

# Extension of non-minimal derivative coupling theory and Hawking radiation in black-hole spacetime

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**ABSTRACT:** We extend the non-minimal derivative coupling theory model to dynamical gravity and use it to study the greybody factor and Hawking radiation in the background of the slowly rotating Kerr-Newman black hole. Our results show that both the absorption probability and luminosity of Hawking radiation of the scalar field increase with the coupling. Moreover, we also find that for the weak coupling  $\eta < \eta_c$ , the absorption probability and luminosity of Hawking radiation decrease when the black hole's Hawking temperature decreases. On the other hand for stronger coupling  $\eta > \eta_c$ , the absorption probability and luminosity of Hawking radiation increase when the black hole's Hawking temperature decreases. This feature is similar to the Hawking radiation in a  $d$ -dimensional static spherically-symmetric black hole surrounded by quintessence [1].

**KEYWORDS:** Black Holes, Classical Theories of Gravity, Strong Coupling Expansion

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**1 Introduction**

Scalar fields in General Relativity has been a topic of great interest in the latest years. One of the main reasons is that the models with scalar fields are relatively simple, which allows us to probe the detailed features of the more complicated physical system. In cosmology, scalar fields can be considered as candidate (inflaton, quintessence, phantom fields, etc.) to explain the inflation of the early Universe [2] and the accelerated expansion of the current Universe [3–14]. In the Standard Model of particle physics, the scalar field presents as the Higgs boson [15], which would help to explain the origin of mass in the Universe. Moreover, it has been found that scalar field plays the important roles in other fundamental physical theories, such as, Jordan-Brans-Dicke theory [16], Kaluza-Klein compactification theory [17] and superstring theory [18], and so on.

In another side, including nonlinear terms of the various curvature tensors (Riemann, Ricci, Weyl) and nonminimally coupled terms in the effective action of gravity has become a very common trend from quantum field theory side and cosmology. These theories cover the  $f(R)$  modified gravity, the Gauss-Bonnet gravity, the tachyon, dilaton, and so on. The nonminimal coupling between scalar field and higher order terms in the curvature (the so-called “scalar-tensor” theory) naturally give rise to inflationary solutions improve the early inflationary models and could contribute to solve the dark matter problem. The new coupling between the derivative of scalar field and the spacetime curvature may appear firstly in some Kaluza-Klein theories [19–21]. Amendola [22] considered the most general theory of gravity with the Lagrangian linear in the Ricci scalar, quadratic in  $\psi$ , in which the coupling terms have the forms as follows

$$R\partial_\mu\psi\partial^\mu\psi, \quad R_{\mu\nu}\partial^\mu\psi\partial^\nu\psi, \quad R\psi\nabla^2\psi, \quad R_{\mu\nu}\psi\partial^\mu\psi\partial^\nu\psi, \quad \partial_\mu R\partial^\mu\psi, \quad \nabla^2 R\psi. \quad (1.1)$$

And then he studied the dynamical evolution of the scalar field in the cosmology by considering only the derivative coupling term  $R_{\mu\nu}\partial^\mu\psi\partial^\nu\psi$  and obtained some analytical inflationary solutions [22]. Capozziello *et al.* [23] investigated a more general model of containing coupling terms  $R\partial_\mu\psi\partial^\mu\psi$  and  $R_{\mu\nu}\partial^\mu\psi\partial^\nu\psi$ , and found that the de Sitter spacetime is an attractor solution in the model. Recently, Daniel and Caldwell [24] obtained the constraints on the theory with the derivative coupling term of  $R_{\mu\nu}\partial^\mu\psi\partial^\nu\psi$  by Solar system tests. In general, a theory with derivative couplings could lead to that both the Einstein equations and the equation of motion for the scalar are the fourth-order differential equations. However, Sushkov [25] studied recently the model in which the kinetic term of the scalar field only coupled with the Einstein's tensor and found that the equation of motion for the scalar field can be reduced to second-order differential equation. This means that the theory is a "good" dynamical theory from the point of view of physics. Gao [26] investigated the cosmic evolution of a scalar field with the kinetic term coupling to more than one Einstein's tensors and found the scalar field presents some very interesting characters. He found that the scalar field behaves exactly as the pressureless matter if the kinetic term is coupled to one Einstein's tensor and acts nearly as a dynamic cosmological constant if it couples with more than one Einstein's tensors. The similar investigations have been considered in refs. [27, 28]. These results will excite more efforts to be focused on the study of the scalar field coupled with tensors in the more general cases.

In this paper we will extend this non-minimal derivative coupling theory model to dynamical gravity and use it to study the greybody factor and Hawking radiation in the background of the slowly rotating Kerr-Newman black hole.

Since black hole is another fascinating object in modern physics, it is of interest to extend the study the properties of the scalar field when it is kinetically coupled to the Einstein's tensors in the background of a black hole. This extension to Reissner-Nordström black hole spacetime is studied by S. Chen *et al* [29]. In this paper, we will investigate the greybody factor and Hawking radiation of the scalar field coupling to the Einstein's tensor  $G^{\mu\nu}$  in the slowly rotating Kerr-Newman black hole spacetime. We find that the presence of the coupling terms enhances both the absorption probability and luminosity of Hawking radiation of the scalar field in the black hole spacetime. Moreover, we also find that for the weak coupling  $\eta < \eta_c$ , the absorption probability and luminosity of Hawking radiation decrease when the black hole's Hawking temperature decreases; while for stronger coupling  $\eta > \eta_c$ , the absorption probability and luminosity of Hawking radiation increase on the contrary when the black hole's Hawking temperature decreases. This feature is similar to the Hawking radiation in a  $d$ -dimensional static spherically-symmetric black hole surrounded by quintessence [1], i.e. when  $0 < -\omega_q < (d-3)/(d-1)$ , Hawking temperature decreases and the luminosity of Hawking radiation both in the bulk and on the brane decreases naturally; when  $(d-3)/(d-1) < -\omega_q < 1$ , Hawking temperature still decreases, but the luminosity of Hawking radiation both in the bulk and on the brane increases conversely.

