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Moments of multiplicity distributions for KNO scaling study using the ATLAS results

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Abstract: The normalised order-q moments of primarily charged-particle multiplicity distributions are studied for KNO scaling investigation in pp collisions as deduced from the results of the ATLAS at the LHC. The normalised moments for the LHC and low-energy experiments are compared for the kinematic region with an absolute pseudorapidity less than 2.5. The normalised moments show a increases linearly with centre-of-mass energies, and therefore the KNO scaling is violated for the full-scaled multiplicity region. The normalised moments for scaled multiplicity greater than one average multiplicity are constant for the highest centre-of-mass energies, and therefore the KNO scaling is concluded to hold.

Keywords: Hadron-Hadron Scattering , Minimum Bias, Proton-Proton Scattering, QCD

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The investigation of charged-particle distributions in proton-proton (pp) collisions probes the strong interaction at the low-momentum transfer (non-perturbative region of quantum chromodynamics). The study of normalised order-q moments, $C_{\mathrm{q}}(\sqrt{s}\,)$, of primary charged-particle multiplicity distributions is sensitive to the KNO scaling.

The KNO scaling hypothesis means that at energy asymptotic the probability distributions $P(n, \sqrt{s}\,)$ of producing n particles in a certain collision process should demonstrate a scaling relation [1–3]. The main assumption of the KNO scaling is Feynman scaling [4], where it was concluded that for asymptotically large centre-of-mass (CM) energies with $\sqrt{s} \to \infty$ the mean total number of any kind of particle logarithmically rises with the CM energy as $\langle n \rangle \propto \ln \sqrt{s}$. For this assumption the multiplicity distribution $P(n, \sqrt{s}\,)$ was represented as

$$P(n, \sqrt{s}\,) = \frac{1}{\langle n(\sqrt{s}\,)\rangle} \Psi(z) + \mathcal{O}\left(\frac{1}{\langle n(\sqrt{s}\,)\rangle^2}\right), \tag{1}$$

where $\langle n(\sqrt{s}\,)\rangle$ is the average multiplicity of primary particles at the CM energy, $\Psi(z)$ is the particle distribution as a function of the scaled multiplicity $z = n(\sqrt{s}\,)/\langle n(\sqrt{s}\,)\rangle$. The first term in (1) results from the leading term in $\ln \sqrt{s}$ (KNO scaling hypothesis), and the second term contains all other terms [5, 6]. The multiplicity distributions become simple rescaled copies of the universal function $\Psi(z)$ depending only on the scaled multiplicity or an energy-independent function. Asymptotically when $\sqrt{s} \to \infty$, the second term in (1) tends to zero, and therefore the KNO scaling holds.

To precisely find the KNO scaling, it is important to study the normalised order-q moments of primary charged-particle multiplicity distributions

$$C_{\mathrm{q}}(\sqrt{s}\,) = \frac{\langle n^q(\sqrt{s}\,)\rangle}{\langle n(\sqrt{s}\,)\rangle^{\mathrm{q}}}, \tag{2}$$

where q is the order of the moment. The moments $C_{\mathrm{q}}(\sqrt{s})$ give an integral characteristic of multiplicity distributions $P(n, \sqrt{s})$, while the KNO scaling distributions, $\Psi(z)$, give differential dependence from normalised multiplicity. The energy independence of $C_{\mathrm{q}}(\sqrt{s}\,)$ of multiplicity distributions of various orders would imply observation of the KNO scaling. Therefore, the investigation of $C_{\mathrm{q}}(\sqrt{s})$ moments is an absolutely essential addition to the study of $\Psi(z)$ functions.

In this paper the normalised order-q moments are studied using the primary charged-particle multiplicity distributions [7–11] by ATLAS [12] at the LHC [13]. Measurements of the primary charged-particle distributions in ATLAS at $\sqrt{s} = 0.9$, 2.36, 7, 8 and 13 TeV were performed for the pseudorapidity region less than 2.5 and for two samples of events: with the primary charged-particle multiplicity greater than or equal to 2 and 1 and with the charged-particle transverse momentum p_{T} greater than 100 and 500 MeV, respectively.

