Published for SISSA by Springer

Received: *August 17, 2010*
Accepted: *October 18, 2010*
Published: *November 8, 2010*

Hidden Yangian symmetry in sigma model on squashed sphere

Io Kawaguchi and Kentaroh Yoshida

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

E-mail: io@gauge.scphys.kyoto-u.ac.jp, kyoshida@gauge.scphys.kyoto-u.ac.jp

Abstract: We discuss a hidden symmetry of a two-dimensional sigma model on a squashed S^3. The SU(2) current can be improved so that it can be regarded as a flat connection. Then we can obtain an infinite number of conserved non-local charges and show the Yangian algebra by directly checking the Serre relations. This symmetry is also deduced from the coset structure of the squashed sphere. The same argument is applicable to the warped AdS_3 spaces by the double Wick rotations.

Keywords: Integrable Field Theories, Sigma Models, AdS-CFT Correspondence

ArXiv ePrint: 1008.0776

doi:10.1007/JHEP11(2010)032

JHEP11(2010)032

Contents

1 Introduction

It is well known that two-dimensional sigma models on *symmetric* spaces are classically integrable [1]. Simple examples are the $O(3)$ non-linear sigma model and the SU(2) principal chiral model. These sigma models are integrable also at quantum level and the physical quantities such as the S-matrix can be computed exactly [2, 3]. Thus, for the integrable models, we can study non-perturbative aspects. Then there is a hidden, infinite-dimensional symmetry [4, 5], called the Yangian symmetry [6, 7], behind the integrable structure.

The integrable structure on the symmetric space AdS$_5 \times S^5$ is inherited to the type IIB string theory on this background [8] and now it has a new perspective and potential applications in the study of the AdS/CFT correspondence [9]. Recently, integrable string backgrounds have been classified based on the symmetric-coset structure [10]. Then an interesting issue is to consider sigma models on *non-symmetric* spaces and discuss a generalization of the AdS/CFT dictionary. Simple examples of non-symmetric spaces are squashed spheres and warped AdS spaces.

In this letter we will mainly consider a sigma model on a squashed S^3. This squashed geometry is realized as a U(1)-fibration (ψ) over S^2 (θ, ϕ) and the metric is given by

$$ds^2 = \frac{L^2}{4}\left[\,d\theta^2 + \cos^2\theta\, d\phi^2 + (1+C)(d\psi + \sin\theta\, d\phi)^2\,\right]\,. \tag{1.1}$$

The constant parameter C measures the deformation of S^3 and $C = 0$ just describes the round S^3 with the radius L. It breaks the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ symmetry of the round S^3 down to $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{R}$. It is known that the sigma model for $C = 0$ has the Yangian symmetry. But it remains to be clarified for $C \neq 0$ whether or not the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{R}$ symmetry can enhance to an infinite-dimenisonal symmetry.

2 Yangian symmetry

We discuss a sigma model on the squashed S^3 with the metric (1.1). Its classical action is given by

$$S = -\frac{1}{2}\iint dtdx\left[(\partial_\mu\theta)^2 + \cos^2\theta(\partial_\mu\phi)^2 + (1+C)(\partial_\mu\psi + \sin\theta\partial_\mu\phi)^2\right]. \tag{2.1}$$

For simplicity, we will not take the Virasoro conditions into account and assume that the base space is two-dimensional Minkowski space with the coordinates $x^\mu = (t,x)$ and the metric $\eta_{\mu\nu} = (-1,1)$.

The isometry of (1.1) is $\mathrm{SU(2)_L} \times \mathrm{U(1)_R}$ and hence the action (2.1) is invariant under the following four transformations:

$$\begin{aligned}
\delta_1(\phi,\psi,\theta) &= \epsilon\,(-1\,,\ 0\,,\ 0)\,, \\
\delta_2(\phi,\psi,\theta) &= \epsilon\left(-\sin\phi\tan\theta\,,\ \frac{\sin\phi}{\cos\theta}\,,\ \ -\cos\phi\right), \\
\delta_3(\phi,\psi,\theta) &= \epsilon\left(\cos\phi\tan\theta\,,\ -\frac{\cos\phi}{\cos\theta}\,,\ \ -\sin\phi\right), \\
\delta_4(\phi,\psi,\theta) &= \epsilon\,(0\,,\ \ -1\,,\ 0)\,.
\end{aligned}$$

