = \frac{(k-1)(D-1)^2(D-1-k)}{8k^2\left[Dk-(D-1)\right]}\Phi^2. \tag{2.17}$$

Consequently, a topological black hole stealth solution of the stealth equation (2.13) is given by

$$\begin{aligned} ds^2 &= -F_{k,D}^{\text{stealth}}(r)dt^2 + \frac{dr^2}{F_{k,D}^{\text{stealth}}(r)} + r^2 d\vec{x}_{D-2}^2, \\ F_{k,D}^{\text{stealth}}(r) &= r^2 - \frac{M}{r^{\frac{D-(2k+1)}{k}}}, \\ \Phi_{k,D}^{\text{stealth}}(r) &= A r^{\frac{(k-1)(1-D)}{2k}}. \end{aligned} \tag{2.18}$$

We may note that in contrast with the previous solution, the black hole stealth solution depends on two integration constants M and A, and in the vanishing M limit, the solution reduces to a stealth solution on the pure AdS background [37]. The GR limit $k=1$ is also well defined yielding to a metric that is noting but the topological Schwarzschild-AdS-Tangherlini spacetime. This is not surprising since in the GR limit case $k=1$, the nonminimal coupling parameter as well as the potential vanish while the scalar field becomes constant, and hence the energy-momentum tensor vanishes $T_{\mu\nu}=0$. In other words, in the GR-limit, the stealth equations (2.13) are equivalent to the pure Einstein equations $G_{\mu\nu}-\frac{(D-1)(D-2)}{2}g_{\mu\nu}=0$. This class of solutions is of particular interest since, up to now, the only black hole stealth solution was the one obtained in [38] in the three-dimensional GR case with a static BTZ metric [35].

Let us go back to the $\xi-$dependent geometry (2.15) allowing the existence of solution of $T_{\mu\nu}=0$. It is clear from the expression of the metric that for a nonminimal coupling parameter $\xi\in[\xi_{D+1},\frac{1}{4}[$ where $\xi_{D+1}=(D-1)/(4D)$ is the conformal coupling in $(D+1)$ dimension, the asymptotic behavior of the metric as $r\to\infty$ is faster than the usual AdS one. However, requiring the metric to be also solution of the gravity part $\mathcal{G}_{\mu\nu}^{(k)}=0$, we have seen that the parameter ξ must take the form (2.16), and it is not difficult to prove that $\xi_{k,D}^{\text{stealth}}<\xi_{D+1}$. To conclude, we would like to point out a certain symmetry between the black hole and stealth solution as reflected by the following relations

$$\xi_{k,D}^{\text{stealth}}=\xi_{k,D+1}^{\text{b.h}}, \quad F_{k,D}^{\text{stealth}}=F_{k,D+1}^{\text{b.h}}, \quad U_{k,D}^{\text{stealth}}=U_{k,D+1}^{\text{b.h}}, \quad \Phi_{k,D}^{\text{stealth}}\propto\Phi_{k,D+1}^{\text{b.h}}.$$

These relations can also be interpreted as follows: a particular two-parametric stealth solution in D dimension given by (2.18) but with a constant A fixed in term of M as

$$A = M^{\frac{k-1}{2}}\sqrt{\frac{4\left[(D-1)k-(D-2)\right]}{(k-1)(D-2)}}$$

can be promoted to the uniparametric black hole solution in $(D+1)$ dimension (2.12).

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3 Adding exact $(D-1)$-forms