The paper is organized as follows: in the following section we will introduce the action of a scalar field coupling to Einstein's tensor and derive its master equation in the slowly rotating Kerr-Newman black hole spacetime. In section 3, we obtain the expression of the

absorption probability in the low-energy limit by using the matching technique. In section 4, we will calculate the absorption probability and the luminosity of Hawking radiation for the coupled scalar field. In section 5, we will include and discuss our conclusions. Appendix is devoted to the extension of the non-minimal derivative coupling theory model to the dynamical gravity.

## 2 Master equation with non-minimal derivative coupling in the slowly rotating black hole spacetime

Let us consider the action of the scalar field coupling to the Einstein's tensor  $G^{\mu\nu}$  in the curved spacetime [25],

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \frac{1}{2} \partial_\mu \psi \partial^\mu \psi + \frac{\eta}{2} G^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \right]. \quad (2.1)$$

The coupling between Einstein's tensor  $G^{\mu\nu}$  and the scalar field  $\psi$  is represented by  $\frac{\eta}{2} G^{\mu\nu} \partial_\mu \psi \partial_\nu \psi$ , where  $\eta$  is the coupling constant with dimensions of length-squared. In general, the presence of such a coupling term brings some effects to the original metric of the background. However, we can treat the scalar field as a perturbation so that the backreaction effects on the background can be ignored, and then we can study the effects of the coupling constant  $\eta$  on the greybody factor and Hawking radiation of the scalar field in a black hole spacetime. In addition, to avoid the kinetic instability for which  $g^{00} + \eta G^{00} > 0$  (the notation  $'-, +, +, +'$  is assumed), we set the coupling constant to be positive, i.e.  $\eta > 0$ .

Varying the action with respect to  $\psi$ , one can obtain the modified Klein-Gordon equation

$$\frac{1}{\sqrt{-g}} \partial_\mu \left[ \sqrt{-g} \left( g^{\mu\nu} + \eta G^{\mu\nu} \right) \partial_\nu \psi \right] = 0, \quad (2.2)$$

which is a second order differential equation. Obviously, all the components of the tensor  $G^{\mu\nu}$  vanish in the Kerr black hole spacetime because it is the vacuum solution of the Einstein's field equation. Thus, we cannot probe the effect of the coupling term on the greybody factor and Hawking radiation in the Kerr black-hole background. The simplest rotating black hole with the non-zero components of the tensor  $G^{\mu\nu}$  is Kerr-Newman one. In this paper, we consider a slowly rotating Kerr-Newman black hole, whose element reads [30]

$$ds^2 = -\frac{\Delta}{r^2} dt^2 - \frac{2(2Mr - Q^2)a \sin^2 \theta}{r^2} dt d\varphi + \frac{r^2}{\Delta} dr^2 + r^2 d\Omega^2 + \mathcal{O}(a^2) \quad (2.3)$$

with

$$\Delta = r^2 - 2Mr + Q^2, \quad (2.4)$$

where  $M$ ,  $a$ ,  $Q$  are the mass, angular momentum and charge of the black hole. The Einstein's tensor  $G^{\mu\nu}$  for the metric (2.3) has a form

$$G^{\mu\nu} = \frac{Q^2}{r^4} \begin{pmatrix} -\frac{r^2}{\Delta} & 0 & 0 & -\frac{a}{r^2\Delta}(r^2 + \Delta) \\ 0 & \frac{\Delta}{r^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ -\frac{a}{r^2\Delta}(r^2 + \Delta) & 0 & 0 & -\frac{1}{r^2 \sin^2 \theta} \end{pmatrix}. \quad (2.5)$$

Adopting to the spherical harmonics

$$\phi(t, r, \theta, \varphi) = e^{-i\omega t} e^{im\varphi} R_{\omega\ell m}(r) T_\ell^m(\theta, a\omega), \quad (2.6)$$

we can obtain the radial part of the equation (2.2)

$$\begin{aligned} & \frac{d}{dr} \left[ \Delta \left( 1 + \frac{\eta Q^2}{r^4} \right) \frac{dR_{\omega\ell m}}{dr} \right] \\ & + \left[ \frac{r^2 \omega (r^2 \omega - 2am)}{\Delta} \left( 1 + \frac{\eta Q^2}{r^4} \right) - [l(l+1) - 2am\omega] \left( 1 - \frac{\eta Q^2}{r^4} \right) \right] R_{\omega\ell m} = 0. \end{aligned} \quad (2.7)$$

Clearly, the radial equation (2.7) contains the coupling constant  $\eta$ , which means that the presence of the coupling term will change the evolution of the scalar field in the Kerr-Newman black hole spacetime.

The solution of the radial function  $R_{\omega\ell m}(r)$  will help us to obtain the absorption probability  $|A_{\ell m}|^2$  and the luminosity of Hawking radiation for a scalar field coupling with Einstein's tensor in the slowly rotating Kerr-Newman black hole spacetime.

### 3 Greybody factor in the low-energy regime

In order to study the effects of the coupling constant  $\eta$  on the absorption probability  $|A_{\ell m}|^2$  and the luminosity of Hawking radiation of a scalar field in the background spacetime, we must first get an analytic solution of the radial equation (2.7). In general, it is very difficult. However, as in ref. [31–69], we can provide an approximated solution of the radial equation (2.7) by employing the matching technique. Firstly, we must derive the analytic solutions in the near horizon ( $r \simeq r_+$ ) and far-field ( $r \gg r_+$ ) regimes in the low-energy limit. Finally, we smoothly match these two solutions in an intermediate region. In this way, we can construct a smooth analytical solution of the radial equation valid throughout the entire spacetime.