Here ϵ is an infinitesimal constant. Note that the transformations are independent of the squashing parameter C. The first three are the $\mathrm{SU(2)_L}$ transformations and the last is the $\mathrm{U(1)_R}$ one. By using the Noether's theorem, we can construct the corresponding conserved current and its componets are given by

$$\begin{aligned}
J^1_\mu &= \partial_\mu\phi + \sin\theta\,\partial_\mu\psi + C\,\sin\theta\,(\partial_\mu\psi + \sin\theta\,\partial_\mu\phi)\,, \\
J^2_\mu &= \cos\phi\,\partial_\mu\theta - \sin\phi\,\cos\theta\,\partial_\mu\psi \\
&\quad -C\sin\phi\,\cos\theta\,(\partial_\mu\,\psi + \sin\theta\,\partial_\mu\phi)\,, \\
J^3_\mu &= \sin\phi\,\partial_\mu\theta + \cos\phi\,\cos\theta\,\partial_\mu\psi \\
&\quad +C\,\cos\phi\,\cos\theta\,(\partial_\mu\psi + \sin\theta\,\partial_\mu\phi)\,, \\
J^4_\mu &= (1+C)(\partial_\mu\psi + \sin\theta\,\partial_\mu\phi)\,.
\end{aligned}$$

After some algebra, for the $\mathrm{SU(2)_L}$ current J^A_μ $(A=1,2,3)$, we obtain that

$$\epsilon^{\mu\nu}\left(\partial_\mu J^A_\nu - \frac{1}{2}\varepsilon_{BC}{}^{A} J^B_\mu\,J^C_\nu\right) = Cn^A\,\epsilon^{\mu\nu}\partial_\mu(\sin\theta)\partial_\nu\phi$$

with the anti-symmetric tensor $\epsilon_{\mu\nu}$ normalized as $\epsilon_{tx} = +1$ and n^A is the unit vector on S^2 given by

$$n^A \equiv (\sin\theta, -\sin\phi\cos\theta, \cos\phi\cos\theta)\,.$$

Thus the $\mathrm{SU(2)_L}$ current J^A_μ does not satisfy the flatness condition.

But it can be improved with the ambiguity of the Noether current so that it satisfies the flatness condition. The improved current j^A_μ is given by a linear combination of the Noether current J^A_μ and the improvement term I^A_μ as follows:

$$j^A_\mu = J^A_\mu + I^A_\mu\,, \qquad I^A_\mu \equiv \pm\sqrt{C}\epsilon_{\mu\nu}\partial^\nu n^A\,. \tag{2.2}$$

It is easy to show that j^A_μ satisfies the conservation law $\partial^\mu j^A_\mu = 0$ and the flatness condition

$$\epsilon^{\mu\nu}\left(\partial_\mu j^A_\nu - \frac{1}{2}\varepsilon_{BC}{}^A j^B_\mu\, j^C_\nu\right) = 0\,. \tag{2.3}$$

Thus the improved current j^A_μ can be regarded as a flat conserved current. We will discuss the geometrical meaning of the improvement term later.

By using the canonical Poisson bracket, the current algebra is computed as

$$\begin{aligned}
\left\{j^A_t(x), j^B_t(y)\right\}_{\rm P} &= \varepsilon^{AB}{}_C\, j^C_t(x)\,\delta(x-y)\,, \\
\left\{j^A_t(x), j^B_x(y)\right\}_{\rm P} &= \varepsilon^{AB}{}_C\, j^C_x(x)\,\delta(x-y) + (1+C)\delta^{AB}\partial_x\delta(x-y)\,, \\
\left\{j^A_x(x), j^B_x(y)\right\}_{\rm P} &= -C\varepsilon^{AB}{}_C\, j^C_t(x)\,\delta(x-y)\,.
\end{aligned} \tag{2.4}$$

The algebra (2.4) contains the squashing parameter C and the last bracket $\{j^A_x, j^B_x\}_{\rm P}$ does not vanish any more. Hence it is not obvious at this stage whether the algebra (2.4) leads to the Yangian algebra or not.