The study of the KNO scaling using the $\Psi(z)$ scaled multiplicity function, which is defined in (1), and calculated on the ATLAS data [7–11] was published in [14] by these authors. A comparison of $C_{\mathrm{q}}(\sqrt{s}\,)$ with the results of the LHC and lower energy experiments for pp interactions is presented. The KNO scaling and $C_{\mathrm{q}}(\sqrt{s})$-moments were studied by

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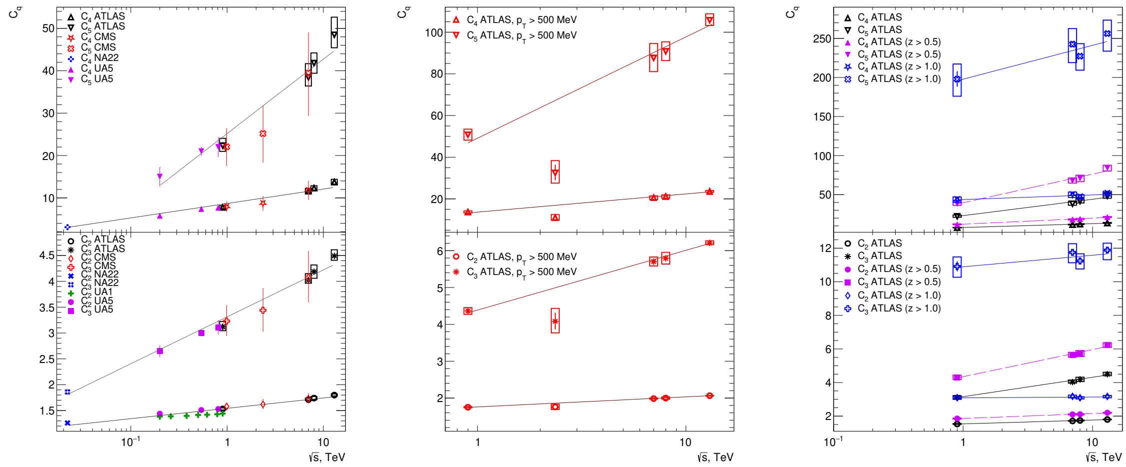

(a) (b) (c)

Figure 1. The normalised order-q moments, $C_q(\sqrt{s})$, in eq. (2) of the primary charged-particle multiplicity distributions measured by ATLAS for events collected at $\sqrt{s} = 0.9,\ 2.36,\ 7, 8$ and 13 TeV for (a) the pseudorapidity region $|\eta| < 2.5$. The results of CMS [15] and lower-energy experiments NA22 [20], UA1 [21], and UA5 [22, 23] are included. (b) The ATLAS results for $|\eta| < 2.5$, $n_{ch} \geq 1$, $p_T > 500$ MeV. (c) The ATLAS results for $|\eta| < 2.5$, $n_{ch} \geq 2$, $p_T > 100$ MeV with additional scaled multiplicity thresholds: $z > 0.5$ and $z > 1.0$. The C_2, C_3 and C_4, C_5 results are shown in the bottom and top panels, respectively. The vertical bars are the statistical uncertainties and the squares are the systematic uncertainties. The coloured symbols are the data. Fits of the log $\sqrt{s}$ dependence of the $C_q(\sqrt{s})$ of the multiplicity distribution (assuming linear dependence) are shown. In (a) for $\sqrt{s} = 0.9$ TeV, data from non-ATLAS experiments have been shifted down $\sqrt{s}$ for clarity. The lines show the results of the fits for $C_q(\sqrt{s})$ with statistical and systematic uncertainties added in quadrature.

the CMS at $\sqrt{s}$ from 0.9 to 7 TeV in central pseudorapidity $|\eta| < 0.5$ and more inclusive $|\eta| < 2.4$ regions [15]. The KNO results were investigated by ALICE at $\sqrt{s}$ from 0.9 to 13 TeV in pseudorapidity regions $|\eta| < 0.5$, $|\eta| < 0.8$, $|\eta| < 1.0$, and $|\eta| < 1.5$ [16–19]. For ALICE and CMS experiments the KNO scaling is violated for energies from 0.9 to 8 TeV if taking into account more inclusive pseudorapidity regions because the $C_q(\sqrt{s})$-moments are increase with energy. The KNO scaling holds for the central pseudorapidity region with $|\eta| < 0.5$ and for the energy region from $\sqrt{s} = 0.9$ to 7 TeV on the CMS data [15]. In this case, the $C_q(\sqrt{s})$-moments are independent of energy. The results of ALICE for $C_q(\sqrt{s})$-moments in central pseudorapidity, $|\eta| < 0.5$, are slightly increased with energy for $q = 4,\ 5$.