Now, we focus on the near-horizon regime and perform the following transformation of the radial variable as in refs. [64–69]

$$r \rightarrow f(r) = \frac{\Delta(r)}{r^2} \implies \frac{df}{dr} = (1-f) \frac{\mathcal{A}(r)}{r}, \quad (3.1)$$

with

$$\mathcal{A} = 1 - \frac{Q^2}{2Mr - Q^2}. \quad (3.2)$$

The equation (2.7) near the horizon ( $r \sim r_+$ ) can be rewritten as

$$f(1-f) \frac{d^2 R(f)}{df^2} + (1-D_* f) \frac{dR(f)}{df} + \left[ \frac{K_*^2}{A_*^2(1-f)f} - \frac{\Lambda_\ell^m}{A_*^2(1-f)} \left( \frac{1 - \eta_* Q_*^2}{1 + \eta_* Q_*^2} \right) \right] R(f) = 0, \quad (3.3)$$

where<sup>1</sup>

$$a_* = a/r_+, \quad Q_* = Q/r_+, \quad \eta_* = \eta/r_+^2, \quad K_* = \omega r_+ - a_* m, \quad A_* = 1 - Q_*^2, \\ \Lambda_\ell^m = l(l+1) - 2ma\omega, \quad D_* = 1 - \frac{2Q_*^2}{(1-Q_*^2)^2} + \frac{4\eta_* Q_*^2}{(1-Q_*^2)[1+\eta_* Q_*^2]}. \quad (3.4)$$

Making the field redefinition  $R(f) = f^\alpha(1-f)^\beta F(f)$ , one can find that the equation (3.3) can be rewritten as a form of the hypergeometric equation

$$f(1-f)\frac{d^2 F(f)}{df^2} + [c - (1 + \tilde{a} + b)f]\frac{dF(f)}{df} - \tilde{a}bF(f) = 0, \quad (3.5)$$

with

$$\tilde{a} = \alpha + \beta + D_* - 1, \quad b = \alpha + \beta, \quad c = 1 + 2\alpha. \quad (3.6)$$

Considering the constraint coming from coefficient of  $F(f)$ , one can easy to obtain that the power coefficients  $\alpha$  and  $\beta$  satisfy

$$\alpha^2 + \frac{K_*^2}{A_*^2} = 0, \quad (3.7)$$

and

$$\beta^2 + \beta(D_* - 2) + \frac{1}{A_*^2} \left[ K_*^2 - \Lambda_\ell^m \left( \frac{1 - \eta_* Q_*^2}{1 + \eta_* Q_*^2} \right) \right] = 0, \quad (3.8)$$

respectively. These two equations admit that the parameters  $\alpha$  and  $\beta$  have the forms

$$\alpha_\pm = \pm \frac{iK_*}{A_*}, \quad (3.9)$$

$$\beta_\pm = \frac{1}{2} \left\{ (2 - D_*) \pm \sqrt{(D_* - 2)^2 - \frac{4}{A_*^2} \left[ K_*^2 - \Lambda_\ell^m \left( \frac{1 - \eta_* Q_*^2}{1 + \eta_* Q_*^2} \right) \right]} \right\}. \quad (3.10)$$

The general solution of eq. (3.5) is

$$R_{NH}(f) = A_- f^\alpha (1-f)^\beta F(\tilde{a}, b, c; f) + A_+ f^{-\alpha} (1-f)^\beta F(\tilde{a} - c + 1, b - c + 1, 2 - c; f), \quad (3.11)$$

where  $A_+, A_-$  are arbitrary constants. Near the horizon,  $r \rightarrow r_h$  and  $f \rightarrow 0$ , the solution has the form

$$R_{NH}(f) = A_- f^{\alpha_\mp} + A_+ f^{\alpha_\pm}. \quad (3.12)$$

Imposing the boundary condition that no outgoing mode exists near the horizon, we are forced to set either  $A_- = 0$  or  $A_+ = 0$ , depending on the choice for  $\alpha_\pm$ . Here we choose  $\alpha = \alpha_-$  and  $A_+ = 0$ . The sign of  $\beta$  will be decided by the criterion for the convergence

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<sup>1</sup>In order to solve the mathematical equation, we should let all coefficients in it dimensionless, so that we define these quantities. Usually we let the numerical value of  $r_+$  equal to unity. However in this paper, we find that to let the numerical value of  $2M$  equal to 1 is more convenient.

of the hypergeometric function  $F(\tilde{a}, b, c; f)$ , i.e.  $\text{Re}(c - \tilde{a} - b) > 0$ , which demands that we choose  $\beta = \beta_-$  [64–69]. Thus the asymptotic solution near horizon has the form

$$R_{NH}(f) = A_- f^\alpha (1-f)^\beta F(\tilde{a}, b, c; f). \quad (3.13)$$

Let us now to stretch smoothly the near horizon solution to the intermediate zone. We can make use of the property of the hypergeometric function [71] and change its argument in the near horizon solution from  $f$  to  $1-f$

$$R_{NH}(f) = A_- f^\alpha (1-f)^\beta \left[ \frac{\Gamma(c)\Gamma(c-\tilde{a}-b)}{\Gamma(c-\tilde{a})\Gamma(c-b)} F(\tilde{a}, b, \tilde{a}+b-c+1; 1-f) + (1-f)^{c-\tilde{a}-b} \frac{\Gamma(c)\Gamma(\tilde{a}+b-c)}{\Gamma(\tilde{a})\Gamma(b)} F(c-\tilde{a}, c-b, c-\tilde{a}-b+1; 1-f) \right]. \quad (3.14)$$

As  $r \gg r_+$ , the function  $(1-f)$  can be approximated as

$$1-f = \frac{2Mr - Q^2}{r^2} \simeq \frac{2M}{r}, \quad (3.15)$$

and then the near horizon solution (3.14) can be simplified further to

$$R_{NH}(r) \simeq C_1 r^{-\beta} + C_2 r^{\beta+D_*-2}, \quad (3.16)$$

with

$$C_1 = A_- (2M)^\beta \frac{\Gamma(c)\Gamma(c-\tilde{a}-b)}{\Gamma(c-\tilde{a})\Gamma(c-b)}, \quad (3.17)$$

$$C_2 = A_- (2M)^{-(\beta+D_*-2)} \frac{\Gamma(c)\Gamma(\tilde{a}+b-c)}{\Gamma(\tilde{a})\Gamma(b)}. \quad (3.18)$$

Nextly in order to obtain a solution in the far field region, we expand the wave equation (2.7) as a power series in  $1/r$  and keep only the leading terms

$$\frac{d^2 R_{FF}(r)}{dr^2} + \frac{2}{r} \frac{dR_{FF}(r)}{dr} + \left( \omega^2 - \frac{l(l+1)}{r^2} \right) R_{FF}(r) = 0. \quad (3.19)$$