After the flat conserved current j^A_μ has been obtained, an infinite number of the conserved charges can be constructed by following [11]. The Noether charge $Q^A_{(0)}$ and the first non-local charge $Q^A_{(1)}$ are defined as, respectively,

$$\begin{aligned}
Q^A_{(0)} &\equiv \int dx\, j^A_t(x)\,, \\
Q^A_{(1)} &\equiv -\int dx\, j^A_x(x) + \frac{1}{4}\iint dxdy\,\epsilon(x-y)\,\varepsilon_{BC}{}^A\, j^B_t(x) j^C_t(y)\,.
\end{aligned}$$

The algebra that $Q^A_{(0)}$ and $Q^A_{(1)}$ satisfy is given by[1]

$$\begin{aligned}
\left\{Q^A_{(0)}, Q^B_{(0)}\right\}_{\rm P} &= \varepsilon^{AB}{}_C Q^C_{(0)}\,, \\
\left\{Q^A_{(0)}, Q^B_{(1)}\right\}_{\rm P} &= \varepsilon^{AB}{}_C Q^C_{(1)}\,, \\
\left\{Q^A_{(1)}, Q^B_{(1)}\right\}_{\rm P} &= \varepsilon^{AB}{}_C\left[Q^C_{(2)} + \frac{1}{12}Q^C_{(0)}\left(Q_{(0)}\right)^2 - CQ^C_{(0)}\right],
\end{aligned}$$

where $Q^A_{(2)}$ is the second non-local charge defined as

$$\begin{aligned}
Q^A_{(2)} \equiv\ & \frac{1}{12}\iiint dxdydz\,\epsilon(x-y)\,\epsilon(y-z) \\
& \times\delta_{BC}\left[\,j^A_t(x)\, j^B_t(y)\, j^C_t(z) - j^B_t(x)\, j^A_t(y)\, j^C_t(z)\,\right] \\
& + \frac{1}{2}\iint dxdy\,\epsilon(x-y)\,\varepsilon_{BC}{}^A\, j^B_t(x)\; j^C_x(y)\,.
\end{aligned}$$

[1] There is an ambiguity concerning the non-ultra local term even in the present case. Its influence is the same as in the principal chiral model because the term is proportional to $1+C$. But demanding the Serre relations leads us to the definite calculus of the Poisson bracket without imposing some artificial prescription by hand. The relation of the Serre relations and the definition of the Poisson bracket is discussed in [5] in the case of the principal chiral model. Here we compute the Poisson bracket of the charges along this line.

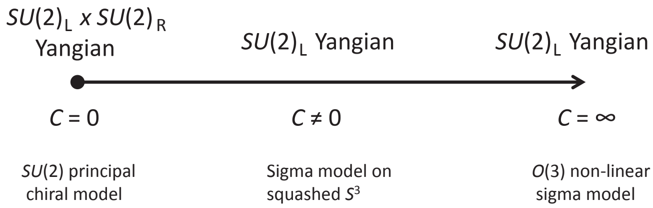

Figure 1. Yangians with respect to the squashing parameter C.

It is straightforward to check the Serre relations,

$$
\begin{aligned}
\{\{Q^{+}_{(1)}, Q^{-}_{(1)}\}_{\rm P}, Q^{z}_{(1)}\}_{\rm P} &= \frac{1}{4} Q^{z}_{(0)} \left[Q^{+}_{(1)} Q^{-}_{(0)} - Q^{-}_{(1)} Q^{+}_{(0)} \right] , \\
\{\{Q^{z}_{(1)}, Q^{\pm}_{(1)}\}_{\rm P}, Q^{\pm}_{(1)}\}_{\rm P} &= \frac{1}{4} Q^{\pm}_{(0)} \left[Q^{z}_{(1)} Q^{\pm}_{(0)} - Q^{\pm}_{(1)} Q^{z}_{(0)} \right] ,
\end{aligned}
$$