In the previous section, we have constructed two classes of topological black hole solutions for a self-interacting nonminimal scalar field with a gravity theory given by a particular Lovelock action. The base manifold of these solutions is planar and these configurations require a particular value of the nonminimal coupling parameter (2.10)–(2.16) which depends on the dimension D and the gravity theory $k \geq 2$. As it will be shown in the appendix, the existence of these solutions is strongly inherent to the presence of the higher-order curvature terms of the Lovelock theory. Indeed in the standard GR case $k = 1$, we will establish that black hole solutions with planar base manifold for a scalar field nonminimally coupled with a possible mass term potential are only possible in three dimensions yielding to the Martinez-Zanelli solution [39].[2] In the standard GR case, it has been shown recently that the inclusion of multiple exact $p-$forms homogenously distributed permits the construction of black holes with planar horizon [40, 41] without any restrictions on the dimension or on the value of the nonminimal parameter [42]. Since, we are interested on such solutions, we now propose to introduce appropriately some exact $p-$forms in order to obtain topological black hole solutions with arbitrary nonminimal coupling parameter. More precisely, we consider the following action in arbitrary D dimension

$$
\begin{aligned}
\mathcal{S}_k = & -\frac{1}{2k(D-3)!}\int \sum_{p=0}^{k} \frac{C_p^k}{(D-2p)}\, L^{(p)} - \int d^Dx\,\sqrt{-g}\left[\frac{1}{2}\,\partial_\mu\Phi\partial^\mu\Phi + \frac{\xi}{2}R\Phi^2 + U(\Phi)\right] \\
& - \int d^Dx\,\sqrt{-g}\left[\frac{\epsilon(\Phi)}{2(D-1)!}\sum_{i=1}^{D-2}\mathcal{H}^{(i)}_{\alpha_1\cdots\alpha_{D-1}}\mathcal{H}^{(i)\alpha_1\cdots\alpha_{D-1}}\right],
\end{aligned}
$$

where we have introduced $(D-2)$- fields $\mathcal{H}^{(i)}$ which are exact $(D-1)$-forms, and where the potential is again the mass term defined in (2.7). The coupling function between the scalar field and the $(D-1)$-forms, $\epsilon(\Phi)$, depends on the scalar field Φ as

$$
\epsilon(\Phi) = \sigma\,\Phi^{\frac{2(2-3k)\xi+k-1}{\xi(k-1)}} \tag{3.1}
$$

where σ is a coupling constant. We stress that the expression of this coupling ϵ is not well-defined in the standard GR case $k = 1$. However, as mentioned before, the solutions in the standard Einstein gravity have been obtained in [42] for a more general class of potential than the one considered here (2.7). Note that there exists another particular value of the nonminimal coupling parameter $\xi = \frac{k-1}{2(3k-2)}$ for which the coupling ϵ becomes constant; we will come to this point below.

The field equations obtained by varying the action with the different dynamical fields $g_{\mu\nu}, \Phi$ and $\mathcal{H}^{(i)}$ read

$$
\mathcal{G}^{(k)}_{\mu\nu} = T_{\mu\nu} + T^{\text{extra}}_{\mu\nu}, \qquad \partial_\alpha\left(\sqrt{-g}\,\epsilon(\Phi)\mathcal{H}^{(i)\alpha\alpha_1\cdots\alpha_{D-2}}\right) = 0, \tag{3.2a}
$$

$$
\Box\Phi = \xi R\Phi + \frac{dU}{d\Phi} + \frac{1}{2}\frac{d\epsilon}{d\Phi}\left[\sum_{i=1}^{D-2}\frac{1}{(D-1)!}\mathcal{H}^{(i)}_{\alpha_1\cdots\alpha_{D-1}}\mathcal{H}^{(i)\alpha_1\cdots\alpha_{D-1}}\right] = 0, \tag{3.2b}
$$

[2] We will obtain this result by considering an Ansatz for the metric that depends on a unique lapse function.

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where the extra piece in the energy-momentum tensor reads

$$T_{\mu\nu}^{\rm extra}=\epsilon(\Phi)\sum_{i=1}^{D-2}\left[\frac{1}{(D-2)!}\mathcal{H}^{(i)}_{\mu\alpha_1\cdots\alpha_{D-2}}\mathcal{H}_{\nu}^{(i)\alpha_1\cdots\alpha_{D-2}}-\frac{g_{\mu\nu}}{2(D-1)!}\mathcal{H}^{(i)}_{\alpha_1\cdots\alpha_{D-1}}\mathcal{H}^{(i)\alpha_1\cdots\alpha_{D-1}}\right]$$