This is the usual Bessel equation. Thus the solution of the radial master equation (2.7) in the far-field limit can be expressed as

$$R_{FF}(r) = \frac{1}{\sqrt{r}} \left[ B_1 J_\nu(\omega r) + B_2 Y_\nu(\omega r) \right], \quad (3.20)$$

where  $J_\nu(\omega r)$  and  $Y_\nu(\omega r)$  are the first and second kind Bessel functions,  $\nu = l + 1/2$ .  $B_1$  and  $B_2$  are integration constants. In order to stretch the far-field solution (3.20) towards small radial coordinate, we take the limit  $r \rightarrow 0$  and obtain

$$R_{FF}(r) \simeq \frac{B_1 (\frac{\omega r}{2})^\nu}{\sqrt{r} \Gamma(\nu+1)} - \frac{B_2 \Gamma(\nu)}{\pi \sqrt{r} (\frac{\omega r}{2})^\nu}. \quad (3.21)$$

In the low-energy and low-angular momentum limit  $(\omega r_+)^2 \ll 1$  and  $(a/r_+)^2 \ll 1$ , the two power coefficients in eq. (3.16) can be approximated as

$$-\beta \simeq l + \mathcal{O}(\omega^2, a^2, a\omega), \tag{3.22}$$

$$(\beta + D_* - 2) \simeq -(l + 1) + \mathcal{O}(\omega^2, a^2, a\omega). \tag{3.23}$$

By using the above results, one can easily show that both eqs. (3.16) and (3.21) reduce to power-law expressions with the same power coefficients,  $r^l$  and  $r^{-(l+1)}$ . By matching the corresponding coefficients between eqs. (3.16) and (3.21), we can obtain two relations between  $C_1, C_2$  and  $B_1, B_2$ . Removing  $A_-$ , we can obtain the ratio between the coefficients  $B_1, B_2$

$$B \equiv \frac{B_1}{B_2} = -\frac{1}{\pi} \left[ \frac{1}{\omega M} \right]^{2l+1} \nu \Gamma^2(\nu) \times \frac{\Gamma(c - \tilde{a} - b) \Gamma(\tilde{a}) \Gamma(b)}{\Gamma(\tilde{a} + b - c) \Gamma(c - \tilde{a}) \Gamma(c - b)}. \tag{3.24}$$

In the asymptotic region  $r \rightarrow \infty$ , the solution in the far-field can be expressed as

$$R_{FF}(r) \simeq \frac{B_1 + iB_2}{\sqrt{2\pi \omega r}} e^{-i\omega r} + \frac{B_1 - iB_2}{\sqrt{2\pi \omega r}} e^{i\omega r} \tag{3.25}$$

$$= A_{\text{in}}^{(\infty)} \frac{e^{-i\omega r}}{r} + A_{\text{out}}^{(\infty)} \frac{e^{i\omega r}}{r}. \tag{3.26}$$

The absorption probability can be calculated by

$$|\mathcal{A}_{\ell m}|^2 = 1 - \left| \frac{A_{\text{out}}^{(\infty)}}{A_{\text{in}}^{(\infty)}} \right|^2 = 1 - \left| \frac{B - i}{B + i} \right|^2 = \frac{2i(B^* - B)}{BB^* + i(B^* - B) + 1}. \tag{3.27}$$

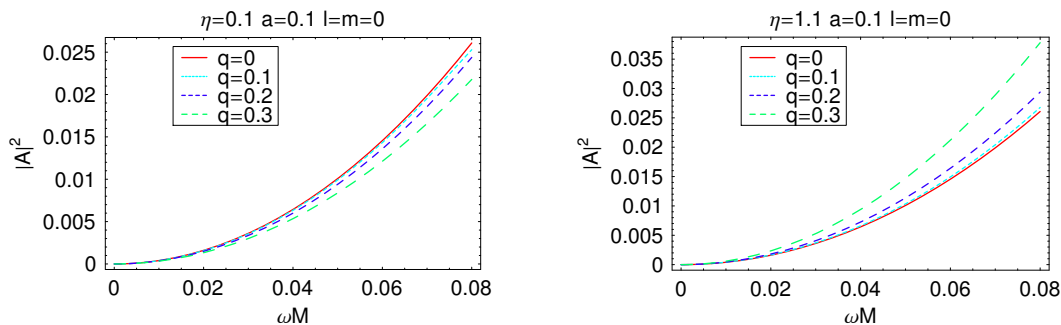
Inserting the expression of  $B$  (3.24) into eq. (3.27), we can probe the properties of absorption probability for the scalar field coupled with Einstein's tensor in the slowly rotating black hole spacetime in the low-energy limit.

#### 4 The absorption probability and Hawking radiation with non-minimal derivative coupling

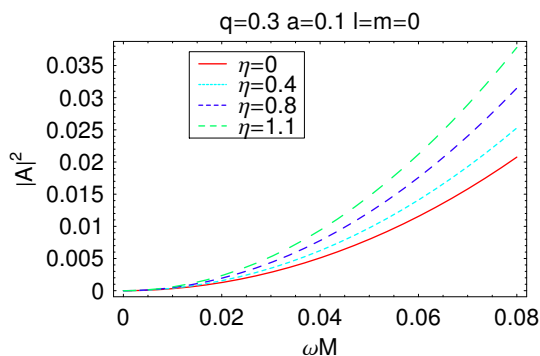
We are now in a position to calculate the absorption probability and discuss Hawking radiation of a scalar field coupling to Einstein's tensor in the background of a slowly rotating Kerr-Newman black hole.

In figure 1, we fix the coupling constant  $\eta$ , and angular momentum  $a$ , and plot the change of the absorption probability of a scalar particle with the charge  $Q$  for the first partial waves ( $\ell = 0$ ) in the slowly rotating Kerr-Newman black hole. One can easily see that for the smaller  $\eta$  the absorption probability  $A_{\ell=0}$  decreases with the charge  $Q$  of the black hole, which is similar to that for the usual scalar field without coupling to Einstein's tensor. However, for the larger  $\eta$ , the absorption probability  $A_{\ell=0}$  increases as the charge  $Q$  increases. These properties mean that the stronger coupling between the scalar field and Einstein's tensor changes the properties of the absorption probability of scalar field in the black hole spacetime. In figure2, we also find that the absorption probability increases





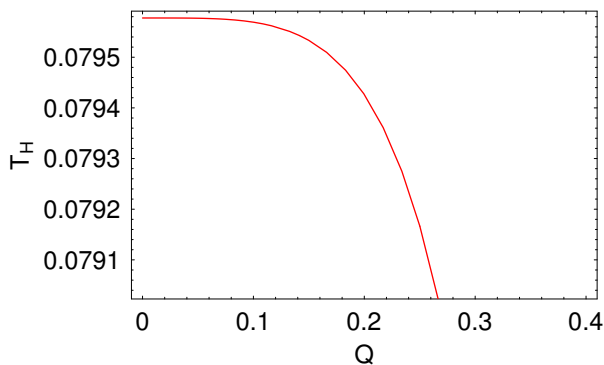
**Figure 1.** Variety of the absorption probability  $|A_{\ell m}|^2$  of a scalar field with the charge  $Q$  and angular momentum in the slowly rotating Kerr-Newman black hole for fixed  $\ell = m = 0$ . The coupling constant  $\eta$  is set by  $\eta = 0.1$  in the left and by  $\eta = 1.1$  in the right.