$$
\{\{Q^{+}_{(1)}, Q^{-}_{(1)}\}_{\rm P}, Q^{\pm}_{(1)}\}_{\rm P} \pm 2 \{\{Q^{z}_{(1)}, Q^{\pm}_{(1)}\}_{\rm P}, Q^{z}_{(1)}\}_{\rm P} = \frac{1}{4} Q^{\pm}_{(0)} \left[Q^{+}_{(1)} Q^{-}_{(0)} - Q^{-}_{(1)} Q^{+}_{(0)} \right] \pm \frac{1}{2} Q^{z}_{(0)} \left[Q^{z}_{(1)} Q^{\pm}_{(0)} - Q^{\pm}_{(1)} Q^{z}_{(0)} \right] ,
$$

where we have introduced the following notation for the group index,

$$
j^{\pm}_{\mu} \equiv j^{2}_{\mu} \pm i j^{3}_{\mu} , \qquad j^{z}_{\mu} \equiv j^{1}_{\mu} .
$$

Thus we have shown that the non-local charges constructed from the improved flat current surely satisfy the Yangian algebra.

This result may seem curious. The Yangian algebra should correspond to the XXX model, while the squashed S^3 is described as a deformation of the round S^3 and the resulting algebra is expected to be an affine quantum algebra related to the XXZ model rather than the XXX model. In fact, the discretized model, which gives rise to the sigma model on the squashed S^3 in a continuum limit, is the XXZ model as discussed in [12].

How can one understand the Yangian algebra? To find a natural interpretation on the result, it would be helpful to consider the $C \to \infty$ limit, where the geometry is reduced to the direct product $S^2 \times S^1$. Then it is easy to find the $\mathrm{SU}(2)_{\rm L}$ Yangian from the $O(3)$ non-linear sigma model. On the other hand, for $C = 0$ there is the $\mathrm{SU}(2)_{\rm L} \times \mathrm{SU}(2)_{\rm R}$ Yangian. A natural interpretation is that the Yangian obtained above is inherited from the $\mathrm{SU}(2)_{\rm L}$ Yangian for $C = \infty$ and interpolates to the $\mathrm{SU}(2)_{\rm L}$ Yangian subalgebra for $C = 0$, as depicted in figure 1. In other words, the $\mathrm{SU}(2)_{\rm L}$ Yangian symmetry of the $O(3)$ non-linear sigma model can survive under the Hopf U(1) fibration for all values of C.

An interesting question here is whether the Yangian symmetry means the complete integrability of the sigma model on the squashed S^3 in the sense of Liouville. There are an infinite number of conserved charges, but which would not imply the complete integrability. Even if the Yangian symmetry does not mean the complete integrability, we may expect some strong constraints coming from the infinite-dimensional symmetry and they may be useful to compute some physical quantities, for example, in the the context of AdS/CFT.

Furthermore it is worth noting the geometrical interpretation of the improvement term. The topological current I^{A}_{μ} given in (2.2) can directly be obtained if the Hopf term is added to the classical action (2.1):

$$\begin{aligned} S_{\text{Hopf}} &= \pm\sqrt{C}\iint dtdx\,\epsilon^{\mu\nu}\,\cos\theta\,\partial_\mu\theta\,\partial_\nu\phi \\ &= \pm\frac{\sqrt{C}}{2}\iint dtdx\,\epsilon^{\mu\nu}\,\varepsilon_{ABC}\,n^A\partial_\mu n^B\partial_\nu n^C\,. \end{aligned}$$

But this term vanishes automatically because there is no topological obstruction in the present setup, namely $\pi_2(S^3) = 0$, and it can be gauged away. Thus it does not change the original theory but it is still sensitive to the flatness condition via the ambiguity of the Noether current.

A similar geometry, that is a squashed S^3 geometry equipped with a two-form flux, can be realized as a squashed giant graviton [13, 14] and hence the hidden symmetry found here may be useful in this issue. For example, it would be possible to consider an open string attaching on the squashed giant graviton by generalizing the works on the maximal giant graviton case [15–18]. This would be an interesting direction to be studied.

3 Coset construction of squashed S^3

It is quite remarkable that the Yangian symmetry has been found in the sigma model on the squashed S^3, because the squashed S^3 is not described as a symmetric coset. Thus it should be interesting to see the coset structure of the squashed S^3.