Looking for a purely electrically homogenous Ansatz for the $(D-1)-$forms as

$$\mathcal{H}^{(i)}=\mathcal{H}^{(i)}_{trx_1\cdots x_{i-1}x_{i+1}\cdots x_{D-2}}(r)dt\wedge dr\wedge\cdots\wedge dx^{i-1}\wedge dx^{i+1}\wedge\cdots\wedge dx^{D-2}, \tag{3.3}$$

a solution of the field equations (3.2) is given by

$$ds^2=-\left(r^2-\frac{M}{r^{\frac{2[2(2k-1)\xi-(k-1)]}{(k-1)(1-4\xi)}}}\right)dt^2+\frac{dr^2}{\left(r^2-\frac{M}{r^{\frac{2[2(2k-1)\xi-(k-1)]}{(k-1)(1-4\xi)}}}\right)}+r^2d\vec{x}^2_{D-2}, \tag{3.4a}$$

$$\Phi(r)=\sqrt{\frac{M^{k-1}\left[2\left(3\,k-2\right)\xi-k+1\right]\left(D-2\right)}{\left\{2\left[2(D-1)k-D\right]\xi-(k-1)(D-2)\right\}k\xi}}\;r^{\frac{2\xi}{4\xi-1}}, \tag{3.4b}$$

$$\mathcal{H}^{(i)}=\frac{p}{\epsilon(\Phi)}r^{D-4}dt\wedge dr\wedge\cdots\wedge dx^{i-1}\wedge dx^{i+1}\wedge\cdots\wedge dx^{D-2}, \tag{3.4c}$$

where the constant p is defined by

$$p=B\sqrt{\frac{-2\,\sigma\left(\xi-\xi_{k,D}^{\rm stealth}\right)\left(\xi-\xi_{k,D}^{\rm b.h}\right)\Big[Dk-(D-1)\Big]\Big[(D-1)k-(D-2)\Big]}{k-1}}, \tag{3.5}$$

and where the constants $\xi_{k,D}^{\rm b.h}$ and $\xi_{k,D}^{\rm stealth}$ are the particular values of the nonminimal parameter for which we have derived the previous solutions in the pure scalar field case (2.10)–(2.16). In this last expression, the constant B reads

$$B=\frac{4\,\xi^{\frac{2(4\,k-3)\xi-k+1}{4\xi\,(k-1)}}\left\{\Big[2\Big(2(D-1)k-D\Big)\xi-(k-1)(D-2)\Big]k\right\}^{\frac{2(2\,k-1)\xi-k+1}{4\xi\,(k-1)}}M^{\frac{(k-1)(1-4\,\xi)}{4\xi}}}{(4\,\xi-1)\left\{\Big[2\left(3\,k-2\right)\xi-k+1\Big]\left(D-2\right)\right\}^{\frac{2(3\,k-2)\xi-k+1}{4\xi\,(k-1)}}}$$

As in the pure scalar field case, many comments can be made concerning the solution obtained in the presence of these $(D-2)$ extra $(D-1)$-forms $\mathcal{H}^{(i)}$. Firstly, it is simple to see that for $\xi=\xi_{k,D}^{\rm stealth}$ or $\xi=\xi_{k,D}^{\rm b.h}$ the constant p becomes zero and the solutions are those found previously considering only a scalar field nonminimally coupled with a mass term potential. From the expression of the metric solution, we can see that for $\xi>1/4$, the asymptotic behavior of the metric is faster than the usual AdS one while for $\xi<1/4$, the dominant term as $r\to\infty$ is given by $F(r)\sim r^2$. From the expression of the scalar field, it is easy to see that for a constant $M>0$, the allowed values of ξ in order to deal with a real solution are

$$\xi\in\,]0,\frac{k-1}{2(3k-2)}]\cup]\xi_{\rm critical},+\infty[,\qquad\quad \xi_{\rm critical}:=\frac{(k-1)(D-2)}{2\big[2(D-1)k-D\big]},$$

while for $M < 0$, the ranges are

$$\xi \in]0, \frac{k-1}{2(3k-2)}] \cup]\xi_{\text{critical}}, +\infty[, \qquad \text{for odd } k,$$
$$\xi \in [\frac{k-1}{2(3k-2)}, \xi_{\text{critical}}[, \qquad \text{for even } k.$$