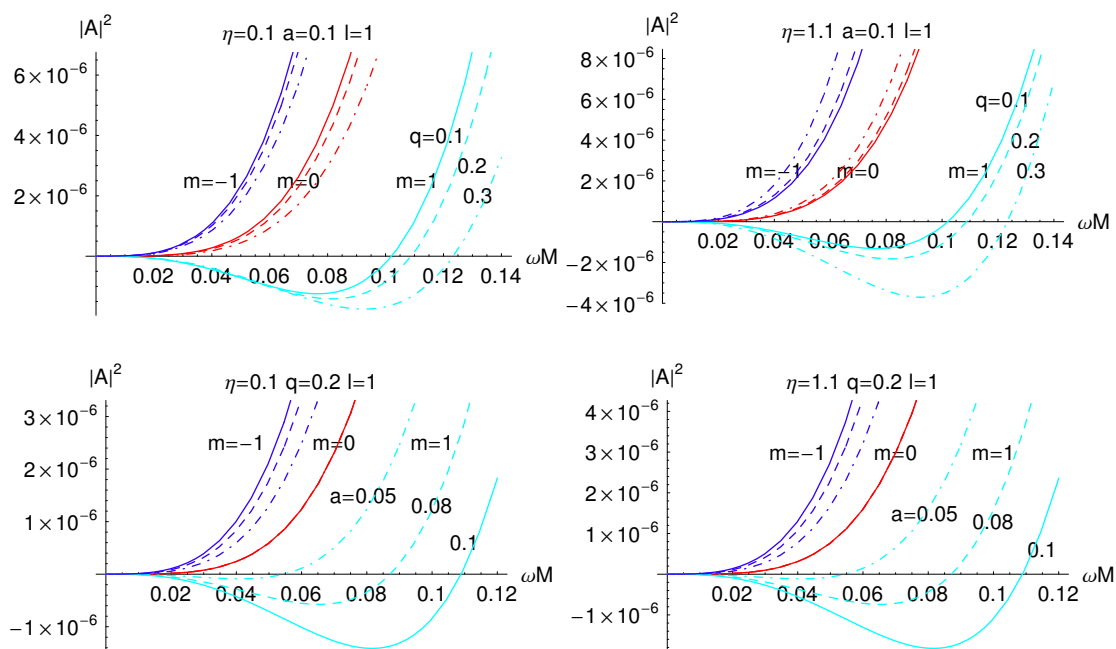
**Figure 2.** The dependence of the absorption probability  $|A_{\ell m}|^2$  of a scalar field on the coupling constant  $\eta$  in the slowly rotating Kerr-Newman black hole for fixed  $\ell = m = 0$  and  $Q = 0.3$ .

with the increase of the coupling constant  $\eta$  for fixed values of charge  $q = 0.3$ , and  $a = 0.1$ . In figure 3, we show the Hawking temperature of the slowly rotating Kerr-Newman black hole with various charge  $Q$ . Therefore, for weak coupling, when Hawking temperature decreases, the greybody factor decreases naturally, but for stronger coupling the greybody factor on the contrary increases.

The above results about the absorption probability also hold true for other values of  $\ell, m$ . In this case, there has superradiation region when  $m = 1, 2, \dots, \ell$ , which is similar to [64]. It is shown in figure 4, in which we plotted the dependence of the absorption probability on the angular index  $\ell$  and  $m$  with different  $\eta, a$  and  $Q$ . From the above two figures in figure 4, we can obtain that the influence of charge  $Q$  on the usual radiation ( $m = -1, 0$ ) is similar to that on the first partial wave. While for the super-radiation, the charge enhance it both for weak and strong coupling. And the angular momentum  $a$  enhance the usual radiation ( $m = -1$ ) and the super-radiation ( $m = 1$ ) both for weak and strong coupling. Moreover, we see the suppression of  $|A_{\ell m}|^2$  as the values of the angular index increase. This means that the first partial wave dominates over all others



**Figure 3.** The Hawking temperature of the slowly rotating Kerr-Newman black hole with various charge  $Q$ .



**Figure 4.** Variety of the absorption probability  $|A_{\ell m}|^2$  of a scalar field with the charge  $Q$  and angular momentum  $a$  in the slowly rotating Kerr-Newman black hole spacetime for fixed  $\ell = 1, m = 1, 0, -1$ . The coupling constant  $\eta$  is set by  $\eta = 0.1$  on the left and  $\eta = 1.1$  on the right. In the above two figures, the solid lines represent  $Q = 0.1$ , the dashed lines represent  $Q = 0.2$ , the dashed-dotted lines represent  $Q = 0.3$ ; in the lower two figures, the solid lines represent  $a = 0.1$ , the dashed lines represent  $a = 0.08$ , the dashed-dotted lines represent  $a = 0.05$ .

in the absorption probability. It is similar to that of the scalar field without coupling to Einstein's tensor as shown in refs. [31–69].