The squashed S^3 has the isometry $G = \text{SU}(2)_{\text{L}} \times \text{U}(1)_{\text{R}}$. The $\text{SU}(2)_{\text{L}}$ generators T_A $(A = 1, 2, 3)$ satisfy $[T_A, T_B] = \varepsilon_{AB}{}^C T_C$. The $\text{U}(1)_{\text{R}}$ generator is represented by T_4.

Let us consider a coset $M = G/H$. When the Lie algebras for M and H are written as $\mathfrak{m}$ and $\mathfrak{h}$, respectively, we may consider the following algebras

$$\mathfrak{m} = \langle T_1, T_2, T_3\rangle\,, \qquad \mathfrak{h} = \langle \alpha T_3 + \beta T_4\rangle\,.$$

Here α and β are constant parameters. The group structure constants related to our argument are represented by

$$f_{[m]\alpha T_3+\beta T_4}{}^{[n]} = \begin{pmatrix} 0 & -\alpha & 0 \\ \alpha & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\,,$$

where the indices $[m], [n]$ are for $\mathfrak{m}$ and are defined up to $\mathfrak{h}$-transformation.

The most general symmetric two-form is given by [19]

$$\Omega_{[m][n]} = \begin{pmatrix} \Omega_{11} & 0 & 0 \\ 0 & \Omega_{11} & 0 \\ 0 & 0 & \Omega_{33} \end{pmatrix}\,.$$

By taking a coset representative as $g = \text{e}^{\phi T_1}\text{e}^{\theta T_2}\text{e}^{\psi T_3}$ and expanding the Maurer-Cartan one-form $J_{\text{L}} \equiv g^{-1}dg$, the vielbeine are computed as

$$\begin{aligned} e^1 &= \cos\psi\,\cos\theta\,d\phi + \sin\psi\,d\theta\,, \\ e^2 &= -\sin\psi\,\cos\theta\,d\phi + \cos\psi\,d\theta\,, \\ e^3 &= d\psi + \sin\theta\,d\phi\,. \end{aligned}$$

Taking $\Omega_{11} = L^2/4$ and $\Omega_{33} = L^2(1+C)/4$, we obtain

$$
\begin{aligned}
ds^2 &= \Omega_{[m][n]} e^{[m]} e^{[n]} \\
&= \frac{L^2}{4} \left[\cos^2\theta d\phi^2 + d\theta^2 + (1+C)(d\psi + \sin\theta\, d\phi)^2 \right] .
\end{aligned}
$$

Thus the metric of the squashed S^3 has been reproduced by the coset construction. Note that the presense of the $\mathrm{U}(1)_\mathrm{R}$ generator T_4 is crucial for the present construction, although it is blind to the infinite-dimensional extension. If not, the coset cannot be defined.

Let us comment on the relation between the coset structure and the non-local charges. The coset we used here is not symmetric but satisfies $[\mathfrak{m},\mathfrak{m}] \subset \mathfrak{m}$. For this case as well as symmetric cosets, an infinite set of non-local charges can be constructed potentially as described in [1]. Thus the coset structure would also be consistent to the previous result.

4 From squashed S^3 to warped AdS_3

We can discuss warped AdS_3 spaces in the same way as in the squashed S^3 case. As summarized in [20], there are three kinds of warped AdS_3 spaces: 1) space-like, 2) time-like and 3) null. The metrics of the first two are reproduced from the metric of the squashed S^3 (1.1) by the double Wick rotations as follows.[2]

First, by taking a Wick rotation $\theta \to i\sigma$, $\phi \to \tau$, $\psi \to iu$ and inverting the overall sign, we obtain the metric describing the space-like warped AdS_3

$$
ds^2 = \frac{L^2}{4} \left[-\cosh^2\sigma\, d\tau^2 + d\sigma^2 + (1+C)(du + \sinh\sigma\, d\tau)^2 \right] .
$$

Similarly, by taking another Wick rotation $\theta \to i\sigma$, $\phi \to iu$, $\psi \to \tau$ and inverting the overall sign, the metric of the time-like warped AdS_3 is obtained as

$$
ds^2 = \frac{L^2}{4} \left[\cosh^2\sigma\, du^2 + d\sigma^2 - (1+C)(d\tau - \sinh\sigma\, du)^2 \right] .
$$

The hidden symmetry of sigma models on space-like and time-like warped AdS_3 spaces can be shown in the same way. The coset structure is also the same.