Solutions of constant scalar curvature $R = -D(D-1)$ are obtained for two values of the nonminimal parameter

$$\xi = \frac{(k-1)D}{4(D(k-1)+1)}, \qquad \xi = \frac{(k-1)(D-1)}{4[(D-1)k-(D-2)]}.$$

For this last value of the parameter ξ, the effective square mass (2.9) becomes

$$m_{\text{eff}}^2 = \frac{(k-1)(k-3)(D-1)^2}{4},$$

and it is intriguing to note that it saturates the Breitenlohner-Freedman bound for $k = 2$ [34] while for $k = 3$, the solution becomes massless.

Pure axionic solution: in order to be complete, we may look for pure axionic solutions. This means a solution of the field equations without considering the contribution of the scalar field,

$$\mathcal{G}_{\mu\nu}^{(k)} = T_{\mu\nu}^{\text{extra}}. \tag{3.6}$$

Considering an Ansatz for the metric involving a unique metric function, the integration of the field equations yields

$$ds^2 = -\left(r^2 - Mr^{\frac{2(k-1)}{k}}\right)dt^2 + \frac{dr^2}{\left(r^2 - Mr^{\frac{2(k-1)}{k}}\right)} + r^2 d\vec{x}_{D-2}^2, \tag{3.7a}$$

$$\mathcal{H}^{(i)} = -M^{\frac{k}{2}}\sqrt{\frac{D-3}{k\sigma}}\, r^{D-4} dt \wedge dr \wedge \cdots \wedge dx^{i-1} \wedge dx^{i+1} \wedge \cdots \wedge dx^{D-2}. \tag{3.7b}$$

We note that this solution can be obtained from the solutions with scalar field (3.4) by taking the well-defined limit $\xi = \frac{k-1}{2(3k-2)}$. This is not surprising owing to the choice of our coupling function ϵ defined in (3.1) which becomes constant for $\xi = \frac{k-1}{2(3k-2)}$. It is also interesting to note that in this case, since the contribution of the scalar field is not longer present, the GR limit $k = 1$ is well-defined yielding a metric function of the BTZ form [40].

4 Comments and conclusions

Here, we have considered a gravity theory given by a particular Lovelock Lagrangian labeled by an integer k for which the coefficients are fixed in order to have an unique AdS vacuum with a fixed value of the cosmological constant. The matter part of our action is concerned with a self-interacting scalar field nonminimally coupled with a potential given by a mass term. For this model labeled by the dimension D and the integer k, we have derived two classes of black hole solutions with planar event horizon topology for particular values of

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the nonminimal coupling parameter depending on D and k. The first class of solutions is uniparametric and reduces to the pure AdS metric without scalar field in the vanishing limit of the parameter. The second class of solutions depends on two parameters and is interpreted as a black hole stealth configuration. To be more precise, we have shown the existence of a nontrivial self-interacting scalar field with a vanishing energy-momentum tensor with a black hole metric solving the pure gravity equations. In the last section, we have added to the starting action exact $(D-1)-$forms minimally coupled to the scalar field. In this case and for an appropriate coupling, we have been able to construct more general black hole solutions with planar event horizon topology. All these solutions generalize for an arbitrary $k \geq 2$ those obtained in [34] in the Einstein-Gauss-Bonnet case $k = 2$. In the appendix, we have established that these solutions may be viewed as originated from a stealth configuration on a pure AdS background through a Kerr-Schild transformation. We have also shown that their standard GR counterpart $k = 1$ can not be obtained along the same lines, and hence the occurrence of such solutions is strongly inherent to the presence of the higher-order curvature terms of the Lovelock gravity theory. It seems then that the emergence of these black hole solutions with planar event horizon topology is a consequence of the higher-order curvature terms combined with the existence of a stealth configuration on the pure AdS metric. Indeed, as shown in [37], static stealth configurations given by a scalar field nonminimally coupled require the base manifold to be planar. Indeed, stealth solutions with spherical or hyperboloid base manifold are possible only in the non static case [37]. It will be interesting to see whether these non static stealth configurations with spherical or hyperboloid base manifold can be promoted as black hole solutions through a similar Kerr-Schild transformation. In this case, since the scalar field depends explicitly on the time as well as on the radial coordinates, the metric function generated through the Kerr-Schild transformation must probably depend also on these two coordinates. This will considerably complicate the task of integrating the metric function.