Now let us turn to study the luminosity of the Hawking radiation for the mode  $\ell =$

$m = 0$  which plays a dominant role in the greybody factor. Performing an analysis similar to that in [64–69], we can obtain that the greybody factor (3.27) in the low-energy limit has a form

$$|A_{\ell=0}|^2 \simeq \frac{4\omega^2 r_+^2 (r_+^2 + Q^2)}{(r_+^2 - Q^2)(2 - D_*)}. \quad (4.1)$$

Combining it with Hawking temperature  $T_H$  [70] of the Kerr-Newman black hole,

$$T_H = \frac{r_+ - M}{2\pi(r_+^2 + a^2)} = \frac{r_+^2 - a^2 - Q^2}{4\pi r_+(r_+^2 + a^2)} = \frac{r_+^2 - Q^2}{4\pi r_+^3} + \mathcal{O}(a^2), \quad (4.2)$$

the luminosity of the Hawking radiation for the scalar field with coupling to Einstein's tensor is given by

$$L = \int_0^\infty \frac{d\omega}{2\pi} |A_{\ell=0}|^2 \frac{\omega}{e^{\omega/T_H} - 1}. \quad (4.3)$$

The integral expressions above are just for the sake of completeness by writing the integral range from 0 to infinity. However, as our analysis has focused only in the low-energy regime of the spectrum, an upper cutoff will be imposed on the energy parameter so that the low-energy conditions  $\omega \ll T_H$  and  $\omega r_+ \ll 1$  are satisfied. In the low-energy limit, the luminosity of the Hawking radiation for the mode  $\ell = 0$  can be approximated as

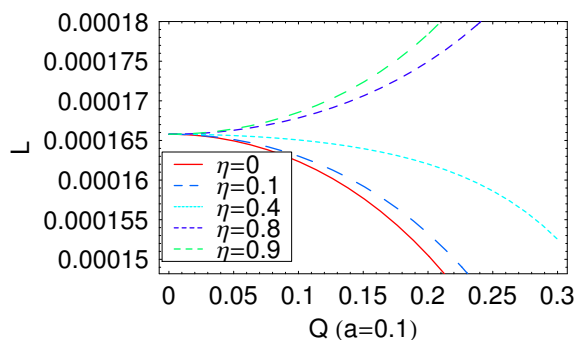
$$L \approx \frac{2\pi^3}{15} G T_H^4, \quad (4.4)$$

with

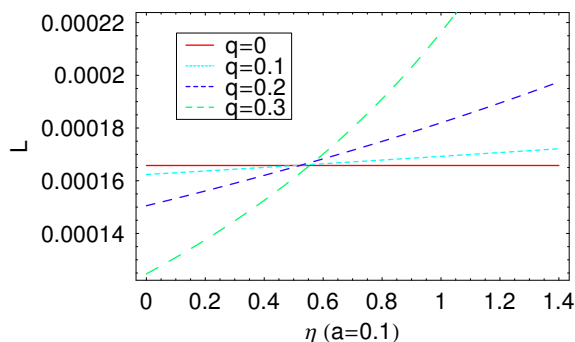
$$G = \frac{r_+^2 (r_+^2 + Q^2)}{(r_+^2 - Q^2)(2 - D_*)}. \quad (4.5)$$

In figure 5 and 6, we show the dependence of the luminosity of Hawking radiation on the charge  $Q$  and the coupling constant  $\eta$ , respectively. From figure 5, one can easily obtain that with increase of  $Q$  the luminosity of Hawking radiation  $L$  decreases for the smaller  $\eta$  and increases for the larger  $\eta$ . In other words, for the weak coupling, the luminosity of Hawking radiation decreases naturally when the black hole's Hawking temperature decreases; while for stronger coupling, the luminosity of Hawking radiation increases on the contrary when the black hole's Hawking temperature decreases. This is similar to the behavior of the absorption probability discussed previously. This feature is similar to the Hawking radiation in a  $d$ -dimensional static spherically-symmetric black hole surrounded by quintessence [1], i.e. when  $0 < -\omega_q < (d-3)/(d-1)$ , Hawking temperature decreases, the luminosity of Hawking radiation both in the bulk and on the brane decrease naturally; when  $(d-3)/(d-1) < -\omega_q < 1$ , Hawking temperature still decreases, the luminosity of Hawking radiation both in the bulk and on the brane increase conversely. In figure 6, we show that the luminosity of Hawking radiation  $L$  increases monotonously with the coupling constant  $\eta$  for the all  $Q$ .

From the figure 5, there should exist a critical coupling constant  $\eta_c$ , so in the table I, we list the critical coupling constant  $\eta_c$  for different  $Q$  by using equation  $\partial L / \partial Q = 0$ . The results show that different charge  $Q$  of the slowly rotating Kerr-Newman black hole correspond to different critical coupling constant  $\eta_c$ .



**Figure 5.** Variety of the luminosity of Hawking radiation  $L$  of scalar particles with the charge  $Q$  and angular momentum  $a$  in the slowly rotating Kerr-Newman black hole for fixed  $\ell = 0$  and different values of  $\eta$ .



**Figure 6.** The dependence of the luminosity of Hawking radiation  $L$  of a scalar field on the coupling constant  $\eta$  in the slowly rotating Kerr-Newman black hole for fixed  $\ell = 0$  and different values of  $Q$  and angular momentum  $a$ .

$Q$	0.1	0.2	0.3
$\eta_c$	0.510	0.541	0.600

**Table 1.** The critical value of coupling constant  $\eta_c$  for different  $Q$ . We set  $\ell = m = 0$ .

## 5 Summary and discussion

In this paper, we have extend the non-minimal derivative coupling theory model to dynamical gravity and studied the greybody factor and Hawking radiation with a non-minimal derivative coupling between a scalar field and the curvature in the background of the slowly rotating Kerr-Newman black hole spacetime. We have found that the presence of the coupling enhances both the absorption probability and the luminosity of Hawking radiation of the scalar field in the black hole spacetime. Moreover, we also find that for the weak cou-

pling  $\eta < \eta_c$ , the absorption probability and the luminosity of Hawking radiation decrease when the black hole's Hawking temperature decreases; while for stronger coupling  $\eta > \eta_c$ , the absorption probability and the luminosity of Hawking radiation increase on the contrary when the black hole's Hawking temperature decreases. This feature is similar to the Hawking radiation in a  $d$ -dimensional static spherically-symmetric black hole surrounded by quintessence [1], i.e. when  $0 < -\omega_q < (d-3)/(d-1)$ , Hawking temperature decreases, the luminosity of Hawking radiation both in the bulk and on the brane decrease naturally; when  $(d-3)/(d-1) < -\omega_q < 1$ , Hawking temperature still decreases, the luminosity of Hawking radiation both in the bulk and on the brane increase conversely.