The results of this analysis of the validity of KNO scaling are shown quantitatively in figure 1 by the $C_q(\sqrt{s})$ of the multiplicity distributions measured by the ATLAS and complemented with the CMS measurements at $\sqrt{s} = 0.9,\ 2.36$ and 7 TeV [15] and results of the lower-energy experiments NA22 [20], UA1 [21], and UA5 [22, 23]. The $C_q(\sqrt{s})$ calculations based on ATLAS results for the kinematic region $|\eta| < 2.5$, $n_{ch} \geq 2$ and $p_T > 100$ MeV are shown in figure 1(a) and in table 1. The ATLAS and CMS results agree within the error range. The values of $C_q(\sqrt{s})$ with $q = 2, \ldots, 5$ for all experiments linearly

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increase with $\log\sqrt{s}$ as illustrated by the fits in figure 1(a) and in table 2 (phase space (A)). Since, as mentioned above, the KNO scaling requires that $C_q(\sqrt{s}\,)$ be independent of energy, we can state that the KNO scaling is violated at least for the full region of scaled multiplicity.

Figure 1(b) shows for the first time the values of $C_q(\sqrt{s}\,)$ calculated using multiplicity distributions measured by ATLAS for the kinematic region $|\,\eta\,| < 2.5$, $n_{ch} \geq 1$ and $p_T >$ 500 MeV. Similarly as in figure 1(a) the values of $C_q(\sqrt{s}\,)$ linearly increase with $\log\sqrt{s}$ as the function

$$C_q(\sqrt{s}\,) = \alpha + \beta(\sqrt{s}\,). \tag{3}$$

The results of the fit are presented in table 2 (phase space (B)). The C_q values at $\sqrt{s} =$ 2.36 TeV in figure 1(b) are much smaller than those for other energies. This is because the region of primary charged-particle multiplicity distributions at 2.36 TeV is smaller (up to $z \approx 3.5$) than that for higher CM energies (up to $z \approx 9$) [14]. Therefore, the C_q values at $\sqrt{s} = 2.36$ TeV were note used in the fits. The $C_q(\sqrt{s}\,)$ for $p_T > 500$ MeV has a higher bias (α) and slope (β) of the fits than those for the minimum p_T threshold, with the bias increasing from 1.1 at $q = 2$ up to 2.1 at $q = 5$, and the slope increasing from 1.4 at $q = 2$ up to 2.6 at $q = 5$. This is the result of stronger interactions with a higher p_T threshold in case (B) than in case (A). This is because the total transverse momentum, $\sum p_T$, for events with $p_T > 500$ MeV (B) is higher than the same observable for events with $p_T > 100$ MeV (A).

Figure 1(c) shows moments C_q for events with $n_{ch} \geq 2$, $p_T > 100$ MeV and for $z > 0.5$ without the fraction of single and double diffraction events, which was accepted by the ATLAS minimum-bias trigger [7–11]. In this case, the values of $C_q(\sqrt{s}\,)$ are systematically higher than those for full distributions with $z > 0$ and show a similar linear increase with $\log\sqrt{s}$ as illustrated in figure 1(c). For multiplicity distributions for $z > 1.0$ the values of $C_q(\sqrt{s}\,)$ at the highest energies $\sqrt{s} = 7$, 8 and 13 TeV are in agreement within error uncertainties, as can be seen in figure 1(c). Therefore, the energy independence of the moments of various orders can be considered as a confirmation of the KNO scaling. This is in agreement with the conclusion in [14] that the KNO scaling holds for the highest energies.

The results of studying $C_q(\sqrt{s}\,)$ of primarily charged-particle multiplicity distributions using the results of the ATLAS at the LHC are presented. The normalised order-q moments a re sensitive to the KNO scaling. The $C_q(\sqrt{s}\,)$ from the ATLAS, CMS and low-energy experiments are compared for the kinematic region with an absolute pseudorapidity less than 2.5 of $\sqrt{s}$ from 0.2 to 13 TeV. For the full-scaled multiplicity region, the $C_q(\sqrt{s}\,)$ moments show a linear increase with $\sqrt{s}$, therefore indicating that KNO scaling is violated. The $C_q(\sqrt{s}\,)$ of the scaled multiplicity larger than one average multiplicity is constant for the highest energies. Thus, for the first time, it can be concluded that for $|\eta| < 2.5$ and $z > 1$, the KNO scaling is valid. Previously, the KNO scaling validity at LHC energies was observed for $|\eta| < 0.5$ by CMS [15].