In this paper, we have also derived a class of black hole stealth configuration whose metric is a black hole solution of the pure gravity equations [8, 9]. These metrics can be promoted to electrically charged black hole solutions with a standard Maxwell source [8, 9]. It is then natural to ask whether one can derive the electrically charged version of the solutions found here. As a first task, it will be useful to derive, if possible, the electrically charged version of the stealth configuration found in [37].

We have seen that the existence of these solutions is strongly inherent to the higher-order curvature terms of the Lovelock theories. As it is well-known, the field equations associated to the Lovelock gravity are of second order in spite of the presence of these terms. In ref. [43], a cubic gravity theory has been constructed in five dimensions by requiring the trace of the field equations to be proportional to the Lagrangian which in turn implies that for an Ansatz metric of the "spherical" form, the field equations are of second-order. It will be interesting to explore if this five-dimensional cubic gravity theory and its generalizations to higher odd dimension can accommodate the classes of solutions found here.

In ref. [8, 9], the authors have done a complete study of the thermodynamical properties of the pure gravity solutions. The black hole stealth solutions obtained here have

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the same lapse metric with a nontrivial matter source, and hence it will be more than interesting to investigate the effects on the thermodynamical quantities of the presence of this nontrivial source. Note that the thermodynamics of general Lovelock gravity has been analyzed in [44].

As a final remark, in ref. [45], the authors constructed conformal coupling to arbitrary higher-order Euler densities. It will be interesting to see whether such matter source can accommodate the kind of solutions derived here.

Acknowledgments

We thank Julio Oliva and Sourya Ray for useful discussions. MB is supported by BECA DOCTORAL CONICYT 21120271. MH was partially supported by grant 1130423 from FONDECYT, by grant ACT 56 from CONICYT and from CONICYT, Departamento de Relaciones Internacionales "Programa Regional MATHAMSUD 13 MATH-05".

A Complements concerning the solutions

A.1 Stealth origin of the solutions

In this appendix, we will show that the AdS black hole solutions obtained here can be viewed as originated from a stealth configuration defined on the pure AdS metric through a Kerr-Schild transformation.

Let us start with a self-interacting scalar field Φ nonminimally coupled whose stress tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = \partial_\mu\Phi\partial_\nu\Phi - g_{\mu\nu}\left(\frac{1}{2}\,\partial_\sigma\Phi\partial^\sigma\Phi + U(\Phi)\right) + \xi\left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + G_{\mu\nu}\right)\Phi^2. \qquad \text{(A.1)}$$

As shown in ref. [37], a solution of the equation $T_{\mu\nu} = 0$ on the pure AdS metric

$$ds^2 = -r^2dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}^2_{D-2} \qquad \text{(A.2)}$$

is given by the following configuration

$$U(\Phi) = \frac{\xi}{(1-4\xi)^2}\left[2\,\xi\, b^2\Phi^{\frac{1-2\xi}{\xi}} - 8(D-1)\,(\xi-\xi_D)\Big(2\,\xi\, b\,\Phi^{\frac{1}{2\xi}} - D\,(\xi-\xi_{D+1})\,\Phi^2\Big)\right], \qquad \text{(A.3a)}$$

$$\Phi(r) = (Ar+b)^{\frac{2\xi}{4\xi-1}}. \qquad \text{(A.3b)}$$

For technical reasons, we will restrict ourselves to the case $b = 0$,[3] which in turn implies that the stealth potential (A.3a) reduces to the mass term considered in this paper (2.7). Let us first operate a Kerr-Schild transformation on the AdS metric (A.2) with a null and