For the null case 3), the metric is known as a three-dimensional Schrödinger spacetime [21, 22] and not directly related to the squashed S^3, though the metric can be obtained by the coset construction [23]. Then the coset proposed in [23] is non-reductive and the hidden symmetry of sigma models on the null warped AdS_3 remains to be clarified.

It is known that the metrics of the warped AdS spaces become solutions of topologically massive gravity [24, 25] by choosing appropriate values of C (depending on the gravitational Chern-Simons coupling and cosmological constant). The hidden symmetry may play an important role in studying topologically massive gravity.

[2]One can definitely discuss the Yangian symmetry in the warped AdS_3 at classical level by performing an analytic continuation. At quantum level, it is not clear whether the analytic continuation works or not, even if the Yangian symmetry could survive under quantum corrections.

JHEP11(2010)032

5 Conclusion and discussion

We have discussed a hidden symmetry of a sigma model on a squashed S^3. By improving the SU(2) current so that it can be regarded as a flat connection, we have constructed an infinite number of non-local charges and shown that the charges satisfy the Yangian algebra, though the squashed S^3 is not a symmetric space. We have also discussed the Yangian symmetry from the coset structure. The same argument is applicable to the warped AdS_3 spaces.

There are many applications. First, the hidden symmetry we found out may be useful in considering a deformation of spin chains in the AdS_3/CFT_2 discussed in [26] (see [27] for an earlier attempt in this direction). In fact, warped AdS_3 and squashed S^3 geometries are realized as string backgrounds [28–33]. It would be a good exercise to show the relation between the Yangian symmetry and the T-duality utilized in [32].

An easy task is to incorporate supersymmetries, for example, based on a super Lie group SU(2|1) and find out a hidden super Yangian symmetry. It is also nice to consider a generalization to the higher-dimensional case. A simple derivation of the metrics of higher-dimensional squashed spheres is discussed in [34] and the result therein would be helpful for this purpose. We are not sure whether the hidden symmetry is intrinsic to Hopf fibrations or not. It would be very interesting to see how it carries over, for example, to the case of the squashed S^7.

While we have discussed here the Yangian symmetry based on the non-local charges, it would also be possible to construct an infinite number of "local" charges though it is left as a future problem. It is interesting to construct the local charges concretely and see the algebra that the local and non-local charges satisfy.

An important task is to check whether the Yangian symmetry can survive under quantum corrections. If it can, its generators can act on the asymptotic states as operators and give some strong constraints for the S-matrix. Then it would be possible to see that the spectrum is also constrained by the Yangian symmetry.

A challenging issue is to try to put the hidden symmetry to practical use in the context of the Kerr/CFT correspondence [35], where a three-dimensional slice of the near-horizon extreme Kerr (NHEK) geometry [36] is described as a warped AdS_3 space. The sigma model on the NHEK geometry might possess a hidden symmetry and if it exists, it would be very helpful to elaborate the Kerr/CFT correspondence.

As another issue, a warped AdS geometry appears also in an application of AdS/CFT to condensed matter physics [37, 38]. The Yangian symmetry may be useful in this direction.

Squashed geometries often appear in various ways and in many cases. We believe that the hidden symmetry discussed here would be an important key ingredient to consider some generalizations of AdS/CFT and open up new research directions.

JHEP11(2010)032

Acknowledgments

We would like to thank S. Detournay, M. Hatsuda, Y. Hatsuda, K. Iwasaki, H. Kawai, D. Orlando, R. Sasaki, M. Sakaguchi, K. Sakai, Y. Yasui and C. A. S. Young for illuminating discussions. This work was supported by the scientific grants from the ministry of education, science, sports, and culture of Japan (No. 22740160), and in part by the Grant-in-Aid for the Global COE Program "The Next Generation of Physics, Spun from Universality and Emergence" from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan. In addition, we thank the Yukawa Institute for Theoretical Physics at Kyoto University. Discussions during the YITP workshop YITP-W-08-04 on "Development of Quantum Field Theory and String Theory" were useful to complete this work.