**Discussion:** this very amazing and interesting similarity shows that the scalar field which is coupled to the Einstein's tensor of a black hole spacetime may behave as a "dark energy model" in the cosmology. In the ref. [26], the authors point out from cosmological side that this scalar field which is coupled to the Einstein's tensor behaves exactly as pressureless matter (without a scalar potential), plays the role of both cold dark matter and dark energy (with a scalar potential). Whether there exists this scalar-tensor coupling or not is still an open question. Because the action (2.1) contains a dimension six operator, which when considered at the effective theory level is an irrelevant operator, and hence should not have any relevance on sub-Planckian scales. All these issues need further studying.

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### A Extending the non-minimal derivative coupling theory model to dynamical gravity

In this appendix, we extend the non-minimal derivative coupling model to the dynamical gravity. The metric tensor is  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , where the components  $h_{\mu\nu}$  be much smaller than 1 and  $\eta_{\mu\nu}$  is the metric tensor of Minkowski. Keeping the leading non-vanishing contributions to the action, and dropping some boundary terms, we get the following action,

$$S = -\frac{1}{16\pi G} \int d^4x \left[ \frac{1}{2} h^{\mu\nu} D_{\mu\nu\rho\sigma} h^{\rho\sigma} + \int d^4x \left[ -\frac{1}{2} \left( \eta^{\mu\nu} + \frac{1}{2} \eta^{\mu\nu} h - h^{\mu\nu} \right) (\partial_\mu \psi)(\partial_\nu \psi) - \frac{\tilde{\eta}}{4} (D_{\rho\sigma}^{\mu\nu} h^{\rho\sigma}) (\partial_\mu \psi)(\partial_\nu \psi) \right] \right], \quad (\text{A.1})$$

where here indices are raised/lowered with  $\eta_{\mu\nu}$  and where  $D_{\mu\nu\rho\sigma}$  represents the well known graviton dynamical operator,

$$D_{\mu\nu\rho\sigma} = \frac{1}{2} \left[ \partial_{(\rho} \eta_{\sigma)(\mu} \partial_{\nu)} - \frac{1}{2} \eta_{\mu(\rho} \eta_{\sigma)\nu} \partial^2 - \frac{1}{2} \eta_{\rho\sigma} \partial_{\mu} \partial_{\nu} - \frac{1}{2} \eta_{\mu\nu} \partial_{\rho} \partial_{\sigma} + \frac{1}{2} \eta_{\mu\nu} \eta_{\rho\sigma} \partial^2 \right], \quad (\text{A.2})$$

and  $\tilde{\eta}$  is a coupling constant. Now varying the action with respect to  $h^{\mu\nu}$  and  $\psi$  gives the following equations of motion,

$$-\frac{1}{16\pi G} D_{\mu\nu\rho\sigma} h^{\rho\sigma} + \frac{1}{2} (\partial_{\mu} \psi) (\partial_{\nu} \psi) - \frac{1}{4} \eta_{\mu\nu} \eta^{\alpha\beta} (\partial_{\alpha} \psi) (\partial_{\beta} \psi) - \frac{1}{4} \tilde{\eta} D_{\rho\sigma}^{\alpha\beta} [(\partial_{\alpha} \psi) (\partial_{\beta} \psi)] = 0$$

$$\partial_{\mu} \left[ \left( \eta^{\mu\nu} + \frac{1}{2} \eta^{\mu\nu} h - h^{\mu\nu} \right) \partial_{\nu} \psi \right] + \frac{\tilde{\eta}}{2} \partial_{\mu} \left[ (D^{\mu\nu}_{\rho\sigma} h^{\rho\sigma}) \partial_{\nu} \psi \right] = 0. \quad (\text{A.3})$$

It seems that both equations contain more than two time derivatives acting on fields (the graviton equation contains three time derivatives acting on the scalar field, while the scalar field equation contains three time derivatives acting on the graviton). But in fact they don't. Then we prove it.

We define the following quantities:

$$X_{\mu\nu\rho\sigma} = \partial_{(\rho} \eta_{\sigma)(\mu} \partial_{\nu)} = \frac{1}{4} [\eta_{\sigma\mu} \partial_{\rho} \partial_{\nu} + \eta_{\rho\mu} \partial_{\sigma} \partial_{\nu} + \eta_{\sigma\nu} \partial_{\rho} \partial_{\mu} + \eta_{\rho\nu} \partial_{\sigma} \partial_{\mu}],$$

$$Y_{\mu\nu\rho\sigma} = -\frac{1}{2} \eta_{\mu(\rho} \eta_{\sigma)\nu} \partial^2 = -\frac{1}{4} [\eta_{\mu\rho} \eta_{\sigma\nu} \partial^{\tau} \partial_{\tau} + \eta_{\mu\sigma} \eta_{\rho\nu} \partial^{\tau} \partial_{\tau}],$$

$$Z_{\mu\nu\rho\sigma} = -\frac{1}{2} \eta_{\rho\sigma} \partial_{\mu} \partial_{\nu},$$

$$K_{\mu\nu\rho\sigma} = -\frac{1}{2} \eta_{\mu\nu} \partial_{\rho} \partial_{\sigma},$$

$$L_{\mu\nu\rho\sigma} = +\frac{1}{2} \eta_{\mu\nu} \eta_{\rho\sigma} \partial^2 = \frac{1}{2} \eta_{\mu\nu} \eta_{\rho\sigma} \partial^{\tau} \partial_{\tau}, \quad (\text{A.4})$$

and firstly calculate the term  $D_{\rho\sigma}^{\alpha\beta} [(\partial_{\alpha} \psi) (\partial_{\beta} \psi)]$ .