[3] In fact, one can follows the analysis with $b \neq 0$, but, the remaining independent Einstein equations will impose $A = 0$ or $b = 0$ in order to deal with a nontrivial metric function. The case $A = 0$ implies that the scalar field is constant and the solution reduces to the pure Lovelock solution with a suitable redefinition of the cosmological constant.

geodesic vector $l = dt - \frac{dr}{r^2}$, and this without affecting the scalar field. The transformed metric becomes after redefining the time coordinate

$$ds^2 = -r^2\Big(1 - f(r)\Big)dt^2 + \frac{dr^2}{r^2\Big(1 - f(r)\Big)} + r^2 d\vec{x}^2_{D-2}. \tag{A.4}$$

It is easy to see that the components on-shell[4] of the energy-momentum tensor (A.1) and the gravity tensor satisfy the following identities

$$\mathcal{G}^{(k)t}_{\ \ t} = \mathcal{G}^{(k)r}_{\ \ r}, \qquad \mathcal{G}^{(k)i}_{\ \ i} = \frac{1}{(D-2)}\left[r\left(\mathcal{G}^{(k)t}_{\ \ t}\right)' + \mathcal{G}^{(k)t}_{\ \ t}(D-2)\right] \tag{A.5a}$$

$$T^t_t = T^r_r, \qquad T^i_i = \frac{(4\xi-1)}{4\xi(D-1)-(D-2)}\left[r\left(T^t_t\right)' + T^t_t(D-2)\right] \tag{A.5b}$$

Because of these relations (A.5), *a necessary condition* for the field equations $\mathcal{G}^{(k)}_{\mu\nu} = T_{\mu\nu}$ to be satisfied is that $\xi = 0$ or $T^i_i = 0 = \mathcal{G}^{(k)i}_{\ \ i}$. The condition $T^i_i = 0$ yields a second-order Cauchy equation for the metric function f whose solution reads

$$f(r) = \frac{M_1}{r^{\frac{4(D-1)\xi-(D-2)}{4\xi-1}}} + \frac{M_2}{r^{\frac{(4\xi-1)D+1}{4\xi-1}}}, \tag{A.6}$$

where M_1 and M_2 are two integration constants. Injecting this metric function (A.6) into the condition $\mathcal{G}^{(k)i}_{\ \ i} = 0$ yields

$$\begin{aligned} M_1^2\left(\xi - \xi^{(1)}\right)\left(\xi - \xi^{\text{b.h}}_{k,D}\right)r^2 + \beta_2 M_1 M_2\left(\xi - \xi^{\text{stealth}}_{k,D}\right)\left(\xi - \xi^{\text{b.h}}_{k,D}\right)r \\ +\beta_3\, M_2^2\left(\xi - \xi^{\text{stealth}}_{k,D}\right)\left(\xi - \xi^{(2)}\right) = 0 \end{aligned} \tag{A.7}$$

where the β_i are non-vanishing constants and where we have defined

$$\xi^{(1)} = \frac{1}{4}\left[\frac{(k-1)D - 2k + 1}{(k-1)(D-1)}\right], \qquad \xi^{(2)} = \frac{1}{4}\left[\frac{(k-1)D - k + 2}{D(k-1)+2}\right].$$

As it can seen from $\xi^{(1)}$, these relations are valid only for $k \geq 2$. In fact, apart from the trivial solution $M_1 = M_2 = 0$ that yields to the pure AdS metric, there exists *a priori* four options to solve the previous constraint (A.7)

$$\text{Option I}: \ \left\{M_1 = 0,\, \xi = \xi^{\text{stealth}}_{k,D}\right\}, \qquad \text{Option II}: \left\{M_1 = 0,\, \xi = \xi^{(2)}\right\},$$

$$\text{Option III}: \ \left\{M_2 = 0,\, \xi = \xi^{\text{b.h}}_{k,D}\right\}, \qquad \text{Option IV}: \left\{M_2 = 0,\, \xi = \xi^{(1)}\right\}.$$