References

[1] E. Abdalla, M. C. Abdalla and K. Rothe, *Non-perturbative methods in two-dimensional quantum field theory*, World Scientific (1991).

[2] M. Lüscher, *Quantum Nonlocal Charges and Absence of Particle Production in the Two-Dimensional Nonlinear σ-model*, *Nucl. Phys.* **B 135** (1978) 1 [SPIRES].

[3] M. Lüscher and K. Pohlmeyer, *Scattering of Massless Lumps and Nonlocal Charges in the Two-Dimensional Classical Nonlinear σ-model*, *Nucl. Phys.* **B 137** (1978) 46 [SPIRES].

[4] D. Bernard, *Hidden Yangians in 2D massive current algebras*, *Commun. Math. Phys.* **137** (1991) 191 [SPIRES].

[5] N.J. MacKay, *On the classical origins of Yangian symmetry in integrable field theory*, *Phys. Lett.* **B 281** (1992) 90 [*Erratum ibid.* **B 308** (1993) 444] [SPIRES].

[6] V. G. Drinfel'd, *Hopf Algebras and the Quantum Yang-Baxter Equation*, *Sov. Math. Dokl.* **32** (1985) 254.

[7] V. G. Drinfel'd, *A New realization of Yangians and quantized affine algebras*, *Sov. Math. Dokl.* **36** (1988) 212

[8] I. Bena, J. Polchinski and R. Roiban, *Hidden symmetries of the $AdS_5 \times S^5$ superstring*, *Phys. Rev.* **D 69** (2004) 046002 [hep-th/0305116] [SPIRES].

[9] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [*Adv. Theor. Math. Phys.* **2** (1998) 231] [hep-th/9711200] [SPIRES].

[10] K. Zarembo, *Strings on Semisymmetric Superspaces*, *JHEP* **05** (2010) 002 [arXiv:1003.0465] [SPIRES].

[11] E. Brézin, C. Itzykson, J. Zinn-Justin and J.B. Zuber, *Remarks About the Existence of Nonlocal Charges in Two-Dimensional Models*, *Phys. Lett.* **B 82** (1979) 442 [SPIRES].

[12] L.D. Faddeev and N.Y. Reshetikhin, *Integrability of the principal chiral field model in* $(1+1)$-*dimension*, *Ann. Phys.* **167** (1986) 227 [SPIRES].

[13] S. Prokushkin and M.M. Sheikh-Jabbari, *Squashed giants: Bound states of giant gravitons*, *JHEP* **07** (2004) 077 [hep-th/0406053] [SPIRES].

[14] M. Ali-Akbari and M.M. Sheikh-Jabbari, *Electrified BPS Giants: BPS configurations on Giant Gravitons with Static Electric Field*, *JHEP* **10** (2007) 043 [arXiv:0708.2058] [SPIRES].

[15] D. Berenstein and S.E. Vazquez, *Integrable open spin chains from giant gravitons*, *JHEP* **06** (2005) 059 [hep-th/0501078] [SPIRES].

[16] A. Agarwal, *Open spin chains in super Yang-Mills at higher loops: Some potential problems with integrability*, *JHEP* **08** (2006) 027 [hep-th/0603067] [SPIRES].

[17] K. Okamura and K. Yoshida, *Higher loop Bethe ansatz for open spin-chains in AdS/CFT*, *JHEP* **09** (2006) 081 [hep-th/0604100] [SPIRES].

[18] D.M. Hofman and J.M. Maldacena, *Reflecting magnons*, *JHEP* **11** (2007) 063 [arXiv:0708.2272] [SPIRES].

[19] C.R. Nappi and E. Witten, *A WZW model based on a nonsemisimple group*, *Phys. Rev. Lett.* **71** (1993) 3751 [hep-th/9310112] [SPIRES].

[20] D. Anninos, W. Li, M. Padi, W. Song and A. Strominger, *Warped AdS_3 Black Holes*, *JHEP* **03** (2009) 130 [arXiv:0807.3040] [SPIRES].