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$\sqrt{s}$	$p_{\mathrm{T}}^{\mathrm{min}}$	C_{q}			
[TeV]	[MeV]	C_2	C_3	C_4	C_5
13	100	$1.799 \pm 0.002^{+0.021}_{-0.016}$	$4.491 \pm 0.008^{+0.119}_{-0.084}$	$13.74 \pm 0.04^{+0.65}_{-0.52}$	$48.49 \pm 0.15^{+1.52}_{-0.66}$
	500	$2.065 \pm 0.002^{+0.008}_{-0.007}$	$6.209 \pm 0.009^{+0.058}_{-0.047}$	$23.57 \pm 0.05^{+0.46}_{-0.34}$	$105.77 \pm 0.29^{+3.05}_{-2.55}$
8	100	$1.741 \pm 0.001^{+0.030}_{-0.029}$	$4.185 \pm 0.007^{+0.131}_{-0.126}$	$12.32 \pm 0.03^{+0.55}_{-0.54}$	$41.81 \pm 0.14^{+2.40}_{-2.40}$
	500	$2.000 \pm 0.001^{+0.030}_{-0.030}$	$5.793 \pm 0.008^{+0.161}_{-0.157}$	$21.12 \pm 0.04^{+0.83}_{-0.81}$	$90.81 \pm 0.21^{+4.56}_{-4.44}$
7	100	$1.712 \pm 0.005^{+0.026}_{-0.016}$	$4.025 \pm 0.022^{+0.124}_{-0.062}$	$11.58 \pm 0.10^{+0.64}_{-0.37}$	$38.43 \pm 0.43^{+3.24}_{-2.04}$
	500	$1.985 \pm 0.004^{+0.018}_{-0.015}$	$5.701 \pm 0.020^{+0.127}_{-0.116}$	$20.59 \pm 0.10^{+0.87}_{-0.88}$	$87.64 \pm 0.52^{+6.90}_{-6.56}$
2.36	500	$1.759 \pm 0.047^{+0.075}_{-0.070}$	$4.082 \pm 0.227^{+0.352}_{-0.313}$	$11.00 \pm 0.95^{+1.46}_{-1.25}$	$32.43 \pm 3.84^{+5.94}_{-4.86}$
0.9	100	$1.530 \pm 0.015^{+0.028}_{-0.021}$	$3.121 \pm 0.057^{+0.102}_{-0.076}$	$7.77 \pm 0.21^{+0.39}_{-0.30}$	$22.33 \pm 0.77^{+1.72}_{-1.36}$
	500	$1.752 \pm 0.011^{+0.016}_{-0.015}$	$4.364 \pm 0.052^{+0.091}_{-0.093}$	$13.72 \pm 0.24^{+0.53}_{-0.51}$	$50.81 \pm 1.20^{+2.64}_{-2.61}$

Table 1. The normalised order-q moments, $C_{\mathrm{q}}(\sqrt{s}\,)$, of the primary charged-particle multiplicity distributions measured by the ATLAS Collaboration for events at center-of-mass energies $\sqrt{s} = 0.9,\ 2.36,\ 7, 8$ and 13 TeV for pseudorapidity region $\mid \eta \mid< 2.5$ and for two different phase spaces $n_{\mathrm{ch}} \geq 2$, $p_{\mathrm{T}} > 100$ MeV and $n_{\mathrm{ch}} \geq 1$, $p_{\mathrm{T}} > 500$ MeV.

C_{q}	Phase Space	α	β
C_2	(A)	1.54 ± 0.01	0.200 ± 0.015
	(B)	1.76 ± 0.02	0.271 ± 0.017
C_3	(A)	3.31 ± 0.03	0.907 ± 0.031
	(B)	4.42 ± 0.10	1.59 ± 0.10
C_4	(A)	8.86 ± 0.18	3.40 ± 0.13
	(B)	14.0 ± 0.5	8.45 ± 0.56
C_5	(A)	25.2 ± 0.7	17.6 ± 1.4
	(B)	52.5 ± 2.7	46.5 ± 3.2

Table 2. The fit parameters of energy dependence of the of normalised order-q moments $C_{\mathrm{q}}(\sqrt{s}\,)$ in eq. (3) for two different phase spaces: (A) $\mid \eta \mid< 2.5$, $n_{\mathrm{ch}} \geq 2$, $p_{\mathrm{T}} > 100$ MeV and (B) $\mid \eta \mid< 2.5$, $n_{\mathrm{ch}} \geq 1$, $p_{\mathrm{T}} > 500$ MeV.

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