The options I and IV give rise to a metric function given by the stealth metric $F^{\text{stealth}}_{k,D}$ while for the options II and III, the metric becomes $F^{\text{b.h}}_{k,D}$. However, it remains one independent Einstein equation to be satisfied, that is $\mathcal{G}^{(k)t}_{\ \ t} = T^{(k)t}_{\ \ t}$. In doing so, the options I and III precisely yield to the two classes of solutions derived in this paper. For the options II, the solution reduces to the stealth configuration on the pure AdS metric [37], and for the option IV besides to the stealth configuration, there exists the possibility with a vanishing scalar field defined on the stealth metric $F^{\text{stealth}}_{k,D}$.

[4]By on-shell, we mean using the expression of the potential (A.3), scalar field (A.3b) and the Ansatz metric (A.4) with $b = 0$.

A.2 Particular case of Einstein gravity $k = 1$

We will now consider the standard GR case $k = 1$, and we will establish that the unique black hole solution with planar base manifold (with a unique metric function) of a scalar field nonminimally coupled with a possible mass term potential is the Martinez-Zanelli solution in $D = 3$ [39].[5] Hence, we consider the following action

$$S = \int d^Dx\,\sqrt{-g}\left[\frac{1}{2}(R-2\Lambda)\right] - \int d^Dx\,\sqrt{-g}\left(\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{\xi}{2}R\Phi^2 + \alpha\Phi^2\right), \quad \text{(A.8)}$$

where $\Lambda = -\frac{1}{2}(D-1)(D-2)$ is the cosmological constant and the potential is given by a mass term $U(\Phi) = \alpha\Phi^2$ where α is a constant. The field equations of (A.8) obtained by varying the action with respect to the different dynamical fields read

$$E_{\mu\nu} := G_{\mu\nu} + \Lambda g_{\mu\nu} - \partial_\mu\Phi\partial_\nu\Phi + g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\Phi\partial^\sigma\Phi + \alpha\Phi^2\right) - \xi\left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + G_{\mu\nu}\right)\Phi^2 = 0,$$

$$\Box\Phi = \xi R\Phi + \frac{dU}{d\Phi}.$$

We look for an Ansatz metric of the form

$$ds^2 = -F(r)\,dt^2 + \frac{dr^2}{F(r)} + r^2 d\vec{x}^2_{D-2}, \quad \text{(A.9)}$$

while the scalar field is assumed to depend only on the radial coordinates, $\Phi = \Phi(r)$. The combination of the Einstein equations combination $E_t^t - E_r^r = 0$ implies that the scalar field must be given by

$$\Phi(r) = \frac{A}{(r+B)^{\frac{2\xi}{1-4\xi}}} \quad \text{(A.10)}$$

where A and B are two integration constants. Substituting this expression (A.10) into the equation $E_t^t = 0$ (or equivalently $E_r^r = 0$), the metric function is obtained as

$$F(r) = \frac{4\,(r+B)^{1+\delta}\left[(D-2)\,(1+\delta)\,r^2 - Cr^{-D+3} - \alpha\,h(r)\right]}{4\,(1+\delta)\,(D-2)\,(r+B)^{1+\delta} + A^2\left[(\delta-D+2)\,r - B\,(D-2)\right]\delta}, \quad \text{(A.11)}$$

where for convenience, we have defined $\delta = 4\xi/(1-4\xi)$ and where C is an integration constant. In this expression, the function $h(r)$ can be given in an integral form as

$$h(r) = 2\,r^{-D+3}A^2\,(1+\delta)\int r^{D-2}\,(r+B)^{-\delta}\,dr$$

or by a finite series

$$h(r) = 2\,r^{-D+3}A^2\,(1+\delta)\left[\frac{r^{D-2}(r+B)^{1-\delta}}{(1-\delta)} + \sum_{k=1}^{D-2}\frac{(-1)^k(D-2)(D-3)\dots(D-k-1)}{(1-\delta)(2-\delta)\dots(k+1-\delta)}\right.$$
$$\left. r^{D-k-2}\,(r+B)^{k+1-\delta}\right],$$

which is only valid for $\delta \neq 1, 2, \dots, (D-1)$.