[21] D.T. Son, *Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry*, *Phys. Rev.* **D 78** (2008) 046003 [arXiv:0804.3972] [SPIRES].

[22] K. Balasubramanian and J. McGreevy, *Gravity duals for non-relativistic CFTs*, *Phys. Rev. Lett.* **101** (2008) 061601 [arXiv:0804.4053] [SPIRES].

[23] S. Schäfer-Nameki, M. Yamazaki and K. Yoshida, *Coset Construction for Duals of Non-relativistic CFTs*, *JHEP* **05** (2009) 038 [arXiv:0903.4245] [SPIRES].

[24] S. Deser, R. Jackiw and S. Templeton, *Three-Dimensional Massive Gauge Theories*, *Phys. Rev. Lett.* **48** (1982) 975 [SPIRES].

[25] S. Deser, R. Jackiw and S. Templeton, *Topologically massive gauge theories*, *Ann. Phys.* **140** (1982) 372 [*Erratum ibid.* **185** (1988) 401] [*Erratum ibid.* **281** (2000) 409] [SPIRES].

[26] A. Babichenko, B. Stefanski, Jr. and K. Zarembo, *Integrability and the AdS_3/CFT_2 correspondence*, *JHEP* **03** (2010) 058 [arXiv:0912.1723] [SPIRES].

[27] W.-Y. Wen, *Spin chain from marginally deformed $AdS_3 \times S^3$*, *Phys. Rev.* **D 75** (2007) 067901 [hep-th/0610147] [SPIRES].

[28] D. Israel, C. Kounnas, D. Orlando and P.M. Petropoulos, *Electric / magnetic deformations of S^3 and AdS_3 and geometric cosets*, *Fortsch. Phys.* **53** (2005) 73 [hep-th/0405213] [SPIRES].

[29] D. Israel, C. Kounnas, D. Orlando and P.M. Petropoulos, *Heterotic strings on homogeneous spaces*, *Fortsch. Phys.* **53** (2005) 1030 [hep-th/0412220] [SPIRES].

[30] S. Detournay, D. Orlando, P.M. Petropoulos and P. Spindel, *Three-dimensional black holes from deformed anti de Sitter*, *JHEP* **07** (2005) 072 [hep-th/0504231] [SPIRES].

[31] G. Compere, S. Detournay and M. Romo, *Supersymmetric Gödel and warped black holes in string theory*, *Phys. Rev.* **D 78** (2008) 104030 [arXiv:0808.1912] [SPIRES].

[32] D. Orlando and L.I. Uruchurtu, *Warped anti-de Sitter spaces from brane intersections in type-II string theory*, *JHEP* **06** (2010) 049 [arXiv:1003.0712] [SPIRES].

[33] S. Detournay, D. Israel, J.M. Lapan and M. Romo, *String Theory on Warped AdS_3 and Virasoro Resonances*, arXiv:1007.2781 [SPIRES].

JHEP11(2010)032

[34] M. Hatsuda and S. Tomizawa, *Coset for Hopf fibration and Squashing*, *Class. Quant. Grav.* **26** (2009) 225007 [arXiv:0906.1025] [SPIRES].

[35] M. Guica, T. Hartman, W. Song and A. Strominger, *The Kerr/CFT Correspondence*, *Phys. Rev.* **D 80** (2009) 124008 [arXiv:0809.4266] [SPIRES].

[36] J.M. Bardeen and G.T. Horowitz, *The extreme Kerr throat geometry: A vacuum analog of $AdS_2 \times S^2$*, *Phys. Rev.* **D 60** (1999) 104030 [hep-th/9905099] [SPIRES].

[37] E. D'Hoker and P. Kraus, *Charged Magnetic Brane Solutions in AdS_5 and the fate of the third law of thermodynamics*, *JHEP* **03** (2010) 095 [arXiv:0911.4518] [SPIRES].

[38] E. D'Hoker and P. Kraus, *Holographic Metamagnetism, Quantum Criticality and Crossover Behavior*, *JHEP* **05** (2010) 083 [arXiv:1003.1302] [SPIRES].

JHEP11(2010)032