$$X_{\rho\sigma}^{\alpha\beta} [(\partial_{\alpha} \psi) (\partial_{\beta} \psi)] = \frac{1}{2} \psi_{,\rho\sigma} \psi^{,\beta} + \frac{1}{4} \psi_{,\rho\alpha} \psi^{,\alpha}_{,\sigma} + \frac{1}{4} \psi_{,\sigma\alpha} \psi^{,\alpha}_{,\rho} + \frac{1}{2} \psi_{,\alpha} \psi^{,\alpha}_{,\rho\sigma} + \frac{1}{4} \psi_{,\sigma} \psi^{,\beta}_{,\rho\beta} + \frac{1}{4} \psi_{,\rho} \psi^{,\beta}_{,\sigma\beta}$$

$$+ \frac{1}{2} \psi_{,\rho\sigma} \psi^{,\alpha}_{,\alpha} + \frac{1}{4} \psi_{,\rho\beta} \psi^{,\beta}_{,\sigma} + \frac{1}{4} \psi_{,\sigma\beta} \psi^{,\beta}_{,\rho} + \frac{1}{2} \psi_{,\beta} \psi^{,\beta}_{,\rho\sigma} + \frac{1}{4} \psi_{,\sigma} \psi^{,\alpha}_{,\rho\alpha} + \frac{1}{4} \psi_{,\rho} \psi^{,\alpha}_{,\sigma\alpha},$$

$$Y_{\rho\sigma}^{\alpha\beta} [(\partial_{\alpha} \psi) (\partial_{\beta} \psi)] = -\frac{1}{2} [\psi^{,\tau}_{,\rho} \psi_{,\sigma\tau} + \psi^{,\tau}_{,\sigma} \psi_{,\rho\tau} + \psi_{,\rho} \psi^{,\tau}_{,\sigma\tau} + \psi_{,\sigma} \psi^{,\tau}_{,\rho\tau}],$$

$$Z_{\rho\sigma}^{\alpha\beta} [(\partial_{\alpha} \psi) (\partial_{\beta} \psi)] = -\frac{1}{2} \eta_{\rho\sigma} [\psi^{,\alpha}_{,\alpha} \psi^{,\beta}_{,\beta} + \psi_{,\alpha} \psi^{,\alpha\beta}_{,\beta} + \psi^{,\beta}_{,\alpha} \psi^{,\alpha}_{,\beta} + \psi_{,\beta} \psi^{,\alpha\beta}_{,\alpha}],$$

$$K_{\rho\sigma}^{\alpha\beta} [(\partial_{\alpha} \psi) (\partial_{\beta} \psi)] = -\frac{1}{2} [\psi_{,\rho\alpha} \psi^{,\alpha}_{,\sigma} + \psi_{,\alpha} \psi^{,\alpha}_{,\rho\sigma} + \psi_{,\sigma\alpha} \psi^{,\alpha}_{,\rho} + \psi_{,\rho\sigma\alpha} \psi^{,\alpha}],$$

$$L_{\rho\sigma}^{\alpha\beta} [(\partial_{\alpha} \psi) (\partial_{\beta} \psi)] = +\frac{1}{2} \eta_{\rho\sigma} [\psi_{,\tau\alpha} \psi^{,\tau\alpha} + \psi_{,\alpha} \psi^{,\alpha\tau}_{,\tau} + \psi^{,\tau}_{,\alpha} \psi^{,\alpha}_{,\tau} + \psi^{,\alpha}_{,\alpha\tau} \psi^{,\tau}_{,\alpha\tau}]. \quad (\text{A.5})$$

So we obtain

$$D_{\rho\sigma}^{\alpha\beta} [(\partial_{\alpha} \psi) (\partial_{\beta} \psi)] = \frac{1}{4} \eta_{\rho\sigma} [\psi_{,\alpha\beta} \psi^{,\alpha\beta} - \psi^{,\alpha}_{,\alpha} \psi^{,\beta}_{,\beta}] - \frac{1}{4} [\psi_{,\rho\alpha} \psi^{,\alpha}_{,\sigma} + \psi_{,\sigma\alpha} \psi^{,\alpha}_{,\rho} + 2\psi_{,\rho\sigma} \psi^{,\alpha}_{,\alpha}], \quad (\text{A.6})$$

which do not contain more than two time derivatives acting on the scalar fields. Secondly we calculate the term  $\partial_\mu(D^{\mu\nu}_{\rho\sigma}h^{\rho\sigma})$ .

$$\begin{aligned}\partial_\mu(X^{\mu\nu}_{\rho\sigma}h^{\rho\sigma}) &= \frac{1}{4}\left(h^{\rho\mu,\nu} + h^{\sigma\mu,\nu} + h^{\rho\nu,\mu} + h^{\sigma\nu,\mu}\right), \\ \partial_\mu(Y^{\mu\nu}_{\rho\sigma}h^{\rho\sigma}) &= -\frac{1}{2}h^{\mu\nu,\tau}, \\ \partial_\mu(Z^{\mu\nu}_{\rho\sigma}h^{\rho\sigma}) &= -\frac{1}{2}h^{\mu\nu}, \\ \partial_\mu(K^{\mu\nu}_{\rho\sigma}h^{\rho\sigma}) &= -\frac{1}{2}h^{\rho\sigma,\nu}, \\ \partial_\mu(L^{\mu\nu}_{\rho\sigma}h^{\rho\sigma}) &= \frac{1}{2}h^{\tau\nu}.\end{aligned}\tag{A.7}$$

Then we obtain

$$\partial_\mu(D^{\mu\nu}_{\rho\sigma}h^{\rho\sigma}) = 0,\tag{A.8}$$

which don't contain three time derivatives acting on the graviton. Lastly we find the motion and graviton equations are

$$\begin{aligned}-\frac{1}{16\pi G}D_{\mu\nu\rho\sigma}h^{\rho\sigma} + \frac{1}{2}(\partial_\mu\psi)(\partial_\nu\psi) - \frac{1}{4}\eta_{\mu\nu}\eta^{\alpha\beta}(\partial_\alpha\psi)(\partial_\beta\psi) \\ - \frac{\tilde{\eta}}{16}\left\{\eta_{\rho\sigma}\left[\psi_{,\alpha\beta}\psi^{,\alpha\beta} - \psi_{,\alpha}^{\alpha}\psi_{,\beta}^{\beta}\right] - \left[\psi_{,\rho\alpha}\psi_{,\sigma}^{\alpha} + \psi_{,\sigma\alpha}\psi_{,\rho}^{\alpha} + 2\psi_{,\rho\sigma}\psi_{,\alpha}^{\alpha}\right]\right\} = 0, \\ \partial_\mu\left[\left(\eta^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}h - h^{\mu\nu}\right)\partial_\nu\psi\right] + \frac{\tilde{\eta}}{2}(D^{\mu\nu}_{\rho\sigma}h^{\rho\sigma})\partial_\mu\partial_\nu\psi = 0.\end{aligned}\tag{A.9}$$

Note that neither the graviton nor the scalar equation of motion contain three time derivatives, suggesting that these type of theories may have, after all, energy bounded from below, and hence represent physically acceptable theories.

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