[5]We also assume that the scalar field as well as the metric function only depends on the coordinate r.

The remaining of the analysis must be divided in two cases depending if the coupling constant associated to the potential α is chosen to be zero or not. For a non-vanishing α, the remaining independent Einstein equation $E_i^i = 0$ will imply that

$$\alpha = \frac{\delta\,(\delta - D + 2)\,(\delta - D + 1)}{8\,(\delta + 1)}, \tag{A.12}$$

and $B = C = 0$. This solution is nothing but a particular stealth solution on the pure AdS metric.

We now turn to the case with a vanishing coupling constant $\alpha = 0$, and we split the analysis in two branches depending on the sign of the constant δ (the option $\delta = 0$ is equivalent to $\xi = 0$). In fact, for $\delta \neq 0$, we note that substituting the metric function (A.11) with $\alpha = 0$ into the equation $E_i^i = 0$, we get the following complicated expression after some algebraic manipulations

$$(y-B)^D\left(\sum_{k=2}^{7}\alpha_k y^{2\delta+k} + \sum_{k=1}^{7}\beta_k y^{\delta+k} + \sum_{k=2}^{7}\gamma_k y^k\right) + C\Bigg[\sum_{k=2}^{8}\epsilon_k y^{2\delta+k} + \sum_{k=1}^{8}\eta_k y^{\delta+k} \tag{A.13}$$
$$+A^4 B\delta^2\left(-\delta + D - 2\right) y^2 \left(-y + B\right)^5\Bigg] = 0.$$

Here, we have defined $y = r + B$ and the different constants appearing in this expression are not reported for simplicity. For $\delta < 0$, the highest power of (A.13) is $D + 7$, and the vanishing of the corresponding coefficient γ_7 implies that $A = 0$ and hence the scalar field $\Phi = 0$. It is also easy to see that the coefficient of $y^{2\delta+D+7}$ never vanishes and finally we conclude that solutions do not exist for $\delta < 0$. Let us now consider the case $\delta > 0$ for which the highest power of (A.13) is $2\delta + D + 7$ and its corresponding coefficient is α_7. In this situation, we have only two possible solutions for a generic dimension D which are $\delta = D - 1$ or $\delta = D - 2$. For the first option $\delta = D - 1$, the constant B must vanish and $C = -\frac{1}{4}A^2(D-1)$, yielding again to a particular stealth configuration on the pure AdS metric. The remaining option $\delta = D - 2$ is the most interesting one. Indeed, the eq. (A.13) becomes now

$$(y-B)^D\left(\sum_{k=2}^{5}\alpha_{k,D}\, y^{2D+k-4} + \sum_{k=1}^{6}\beta_{k,D}\, y^{D+k-2} + \sum_{k=2}^{5}\gamma_{k,D}\, y^k\right) + \tag{A.14}$$
$$C\left(\sum_{k=2}^{8}\epsilon_{k,D}\, y^{2D+k-4} + \sum_{k=1}^{7}\eta_{k,D}\, y^{D+k-2}\right) = 0.$$

The highest possible powers are given by y^{3D+1} and y^{2D+4} and these latter coincide only in three dimension $D = 3$. Otherwise for $D > 3$, the highest power is y^{3D+1}, and after some computations we obtain that $B = C = 0$ yielding again a particular stealth configuration on the pure AdS metric. The case $D = 3$ has already been analyzed in details in [46], and the only black hole solution is the Martinez-Zanelli solution [39].

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